Two-loop QCD corrections to $Wb\bar{b}$ production at hadron colliders

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based on arXiv:2102.02516
with Simon Badger and Simone Zoia

RADCOR-LoopFest XIX, FSU (Virtual)
May 19, 2021
Precise prediction for the LHC

⇒ QCD corrections are important at the LHC

\[ d\sigma = d\sigma^{\text{LO}} + d\sigma^{\text{NLO}} + d\sigma^{\text{NNLO}} + \ldots \]

10–30% \quad 1–10%

NNLO frontier: 2 to 3 scattering

- \( pp \to jbjj \): \( R_{3/2}, m_{jjj} \Rightarrow \alpha_s \) determination at multi-TeV range
- \( pp \to \gamma\gamma j \): background to Higgs \( p_T \), signal/background interference effects
- \( pp \to Hjj \): Higgs \( p_T \), background to VBF (probes Higgs coupling)
- \( pp \to Vjj \): Vector boson \( p_T \), \( W^+ / W^- \) ratios, multiplicity scaling
- \( pp \to VVj \): background for new physics
NNLO cross sections for 2 → 3 processes

\[ d\sigma_{\text{NNLO}} = \sum (\text{rational coefficients}) \times (\text{integral/special functions}) \]

finite remainder = loop amplitude − poles
Introduction

Massive progress in massless 2-loop 5-particle scattering

► All 2-loop 5-particle integrals are known talk by Vasily

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► Many 2-loop 5-particle QCD amplitudes known analytically talks by Federico, Herschel, Vasily

Leading colour ⇒ $5g, 2q3g, 4q1g, 2q3\gamma, 2q1g2\gamma$


Full colour ⇒ $5g$ all-plus, $2q1g2\gamma$

[Badger, Chicherin, Gehrmann, Heinrich, Henn, Peraro, Wasser, Zhang, Zoia (2019)] [Agarwal, Buccioni, Tancredi, von Manteuffel (2021)]

► NNLO QCD calculations for $2 \rightarrow 3$ processes talk by Rene

$pp \rightarrow \gamma\gamma\gamma$ [Chawdhry, Czakon, Mitov, Poncelet (2019)] [Kallweit, Sotnikov, Wiesemann (2020)]

$pp \rightarrow \gamma\gamma j$ [Chawdhry, Czakon, Mitov, Poncelet (2021)]
Scattering with an off-shell leg

- rich potential phenomenology
- massless internal particles, focus on QCD corrections
- high algebraic and analytic complexity
  ⇒ six independent variables
  ⇒ 3 square roots
A first look: two-loop $W+4$ parton amplitudes

Numerical evaluation of leading colour $q\bar{Q}Q\bar{q}'\bar{\nu}\ell$ and $qg\bar{q}'\bar{\nu}\ell$ helicity amplitudes at two loops

Feynman diagrams integrand reduction IBP reduction finite-field sampling

- Full solutions of the master integrals were not available back then
- unknown MIs are evaluated numerically using pySecDec/Fiesta

\[
I \left( \begin{array}{c} 6 \ 2 \ 1 \\ 4 \ 1 \ 3 \end{array} \right) \left[ \langle 4|k_2|p_{56}|4\rangle \mu_{11} \right] \sim O(\epsilon) \quad I \left( \begin{array}{c} 6 \ 2 \ 1 \\ 4 \ 1 \ 3 \end{array} \right) \left[ [4|k_2|p_{56}|4] \mu_{11} \right] \sim O(\epsilon)
\]

\[
I \left( \begin{array}{c} 6 \ 2 \ 1 \\ 4 \ 1 \ 3 \end{array} \right) \left[ \text{tr} - (1(k_1 - p_1)(k_1 - p_{12})3)\langle 4|k_2|p_{56}|4\rangle \right] \sim O(1)
\]

- Use momentum twistor parametrisation for $2 \to 4$ massless scattering: 8 variables
- Coefficient of master integrals are computed numerically over finite fields
Two-loop master integrals for 5-point 1-mass process

4-point sub-topologies known from $pp \rightarrow V_1^* V_2^*$

[Gehrman, Remiddi (2000)] [Henn, Melnikov, Smirnov (2014)]
[Gehrmann, von Manteuffel, Tancredi (2015)]

All planar 2-loop integrals are available

[Abreu, Ita, Moriello, Page, Tschernow, Zeng (2020)]
[Canko, Papadopoulos, Syrrakos (2020)] [Syrrakos (2020)]

Non-planar integrals in progress talks by Ben & Costas
Leading colour $Wb\bar{b}$ amplitude

$$\bar{d}(p_1) + u(p_2) \rightarrow b(p_3) + \bar{b}(p_4) + W^+(p_5)$$

- colour decomposition at leading colour $\rightarrow$ only planar contribution

$$A^{(2)}(1\bar{d}, 2u, 3b, 4\bar{b}, 5W) \sim g_s^6 g_W \mathcal{N}_c^2 \delta_{i_1}^{i_4} \delta_{i_3}^{i_2} A^{(2)}(1\bar{d}, 2u, 3b, 4\bar{b}, 5W)$$

- massless $b$ quarks, $p_3^2 = p_4^2 = 0$

- onshell $W$ boson

$$p_5^2 = m_W^2, \quad \sum_\lambda \varepsilon_{W}^{\mu*}(p_5, \lambda) \varepsilon_{W}^{\nu}(p_5, \lambda) = -g^{\mu\nu} + \frac{p_5^\mu p_5^\nu}{m_W^2}$$

Invariants:

$$s_{12} = (p_1 + p_2)^2, \quad s_{23} = (p_2 - p_3)^2, \quad s_{34} = (p_3 + p_4)^2, \quad s_{45} = (p_4 + p_5)^2, \quad s_{15} = (p_1 - p_5)^2, \quad s_5 = p_5^2,$$

$$\text{tr}_5 = 4i \epsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu p_3^\rho p_4^\sigma.$$
Integrand construction

Feynman diagrams generated using QGRAF [Nogueira(1993)]

\[
\begin{align*}
&W^+ + u \bar{b}^*\bar{b} - \bar{d} u
\end{align*}
\]

\[
M^{(2)} = \sum_{\text{spin}} A^{(0)*} A^{(2)} = M_{\text{even}} + \text{tr}_5 M_{\text{odd}}^{(2)}
\]

Numerators containing: \(\text{tr}(\cdots)\) and \(\text{tr}(\cdots \gamma_5 \cdots \gamma_5 \cdots)\) ⇒ anti-commuting \(\gamma_5\) prescription
\(\text{tr}(\cdots \gamma_5 \cdots)\) ⇒ Larin’s prescription [Larin(1993)]

Amplitudes in terms of scalar integrals

\[
M_k^{(2)}(\{p\}) = \sum_i c_k,i(\epsilon, \{p\}) I_{k,i}(\epsilon, \{p\}), \quad k \in \{\text{even, odd}\}
\]
Integration-by-parts (IBP) reduction to master integrals

- Map each topology to a set of maximal topologies
- IBP systems for $T_1 - T_{10}$

Entire workflow on finite fields

\[
M^{(2)}_k(\{p\}, \epsilon) = \sum_i c_{k,i}(\{p\}, \epsilon) I_{k,i}(\{p\}, \epsilon)
\]

\[
M^{(2)}_k(\{p\}, \epsilon) = \sum_i d_{k,i}(\{p\}, \epsilon) MI_{k,i}(\{p\}, \epsilon)
\]

$k \in \{\text{even, odd}\}$

- IBP reduction directly to canonical MIs [Abreu, et al. (2020)]
- IBP systems generated with \textsc{LiteRed} [Lee (2012)], solved with \textsc{FiniteFlow} [Peraro (2019)] using Laporta algorithm [Laporta (2000)]
- IBP tables known numerically at each value of $(\{p\}, \epsilon)$
- Numerically compute $d_{k,i}$ over finite fields
Plugging in the master integrals

1. [Abreu, Ita, Moriello, Page, Tschernow, Zeng (2020)] talk by Ben
   - Planar alphabet identified (58 letters, 3 square-roots), canonical DEs derived
   - Integrate DEs numerically using generalised series expansions [Moriello (2019)]

2. [Canko, Papadopoulos, Syrrakos (2020)] talks by Costas, Nikolaos
   - Construct Simplified Differential Equations (SDEs) using known canonical basis
   - Analytic solutions in term of Goncharov PolyLogarithms (GPLs)

1 ⇒ analytically reconstructing MI coefficients is still too complicated
2 ⇒ GPLs not linearly independent: no analytic pole cancellations
A basis of special functions

- use the components of the $\epsilon$-expansion of the MIs as special functions

$$\text{MI}_i(s) = \sum_{w \geq 0} \epsilon^w \text{MI}_i^{(w)}(s)$$

- starting from canonical DEs [Abreu, et al. (2020)] write MIs in terms of Chen’s iterated integrals [Chen (1977)] for example:

$$\text{MI}_i^{(2)} = [w_1, w_2]_{s_0} + [w_1, w_3]_{s_0} + \cdots + t c_j^{(2)}(s_0)$$

where

$$[w_1, \ldots, w_i]_{s_0}(s) = \int_{\gamma} d \log w_i(n)(s') [w_1, \ldots, w_{i-1}]_{s_0}(s')$$

- Use GPL expressions [Canko, et al. (2020)] [Syrrakos (2020)] + PSLQ algorithm to prepare the boundary values

- Shuffle algebra to remove products of lower-weight functions + linear algebra to extract linearly independent functions talk by Vasily

$$\left\{ \text{MI}_i^{(w)}(s) \right\} \implies \left\{ f_i^{(w)}(s) \right\}$$
Reconstructing the finite remainders

\[
M_{k}^{(2)}\left(\{p\}, \epsilon\right) = \sum_{i} c_{k,i} \left(\{p\}, \epsilon\right) \mathcal{I}_{k,i} \left(\{p\}, \epsilon\right)
\]

\[
\downarrow \quad \text{IBP reduction}
\]

\[
M_{k}^{(2)}\left(\{p\}, \epsilon\right) = \sum_{i} d_{k,i} \left(\{p\}, \epsilon\right) M_{k,i} \left(\{p\}, \epsilon\right)
\]

\[
\downarrow \quad \text{map to special function basis}
\]

\[
\downarrow \quad \text{subtract UV/IR poles}
\]

\[
\downarrow \quad \epsilon \text{ expansion}
\]

\[
F_{k}^{(2)}\left(\{p\}\right) = \sum_{i} e_{k,i} \left(\{p\}\right) m_{k,i} \left(f\right) + \mathcal{O}\left(\epsilon\right)
\]

\[k \in \{\text{even, odd}\}\]

- Finite remainders

\[
F_{k}^{(2)} = M_{k}^{(2)} - \sum_{j=1}^{2} I^{(j)} M_{k}^{(2-j)}
\]

\[I^{(L)} \rightarrow L\text{-loop universal UV/IR poles}\]

- Numerically compute \(e_{k,i}\) over finite fields
- Analytic pole cancellation
- Drop in polynomial complexities for \(e_{k,i}\)
- Reconstruct analytic expressions of \(e_{k,i}\) from several numerical evaluations [Peraro(2016)]
Reconstructing the finite remainders

\[
F_k^{(2)}(\{p\}) = \sum_i e_{k,i}(\{p\}) \; m_{k,i}(f) + O(\epsilon), \quad k \in \{\text{even, odd}\}
\]

- set \( s_{12} = 1 \)

- Not all \( e_{k,i} \) coefficients independent
  - find linear relations between coefficients and reconstruct the simpler ones

\[
\sum_i y_i e_i = 0, \quad y_i \in \mathbb{Q}
\]

- allow to supply known/candidate coefficients \( \tilde{e}_j \)

\[
\sum_i y_i e_i + \sum_j \tilde{y}_j \tilde{e}_j = 0, \quad y_i, \tilde{y}_j \in \mathbb{Q}
\]

- guess the denominator \( \rightarrow \) from letters \cite{Abreuetal2019,Abreuetal2020}

- partial fraction in one variable \( s_{23} \) and reconstruct in the remaining variables \( s_{34}, s_{45}, s_{15}, s_5 \)

\( \Rightarrow \sim 4 \) times speed up \( \Rightarrow \) 2 prime fields needed

- Reconstructed analytic expressions are simplified using MultivariateApart \cite{Heller, von Manteuffel2021}

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Numerical evaluation

- Only 19 linear combinations of $f_i^{(4)}$ appear in the two-loop finite remainder
  \[ \Rightarrow \text{define a new basis } g_i^{(w)} \]
  \[ \{ f_i^{(w)}(s) \} \Rightarrow \{ g_i^{(w)}(s) \} \]

- Apply generalised series expansion method directly to the $g_i^{(w)}$ basis
  \[ \vec{g} = \begin{pmatrix} \epsilon^4 g_i^{(4)} \\ \epsilon^3 g_i^{(3)} \\ \epsilon^2 g_i^{(2)} \\ \epsilon g_i^{(1)} \\ 1 \end{pmatrix} \]
  \[ d\vec{g} = \epsilon d\vec{B} \cdot \vec{g} \]

- Much simpler than the DEs for the master integrals
- Use generalised series expansion approach [Moriello(2019)] as implemented in DIFFEXP [Hidding(2020)] talk by Martijn
Numerical evaluation

Evaluation on a univariate slice of the physical phase space

\[ p_1 = \frac{\sqrt{5}}{2}(1, 0, 0, 1), \]
\[ p_2 = \frac{\sqrt{5}}{2}(1, 0, 0, -1), \]
\[ p_3 = \frac{\sqrt{5}}{2}(1, 1, 0, 0), \]
\[ p_4 = \frac{\sqrt{5}}{2}(1, \cos \theta, -\sin \phi \sin \theta, -\cos \phi \sin \theta), \]
\[ p_5 = \sqrt{5}(1, 0, 0, 0) - p_3 - p_4 \]

\[ s = 1, \; m_W^2 = 0.1, \; \phi = 0.1, \; x_1 = 0.6 \]

- 1100 points → average 260 s/point
- Reasonable evaluation time with basic DIFFEXP setup
- further optimisation is possible

[Abreu,etal(2020)][Becchetti,etal(2020)]
Summary

✔ First analytic result for 2-loop 5-point amplitude with one massive leg
  ⇒ leading colour $u\bar{d} \rightarrow W^+ b\bar{b}$

✔ Basis of special functions for leading colour 5-particle amplitudes with 1 off-shell leg up to 2 loops

✗ Include $W$-boson decay

✗ Application to other processes

✗ Full colour (need non-planar integrals)
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- Application to other processes
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THANK YOU!!!