TWO-LOOP QCD AMPLITUDES FOR 2→3 PROCESSES

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RADCOR-Loopfest 2021 (Florida State University, 19th May 2021, via Zoom)

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Based on arXiv:1911.00479, 2012.13553, 2103.04319, 2105.06940

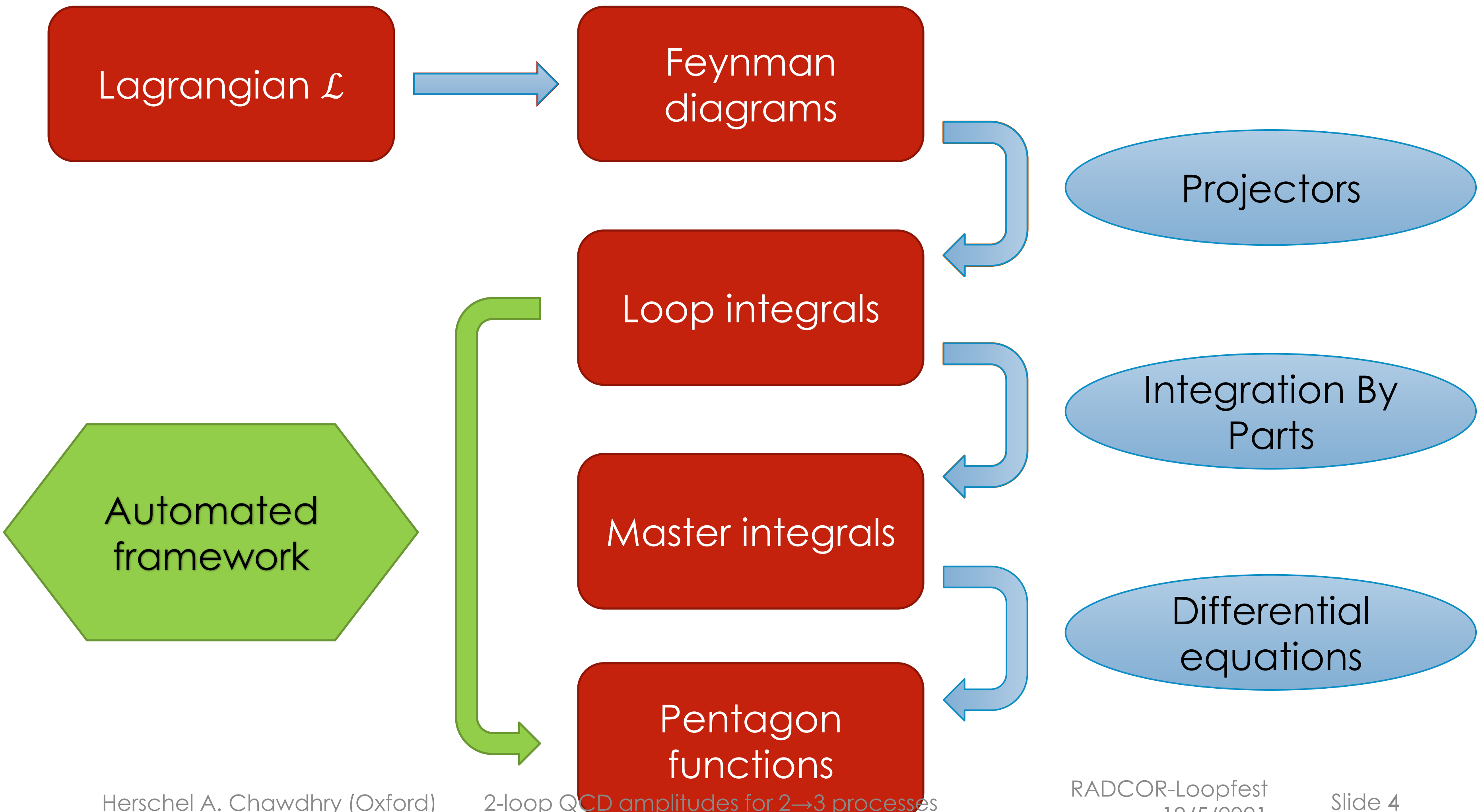
MOTIVATION

- Precise predictions for collider observables require higher-order QCD calculations
- NNLO QCD: $2 \to 1$ and $2 \to 2$ processes mostly done
  - Consistent with LHC measurements
- $2 \to 3$ is next frontier in NNLO QCD calculations
- 2-loop amplitudes are a key ingredient
  - Lots of work by many research groups (see summary slide by Vasily Sotnikov)
- $2 \to 3$ cross-sections at NNLO QCD
  - $pp \to \gamma\gamma\gamma$
    - 1911.00479 (HAC, Czakon, Mitov, Poncelet)
    - 2010.04681 (Kallweit, Sotnikov, Wiesemann)
  - $pp \to \gamma\gamma + j$
    - 2105.06940 (HAC, Czakon, Mitov, Poncelet)

OUTLINE

- Introduction
- Techniques
  - polarisation projectors
  - improved IBPs
  - automated framework for assembling amplitudes
- Results:
  - amplitudes for 3-photon production
    - cross-sections: 3-photon production at NNLO in QCD
  - amplitudes for 2-photon + jet production
    - cross-sections: 2-photon + jet production at NNLO in QCD
- Conclusions

OUR MULTI-LOOP TOOLBOX



POLARISATION PROJECTORS

1904.00705 (Chen)

$$\mathcal{M}^{\bar{h}} = \varepsilon^{\mu}_{3,h_3}\varepsilon^{\nu}_{4,h_4}\varepsilon^{\rho}_{5,h_5}\,\mathrm{Tr}\Big\{\big(u(h_1)\otimes\bar{v}(h_2)\big)\Gamma_{\mu\nu\rho}\Big\}$$

- Construct explicit representations of the external wavefunctions for a given helicity or polarisation

$$\varepsilon^{\mu}_{i,X} = c^{X}_{i,1}p^{\mu}_1 + c^{X}_{i,2}p^{\mu}_2 + c^{X}_{i,3}p^{\mu}_i$$

$$(\varepsilon_{i,X})^2 = -1 \quad , \quad \varepsilon_{i,X}\cdot q = 0 \quad , \quad \varepsilon_{i,X}\cdot p_i = 0$$

$$\varepsilon^{\mu}_{i,h} = \frac{1}{\sqrt{2}}(\varepsilon^{\mu}_{i,X} + hi\varepsilon^{\mu}_{i,Y})$$

$$(u\otimes\bar{v})_{\alpha\beta} = \frac{\bar{u}Nv}{\bar{u}Nv}(u\otimes\bar{v})_{\alpha\beta} = \frac{1}{\bar{u}Nv}(u\otimes\bar{u})_{\alpha\gamma}N_{\gamma\delta}(v\otimes\bar{v})_{\delta\beta} = \frac{1}{\mathcal{N}}[(u\otimes\bar{u})N(v\otimes\bar{v})]_{\alpha\beta}$$

- Construction is in $d = 4$
  - Bare amplitudes are scheme-dependent
  - Dependence cancels in finite remainder

IBPS: A NEAT STRATEGY

1805.09182 (HAC, Lim, Mitov). See also 1812.01491 (Maierhöfer, Usovitsch)

- Loop integrals live in a vector space over $\mathbb{Q}(\{s_{ij}\}, d)$
- Master integrals $\{M_n\}$ form a basis for this space
- Can therefore simplify the IBP equations by *taking coefficients w.r.t. the basis vectors*
- In practice:
  - Set $M_n = 0$ for all $n \neq 1$
  - Solve simplified IBPs
  - Repeat in turn for $n = 2$, $n = 3$, … to obtain complete IBP solution
- Correct because in any vector space, a vector can be uniquely written as a vector superposition of a basis
  - Equivalently, correct because IBP equations - and solutions - are linear (in the integrals) and homogeneous
- Advantage: one simplified IBP system per M_j
  - Independent → can parallelise
  - Simpler → faster to solve, requires less RAM

AUTOMATED ASSEMBLY OF THE AMPLITUDES

- Ingredients
  - Coefficients of unreduced integrals in amplitude
  - IBP solutions
  - master integral solutions
  - IR counterterms (if seeking finite remainder)
- All these ingredients are calculated analytically, but assembling them is non-trivial
- Our automated framework uses finite fields for the assembly
  - "Numerically" evaluate each of the ingredients
  - Calculate numerical value of the amplitude/finite remainder
  - Interpolate to obtain analytical amplitude
- Complications (see later)
  - Momentum crossings
  - Expansion in $\varepsilon \equiv (4-d)/2$
    - $A^{(2-\mathrm{loop})} = \sum_n \varepsilon^n \sum_k r_{n,k}(s_{ij})\, g_k(s_{ij})$, where $g_k(s_{ij})$ are pentagon functions
- Simultaneously calculate multiple amplitude structures A_n by sharing finite-field samples of IBPs and masters
  - useful for processes with many partonic channels, and/or helicity amplitudes

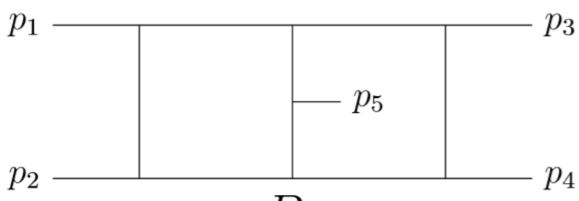
FINITE-FIELD METHODS

- Long-used in computer algebra (e.g. Mathematica), now also used by physicists
  - e.g. [1406.4513 – Manteuffel, Schabinger], [1608.01902 – Peraro]
- Speed largely determined by the polynomial degrees of the expressions in the final result
- Reconstruct analytical results using interpolation and Chinese remainder theorem
  - we use FireFly library [1904.00009 , 2004.01463 – Klappert, Klein, Lange]

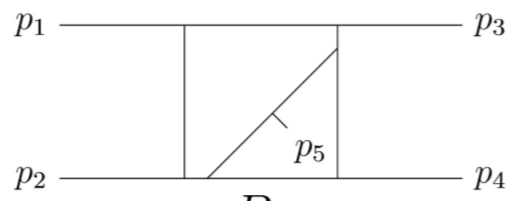
IBP SOLUTIONS

- Using our analytical solutions from 1805.09182 (HAC, Lim, Mitov)
- Preparatory step: analytically expand IBPs to 5 orders in $\varepsilon \equiv (4-d)/2$
  - Insert book-keeping terms to account for truncated higher-orders in ε
- Analytical momentum crossings not required
  - During finite-field sampling, apply momentum crossings numerically to the finite-field values of $\{s_{ij}\}$
- Time needed to evaluate all required IBPs: <10s per crossing (without any optimisations)

INTEGRAL FAMILIES



B_1



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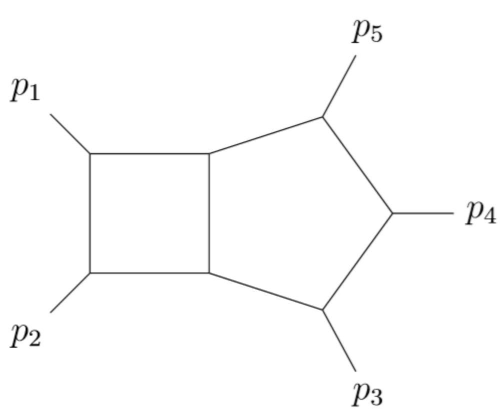

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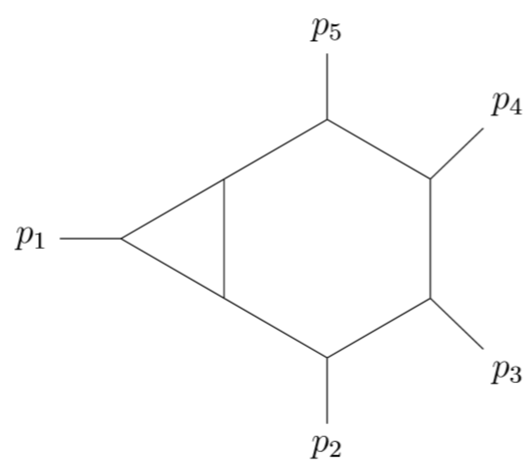

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MASTER INTEGRALS

- Solved several times:
  - 1511.09404 (Papadopoulos, Tommasini, Wever)
  - 1807.09812 (Gehrmann, Henn, Lo Presti)
  - 2009.07803 (Chicherin, Sotnikov)
- C1 masters → UT masters → pentagon functions
  - produced analytically for each momentum crossing
    - minimal basis of pentagon functions
  - analytically expanded to 5 orders in ε
    - inserted book-keeping terms (as with IBPs)
- mapped C2 masters → C1 masters
- Finite-field evaluation time < 1s per crossing

SIMPLIFYING THE FINAL RESULTS

- $\mathbb{Q}$-linear relations between the coefficients r_k of pentagon functions g_k
  - $\sum_k \lambda_k r_k = 0$, with $\lambda_k \in \mathbb{Q}$ recall: $A = \sum r_k g_k$, with $r_k \in \mathbb{Q}(\{s_{ij}\}, d)$
  - Greatly reduces number - and total size - of coefficients
  - Used by e.g.
    - [1904.00945 – Abreu, Dormans, Febres Cordero, Ita, Page, Sotnikov]
    - [1911.00479 – HAC, Czakon, Mitov, Poncelet]
    - [2102.01820 – Agarwal, Buccioni, von Manteuffel, Tancredi]
    - [2102.02516 – Badger, Hartanto, Zoia]
    - [2102.13609 - Abreu, Febres Cordero, Ita, Page, Sotnikov]
  - Exploration of origin and further applications?
- partial fractioning
  - e.g. MultivariateApart [2101.08283 – Heller, von Manteuffel]
  - Further reduces sizes of coefficients by factor of ~2–4

RESULTS: LEADING-COLOUR $q\bar{q} \to \gamma\gamma\gamma$

✓ Spin-averaged $2\mathrm{Re}\langle A^{(0)}|A^{(2)}\rangle$ [1911.00479] (HAC, Czakon, Mitov, Poncelet)

- Automated results checked against by-hand calculation
- Pentagon functions' coefficients were large (but fast)
  - For pheno, evaluate coefficients using exact rational arithmetic
  - Takes 1-2 seconds per kinematic point (full set of coefficients)
- Pentagon functions themselves are much slower
  - 10-50 min per kinematic point (average 17 min) to evaluate functions from 1807.09812 (Gehrmann, Henn, Lo Presti)
  - 2009.07803 (Chicherin, Sotnikov) offers large improvements

✓ Full set of 16 linearly-polarised amplitudes

- Checked against unpolarised $2\mathrm{Re}\langle A^{(0)}|A^{(2)}\rangle$

✓ Full set of helicity amplitudes [2012.13553] (HAC, Czakon, Mitov, Poncelet)

- Much lower polynomial degrees than in linearly-polarised
  - Raw results ~100 MB (reduces to 1.8 MB using $\mathbb{Q}$-linear relations and partial fractioning)

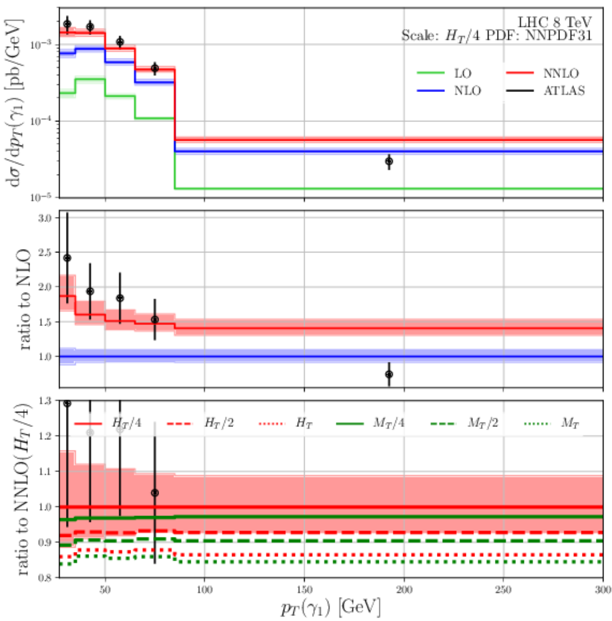
RESULTS:
LEADING-COLOUR $\gamma\gamma + j$

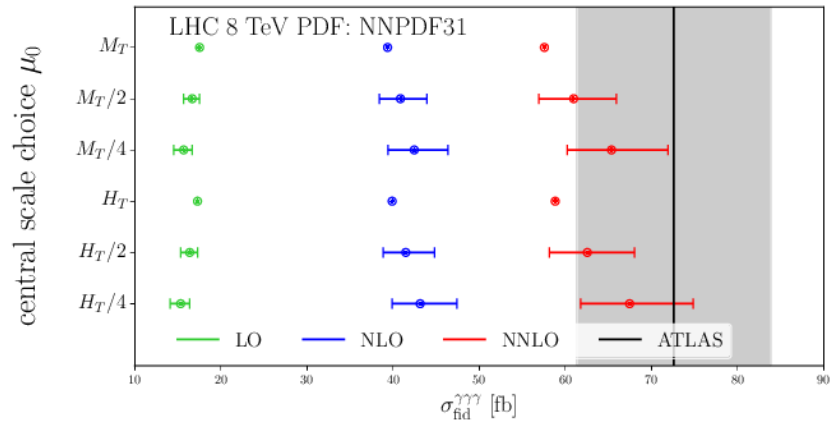
- Full set of helicity amplitudes [2103.04319] (HAC, Czakon, Mitov, Poncelet)
- 2 partonic channels, 9 helicity amplitudes
  - $q\bar{q} \to g\gamma\gamma$ (3 independent helicities)
  - $qg \to q\gamma\gamma$ (6 independent helicities)
- Automated calculation took O(1 day)
- Raw results ~ a few hundred MB (reduces to ~4 MB after $\mathbb{Q}$-linear relations and partial fractioning)
- Obtained the first NNLO QCD cross-sections for $\gamma\gamma + j$ production at the LHC (see later) [2105.06940](HAC, Czakon, Mitov, Poncelet)

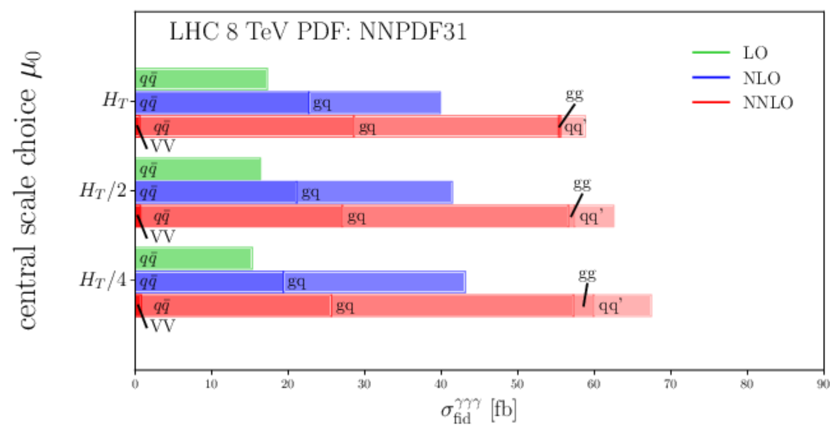
PHENO RESULTS: 3-PHOTON

1911.00479 (HAC, Czakon, Mitov, Poncelet)

- NNLO gives good description of LHC data
- scale-independent leading-colour 2-loop finite remainder contribution is at % level







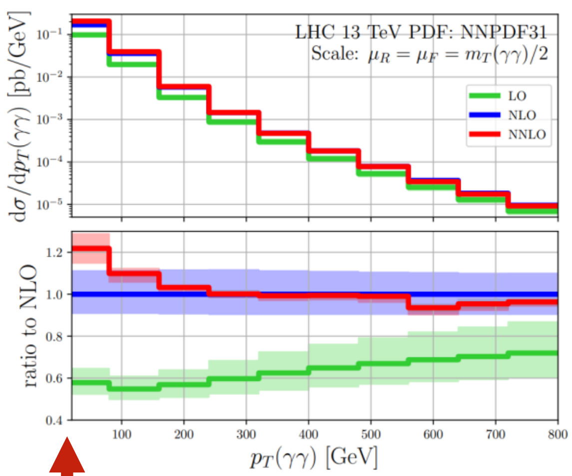
See talk by Rene Poncelet on Friday at 15:00 (Florida time)

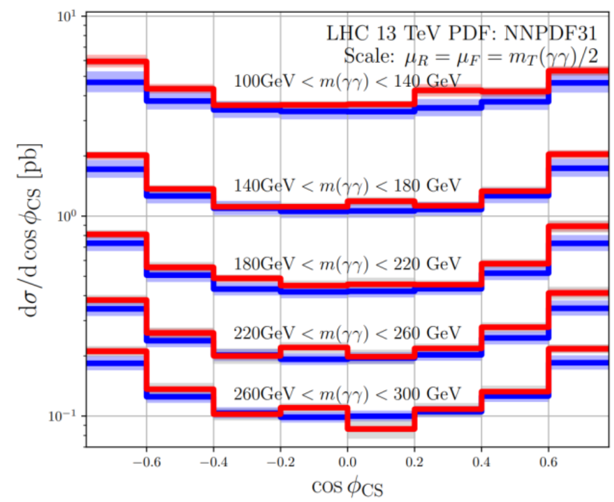
PHENO RESULTS: 2-PHOTON +JET

2105.06940 (HAC, Czakon, Mitov, Poncelet)

See talk by Rene Poncelet on Friday at 15:00 (Florida time)

- First NNLO-accurate prediction for $p_T(\gamma\gamma)$ distribution
- Main background for Higgs production at high p_T





Require $p_T(\gamma\gamma) > 20$ GeV

- 2-loop subleading-colour effects likely to be 1-2% at most
- Future work: calculate 2-loop amplitude for loop-induced process $gg \to g\gamma\gamma$ (gives a partial N^{3}LO contribution)

CONCLUSIONS

- Fast automated framework for assembling amplitudes
  - well-suited for processes with several partonic channels and/or helicities and/or colour-structures
- Several 2-loop QCD amplitudes for $2 \to 3$ processes
  - $q\bar{q} \to \gamma\gamma\gamma$ (leading colour)
    - ✓ spin-averaged $2\mathrm{Re}\left[\langle A^{(0)} | A^{(2)} \rangle\right]$
      - evaluation time dominated by pentagon functions, not their coefficients
    - ✓ complete set of 16 linearly-polarised amplitudes
    - ✓ complete set of helicity amplitudes
    - ✓ NNLO QCD cross-sections for 3-photon production
  - $\gamma\gamma$+jet production
    - ✓ complete set of helicity amplitudes (leading colour)
    - ✓ NNLO QCD cross-sections for 2-photon+jet production
- More details: [1911.00479, 2012.13553, 2103.04319, 2105.06940]