NNLO EW-QCD corrections to on-shell Z production

Narayan Rana

19th May, 2021

RADCOR-LoopFest 2021

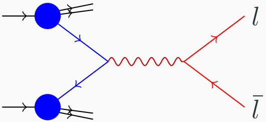
UNIVERSITÀ DEGLI STUDI DI MILANO

R. Bonciani, F. Buccioni, NR, A. Vicini
`NNLO QCDxEW corrections to on-shell Z production`,
Phys.Rev.Lett. 125 (2020) 232004 (arXiv:2007.06518 [hep-ph]).

R. Bonciani, F. Buccioni, NR, I. Triscari, A. Vicini
`NNLO QCDxEW corrections to Z production in the qq¯ channel`,
Phys.Rev. D101 (2020) 031301 (arXiv:1911.06200 [hep-ph]).

R. Bonciani, F. Buccioni, R. Mondini, A. Vicini
`Double-real corrections at` $\mathcal{O}(\alpha\alpha_s)$ `to single gauge boson production`,
Eur.Phys.J.C 77 (2017) 3, 187 (arXiv:1611.00645 [hep-ph]).

Drell-Yan



- ✓ **One of the standard candle processes**
  - Large cross section and clean experimental signature - important for detector calibration and constraining parton distribution functions
- ✓ **Precise predictions for electroweak parameter**
  - W boson mass (m_W), Weak mixing angle $(\sin^2\theta_{\rm eff})$ ... $(\delta m_W <$ 5 MeV and $\delta\sin^2\theta_{\rm eff} <$ 0.0001 would provide very stringent test of the SM likelihood. $)$
- ✓ **New physics potential**
  - Many BSM scenarios with same final states - W', Z', KK modes *etc.*

Chronicles of the inclusive Drell-Yan

NLO QCD & NLO EW

Politzer (1977)

Sachrajda (1978)

Altarelli, Ellis, Martinelli (1979)

Humpert, van Neerven (1979)

Dittmaier, Krämer (2002)

Baur, Brein, Hollik, Schappacher, Wackeroth (2002)

$\vdots$

NNLO QCD

Hamberg, Matsuura, van Neerven (1991)

Harlander, Kilgore (2002)

$\vdots$

N^{3}LO QCD

$\vdots$

Duhr, Dulat, Mistlberger (2020)

Progress in obtaining the NNLO QCDxEW corrections

On-shell Z/W production - first step towards full Drell-Yan

- Pole approximation : Dittmaier, Huss, Schwinn;
- Analytic QCDxQED corrections : de Florian, Der, Fabre;
- p_T^Z distribution in QCDxQED including p_T resummation : Cieri, Ferrera, Sborlini;
- Differential on-shell Z production including QCDxQED : Delto, Jaquier, Melnikov, Roentsch;
- Total QCDxEW corrections to Z production (fully analytic):

Bonciani, Buccioni, NR, Triscari, Vicini; Bonciani, Buccioni, NR, Vicini; $\Longleftarrow$ This Talk

- Differential on-shell Z/W production including QCDxEW :

Behring, Buccioni, Caola, Delto, Jaquier, Melnikov, Roentsch; $\Longleftarrow$ Talk by A. Behring

- QCDxQED corrections beyond SV at N^{3}LO :

A.H, Mukherjee, Ravindran, Sankar, Tiwari; $\Longleftarrow$ Talk by A. Sankar

Complete Drell-Yan

- neutrino pair production in QCDxQED : Cieri, de Florian, Der, Mazzitelli;
- $pp \to l\nu_l + X$ in QCDxEW :

Buonocore, Grazzini, Kallweit, Savoini, Tramontano; $\Longleftarrow$ Talk by L. Buonocore

- two-loop amplitudes:

Heller, von Manteuffel, Schabinger; $\Longleftarrow$ Talk by A. von Manteuffel

$\alpha_s(m_Z) \simeq 0.118$	$\alpha(m_Z) \simeq 0.0078$	$\frac{\alpha_s(m_Z)}{\alpha(m_Z)} \simeq 15.1$	$\frac{\alpha_s^2(m_Z)}{\alpha(m_Z)} \simeq 1.8$

1. From naive argument of coupling strength, N^{3}LO QCD $\sim$ mixed NNLO QCD$\otimes$EW.
2. However, in specific phase-space points, fixed order EW corrections can become very large because of logarithmic (weak and QED Sudakov type) enhancement. These effects are large for W mass measurements. On the other hand, these corrections suffer from large uncertainties coming from unphysical scales.
3. N^{3}LO QCD corrections control the uncertainties arising from the unphysical scales, but they lack the large EW effects.
4. The appearance of photon induced processes $\Rightarrow$ photon PDFs.

The NNLO mixed QCD-EW corrections

- have similar magnitude as N^{3}LO QCD,
- contain the large EW effects,
- and also reduce the dependency on unphysical scales.

NNLO QCD$\otimes$EW corrections extremely important for high ($\mathcal{O}(10^{-4})$) precision pheno.

Another motivation : Electroweak scheme dependence

The Lagrangian has 3 inputs (g, g', v). More observables (like $G_\mu, \alpha, m_W, m_Z, \sin\theta_W$) are experimentally measured and can be considered as input parameters in different schemes. Such two schemes are

1. G_μ-scheme : where (G_μ, m_W, m_Z) are considered as input
2. $\alpha(0)$-scheme : where (α, m_W, m_Z) are considered as input

The relation between G_μ and α gets EW and mixed QCD⊗EW corrections.

$$\frac{G_\mu}{\sqrt{2}} = \frac{\pi\alpha}{2\sin^2\theta_W \cos^2\theta_W m_Z^2}(1 + \Delta r)$$

At LO, $\alpha(G_\mu)$ and $\alpha(0)$ differs by 3.53%.

order	G_μ-scheme	$\alpha(0)$-scheme	$\delta_{G_\mu-\alpha(0)}$ (%)
LO	48882	47215	3.53
NLO QCD (LO + Δ_{10})	55732	53831	3.53
NNLO QCD (LO + Δ_{10} + Δ_{20})	55651	53753	3.53
NLO EW (LO + Δ_{01})	48732	48477	0.53
LO + Δ_{10} + Δ_{01}	55582	55093	0.89

In this talk, I present the NNLO mixed QCD$\times$EW corrections to Z boson production. I outline the computational details and technical challenges to obtain such contributions.

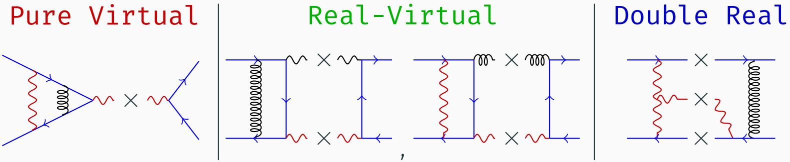
Notation

$$\sigma_{tot}(z) = \sum_{i,j\in q,\bar{q},g,\gamma} \int \mathrm{d}x_1 \mathrm{d}x_2 \; f_i(x_1,\mu_F) f_j(x_2,\mu_F) \sigma_{ij}(z,\varepsilon,\mu_F)$$

In the full QCD-EW SM, we have a double expansion of the partonic cross sections in the electromagnetic and strong coupling constants, α and α_s, respectively:

$$\begin{aligned}
\sigma_{ij}(z) &= \sigma_{ij}^{(0)} \sum_{m,n=0}^{\infty} \alpha_s^m \alpha^n \; \sigma_{ij}^{(m,n)}(z) \\
&= \sigma_{ij}^{(0)} \Big[\sigma_{ij}^{(0,0)}(z) \\
&\qquad + \alpha_s \sigma_{ij}^{(1,0)}(z) + \alpha \sigma_{ij}^{(0,1)}(z) \\
&\qquad + \alpha_s^2 \sigma_{ij}^{(2,0)}(z) + \alpha \alpha_s \sigma_{ij}^{(1,1)}(z) + \alpha^2 \sigma_{ij}^{(0,2)}(z) \\
&\qquad + \alpha_s^3 \sigma_{ij}^{(3,0)}(z) + \alpha \alpha_s^2 \sigma_{ij}^{(2,1)}(z) + \alpha^2 \alpha_s \sigma_{ij}^{(1,2)}(z) + \cdots \Big]
\end{aligned}$$

Anatomy of NNLO contributions for Z production



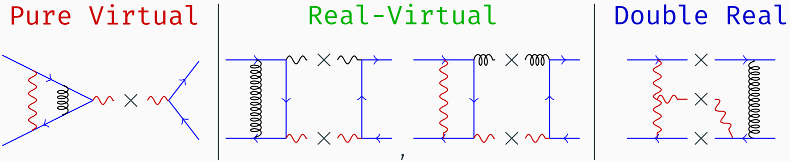
Loop integrals

- Integrating the virtual loop momenta, widely studied and understood
- The integrals result in constants (MZVs and cyclotomic constants)

Phase-space integrals

- Integrating the momenta of real-emitted particles
- Often performed numerically
- To obtain inclusive production cross-section, we require an analytic computation
- These integrals contain standard HPLs and elliptic polylogarithms

Anatomy of NNLO contributions for Z production



For different vector bosons, the contribution can be organized into four types

- QCD⊗QED : γ propagator in the loop / emission of γ
- EW1 : single Z propagator in the loop
- EW2 : single W propagator in the loop
- EW3 : Contributions with WWZ vertex

Emission of massive boson is infrared finite, hence, is treated as separate process.

gauge invariant and finite : QCD⊗QED, EW1, EW2+EW3

The generic procedure

$d = 4 - 2\epsilon$

- Diagrammatic approach -> QGRAF to generate diagrams $(10^2 - 10^3)$
- In-house FORM routines for algebraic manipulation :

  Lorentz, Dirac and Color algebra

- Reverse unitarity : phase-space integrals to loop integrals

$$\delta(k^2 - m^2) \sim \frac{1}{2\pi i}\left(\frac{1}{k^2 - m^2 + i0} - \frac{1}{k^2 - m^2 - i0}\right)$$

- Decomposition of the dot products to obtain scalar integrals $(10^5 - 10^6)$

$$\frac{2l.p}{l^2(l-p)^2} = \frac{l^2 - (l-p)^2 + p^2}{l^2(l-p)^2} = \frac{1}{(l-p)^2} - \frac{1}{l^2} + \frac{p^2}{l^2(l-p)^2}$$

- Identity relations $(10^5 - 10^6)$ among scalar integrals : *IBPs, LIs & SRs*
- Algebraic linear system of equations relating the integrals

$$\Downarrow$$

Master integrals (MIs) $(10 - 10^2)$

- Computation of MIs : *Differential equations*

Comments

on the method of differential equations :

1. We do not use canonical form for the system of differential equations.
2. Hence, we find coupled differential equations which we decouple at each order in ϵ.

on the dependence on m_W and m_Z:

1. For convenience, the calculation of the MIs that depend on two different masses (m_Z and m_W) is done performing an expansion of the integrand in powers of the ratio $\delta_{m^2} = (m_Z^2 - m_W^2)/m_Z^2$.

More notations

The variable z and its various forms

$$z = \frac{t}{(1+t)^2} = \frac{\rho}{(1-\rho+\rho^2)} = \frac{w}{1-w^2}\,.$$

Appearing kernels and the corresponding letters

$$\left\{-1, -\frac{1}{2}, 0, \frac{1}{2}, 1\right\} \equiv \left\{\frac{1}{1+x}, \frac{1}{\frac{1}{2}+x}, \frac{1}{x}, \frac{1}{\frac{1}{2}-x}, \frac{1}{1-x}\right\}$$

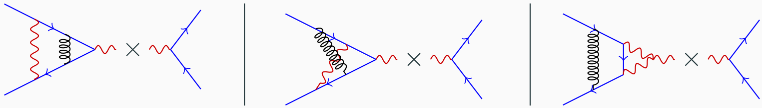
$$\left\{\{3,0\}, \{3,1\}, \{6,0\}, \{6,1\}\right\} \equiv \left\{\frac{1}{1+x+x^2}, \frac{x}{1+x+x^2}, \frac{1}{1-x+x^2}, \frac{x}{1-x+x^2}\right\}$$

$$\left\{\{4,1\}, i_1, -i_2\right\} \equiv \left\{\frac{x}{1+x^2}, \frac{1}{i_1-x}, \frac{1}{i_2+x}\right\}$$

where i_1 and i_2 are given by

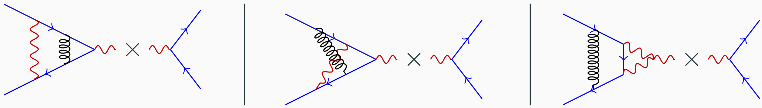
$$i_1 = \frac{\sqrt{5}-1}{2} \equiv 0.618034\ldots, \qquad i_2 = \frac{\sqrt{5}+1}{2} \equiv 1.618034\ldots$$

Computing the double-virtual



1. We compute the integrals considering off-shell Z.
2. We define the variables x and x_L, for single and double mass case, respectively as $x = -\frac{q^2}{m^2} = \frac{(1-x_L)^2}{x_L}$.
3. The boundary conditions are obtained for $x, x_L = 1$.
4. The result is written in HPLs with alphabet $\{-1, 0, 1, \{6, 0\}, \{6, 1\}\}$.

Computing the double-virtual



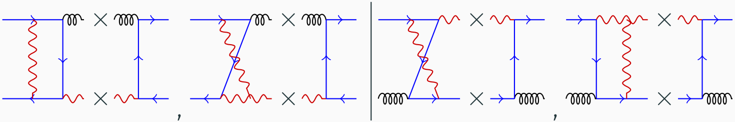
To achieve the on-shell result, appropriate limit needs to be taken.

- For EW1, the limit is $x \to -1$.
- For EW3, instead of taking the limit $x_L \to 1 - \frac{m_Z^2}{2m_W^2} - \frac{1}{2}\sqrt{\frac{m_Z^2}{m_W^2}\left(\frac{m_Z^2}{m_W^2} - 4\right)}$, we do a Taylor series expansion around $\delta_{m^2} = 0$.
- This produces HPLs (constants) with argument $r_2 = \frac{1}{2} - i\frac{\sqrt{3}}{2}$.
- Finally, we reduce all these constants to a basis (introduced by [Henn, Smirnov, Smirnov]).

$$H[\_\_, -1], H[\_\_, r_2] \quad \Rightarrow \quad \{\pi, \ln 2, \ln 3, \zeta_2, \zeta_3, \ldots, G_R[\_\_], G_I[\_\_]\}$$

`The basis is very important for analytic cancellation of singularities.`

Computing the real-virtual



- Reverse unitarity $\to$ IBP $\to$ MIs $\to$ $\frac{d}{dz}$ $\to$ Solve the diff. eqns.
- The following kernels appear

$$\frac{1}{1+z}, \frac{1}{z}, \frac{1}{\frac{1}{2}-z}, \frac{1}{1-z}, \frac{1}{1-z+z^2}, \frac{z}{1-z+z^2}, \frac{1}{z\sqrt{1-z}\sqrt{1+3z}}, \frac{1}{z\sqrt{1+4z^2}}$$

- However, the numerical evaluation of the iterated integrals with square-root letters is not efficient.
- Instead of using a single transformation rule to rationalize them, we write the system (each MI) as sum of functions of dependent variables and separately treat them. As a result, each sub-system has alphabet with 'good' letters $(-1, 0, \frac{1}{2}, 1, \{6, 0\}, \{6, 1\}, i_1, -i_2)$ with different argument (z, ρ, w).

Computing the real-virtual (example)

Let's consider two integrals $\{J_1, J_2\}$ such that

$$J_1' = a_1(d,z)J_1 + r_1(d,z);\; J_2' = a_2(d,z)J_2 + b_2(d,z)J_1 + r_2(d,z)$$

- the solution of J_1 involves a square-root letter,
- the homogeneous solution of J_2 contains standard kernel.

Expecting that all the coefficient of the poles should have a simpler/standard HPLs, we look for a combination $J_0 \equiv f_1(z)J_1 + f_2(z)J_2$, such that

$$J_0' = a_0(d,z)J_0 + (d-4)b_0(d,z)J_1 + r_0(d,z)\,.$$

This allows poles with simpler HPLs.

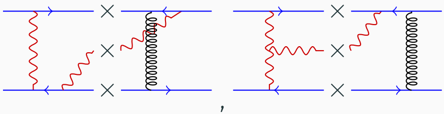
For the finite contributions from J_0, of course, the iterative integral over square-root will be present, along with standard HPLs. We perform variable transformation for only the square-root and associated terms to rationalize and write the non-homogeneous part as the following sum $\mathrm{nonh} = \mathrm{nonh}(z) + \mathrm{nonh}(w)$. Thus we avoid square-root letters in the alphabet which allows a smooth numerical evaluation.

Computing the real-virtual (example)

$$J_1^{(-1)} = \frac{w^2}{1-w^4}\Big(i\pi\big(3H_0(w)+H_1(w)+H_{-1}(w)\big) - 7H_{0,0}(w) - 4H_{0,1}(w)$$
$$+ 3H_{0,-i_2}(w) - 3H_{0,i_1}(w) + 4H_{0,-1}(w) - H_{1,0}(w) - H_{1,i_1}(w) - H_{-1,0}(w)$$
$$- H_{-1,i_1}(w) + 3\zeta_2 + H_{1,-i_2}(w) + H_{-1,-i_2}(w)\Big)$$

$$J_0^{(0)} = z^2\Big(-9\zeta_3 + \cdots\cdots - 2H_{\frac{1}{2},\frac{1}{2},0}(z) - 2H_{\frac{1}{2},\frac{1}{2},1}(z) - 5H_{\frac{1}{2},0,0}(z) - 6H_{\frac{1}{2},0,1}(z)$$
$$- 4H_{\frac{1}{2},1,0}(z) - 5H_{\frac{1}{2},1,1}(z) + 12H_{1,0,0}(z) + 14H_{1,1,0}(z) + 19H_{1,1,1}(z) + \cdots\cdots\Big)$$
$$+ \frac{w^2}{(1-w^2)^2}\Big(-3H_{-1}(w)\zeta_2 + \cdots\cdots + H_{1,-1,0}(w) + 3H_{1,0,i_1}(w) - 3H_{1,0,-i_2}(w)$$
$$+ H_{1,1,i_1}(w) + H_{1,-1,i_1}(w) - H_{1,1,-i_2}(w) - H_{1,-1,-i_2}(w) + \cdots\cdots\Big)$$

Computing the double-real

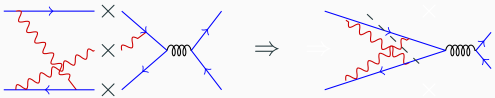


- Reverse unitarity $\to$ IBP $\to$ MIs $\to$ $\frac{d}{dz}$ $\to$ Solve the diff. eqns.
- The following kernels appear

$$\frac{1}{1+z}, \frac{1}{\frac{1}{2}+z}, \frac{1}{z}, \frac{1}{\frac{1}{2}-z}, \frac{1}{1-z},$$
$$\frac{1}{1+z+z^2}, \frac{z}{1+z+z^2}, \frac{1}{1-z+z^2}, \frac{z}{1-z+z^2}, \frac{z}{1+z^2}, \frac{1}{z\sqrt{1-z}\sqrt{1+3z}}$$

- Similar to RV, we write the system (each MI) as sum of functions of dependent variables and separately treat them. As a result, each sub-system has alphabet with 'good' letters $(-1, -\frac{1}{2}, 0, \frac{1}{2}, 1, \{3,0\}, \{3,1\}, \{6,0\}, \{6,1\}, \{4,1\})$ with different argument (z, ρ, t).

Computing the double-real (Elliptic)



This topology produces a 3×3 system which is not first-order factorizable, giving a set of elliptic integrals $\{I_1, I_2, I_3\}$, the homogeneous part of which is the same as the one studied for the corresponding virtual diagram by Aglietti, Bonciani, Grassi & Remiddi and Broedel, Duhr, Dulat, Penante & Tancredi to obtain the results in terms of elliptic integrals of the first kind and eMPLs, respectively.

In each order of ϵ-expansion, the 3×3 system reduces to 2×2 and 1×1 sub-systems.

The system can be solved with standard HPLs in the poles and eMPLs in the finite part and higher ϵ orders.

Computing the double-real (Elliptic)

However, the IBP reduction introduces a $\frac{1}{\epsilon}$ in the coefficient of these integrals which implies the integrals need to be computed up to $\mathcal{O}(\epsilon)$ and the finite part of the integrals (contain eMPLs) contribute to single pole of the matrix element.

Expecting the simpler polylogarithmic structure (no eMPL) of the single pole of the matrix element, we find the following combination of the elliptic masters (contributing to single pole) and solve for the ordinary d.e. in terms of HPLs.

$$I_0^{(n)} = z(1+2z)I_1^{(n)} + z(1-4z)I_2^{(n)} - (1+5z)I_3^{(n)}\,.$$

We find the solutions

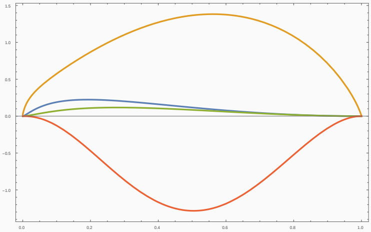
$$I_0^{(-1)} = \frac{1}{2}z^2(-1+4z)H_0(z)$$

$$I_0^{(0)} = \left(-\frac{5z^2}{2} + \frac{6z^4}{-1+z}\right)H_{0,0}(z) + 2z^2(-1+4z)H_{0,1}(z) + 2(1-4z)z^2\zeta_2$$

The solutions provide analytic cancellation of the single pole. Of course, the combination does not remain independent of I_2, I_3 for $I_0^{(1)}$, which contribute with eMPLs to the finite part of the matrix element.

Computing the double-real (Elliptic)

- To avoid the numerical evaluation of the eMPLs, we expand in logarithmic series the solutions for I_2 and I_3 around $z = 1, \frac{1}{2}$ and 0, imposing initial conditions in $z \to 1$ and matching the different series in intermediate points.
- We replace I_0 for I_1 and compute the HPL-dependent part of I_0 in closed form and the rest in expansion.
- In the end, we have part of the result of these elliptic integrals in closed form and the rest in expansion which enables us for a smooth numerical evaluation.

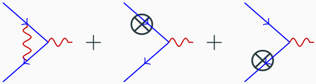


$I_1^{(0)}$ in blue, $I_2^{(0)}$ in yellow, $I_3^{(0)}$ in green, $I_0^{(1-expanded)}$ in red

Ultraviolet renormalization

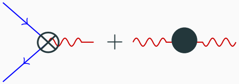
⊛ The Born contribution is zeroth order in α_s, hence no α_s renormalization is needed.

⊛ Renormalization of quark wave function receives one-loop EW and two-loop mixed QCD⊗EW contributions in the on-shell scheme.



$\Rightarrow$ UV finite

⊛ The neutral current vertex is renormalized using background field gauge, with the advantage that the vertex and propagator contributions are separately UV finite.



$\Rightarrow$ UV finite

⊛ The UV counter terms get contributions from two-point functions.

The UV renormalized matrix-elements are finally combined with appropriate mass counter terms to obtain the finite partonic cross sections ($\sigma_{ij}^{(1,1)}$).

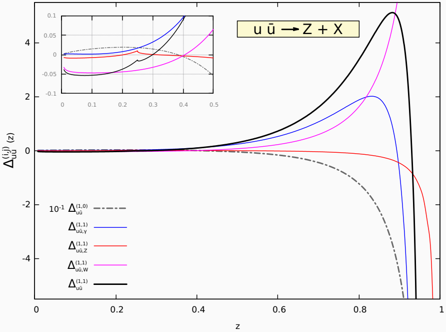
Numerical evaluation

We perform the convolution of the physical parton densities with the finite partonic cross-sections through two parallel `FORTRAN` codes to obtain the inclusive production cross-section.

In one code, we use `HarmonicSums` and `GiNaC` to evaluate $\sigma_{ij}^{(1,1)}(z)$ and save them as grids. Next, we use an interpolation routine to perform the convolution. Each $\sigma_{ij}^{(1,1)}(z)$ can be evaluated for 1000 points in a single-core in minutes, due to the compact structure.

In the other, we use `handyG` to evaluate $\sigma_{ij}^{(1,1)}(z)$ during convolution integration.

Results!



The finite partonic cross-sections for the particular process $u\bar{u} \to Z + X$.

It is interesting to note the 'kink' at $z = \frac{1}{4}$ in the weak contributions, arising from the di-boson production threshold.

Results!

Inclusive production cross-section for Z boson at 13 TeV

NNLO QCD +	G_μ	$\alpha(0)$	$\delta_{G_\mu-\alpha(0)}$ (%)
	55651	53753	3.53
$\delta_{\mathrm{NLO-EW}}$	55501	55015	0.88
$\delta_{\mathrm{NLO-EW}} + \delta_{\mathrm{NNLO-QCD\times QED}}$	55516	55029	0.88
$\delta_{\mathrm{NLO-EW}} + \delta_{\mathrm{NNLO-QCD\times EW}}$	55469	55340	0.23

- We use NNPDF31_nnlo_as_0118_luxqed_nf_4 pdfset.
- The mixed NNLO QCD$\times$QED correction is 0.03% of the Born, while the mixed NNLO QCD$\times$EW correction is negative and larger than the earlier by almost a factor of 3, providing per mille correction to the Born.
- After including the mixed NNLO QCD$\times$EW correction, the spread between two schemes reduces to 0.23%.

Results!

Inclusive production cross-section for Z boson at 13 TeV

Definitions of best prediction

PDF with DGLAP-QCD evolution `NNPDF31_nnlo_as_0118_nf_4`	σ_A = NNLO QCD
PDF with DGLAP-(QCDxQED) evolution `NNPDF31_nnlo_as_0118_luxqed_nf_4`	σ_B = NNLO QCD + $\delta_{\mathrm{NLO-EW}} + \delta_{\mathrm{NNLO-QCD\times EW}}$

In pure QCD model, the PDF is evolved with DGLAP-QCD

$$\sigma_A = 55787 \text{ pb}$$

In case of mixed corrections, PDF must be evolved with DGLAP-(QCDxQED)

$$\sigma_B = 55469 \text{ pb}$$

Both models are legitimate and differ by $\sim$0.57%.
However the full QCD-EW model is the logical choice for high precision studies!

Summarizing

- We have obtained analytic results for mixed NNLO QCD$\times$EW corrections to on-shell Z boson production.
- The method of reverse unitarity allows us to use the techniques (IBP, DE) of loop calculation for the phase-space integrals.
- We have computed two-loop virtual & phase-space integrals with massive lines.
- The solutions are obtained mostly in terms of HPLs and special constants (MZV and cyclotomic HPL at 1). The contributions from eMPLs are obtained as expansion.
- Cross checks
  - analytically and numerically with available QCD$\times$QED results.
  - within expected numerical accuracy with the Monte-Carlo computation.

Thank you for your attention!