Time-like Transition Form Factor for CP-odd Higgs Boson Production

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Plan of the talk

1. Introduction and Motivation
2. Space-like and Time-like Transition Form Factor
3. CP-odd Higgs $A^0$ Production in $e^+e^-$ Collisions
4. Concluding remarks

1. Introduction and Motivation

- The Transition Form Factor (TFF) has been studied for pion ($\pi^0$) production in $e^+e^-$ or $e\gamma$ collisions.
- The coupling of the CP-odd Higgs boson $A^0$ to fermions is pseudo-scalar, so it is similar to $\pi^0$.
- The question is whether the TFF is useful or not, for describing the $A^0$ production in $e^+e^-$ or $e\gamma$ collisions.
- In our previous work, we discussed $e\gamma \rightarrow eA^0$ in terms of space-like TFF. Now we investigate the $e^+e^- \rightarrow A^0\gamma$ by the time-like TFF which is related to space-like TFF in $e\gamma \rightarrow eA^0$ by the analytic continuation.
Pion Transition Form Factor

\[ F(Q^2) \sim 2f_\pi / Q^2 \]

Brodsky et al.

\[ \langle \pi^0 (k) | T | \gamma(p)\gamma^*(q) \rangle = \epsilon_\mu(p) \epsilon_\nu(q) T^{\mu\nu}(p, q) \]

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\[ T^{\mu\nu} = e^2 F(Q^2) \epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta \]
Space-like and Time-like Transition Form Factor

\[ e\gamma \rightarrow eA^0 \]

Space-like region \( q^2 < 0 \)

\[ e^+e^- \rightarrow \gamma A^0 \]

Time-like region \( q^2 > 0 \)

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Time-like Transition Form Factor for CP-odd Higgs Production
CP-odd Higgs boson $A^0$

- Minimal extension of Higgs Sector
  Here, we consider 2HDM type II or MSSM

  Charged $H^+, H^-; \text{CP-even } h^0, H^0; \text{CP-odd } A^0$

- $A^0$ does not couple to $W^+W^- \ ZZ$ pairs at tree level in contrast to CP-even Higgs $h^0$ or $H^0$

- $A^0$ does not couple to 2 physical Higgs bosons in the cubic interaction

- Coupling of $A^0$ with a fermion is proportional to the fermion mass:
  \[ -\frac{g m_u \cot \beta}{2m_W} \gamma_5 \text{ (up-type)} \quad -\frac{g m_d \tan \beta}{2m_W} \gamma_5 \text{ (down-type)} \]

- we consider the top quark contributions
Two processes related by s-t crossing

\[ A^t_{\mu\nu} = -\frac{e^2 g}{(4\pi)^2} N_C q_t^2 \cot \beta \frac{F_t(\rho, \tau) \epsilon_{\mu\nu\rho\sigma} q^\rho p^\sigma}{2m_W} \]

Transition form factor

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2. Space-like and Time-like Transition Form Factor

- In our previous work on $e\gamma \to eA^0$ process we have introduced the space-like transition form factor as

$$F_{t}^{e\gamma\to eA^0}(\rho, \tau) = \frac{\tau}{1 + \rho \tau} [g(\rho) + 4f(\tau)]$$

where

$$\rho \equiv \frac{-q^2}{4m_t^2}, \quad \tau \equiv \frac{4m_t^2}{m_A^2} \quad (q^2 < 0)$$

- In the case of $e^+e^- \to A^0\gamma$ we introduce time-like TFF now as

$$F_{t}^{e^+e^-\to \gamma A^0}(\rho, \tau) = \frac{\tau}{1 - \rho \tau} [-g(\rho) + 4f(\tau)]$$

$$\rho \equiv \frac{q^2}{4m_t^2}, \quad \tau \equiv \frac{4m_t^2}{m_A^2} \quad (q^2 > 0)$$
• For the space-like case in $e\gamma$ collisions, the $g(\rho)$ is

$$g^{e\gamma}(\rho) = \left[ \log \frac{\sqrt{\rho + 1} + \sqrt{\rho}}{\sqrt{\rho + 1} - \sqrt{\rho}} \right]^2$$

• While in the case of $e^+e^- \rightarrow A_0^0 \gamma$ $g(\rho)$ given by

$$g^{e^+e^-}(\rho) = -\left[ \log \frac{\sqrt{\rho + \sqrt{\rho - 1}}}{\sqrt{\rho - \sqrt{\rho - 1}} - i\pi} \right]^2$$

$q^2 \rightarrow q^2 + i\epsilon$

• We have the following analytic continuation:

$$g^{e\gamma}\left(\frac{-q^2}{4m_t^2} - i\epsilon\right) = -g^{e^+e^-}\left(\frac{q^2}{4m_t^2} + i\epsilon\right) \text{ i.e. } g^{e\gamma}(-\rho) = -g^{e^+e^-}(\rho)$$

$$F_t^{e\gamma\rightarrow eA_0^0}(-\rho, \tau) = \frac{\tau}{1 - \rho\tau}[g^{e\gamma}(-\rho) + 4f(\tau)]$$

$$= \frac{\tau}{1 - \rho\tau}[-g^{e^+e^-}(\rho) + 4f(\tau)] = F_t^{e^+e^-\rightarrow \gamma A_0^0}(\rho, \tau)$$
Transition form factor $e^+e^- \rightarrow A^0\gamma$

\[ \rho = \frac{q^2}{4m_t^2\sqrt{q^2}} = 2m_t \]

$\tau = 4m_t^2/m_A^2$

$\mathcal{M} = 400$ GeV

$m_t = 173$ GeV
Transition form factor $F_t(\rho, \tau)$ for $e^+e^- \rightarrow A^0\gamma$

$m_A = 400 \text{ GeV}$
$m_t = 173 \text{ GeV}$

Time-like region

$0 < q^2 < 4m_t^2$
$q^2 < 0$
$q^2 > 0$

$\sqrt{q^2} = 2m_t$
we also note no contribution from squark-loop for $A^0$ production
3. CP-odd Higgs $A^0$ Production in $e^+e^-$ Collisions


- Top-quark triangle loop diagram through $\gamma^* \& Z^*$
- No QCD radiative corrections by non-renormalization theorem of anomaly, in the case $m_A \ll m_t$.
- If extra contributions are negligible, TFF description makes sense.

**Kinematical variables**

\[ s = (l_+ + l_-)^2 = 4E^2 = q^2 > 0 \quad \text{s} \rightarrow \]
\[ t = (l_- - p)^2 = -2l_- \cdot p \]
\[ u = (l_+ - p)^2 = 2l_+ \cdot p \]
Scattering amplitudes and transition form factor

\[ \langle \gamma A^0 | T | e^- e^+ \rangle_{\gamma^*}^t = [\bar{\nu}(l_+) (-ie\gamma_\rho) u(l_-)] \frac{-ig^{\rho\mu}}{q^2 + i\epsilon} A_{\mu\nu}^t \epsilon^\nu(p) \]

\[ \langle \gamma A^0 | T | e^- e^+ \rangle_{Z^*}^t = \frac{g}{4\cos\theta_W} [\bar{\nu}(l_+) (i\gamma_\mu)(f_{Ze} + \gamma_5) u(l_-)] \frac{-i}{q^2 - m_Z^2} \tilde{A}_{\mu\nu}^t \epsilon^\nu(p) \]

\[ A_{\mu\nu}^t = -\frac{e^2 g}{(4\pi)^2} N_C q_t^2 \frac{\cot\beta}{2m_W} F_t(\rho, \tau) \epsilon_{\mu\nu\rho\sigma} q^\rho p^\sigma \]

\[ \tilde{A}_{\mu\nu}^t = -\frac{e^2 g}{(4\pi)^2} \frac{N_C q_t f_{Zt}}{4\cos\theta_W} \frac{\cot\beta}{2m_W} F_t(\rho, \tau) \epsilon_{\mu\nu\rho\sigma} q^\rho p^\sigma \]

Transition Form Factor

\[ F_t(\rho, \tau) = \frac{\tau}{1 - \rho\tau} [-g(\rho) + 4f(\tau)] , \quad \rho = \frac{q^2}{4m_t^2} , \quad \tau = \frac{4m_t^2}{m_A^2} \]

\[ f(\tau) = \left[ \sin^{-1} \sqrt{\frac{1}{\tau}} \right]^2 \quad \tau \geq 1 , \quad g(\rho) = -\left[ \log \frac{\sqrt{\rho} + \sqrt{\rho - 1}}{\sqrt{\rho} - \sqrt{\rho - 1}} - i\pi \right]^2 \quad \rho \geq 1 \]

\[ = -\frac{1}{4} \left[ \log \frac{1 + \sqrt{1 - \tau}}{1 - \sqrt{1 - \tau}} - i\pi \right]^2 \quad 0 < \tau < 1 \]

\[ = 4 \left[ \sin^{-1} \sqrt{\rho} \right]^2 \quad 0 < \rho < 1 . \]
Cross sections through $\gamma^*$ and $Z^*$

$$\left( \frac{d\sigma}{dt} \right)_{\gamma^*} = \frac{\alpha_{em}^3}{64\pi} \frac{g^2}{4\pi} \left( \frac{\cot \beta}{2m_W} \right)^2 \frac{1}{s} \left( \frac{t^2 + u^2}{s^2} \right) \left| N_C^t q_t^2 F_t(q^2, m_A^2, m_t^2) \right|^2$$

$$\left( \frac{d\sigma}{dt} \right)_{Z^*} = \frac{\alpha_{em}}{64\pi} \left( \frac{g^2}{4\pi} \right)^3 \left( \frac{\cot \beta}{2m_W} \right)^2 \left( \frac{1}{16 \cos^2 \theta_W} \right)^2 \frac{s}{(s - m_Z^2)^2} \times f^2_{Zt} (f_{Ze}^2 + 1) \left( \frac{t^2 + u^2}{s^2} \right) \left| N_C^t q_t F_t(q^2, m_A^2, m_t^2) \right|^2,$$

$$\left( \frac{d\sigma}{dt} \right)_{int} = -2 \times \frac{\alpha_{em}^2}{64\pi} \left( \frac{g^2}{4\pi} \right)^2 \left( \frac{\cot \beta}{2m_W} \right)^2 \frac{1}{16 \cos^2 \theta_W} \frac{1}{s - m_Z^2} \times f_{Zt} f_{Ze} q_t \left( \frac{t^2 + u^2}{s^2} \right) \left| N_C^t q_t F_t(q^2, m_A^2, m_t^2) \right|^2,$$

$$f_{Ze} = -1 + 4 \sin^2 \theta_W \quad f_{Zt} = 1 - \frac{8}{3} \sin^2 \theta_W$$
The cross section is proportional to $\cot^2 \beta$. Here we consider the case $\tan \beta$ not large and $A^0$ is rather light $m_A \leq 500$ GeV.
Total Cross Section through $\gamma^*$, $Z^*$ and Interference term

\[
cot \beta = 1
\]

components of cross section

A$^0$ mass dependence

$\sigma_{\text{tot}(\gamma^*)}$
$\sigma_{\text{tot}(Z^*)}$
$\sigma_{\text{tot}(\text{int})}$

$2m_t$

$\sqrt{s}$ [GeV]

$\sigma_{\text{tot}}$ [fb]

$\sigma_{\text{tot}}$ [fb]
Triangle vs. Box diagrams and Transition FF

Diagrams \( \{ \begin{align*}
\text{T-type} & \quad \text{Triangle diagrams} \\
\text{B-type} & \quad \text{Box diagrams}
\end{align*} \)

If T-type >> B-type

\[ \frac{d\sigma}{dQ^2} \propto |T|^2 \]

Transition Form Factor interpretation!
We are only interested in the order of magnitudes of their contributions.
Chargino-sneutrino process

Amplitude

\[ A^{(\tilde{\chi} \tilde{\nu})}_{e^+e^- \rightarrow \gamma A^0} = \left( \frac{eg^3\kappa_1 |V_{11}|^2}{16\pi^2} \right) \frac{m_{\tilde{\chi}}}{4} \epsilon^*(p)\beta [\bar{\nu}(l_+) F(\tilde{\chi} \tilde{\nu})\beta (1 - \gamma_5) u(l_-)] \]

Gauge invariant decomposition

\[ \kappa_1 = \frac{1}{\sqrt{2}} (\sin \beta U_{12} V_{11} + \cos \beta U_{11} V_{12}) \]

\[ F(\tilde{\chi} \tilde{\nu})\beta = \left( \frac{2l_+ - \beta p^{'t}}{t} + \gamma_\beta \right) S_1^{\tilde{\chi} \tilde{\nu}} (s, t, m_{A^0}^2, m_{\tilde{\chi}}^2, m_{\tilde{\nu}}^2) \]
\[ + \left( \frac{2l_+ + \beta p^{'u}}{u} + \gamma_\beta \right) S_2^{\tilde{\chi} \tilde{\nu}} (s, t, m_{A^0}^2, m_{\tilde{\chi}}^2, m_{\tilde{\nu}}^2) \]

Differential cross section

\[ \frac{d\sigma_{\tilde{\chi} \tilde{\nu}}}{dt} = \frac{1}{16\pi s^2} \left( \frac{eg^3\kappa_1 |V_{11}|^2}{16\pi^2} \right)^2 \cdot \frac{m_{\tilde{\chi}}^2}{8} s \times \left\{ |S_1^{\tilde{\chi} \tilde{\nu}}|^2 + |S_2^{\tilde{\chi} \tilde{\nu}}|^2 \right\} \]

\[ S_1^{\tilde{\chi} \tilde{\nu}} \text{ and } S_2^{\tilde{\chi} \tilde{\nu}} \text{ are given in terms of Passarino and Veltman’s scalar integrals} \]
\[ C_0’s \text{ and } D_0’s \text{ we assume } \kappa_1 |V_{11}|^2 \sim \mathcal{O}(1) \]
Chargino-sneutrino-box -two form factors-

\[
S_1^{\tilde{\chi} \tilde{\nu}}(s, t, m_{A_0}^2, m_{\tilde{\chi}}^2, m_{\tilde{\nu}}^2) \\
= -\frac{t-s}{s} C_0(0, u, m_{A_0}^2, m_{\tilde{\chi}}^2, m_{\tilde{\nu}}^2) - \frac{u}{s} C_0(0, 0, u, m_{\tilde{\chi}}^2, m_{\tilde{\nu}}^2) \\
+ \frac{2(t+u)}{s} C_0(0, s, m_{A_0}^2, m_{\tilde{\chi}}^2, m_{\tilde{\nu}}^2) \\
- \frac{(m_{\tilde{\chi}}^2(t+u) - m_{\tilde{\nu}}^2(t+u) - su)}{s} D_0(0, 0, 0, m_{A_0}^2, s, u, m_{\tilde{\chi}}^2, m_{\tilde{\nu}}^2, m_{\tilde{\nu}}^2) \\
- \frac{s + u}{s} C_0(0, t, m_{A_0}^2, m_{\tilde{\chi}}^2, m_{\tilde{\nu}}^2) - \frac{t}{s} C_0(0, 0, t, m_{\tilde{\chi}}^2, m_{\tilde{\nu}}^2) \\
- \frac{(m_{\tilde{\chi}}^2(t+u) - m_{\tilde{\nu}}^2(t+u) + st)}{s} D_0(0, 0, 0, m_{A_0}^2, t, s, m_{\tilde{\chi}}^2, m_{\tilde{\nu}}^2, m_{\tilde{\nu}}^2, m_{\tilde{\nu}}^2)
\]

and

\[
S_2^{\tilde{\chi} \tilde{\nu}}(s, t, m_{A_0}^2, m_{\tilde{\chi}}^2, m_{\tilde{\nu}}^2) \\
= -\frac{u-s}{s} C_0(0, t, m_{A_0}^2, m_{\tilde{\chi}}^2, m_{\tilde{\nu}}^2) - \frac{t}{s} C_0(0, 0, t, m_{\tilde{\chi}}^2, m_{\tilde{\nu}}^2) \\
+ \frac{2(t+u)}{s} C_0(0, s, m_{A_0}^2, m_{\tilde{\chi}}^2, m_{\tilde{\nu}}^2) \\
- \frac{(m_{\tilde{\chi}}^2(t+u) - m_{\tilde{\nu}}^2(t+u) - st)}{s} D_0(0, 0, 0, m_{A_0}^2, t, s, m_{\tilde{\chi}}^2, m_{\tilde{\nu}}^2, m_{\tilde{\nu}}^2, m_{\tilde{\nu}}^2) \\
- \frac{t+s}{s} C_0(0, u, m_{A_0}^2, m_{\tilde{\chi}}^2, m_{\tilde{\nu}}^2) - \frac{u}{s} C_0(0, 0, u, m_{\tilde{\chi}}^2, m_{\tilde{\nu}}^2) \\
- \frac{(m_{\tilde{\chi}}^2(t+u) - m_{\tilde{\nu}}^2(t+u) + su)}{s} D_0(0, 0, 0, m_{A_0}^2, s, u, m_{\tilde{\chi}}^2, m_{\tilde{\nu}}^2, m_{\tilde{\nu}}^2, m_{\tilde{\nu}}^2)
\]
Differential cross section

\[ \sqrt{s} = 500\text{GeV} \]

\[ m_A = 200\text{GeV} \]

\[ m_{\text{chargino}} = 300\text{GeV} \]

\[ \cos \theta \]

\[ \sqrt{s} = 500\text{GeV} \]

\[ m_A = 200\text{GeV} \]

\[ m_{\text{chargino}} = 1000\text{GeV} \]

\[ \cos \theta \]

\[ \sqrt{s} = 500\text{GeV} \]

\[ m_{\text{chargino}} = 300\text{GeV} \]

\[ \sqrt{s} = 250\text{GeV} \]

\[ \cos \theta \]

\[ \sqrt{s} = 500\text{GeV} \]

\[ m_{\text{chargino}} = 1000\text{GeV} \]

\[ \sqrt{s} = 250\text{GeV} \]

\[ \cos \theta \]
Neutralino-selectron process

We are only interested in the order of magnitudes, assuming that the diagrams of one mass eigenstate of neutralino $\tilde{\chi}_1^0$ and one mass eigenstate of selectron $\tilde{e}_1$ dominantly contribute.

$$A_{\gamma e^+e^-\rightarrow A^0} = \frac{ige^3\eta_1|\tilde{N}_{12}|^2}{16\pi^2} \frac{m_{\tilde{e}_1}}{4} \epsilon^*(p)^\beta \overline{u}(l_+) F^{(\tilde{\chi}_1^0 \tilde{e}_1)}_{2\beta}(1 - \gamma_5) u(l_-)$$

Similar gauge-invariant decomposition leads to $S_1^{\tilde{\chi}\tilde{e}}$ and $S_2^{\tilde{\chi}\tilde{e}}$

Differential cross section

$$\frac{d\sigma_{\tilde{\chi}\tilde{e}}}{dt} = \frac{1}{16\pi s^2} \left( \frac{e g^3 \eta_1 |\tilde{N}_{12}|^2}{16\pi^2} \right)^2 \times \frac{m_{\tilde{\chi}_1}^2}{8} S_1^{\tilde{\chi}\tilde{e}} S_2^{\tilde{\chi}\tilde{e}}$$

$S_1^{\tilde{\chi}\tilde{e}}$ and $S_2^{\tilde{\chi}\tilde{e}}$ are given in terms of Passarino and Veltman’s scalar integrals $C_0$’s and $D_0$’s we also assume $\eta_1 |\tilde{N}_{12}|^2 \sim \mathcal{O}(1)$
Neutralino-selectron-box\,-two form factors-

\[ S_1^{\tilde{\chi}\tilde{e}}(s, t, m_{A^0}^2, m_{\tilde{\chi}}^2, m_{\tilde{\nu}}^2) \]
\[ = \frac{s + u}{2s} C_0(0, t, m_{A^0}^2, m_{\tilde{\chi}}^2, m_{\tilde{e}}^2, m_{\tilde{\chi}}^0) + \frac{t - s}{2s} C_0(0, u, m_{A^0}^2, m_{\tilde{\chi}}^2, m_{\tilde{e}}^2, m_{\tilde{\chi}}^0) \]
\[ - \frac{t}{2s} C_0(0, 0, t, m_{\tilde{e}}^2, m_{\tilde{e}}^2, m_{\tilde{\chi}}^0) - \frac{u}{2s} C_0(0, 0, u, m_{\tilde{e}}^2, m_{\tilde{e}}^2, m_{\tilde{\chi}}^0) \]
\[ + \frac{(m_{\tilde{e}}^2 - m_{\tilde{\chi}}^0)(t + u) + tu}{2s} D_0(0, 0, 0, m_{A^0}^2, t, u, m_{\tilde{\chi}}^2, m_{\tilde{e}}^2, m_{\tilde{e}}^2, m_{\tilde{\chi}}^0) \]

and

\[ S_2^{\tilde{\chi}\tilde{e}}(s, t, m_{A^0}^2, m_{\tilde{\chi}}^2, m_{\tilde{\nu}}^2) \]
\[ = \frac{u - s}{2s} C_0(0, t, m_{A^0}^2, m_{\tilde{\chi}}^2, m_{\tilde{e}}^2, m_{\tilde{\chi}}^0) + \frac{t + s}{2s} C_0(0, u, m_{A^0}^2, m_{\tilde{\chi}}^2, m_{\tilde{e}}^2, m_{\tilde{\chi}}^0) \]
\[ - \frac{t}{2s} C_0(0, 0, t, m_{\tilde{e}}^2, m_{\tilde{e}}^2, m_{\tilde{\chi}}^0) - \frac{u}{2s} C_0(0, 0, u, m_{\tilde{e}}^2, m_{\tilde{e}}^2, m_{\tilde{\chi}}^0) \]
\[ + \frac{(m_{\tilde{e}}^2 - m_{\tilde{\chi}}^0)(t + u) + tu}{2s} D_0(0, 0, 0, m_{A^0}^2, t, u, m_{\tilde{\chi}}^2, m_{\tilde{e}}^2, m_{\tilde{e}}^2, m_{\tilde{\chi}}^0) \]
Total cross section \( e^+e^- \rightarrow A^0\gamma \)

Top-quark-loop vs. Box diagram contributions

Interference between top-quark-loop and box diagram is found to be very small.
4. Concluding Remarks

• We have investigated production of CP-odd Higgs $A^0$ associated with a real photon $\gamma$ in $e^+e^-$ collisions, in terms of time-like transition form factor.

• The dominant contribution coming from top-quark one-loop diagrams. The $\gamma^*$ process is far more dominant over $Z^*$ process.

• The Box contributions, from chargino-sneutrino and neutralino-selectron related processes do not give sizable effects, in the parameter space we have studied. If $\tan\beta$ is not large and chargino very heavy, their contributions are negligible. Then TFF description makes sense, but more thorough analysis is needed.
Back up slides
Total cross section $e^+e^- \rightarrow A^0\gamma$

mass dependence

$\sqrt{s} = 500\text{GeV}$

$\sigma_{\text{tot}}(s)$ vs $m_A[\text{GeV}]$
Differential Cross Section

$e\gamma \rightarrow eA^0$ process

$\sqrt{s} = 500 \text{ GeV} \quad m_t = 173 \text{ GeV}$

$\cot \beta = 1$

- $m_A = 400 \text{ GeV}$
- $m_A = 300 \text{ GeV}$
- $m_A = 200 \text{ GeV}$
Cross section: chargino-sneutrino process

$e \gamma \rightarrow e A^0$ process

\[ \sqrt{s} = 500 \text{ GeV} \]
\[ m_A = 400 \text{ GeV} \]

Chargino 200 GeV
Sneutrino 300 GeV

Top-quark loop

Chargino-sneutrino box
Cross section: Neutralino-selectron process

$e \gamma \rightarrow e A^0$ process

$\sqrt{s} = 500$ GeV
$m_A = 400$ GeV

Neutralino 200 GeV
selectron 300 GeV

Top-quark loop

Neutralino-selectron box