The pion-photon transition form factor at two loops in QCD

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Intro / motivation

- Pion-photon transition form factor: theoretically (one of) the simplest hadronic matrix elements

$$\langle \pi(p) | j_\mu^{\rm em} | \gamma(p') \rangle = g_{\rm em}^2 \, \epsilon_{\mu\nu\alpha\beta} \, q^\alpha \, p^\beta \, \epsilon^\nu(p') F_{\gamma^*\gamma\to\pi^0}(Q^2)$$

- Ideally suited for
  - precision studies of the partonic landscape of composite hadrons
  - investigating the factorization properties of hard exclusive QCD reactions

- First studies date back half a century (before QCD!)

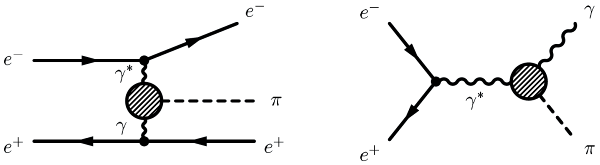
  [Cornwall'66; Gross,Treiman'71; Brodsky,Kinoshita,Terazawa'71]
  - Later one-loop and subleading power-corrections

    [del Aguila,Chase'81; Braaten'83; Kadantseva,Mikhailov,Radyushkin'86; Shen,Wang'17]

- Also with connections to
  - HLbL contribution to $(g-2)_\mu$ in the dispersive framework (double-virtual FF)
  - exclusive B-meson decays such as $B \to \pi \ell \nu$ or $B \to \pi\pi$ (pion LCDA)

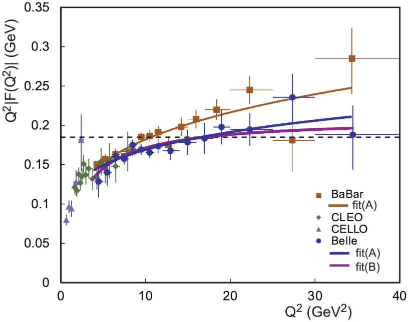
Intro / motivation

- Experimental measurements in e^+e^- collisions



- Status of experimental measurements

[figures from Wang'18]



- Asymptotic limit (dashed line)

$$\lim_{Q^2\to\infty} Q^2 F_{\gamma^*\gamma\to\pi^0}(Q^2) = \sqrt{2}\, f_\pi$$

[Brodsky,Lepage'80]

- Scaling violation?

The pion-photon transition form factor

- Pion-photon transition form factor $F_{\gamma^*\gamma\to\pi^0}$
- extracted from matrix element of e.m. current $j_\mu^{\rm em} = \sum_q g_{\rm em}\, Q_q\, \bar{q}\, \gamma_\mu\, q$

  between on-shell photon of momentum p' and pion of momentum p

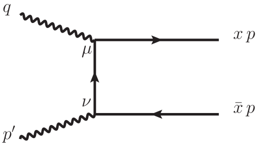
$$\langle \pi(p) | j_\mu^{\rm em} | \gamma(p') \rangle = g_{\rm em}^2\, \epsilon_{\mu\nu\alpha\beta}\, q^\alpha\, p^\beta\, \epsilon^\nu(p')\, F_{\gamma^*\gamma\to\pi^0}(Q^2)$$

- Have $q = p - p'$ and $Q^2 = -q^2$
- Kinematics at leading power

$$p'_\mu = (n\, p')\frac{\bar{n}_\mu}{2} \qquad p_\mu = (\bar{n}\, p)\frac{n_\mu}{2} \qquad (n\, p') \sim (\bar{n}\, p) \sim \mathcal{O}(\sqrt{Q^2})$$

- Goal: Establish QCD factorization formula for $F_{\gamma^*\gamma\to\pi^0}(Q^2)$ at leading power

The pion-photon transition form factor



- Consider four-point QCD matrix element (x: momentum fraction, $\bar{x} \equiv 1 - x$)

$$\Pi_\mu = \langle q(x\,p)\,\bar{q}(\bar{x}\,p)|j_\mu^{\rm em}|\gamma(p')\rangle$$

- Allows to derive factorization formula for $F_{\gamma^*\gamma\to\pi^0}$ at leading power

$$F^{\rm LP}_{\gamma^*\gamma\to\pi^0}(Q^2) = \frac{(Q_u^2 - Q_d^2)\, f_\pi}{\sqrt{2}\,Q^2} \int_0^1 dx\; T_2(x)\,\phi_\pi(x,\mu)$$

- $T_2(x)$: hard function, computable in perturbation theory
- $\phi_\pi(x,\mu)$: Leading twist pion light-cone distribution amplitude (LCDA)

$$\langle\pi(p)|\bar{\xi}(y)\,W_c(y,0)\,\gamma_\mu\,\gamma_5\,\xi(0)|0\rangle = -i\,f_\pi\,p_\mu \int_0^1 du\,e^{i\,u\,p\cdot y}\,\phi_\pi(u,\mu) + \mathcal{O}(y^2)$$

Matching onto SCET

- Goal: Extract hard function $T_2^{(L)}$ at given loop-order L.
- Introduce SCET operator basis: $\{O_1^{\mu\nu}(x),\, O_2^{\mu\nu}(x),\, O_E^{\mu\nu}(x)\}$

$$O_j^{\mu\nu}(x) = \frac{\bar{n}\cdot p}{2\pi} \int d\tau\, e^{i\,x\,\tau\,\bar{n}\cdot p}\; \bar{\xi}(\tau\bar{n})\, W_c(\tau\bar{n},0)\, \Gamma_j^{\mu\nu}\, \xi(0)\,,$$

- Dirac structures

$$\Gamma_{1,\,\mu\nu} = g_{\mu\nu}^{\perp}\;\; \not{n}\,, \qquad \Gamma_{2,\,\mu\nu} = i\,\epsilon_{\mu\nu}^{\perp}\;\; \not{n}\,\gamma_5\,,$$

$$\Gamma_{E,\,\mu\nu} = \not{n}\left(\tfrac{1}{2}[\gamma_{\mu,\perp},\gamma_{\nu,\perp}] - i\,\epsilon_{\mu\nu}^{\perp}\,\gamma_5\right)\,.$$

- For our calculation $O_2^{\mu\nu}(x)$ is the relevant operator
  - $O_E^{\mu\nu}(x)$ is evanescent
  - $O_1^{\mu\nu}(x)$ cannot couple to a collinear pion state (parity). Relevant for DVCS.

Matching onto SCET

- Start with

$$\langle j^{\rm em} \rangle = \sum_{a=1,2,E} T_a \, \langle O_a \rangle \qquad (1)$$

- Expand both sides in $\widetilde{\alpha}_s = \alpha_s/(4\pi)$, superscripts give loop order

$$T_a = T_a^{(0)} + \widetilde{\alpha}_s \, T_a^{(1)} + \widetilde{\alpha}_s^2 \, T_a^{(2)} + \ldots$$

$$\langle j^{\rm em} \rangle = \sum_{a=1,2,E} \left\{ A_a^{(0)} + \widetilde{\alpha}_s A_a^{(1)} + \widetilde{\alpha}_s^2 \left[A_a^{(2)} + Z_\alpha^{(1)} A_a^{(1)} \right] + \ldots \right\} \langle O_a \rangle^{(0)}$$

- On-shell matrix elements of $\langle O_a \rangle$ simplify due to scaleless integrals

$$\langle O_a \rangle = \sum_{b=1,2,E} \left\{ \delta_{ab} + \widetilde{\alpha}_s \, Z_{ab}^{(1)} + \widetilde{\alpha}_s^2 \, Z_{ab}^{(2)} + \ldots \right\} \langle O_b \rangle^{(0)}$$

- Compare coefficients of $\langle O_i \rangle^{(0)}$ on both sides of (1) at given order in $\widetilde{\alpha}_s$.

Matching onto SCET: Master formulas

- Tree level $T_a^{(0)} = A_a^{(0)}$
- One loop

$$T_2^{(1)} = A_2^{(1)} - T_2^{(0)} * Z_{22}^{(1)} - T_E^{(0)} * Z_{E2}^{(1)}$$

$$T_E^{(1)} = A_2^{(1)} - T_2^{(0)} * Z_{22}^{(1)} = T_2^{(1)} + \underbrace{T_E^{(0)} * Z_{E2}^{(1)}}_{\text{finite shift}}$$

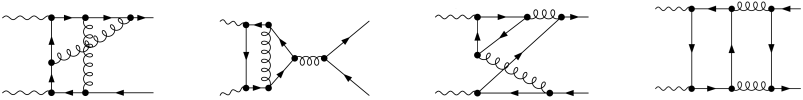
  - Asterisk denotes convolution, due to non-locality of SCET operators O_a.
  - Have used that

$$A_2^{(L)} = A_E^{(L)} \qquad Z_{1E}^{(1)} = Z_{2E}^{(1)} = 0$$

$$Z_{12}^{(1)} = 0 \text{ (parity)} \qquad Z_{EE}^{(1)} = Z_{22}^{(1)}$$

- Two loops

$$T_2^{(2)} = A_2^{(2)} + Z_\alpha^{(1)} A_2^{(1)} - \sum_{a=2,E} \left[T_a^{(1)} * Z_{a2}^{(1)} + T_a^{(0)} * Z_{a2}^{(2)} \right]$$



- Generate diagrams with FeynArts and in-house routine [Hahn'00+]
- Many diagrams vanish
  - Zero color factor
  - Furry's theorem
  - projection onto the π^0 wave-function $\propto |u\bar{u}\rangle - |d\bar{d}\rangle$
- 42 diagrams (plus $\gamma \leftrightarrow \gamma^*$) remain
- Use dimensional regularization with $D = 4 - 2\epsilon$, NDR for γ_5
- Perform Dirac reduction to $\Gamma^{\mu\nu}_{1,2,E}$
  - In individual diagrams two additional Dirac structures appear, $n^\mu n^\nu \not{\bar{n}}$ and $\bar{n}^\mu n^\nu \not{\bar{n}}$

Two-loop calculation

- Reduction of scalar integrals: IBP relations, Laporta algorithm in FIRE

[Tkachov'81; Chetyrkin,Tkachov'81] [Laporta'01; Smirnov'08]

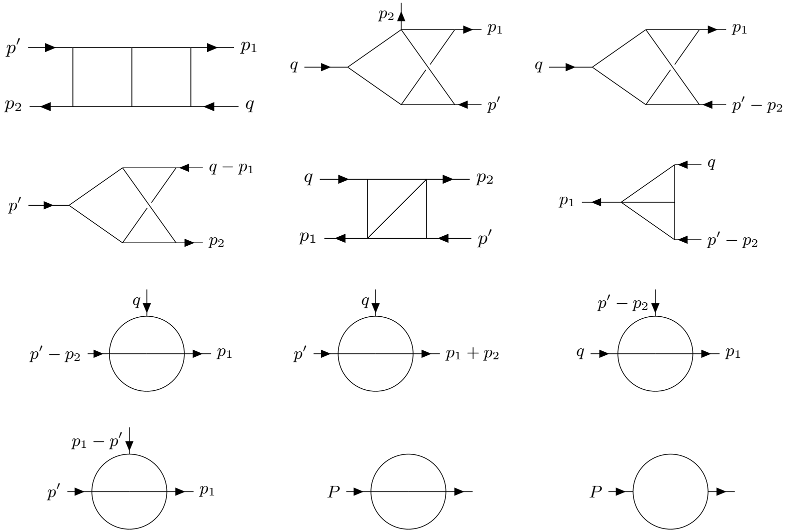
- Reduction of most complicated diagram takes $\sim$ 1 day
- In addition, exploit relations based on momentum conservation
  - Required since $p_1 = x\,p$ and $p_2 = \bar{x}\,p$ are parallel

$$\bar{x}k_1^2 - \bar{x}(k_1+p_1)^2 + xk_2^2 + x(k_2-k_1-p_2)^2 - x(k_1-k_2)^2 - x(k_2-p_2)^2 = 0$$

$$k_2^2 - x(k_2-p_2)^2 - \bar{x}(k_2+p_1)^2 = 0$$

- Reduction yields (only) 12 master integrals
- After IBP reduction, Dirac structures $n^\mu n^\nu \not{\bar{n}}$ and $\bar{n}^\mu n^\nu \not{\bar{n}}$ drop out identically
  - Manifestation of QED Ward identity

Master integrals



Evaluating Master integrals

- Closed forms, valid to all orders in ϵ. Gives Γ- and hypergeometric functions.
  - Expand with `HypExp` [Maître,TH'05'07]
- Method of differential equations [Kotikov'90'91; Remiddi'97]
  - Partially in canonical form [Henn'13]
- Mellin Barnes representations [Czakon'05; Gluza,Riemann et al.'07+; Kosower'09]
  - compute boundary conditions for DEs as $x \to 0$ or $x \to 1$ [Czakon'06]
  - derive full x-dependence of analytic functions
- Numerical checks with FIESTA [Tentyukov,Smirnov'08+]
- Obtain analytic ϵ-expansion of all master integrals
  - Harmonic polylogarithms (HPLs) of weights $0, 1$ [Remiddi,Vermaseren'99]
  - HPLs of at most weight four appear in the amplitude.

Infrared subtraction

- Recall master formula

$$T_2^{(2)} = A_2^{(2)} + Z_\alpha^{(1)}\, A_2^{(1)} - \sum_{a=2,E} \left[T_a^{(1)} * Z_{a2}^{(1)} + T_a^{(0)} * Z_{a2}^{(2)} \right]$$

- $Z_{22}^{(1)}$ is the one-loop ERBL kernel [Efremov,Radyushkin'80; Brodsky,Lepage'80]

$$Z_{22}^{(1)}(x,x') = -\frac{2C_F}{\epsilon} \left[\frac{\bar{x}}{\bar{x}'} \Big(1 + \frac{1}{x-x'}\Big) \theta(x-x') + (x \leftrightarrow \bar{x} \;,\; x' \leftrightarrow \bar{x}') \right]_+$$

- $Z_{22}^{(2)}$ is the two-loop ERBL kernel

$$Z_{22}^{(2)}(x,x') = \frac{1}{2\epsilon^2} \left\{ Z_{22}^{(1)}(x,x'') * Z_{22}^{(1)}(x'',x') - \beta_0\, Z_{22}^{(1)}(x,x') \right\}$$
$$+ \frac{1}{2\epsilon} \left\{ 2\, n_f\, C_F\, V_F + 2\, C_F\, C_A\, V_G + C_F^2\, V_F \right\}_+$$

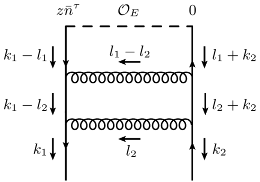
- Need lower-loop quantities beyond $\mathcal{O}(\epsilon^0)$.

Infrared subtraction

- Most delicate point:

  Evanescent $\longrightarrow$ physical mixing

  at two loops, $Z_{E2}^{(2)}$



- Insertion of non-local evanescent operator, regularize IR divergences

$$I^{\mu\nu}_{(2),A} = C_F^2(\bar{n}\cdot k)\int_{-\infty}^{\infty}\frac{dz}{2\pi}\,\mathrm{e}^{-itzk\cdot\bar{n}}\int\frac{d^dl_1 d^dl_2}{(2\pi)^{2d}}\bar{u}(k_1)(ig\gamma^\rho)\frac{i(\not{k}_1-\not{l}_2)}{(k_1-l_2)^2}(ig\gamma^\alpha)\frac{i(\not{k}_1-\not{l}_1)}{(k_1-l_1)^2}$$
$$\times\,\mathcal{O}_E^{\mu\nu}\frac{-i(\not{l}_1+\not{k}_2)}{(l_1+k_2)^2}(ig\gamma^\beta)\frac{-i(\not{l}_2+\not{k}_2)}{(l_2+k_2)^2}(ig\gamma^\sigma)v(k_2)\frac{-ig_{\rho\sigma}}{l_2^2}\frac{-ig_{\alpha\beta}}{(l_1-l_2)^2}\,\mathrm{e}^{iz(k_1-l_1)\cdot\bar{n}}$$

- Take coefficient of $O_2^{\mu\nu}$, Fourier transform

$$I^{\mu\nu}_{(2),A} = 8a_s^2C_F^2\left\{\frac{\bar{t}}{\bar{x}}\,[3+2L_m]+\frac{t\ln t}{\bar{x}}\,[2(1+L_m)+\ln t]+\frac{\bar{t}}{x}\ln\bar{x}\,[2(1+L_m+\ln\bar{t})-\ln\bar{x}]\right\}\theta(t-x)+(t\to\bar{t},x\to\bar{x})$$

- IR regulator drops in sum of all diagrams

Analytic results

- Sum everything up, poles in ϵ cancel, extract $T_2^{(2)}$

$$T_2^{(2)} = \beta_0\, C_F \left(\mathcal{K}_\beta^{(2)}(x)/x + \mathcal{K}_\beta^{(2)}(\bar{x})/\bar{x}\right) + C_F^2 \left(\mathcal{K}_F^{(2)}(x)/x + \mathcal{K}_F^{(2)}(\bar{x})/\bar{x}\right) + C_F/N_c \left(\mathcal{K}_N^{(2)}(x)/x + \mathcal{K}_N^{(2)}(\bar{x})/\bar{x}\right)$$

[agrees with Braun,Manashov,Moch,Schoenleber'21]

where

$$\begin{aligned}\mathcal{K}_\beta^{(2)}(x) =& - L^2 \left(H_0(x) + \frac{3}{2}\right) + L\left(-\frac{10}{3}H_0(x) - H_1(x) + 2H_{0,0}(x) - 2H_{1,0}(x) - 2\zeta_2 - \frac{19}{2}\right) - \zeta_2 H_1(x) - \frac{19}{9}H_0(x) \\ & - \frac{1}{2}H_1(x) + \frac{10}{3}H_{0,0}(x) - \frac{14}{3}H_{1,0}(x) - H_{1,1}(x) - 2H_{0,0,0}(x) + 2H_{1,0,0}(x) - H_{1,1,0}(x) - \frac{14}{3}\zeta_2 - \zeta_3 - \frac{457}{24}\end{aligned}$$

$$\begin{aligned}\mathcal{K}_F^{(2)}(x) =& L^2 \left(6H_0(x) - 2H_1(x) + 4H_{0,0}(x) + 2H_{1,0}(x) + \frac{9}{2}\right) + L\left(8\zeta_2 H_0(x) + \frac{38}{3}H_0(x) + 4\zeta_2 H_1(x) - 17H_1(x)\right. \\ & \left. -6H_{0,0}(x) + 8H_{1,0}(x) - 2H_{1,1}(x) - 12H_{0,0,0}(x) + 4H_{1,1,0}(x) - 4\zeta_3 + 6\zeta_2 + \frac{47}{2}\right) + 6\zeta_2 H_0(x) + 4\zeta_2 H_1(x) \\ & - 2\zeta_2 H_2(x) - 8\zeta_2 H_{0,0}(x) - 2\zeta_2 H_{1,0}(x) + 2\zeta_2 H_{1,1}(x) + 32\zeta_3 H_0(x) - 4\zeta_3 H_1(x) - \frac{64}{9}H_0(x) - \frac{71}{2}H_1(x) \\ & - \frac{38}{3}H_{0,0}(x) + \frac{34}{3}H_{1,0}(x) - 11H_{1,1}(x) - 2H_{1,2}(x) - 8H_{1,0,0}(x) + 4H_{1,1,0}(x) - 2H_{1,1,1}(x) \\ & - 2H_{1,2,0}(x) - 4H_{2,0,0}(x) - 2H_{2,1,0}(x) + 12H_{0,0,0,0}(x) - 2H_{1,0,0,0}(x) - 2H_{1,1,0,0}(x) \\ & + 2H_{1,1,1,0}(x) + 3\zeta_2^2 + \frac{34}{3}\zeta_2 + 39\zeta_3 + \frac{701}{24}\end{aligned}$$

Analytic results

- Sum everything up, poles in ϵ cancel, extract $T_2^{(2)}$

$$T_2^{(2)} = \beta_0 \, C_F \left(\mathcal{K}_\beta^{(2)}(x)/x + \mathcal{K}_\beta^{(2)}(\bar{x})/\bar{x} \right) + C_F^2 \left(\mathcal{K}_F^{(2)}(x)/x + \mathcal{K}_F^{(2)}(\bar{x})/\bar{x} \right) + C_F/N_c \left(\mathcal{K}_N^{(2)}(x)/x + \mathcal{K}_N^{(2)}(\bar{x})/\bar{x} \right)$$

[agrees with Braun,Manashov,Moch,Schoenleber'21]

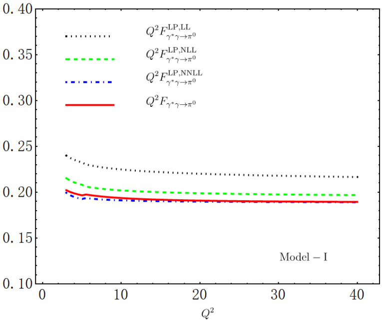
where

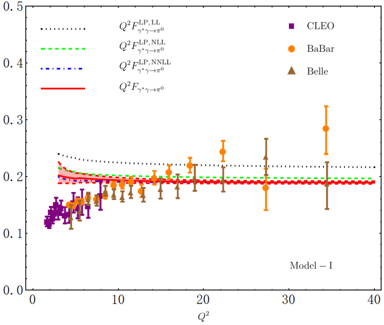
$$\begin{aligned}
\mathcal{K}_N^{(2)}(x) =& L \left(4\zeta_2 H_0(x) - \frac{8}{3} H_0(x) - 4H_1(x) - 4H_3(x) + 4H_{2,0}(x) + 12\zeta_3 - 1 \right) + 12\, x \left(\zeta_2 H_0(x) - H_3(x) + H_{2,0}(x)\right) \\
& - 6\zeta_2 H_1(x) - 4\zeta_2 H_2(x) - 4\zeta_2 H_{0,0}(x) + 4\zeta_2 H_{1,0}(x) + 2\zeta_2 H_{1,1}(x) + 14\zeta_3 H_0(x) - \frac{32}{9} H_0(x) + 11 H_1(x) \\
& + 4H_2(x) + 8H_4(x) + \frac{8}{3} H_{0,0}(x) + \frac{2}{3} H_{1,0}(x) + 2H_{1,1}(x) + 6H_{1,2}(x) - 2H_{1,3}(x) + 6H_{2,2}(x) - 4H_{3,0}(x) \\
& - 4H_{3,1}(x) - 6H_{1,1,0}(x) - 2H_{1,1,2}(x) + 4H_{1,2,0}(x) - 6H_{2,0,0}(x) - 4H_{2,1,0}(x) - 2H_{1,1,0,0}(x) \\
& + 2H_{1,1,1,0}(x) + \frac{1}{5}\zeta_2^2 - \frac{22}{3}\zeta_2 + 54\zeta_3 - \frac{73}{12}
\end{aligned}$$

Numerical results

- Need to model the pion LCDA, choose five models
- Use three-loop evolution of pion LCDA, expand to first 12 Gegenbauer moments
- Model I: $\phi_\pi(x,\mu_0) = \frac{\Gamma(2+2\alpha_\pi(\mu_0))}{\Gamma^2(1+\alpha_\pi(\mu_0))}\,(x\,\bar{x})^{\alpha_\pi(\mu_0)}, \quad \alpha_\pi(2\text{ GeV}) = 0.585^{+0.061}_{-0.055}$

[Khodjamirian,Melic,Wang,Wei'20]





- Red line includes subleading power corrections (twist 4, hadronic photon effect)

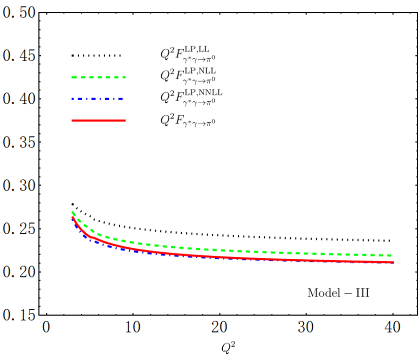
[Shen,Wang'17]

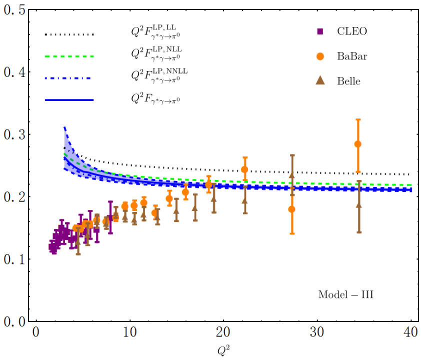
- Only perturbative uncertainties are shown

Numerical results

- Model III: $a_2(1\text{ GeV}) = 0.14 \qquad a_4(1\text{ GeV}) = 0.23$
  $a_6(1\text{ GeV}) = 0.18 \qquad a_8(1\text{ GeV}) = 0.05$

[Agaev,Braun,Offen,Porkert'10]

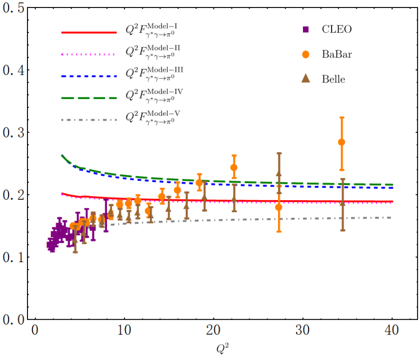


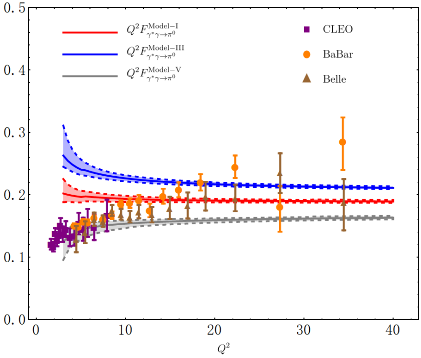


- Only perturbative uncertainties are shown

Numerical results

- Comparison between models
- Only perturbative uncertainties are shown





- Belle II data will allow to distinguish between LCDA models

Conclusion and Outlook

- We computed the pion-photon transition form factor to two loops in QCD
  - Prime example of QCD factorization in hard exclusive processes
  - Two-loop computation of bare amplitude uses standard multi-loop methods
  - IR subtraction non-trivial due to evanescent-physical mixing at two loops
  - Studied models of pion LCDA, comparison to data
- Future plans, e.g.
  - Include massive quarks in fermion loops
  - Take second photon off-shell