SUBLEADING POWER RESUMMATION AND THE ENDPOINT DIVERGENT CONTRIBUTION IN DIS

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NEXT-TO-LEADING POWER

This talk is based on

[M. Beneke, M. Garny, S. Jaskiewicz, **RS**, L. Vernazza, J. Wang: 2008.04943]

NLP has been a topic of intensive studies in SCET community and beyond:

- *Higgs at threshold*
  [M. Beneke, M. Garny, S. Jaskiewicz, **RS**, L. Vernazza, J. Wang: 1910.12685]
- *Drell-Yan at threshold*
  [M. Beneke, A. Broggio, M. Garny, S. Jaskiewicz, **RS**, L. Vernazza, J. Wang: 1809.10631]
  [M. Beneke, A. Broggio, S. Jaskiewicz, L. Vernazza: 1912.01585]
- *Thrust*
  [I. Moult, I. Stewart, G. Vita, H. X. Zhu: 1804.04665,1910.14038]
- *Rapidity logs*
  [I. Moult, G. Vita, K. Yan 1912.02188]
- *Mass effects* ($\mathrm{SCET}_{\mathrm{II}}$, *NLL!*)
  [Z. Liu, B. Mecaj, M. Neubert, X. Wang: 2009.04456, 2009.06779]
  [Z. Liu, M. Neubert: 1908.11379] *(refactorization!)*
  [C. Anastasiou, A. Penin: 2004.03602]

RESUMMATION IN SCET

Unlike traditional diagrammatic approach, SCET allows for systematic, more intuitive derivation of factorization theorems. Each function is associated with an operatorial definition and resummation is performed using renormalization group equations

$$\sigma \sim H \otimes J_1 \otimes \ldots \otimes J_n \otimes S$$

Hard matching coefficient

Jet functions - matrix elements of collinear fields

Soft function - matrix element of soft fields

1. *Define functions*
2. *Renormalize UV divergences of EFT operators*
3. *Solve RGE*

Key point:
Each function is a single scale object - RGE resums large logs

SCET 101

Systematic expansion in $\lambda \sim \dfrac{p_\perp}{n_+ p}$

Hard-Collinear momentum	$n_+ p \sim Q$	$p_\perp \sim \lambda Q$	$n\_p \sim \lambda^2 Q$
Soft momentum	$n_+ p \sim \lambda^2 Q$	$p_\perp \sim \lambda^2 Q$	$n\_p \sim \lambda^2 Q$

Every field has a well defined scaling

Hard-collinear quark: $\chi \sim \lambda$ *Soft quark:* $q \sim \lambda^3$

Hard-collinear gluon: $\mathscr{A}_\perp \sim \lambda$

Operators are non-local along the light-cone direction

$$\int dt_1 dt_2\, C^{B1}(t_1, t_2)\, \chi(t_1 n_+) \mathscr{A}_\perp(t_2 n_+)$$

$$\frac{d\sigma}{d\lambda} = \sum_{k=0} \sum_{l=0}^{2k-1} \left(\frac{1}{\lambda^2} c_{kl}^{\mathrm{LP}} + d_{kl}^{\mathrm{NLP}} \right) \alpha^k \ln^l \lambda$$

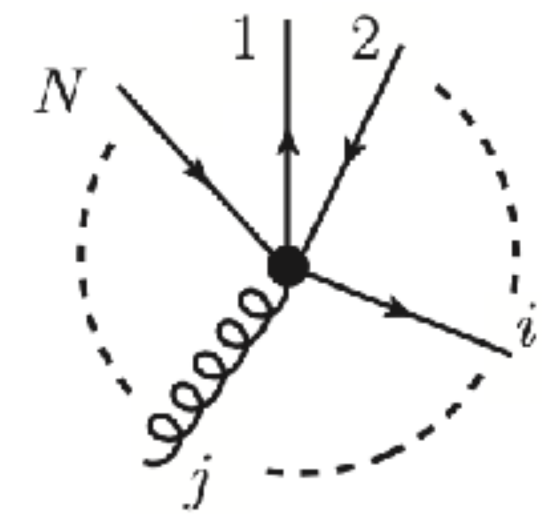
BEYOND LEADING POWER

SCET allows for systematic expansion

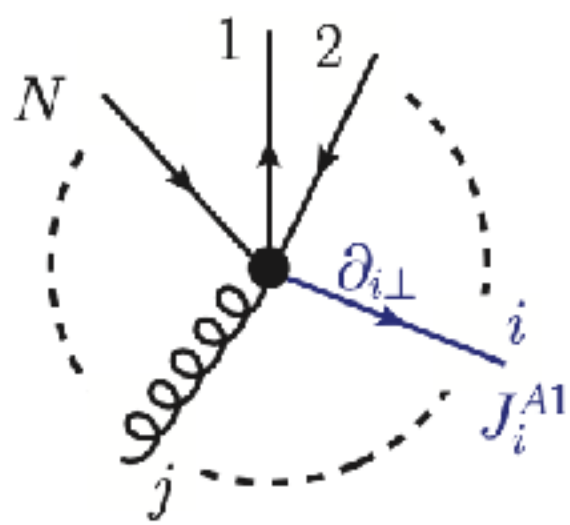
1. Power-suppressed operators

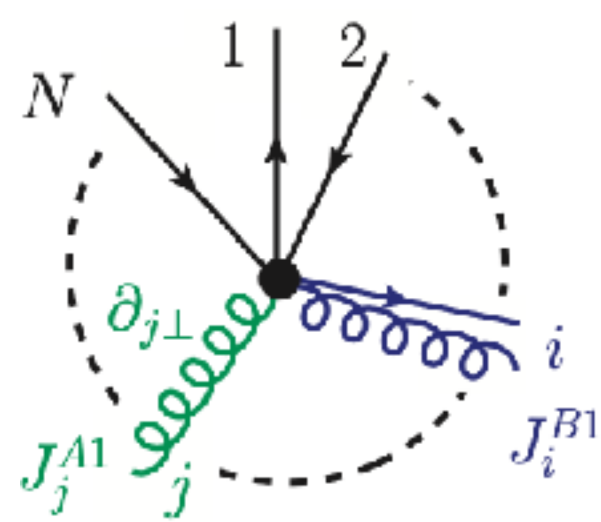
2. Subleading Lagrangian insertions

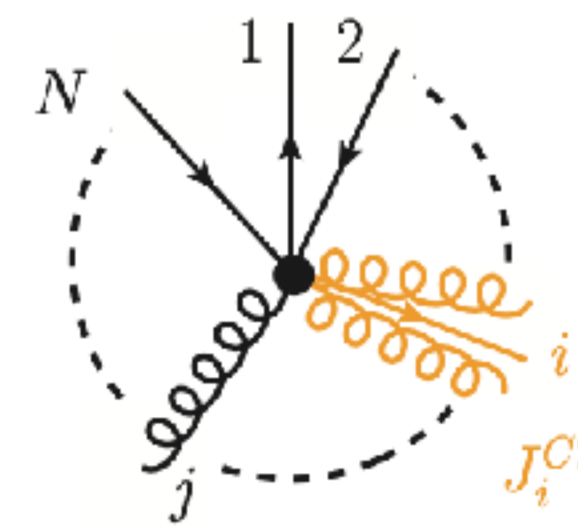
LP-N-jet



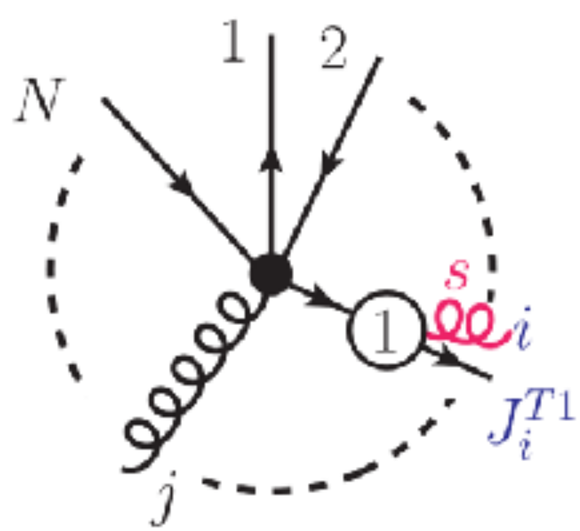
NLP-N-jet Current





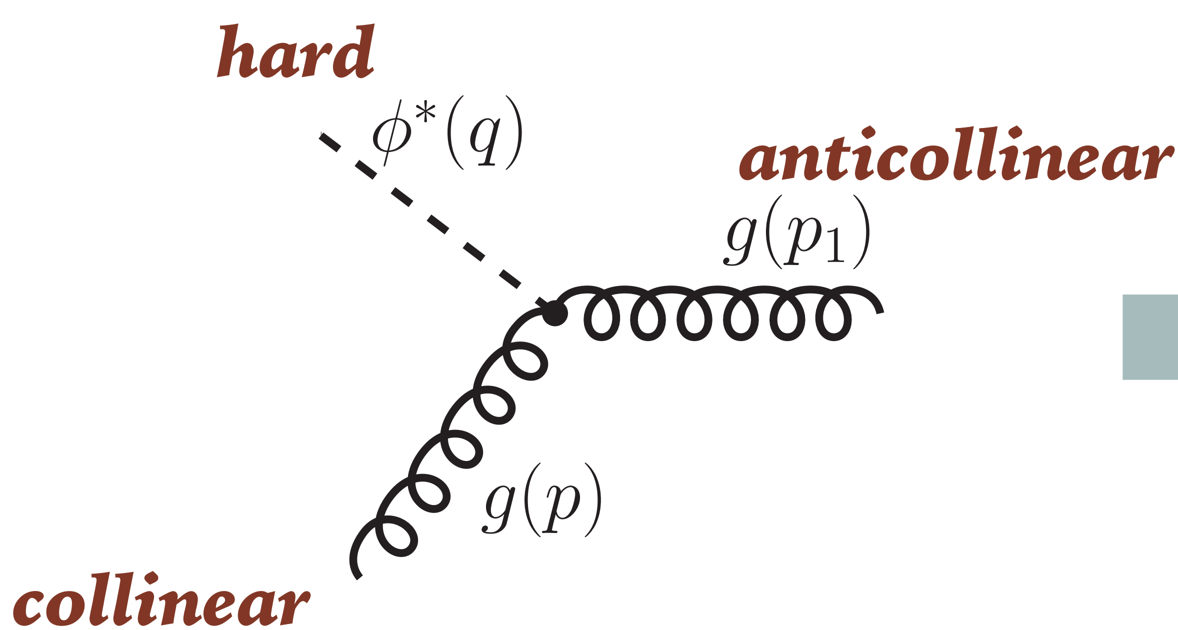


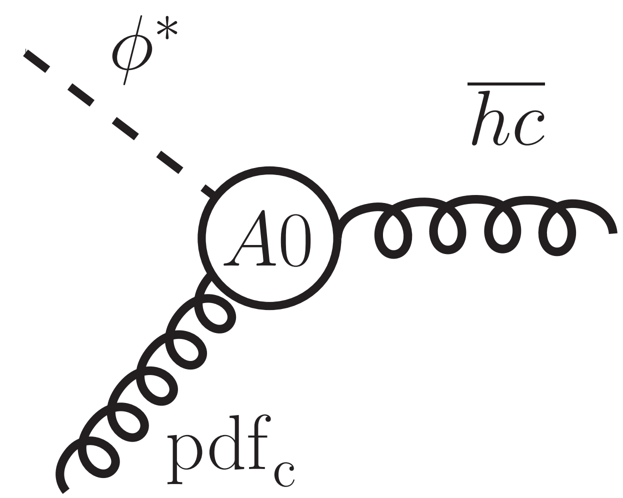
NLP-N-jet Lagrangian insertion



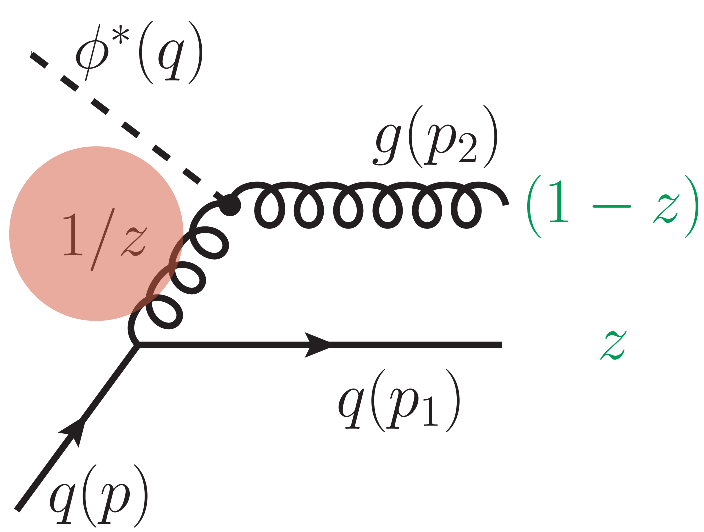
[M. Beneke, M. Garny, **RS**, J. Wang: 1712.04416, 1808.04742]

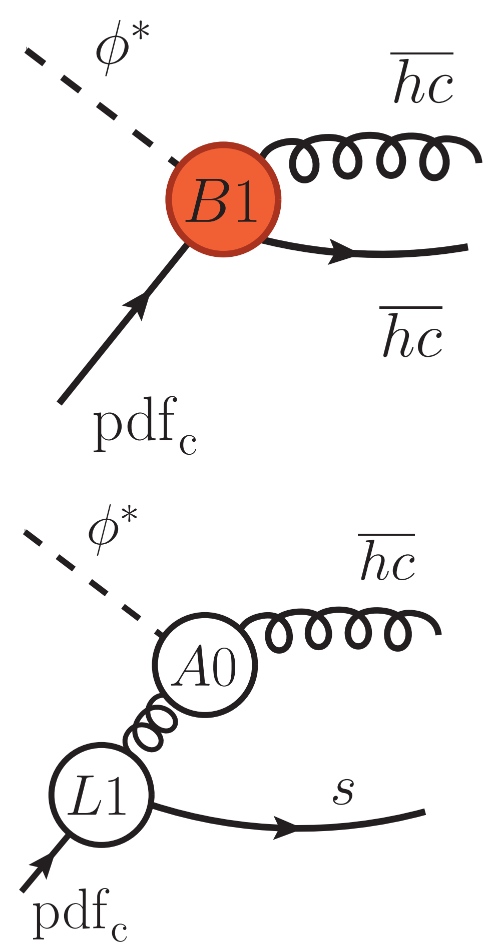
BREAKDOWN OF THE FACTORIZATION NEAR THE ENDPOINT





For $z \sim 1$ intermediate propagator is hard





See also

[M. Beneke, M. Garny, **RS**, J. Wang: 1907.05463]

$$\boxed{(p-p_1)^2 = -2p\cdot p_1 \approx -n_+ p\, n_- p_2 \sim zQ^2}$$

For $z \ll 1$ soft and collinear modes overlap

"Z-SCET"

[M. Beneke, M. Garny, S. Jaskiewicz, **RS**, L. Vernazza, J. Wang: 2008.04943]

New scale emerging: zQ^2

In the endpoint region, new counting parameter $\lambda^2 \ll z \ll 1$

This leads to new modes

Name	$(n_+l\,,\, l_\perp\,,\, n_-l)$	virtuality l^2
hard $[h]$	$Q(1\,,\,1\,,\,1)$	Q^2
z-hardcollinear $[z-hc]$	$Q(1\,,\,\sqrt{z}\,,\,z)$	$z\,Q^2$
z-anti-hardcollinear $[z-\overline{hc}]$	$Q(z\,,\,\sqrt{z}\,,\,1)$	$z\,Q^2$
z-soft $[z-s]$	$Q(z\,,\,z\,,\,z)$	$z^2\,Q^2$
z-anti-softcollinear $[z-\overline{sc}]$	$Q(\lambda^2\,,\,\sqrt{z}\,\lambda\,,\,z)$	$z\,\lambda^2\,Q^2$

z-scales are not physical!

Similar to $\mathrm{SCET_I}$ *and* $\mathrm{SCET_{II}}$ *problems*

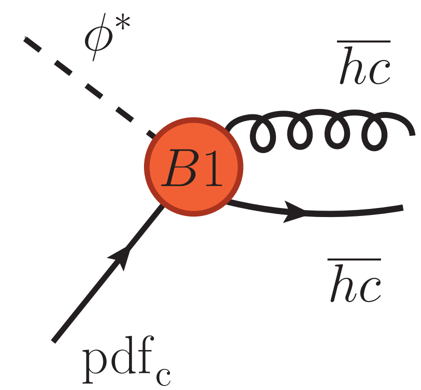
[Z. Liu, M. Neubert: 1908.11379]

[Z. Liu, B. Mecaj, M. Neubert, X. Wang: 2009.04456, 2009.06779]

RE-FACTORIZATION

[M. Beneke, M. Garny, S. Jaskiewicz, **RS**, L. Vernazza, J. Wang: 2008.04943]

$$\int_0^1 dz C^{B1}(Q,z) \quad \times$$



In the region $z \ll 1$, $\ln z$ becomes parametrically large and needs to be resummed

Multi-scale object

$$C^{B1}(Q,z) J^{B1}(z) \xrightarrow{z\to 0} C^{A0}(Q^2) \int d^4x\, T\left\{J^{A0}, \mathscr{L}^{(1)}_{\xi q_{z-\overline{sc}}}(x)\right\} = C^{A0}(Q^2)\, D^{B1}(zQ^2,\mu^2)\, J^{B1}_{z-\overline{sc}}$$

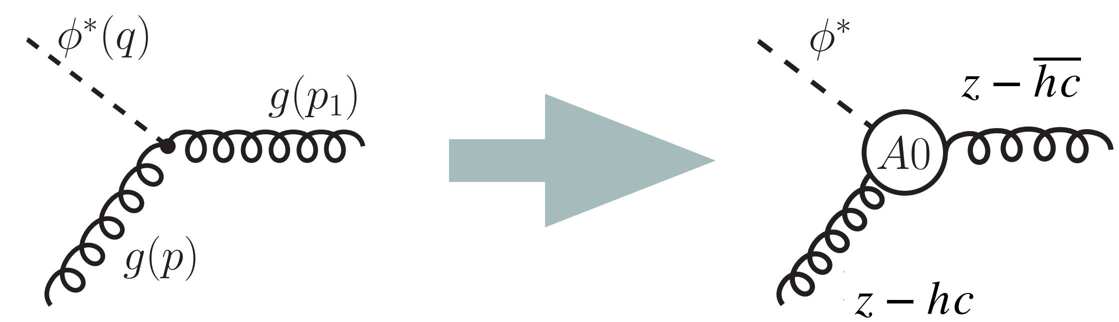
Single-scale objects

Similar re-factorization has been proven in

[Z. L. Liu, B. Mecaj, M.Neubert, X.Wang: 2009.06779]

TWO STEP MATCHING

Integrate out hard modes

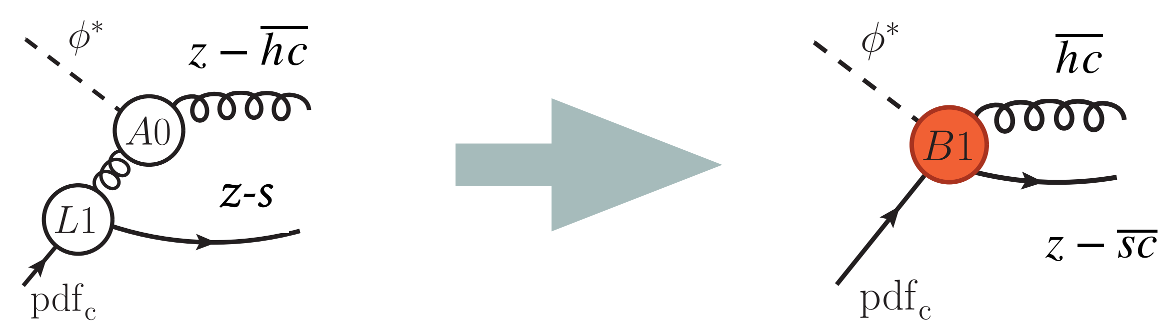


Q^2

$$\frac{d}{d\ln\mu} C^{A0}(Q^2,\mu^2) = \frac{\alpha}{\pi} C_A \ln\frac{Q^2}{-\mu^2} C^{A0}(Q^2,\mu^2)$$

zQ^2

Integrate out z-hard-collinear modes



$\lambda^2 Q^2$

$$\frac{d}{d\ln\mu} D^{B1}\left(zQ^2,\mu^2\right) = \frac{\alpha}{\pi}\left(C_F - C_A\right) \ln\frac{zQ^2}{-\mu^2} D^{B1}\left(zQ^2,\mu^2\right)$$

SOFT QUARK SUDAKOV

We can solve RGE in d-dimensions for bare objects

$$\left[C^{A0}\left(zQ^2,\mu^2\right)\right]_{\text{bare}} = C^{A0}\left(Q^2,Q^2\right)\exp\left[-\frac{\alpha_s C_A}{2\pi}\frac{1}{\epsilon^2}\left(\frac{Q^2}{\mu^2}\right)^{-\epsilon}\right]$$

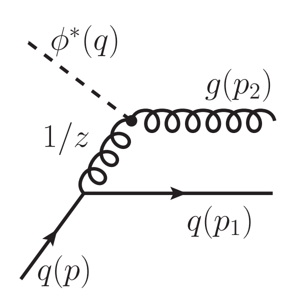
Note $C_F - C_A$ structure!

$$\left[D^{B1}\left(zQ^2,\mu^2\right)\right]_{\text{bare}} = D^{B1}\left(zQ^2,zQ^2\right)\exp\left[-\frac{\alpha_s}{2\pi}\left(C_F - C_A\right)\frac{1}{\epsilon^2}\left(\frac{zQ^2}{\mu^2}\right)^{-\epsilon}\right]$$

Confirms earlier "soft quark Sudakov" conjecture by

[I. Moult, I. Stewart, G. Vita, H.X. Zhu: 1910.14038]

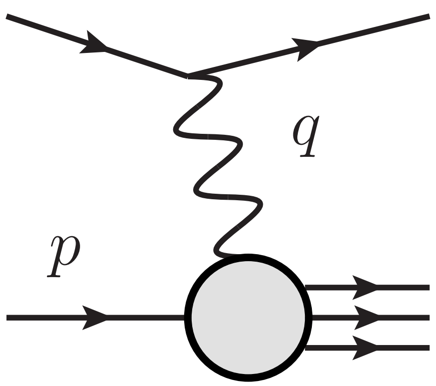
$$\mathscr{P}_{qg}(z) = \left| \quad \right|^2$$



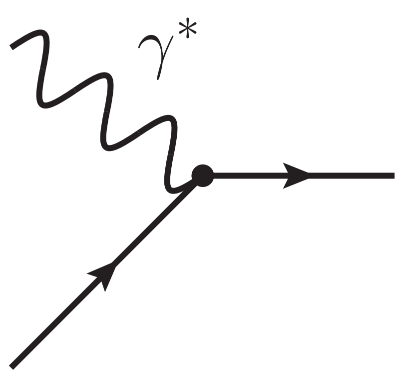
$$\sim \left|C^{B1}(z)\right|^2 = \left|C^{A0}D^{B1}\right|^2$$

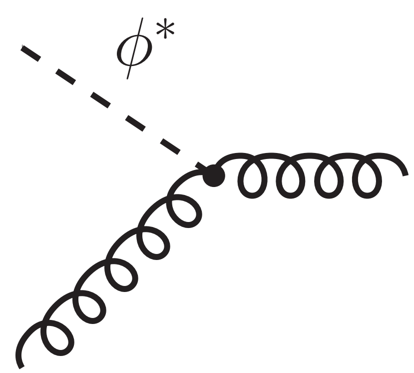
$$\mathscr{P}_{qg}(z) = \frac{\alpha_s C_F}{2\pi}\frac{1}{z}\exp\left[\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\left(-C_A\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} + (C_A - C_F)\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon}\right)\right]$$

WE FOCUS ON THE DIS



Deep inelastic scattering (DIS) at threshold develops a hierarchy of scales: $Q^2 \gg P_X^2 \sim Q^2(1-x)$

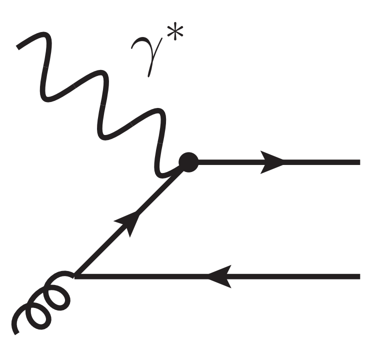


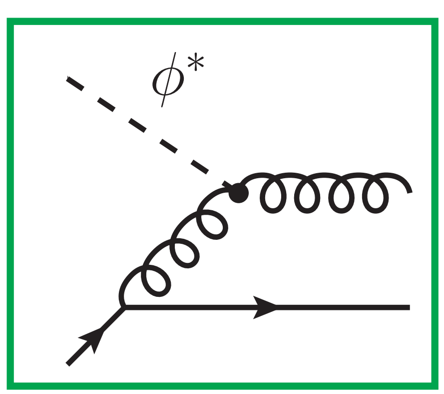


LP factorization and resummation is well understood

[S. Moch, J. Vermaseren, A. Vogt: hep-ph/0506288]

[T. Becher, M. Neubert, B. Pecjak: hep-ph/0607228]





Less is known about off-diagonal channel, which start at NLO

NLP FACTORIZATION

Modes

T H

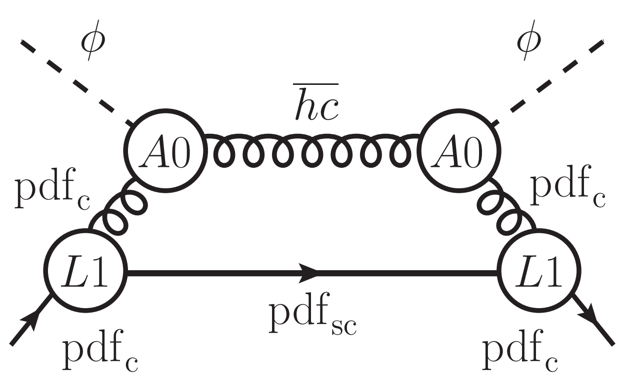
- *Hard*, $p^2 = Q^2$
- *Anti-hardcollinear*, $p^2 = Q^2\lambda^2 = Q^2/N$

P D F

- *Collinear*, $p^2 = \Lambda^2$
- *Soft-Collinear*, $p^2 = \Lambda^2\lambda^2 = \Lambda^2/N$

$$g(N) \equiv \int_0^1 dx x^{N-1} g(x)$$

$$x \to 1 \leftrightarrow N \to \infty$$



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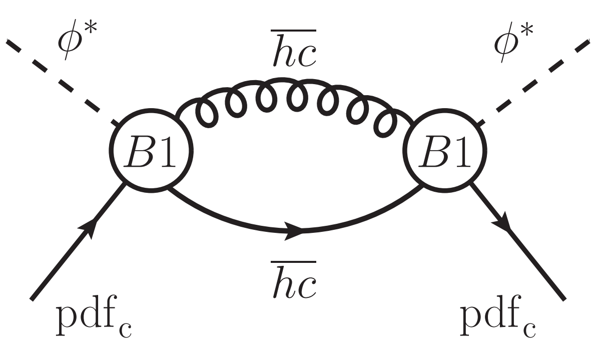


Time-ordered product contribution

B-type current contribution

Both terms contain endpoint divergence in the convolution integral

We could reshuffle factorization theorem, see [Z. Liu, M. Neubert: 1912.08818]

Instead, we apply d-dimensional consistency conditions

D-DIMENSIONAL CONSISTENCY CONDITIONS

Instead of completing re-factorization, we can work in d-dimensions — ε regularizes endpoint divergence in the convolution integral

Take hadronic "tensor":

$$W = \sum_i W_{\phi,i}\, f_i\,,$$

Bare partonic coefficients *Bare PDFs*

Introduce renormalized (d-dimensional) objects $\tilde{f}_k = Z_{ki} f_i, \qquad W_{\phi,i} = \tilde{C}_{\phi,k} Z_{ki}$

$$\sum_i W_{\phi,i} f_i = \sum_k \tilde{C}_{\phi,k} \tilde{f}_k\,.$$

Splitting kernels are defined

$$P_{ij} = -\gamma_{ij} = \frac{dZ_{ik}}{d\ln\mu} (Z^{-1})_{kj}\,.$$

D-DIMENSIONAL CONSISTENCY CONDITIONS AT NLP

$$\sum_i (W_{\phi,i} f_i)^{NLP} = W_{\phi,q}^{NLP} f_q^{LP} + W_{\phi,\bar{q}}^{NLP} f_{\bar{q}}^{LP} + W_{\phi,g}^{NLP} f_g^{LP} + W_{\phi,g}^{LP} f_g^{NLP}$$

$$\sum_i (W_{\phi,i} f_i)^{NLP} = f_q(\Lambda) \times \frac{1}{N} \sum_{n=1} \left(\frac{\alpha_s}{4\pi} \right)^n \frac{1}{\epsilon^{2n-1}} \sum_{k=0}^{n} \sum_{j=0}^{n} c_{kj}^{(n)}(\epsilon) \left(\frac{\mu^{2n} N^j}{Q^{2k} \Lambda^{2(n-k)}} \right)^{\epsilon}$$

Each hard-collinear loop gives $\left(\frac{\mu^2}{Q^2} N \right)^{\epsilon}$ *Each collinear loop gives* $\left(\frac{\mu^2}{\Lambda^2} \right)^{\epsilon}$

Each soft-collinear loop gives $\left(\frac{\mu^2}{\Lambda^2} N \right)^{\epsilon}$ *Each hard loop gives* $\left(\frac{\mu^2}{Q^2} \right)^{\epsilon}$

$k = h + \overline{hc}$ $j = sc + \overline{hc}$ $n - k = c + sc$

SOLVING CONSISTENCY

Cancellation of poles requires $\sum_{k=0}^{n}\sum_{j=0}^{n} j^r k^s c_{kj}^{(n)} = 0$ *for* $s + r < 2n - 1,\ r, s \geq 0$

$$\sum_i (W_{\phi,i} f_i)^{NLP} = f_q(\Lambda) \times \frac{1}{N} \sum_{n=1} \left(\frac{\alpha_s}{4\pi}\right)^n \frac{1}{\epsilon^{2n-1}} \sum_{k=0}^{n}\sum_{j=0}^{n} c_{kj}^{(n)}(\epsilon) \left(\frac{\mu^{2n} N^j}{Q^{2k}\Lambda^{2(n-k)}}\right)^{\epsilon}$$

$c_{n0}^{(n)} = 0$ *finial state cannot be purely hard*

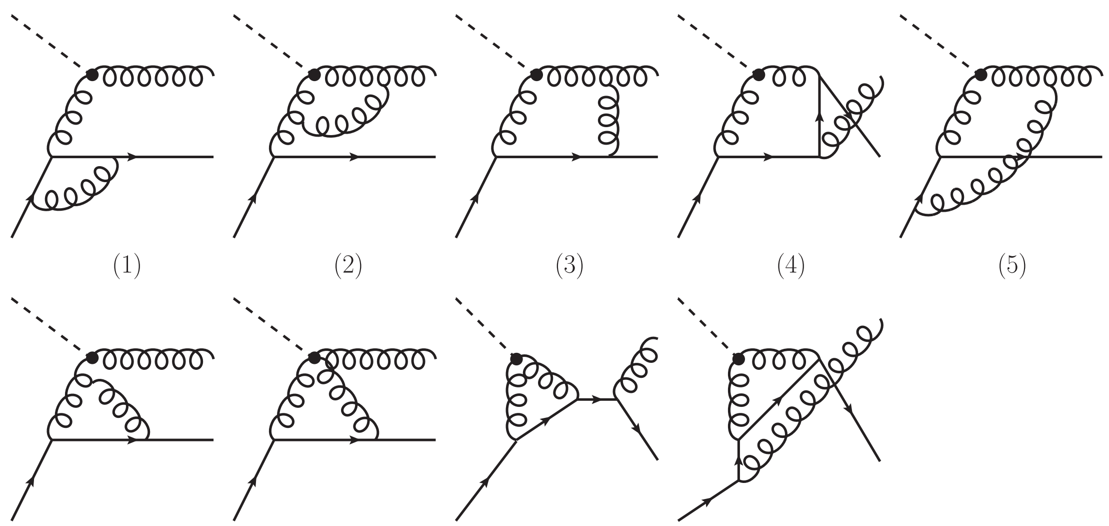
$c_{00}^{(n)} = 0$ *without any hard or anti-hardcollinear loops* $(k = 0)$ *there must be at least one softcollinear loop*

Non-trivial initial condition is $c_{n1}^{(n)}$ given by Soft quark Sudakov derived before

$$W_{\phi,q}\Big|_{q\phi^* \to qg} = \int_0^1 dz \left(\frac{\mu^2}{s_{qg} z \bar{z}}\right)^{\epsilon} \mathcal{P}_{qg}(z)\Big|_{s_{qg} = Q^2 \frac{1-x}{x}}$$

$$\mathcal{P}_{qg}(z) = \frac{\alpha_s C_F}{2\pi} \frac{1}{z} \exp\left[\frac{\alpha_s}{\pi} \frac{1}{\epsilon^2} \left(- C_A \left(\frac{\mu^2}{Q^2}\right)^{\epsilon} + (C_A - C_F) \left(\frac{\mu^2}{zQ^2}\right)^{\epsilon} \right)\right]$$

INTERLUDE: DID WE REALLY NEED TO RESUM LOG(Z)?



$$W_{\phi,q}\Big|_{q\phi^*\to qg} = \int_0^1 dz \left(\frac{\mu^2}{s_{qg} z\bar{z}}\right)^{\epsilon} \mathscr{P}_{qg}(z)\Big|_{s_{qg}=Q^2\frac{1-x}{x}}$$

Formally single pole

$$\mathscr{P}_{qg}(z)\,|_{1-\text{loop}} = \mathscr{P}_{qg}(z)\,|_{\text{tree}} \frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\left\{T_1\cdot T_0\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon} + T_2\cdot T_0\left(\frac{\mu^2}{\bar{z}Q^2}\right)^{\epsilon} + T_1\cdot T_2\left[\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} - \left(\frac{\mu^2}{zQ^2}\right)^{\epsilon}\right]\right\}$$

Promoted to leading pole after integration

$$\frac{1}{\epsilon^2}\int_0^1 dz\,\frac{1}{z^{1+\epsilon}}(1-z^{-\epsilon}) = -\frac{1}{2\epsilon^3}$$

$$\frac{1}{\epsilon^2}\int_0^1 dz\,\frac{1}{z^{1+\epsilon}}\left(\epsilon\ln z - \frac{\epsilon^2}{2!}\ln^2 z + \frac{\epsilon^2}{3!}\ln^3 z + \cdots\right) = -\frac{1}{\epsilon^3} + \frac{1}{\epsilon^3} - \frac{1}{\epsilon^3} + \cdots$$

SOLUTION

Discussed set of boundary conditions is enough to fix a unique solution

$$\tilde{C}^{NLP,LL}_{\phi,q}\Big|_{\epsilon\to 0} = \frac{1}{2N\ln N}\frac{C_F}{C_F - C_A}\left(\mathscr{B}_0(a)\exp\left[C_A\frac{\alpha_s}{\pi}\left(\frac{1}{2}\ln^2 N + \ln N\ln\frac{\mu^2}{Q^2}\right)\right] - \exp\left[\frac{\alpha_s C_F}{\pi}\left(\frac{1}{2}\ln^2 N + \ln N\ln\frac{\mu^2}{Q^2}\right)\right]\right)$$

$$\gamma^{NLP,LL}_{gq}(N) = -\frac{1}{N}\frac{\alpha_s C_F}{\pi}\mathscr{B}_0(a)\,,$$

$$a = \frac{\alpha_s}{\pi}(C_F - C_A)\ln^2 N$$

$$\boxed{\mathscr{B}_0(x) = \sum_{n=0}^{\infty}\frac{B_n}{(n!)^2}x^n}$$

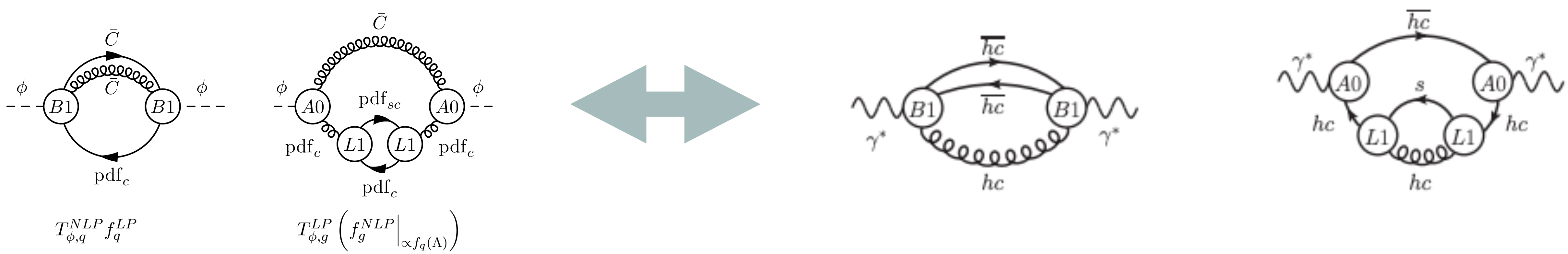
In agreement with a conjecture

[A. Vogt, 1005.1606]

with Bernoulli numbers $B_0 = 1,\, B_1 = -1/2, \ldots$

COMMENTS ON THE FINAL RESULTS

- *The anomalous dimension is a two-scale object in the off-diagonal channel*
- *The double logarithms are associated with the color charge change $C_F - C_A$ of the partons that carry large momentum*
- *Bernoulli numbers appear in $\overline{MS}$ scheme due to separation of the pole part and short distance coefficients*
- *Similar situation appears for thrust*



$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau}\bigg|_{II}^{NLP,leading\ poles} = \frac{C_F}{C_F - C_A}\frac{\epsilon\tau^{-\epsilon}}{\tau^{-\epsilon}-1}\left\{\exp\left[-\frac{\alpha_s C_F}{\pi}\frac{1}{\epsilon^2}\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}(1-\tau^{-\epsilon})^2\right] - \exp\left[-\frac{\alpha_s C_A}{\pi}\frac{1}{\epsilon^2}\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}(1-\tau^{-\epsilon})^2\right]\right\}$$

SUMMARY AND OUTLOOK

- Divergences in the convolution integral require additional refactorization
- New "internal" modes appear due to endpoint divergences
- Rigorous factorization and resummation still possible but highly non trivial
- Relationships between bare and renormalized objects need to be better understood
- A lot of interesting NLP results are still waiting to be discovered
- Interesting applications beyond QCD

[M. Beneke, C. Bobeth , **RS**: 1908.07011]