Emittance Exchange in MICE

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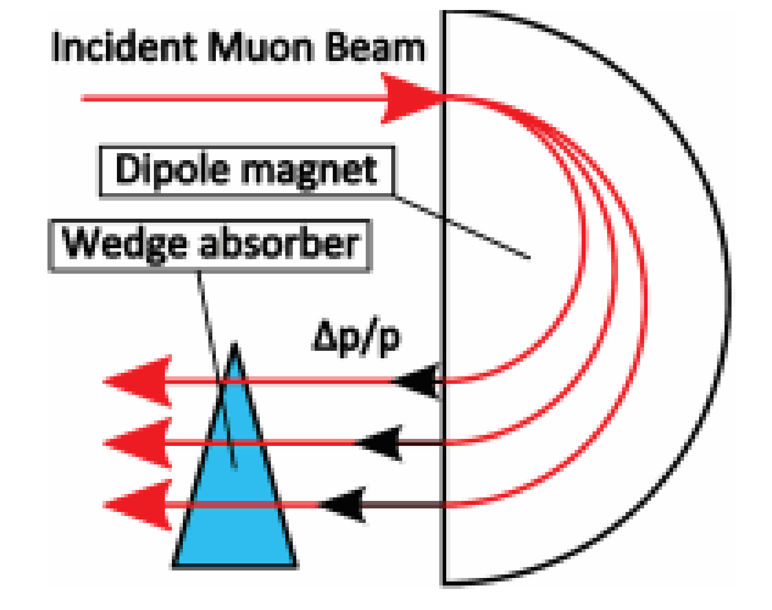
22 October 2020

Today's Menu

- Emittance Exchange – What is it?
- How to measure it – Emittance, Amplitude and Phase Space Density
  - Covariance Matrix
  - Missing Data - Transmission losses
  - Transverse Phase Space Density, Amplitude and Emittance
  - Liouville, Introducing Time and Longitudinal Density, Amplitude and Emittance
- Recon Bias – TKU and TKD are different
  - PT bias, PZ bias, Energy addition and a non-homogenous magnetic field
- Magnetic field
  - Potential sources of error
- Conclusion

Aims



- Demonstrate Emittance Exchange and Reverse Emittance Exchange in the Wedge using MICE data
- Emittance Exchange can be demonstrated by looking at the change in phase space density of the particle selection before and after having passed through a Wedge absorber
- Emittance Exchange is shown by a decreased transverse phase space density (x, px, y, py) and increased longitudinal phase space density (z, pz), (and vice versa for Reverse Emittance Exchange)
- Can use a number of techniques to calculate phase space density: KDE, KNN, Voronoi Tessellations, etc.
- MICE beam only has a small natural dispersion → Use beam reweighing techniques to select beams with desired dispersion

MICE – 3 Cooling measurements

- MICE measures cooling using three techniques

- 1. Emittance:

$$\varepsilon_d = \frac{\sqrt[d]{|\Sigma|}}{m_\mu c}$$

- 2. Amplitude:

$$A_d = \varepsilon_d \boldsymbol{x}^T \Sigma^{-1} \boldsymbol{x} \quad = \quad \frac{\sqrt[d]{|\Sigma|} \boldsymbol{x}^T \Sigma^{-1} \boldsymbol{x}}{m_\mu c}$$

- 3. Phase-Space Density:

  - Kernel Density Estimation (KDE)

$$\hat{\rho}(\vec{x}) = \frac{1}{n} \sum_{i=1}^{n} K_{\boldsymbol{H}}(\boldsymbol{x}) = \frac{1}{nh^d} \sum_{i=1}^{n} K\left(\frac{\boldsymbol{x}}{h}\right) = \frac{\sum_{i=1}^{n} exp[-\frac{1}{2}\boldsymbol{x}^T \Sigma^{-1} \boldsymbol{x}]}{n(2\pi)^{d/2}|\Sigma|^{1/2}}$$

  - k-nearest neighbour (KNN)

$$\vec{\rho}(x) = \frac{k}{n\kappa_d R_k^d} = \frac{k\Gamma(\frac{d}{2}+1)}{n\pi^{\frac{d}{2}} R_k^d}$$

MICE – 3 Cooling measurements

- MICE measures cooling using three techniques

- 1. Emittance:

$$\varepsilon_d = \frac{\sqrt[d]{|\Sigma|}}{m_\mu c}$$

d = dimension
Σ = covariance matrix
m_μ = muon mass
c = speed of light

- 2. Amplitude:

$$A_d = \varepsilon_d \boldsymbol{x}^T \Sigma^{-1} \boldsymbol{x} \quad = \quad \frac{\sqrt[d]{|\Sigma|}\boldsymbol{x}^T \Sigma^{-1} \boldsymbol{x}}{m_\mu c}$$

$\boldsymbol{x}^T \Sigma^{-1} \boldsymbol{x}$ is the Mahalanobis distance

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K = Kernel Choice
h = Kernel bandwidth
n = no. pf particles
Last part shows a Gaussian kernel

  - k-nearest neighbour (KNN)

$$\vec{\rho}(x) = \frac{k}{n\kappa_d R_k^d} = \frac{k\Gamma(\frac{d}{2}+1)}{n\pi^{\frac{d}{2}} R_k^d}$$

k = no. of neighbours
R_k^d = Euclidean distance
κ_d = Volume of a unit d-ball
$\Gamma(\frac{d}{2}+1)$ = Euler-Gamma function

MICE – 3 Cooling measurements

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What do all 3 share?
Dependence on the Covariance Matrix

Covariance Matrix

- Covariance Matrix representation of the distribution of your particles in your beam
- In linear optics (e.g. rotations, translations) the determinant of the covariance matrix remains constant, hence emittance, amplitude and density remain constant.
- In linear optics the transverse and longitudinal planes can be decoupled in a solenoid (where we measure)
- Hence we can look at the transverse planes separately and thus look at the transverse and longitudinal emittance, amplitude and density
- Problems for some of these techniques arise when there are non-linear effects (e.g. emittance growth already extensively seen) or misalignments
- Or when there are transmission losses – This results in a downstream selection bias as we are now comparing different particle distribution functions.
- If we are looking at different particle distribution functions, then we will have changing covariance matrices (determinant is no longer constant)

Missing Data - Missing particles

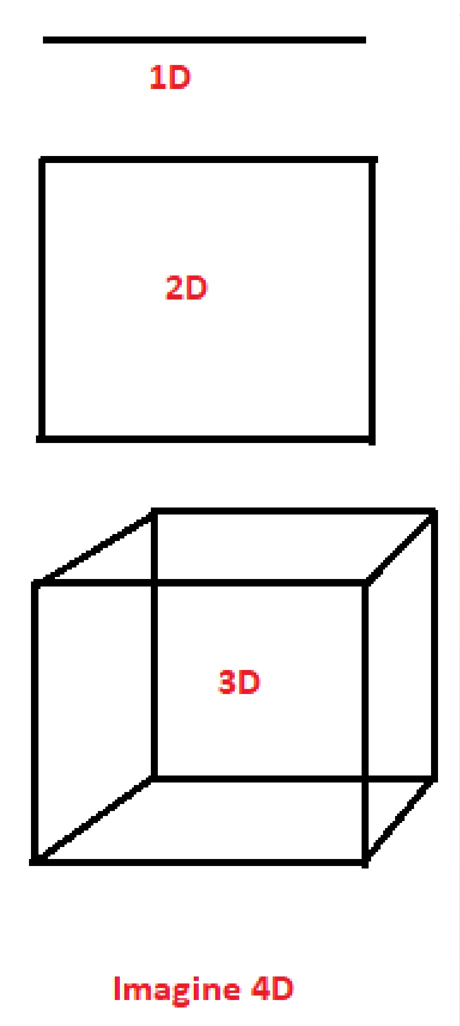
- Missing Data can be classified in three different ways
  - \- Missing Completely At Random (MCAR)
    - Particle Distribution Functions remain the same
    - E.g. 1000 vs 2000 coin tosses
  - \- Missing At Random (MAR)
    - Particle Distribution Functions change, but can be corrected for
    - E.g. Sensor defect. 10 sensors measure the number of cars passing along a road, but one sensor only makes readings 95% of the time
  - \- Missing Not At Random (MNAR)
    - Data is missing due to the experiment
    - Common problem in medicine e.g. COVID-19 death rate with low testing
    - In MICE data is MNAR due to aperture of the MICE beamline, and the reasons for particles to escape the beamline, e.g. magnetic field, scattering

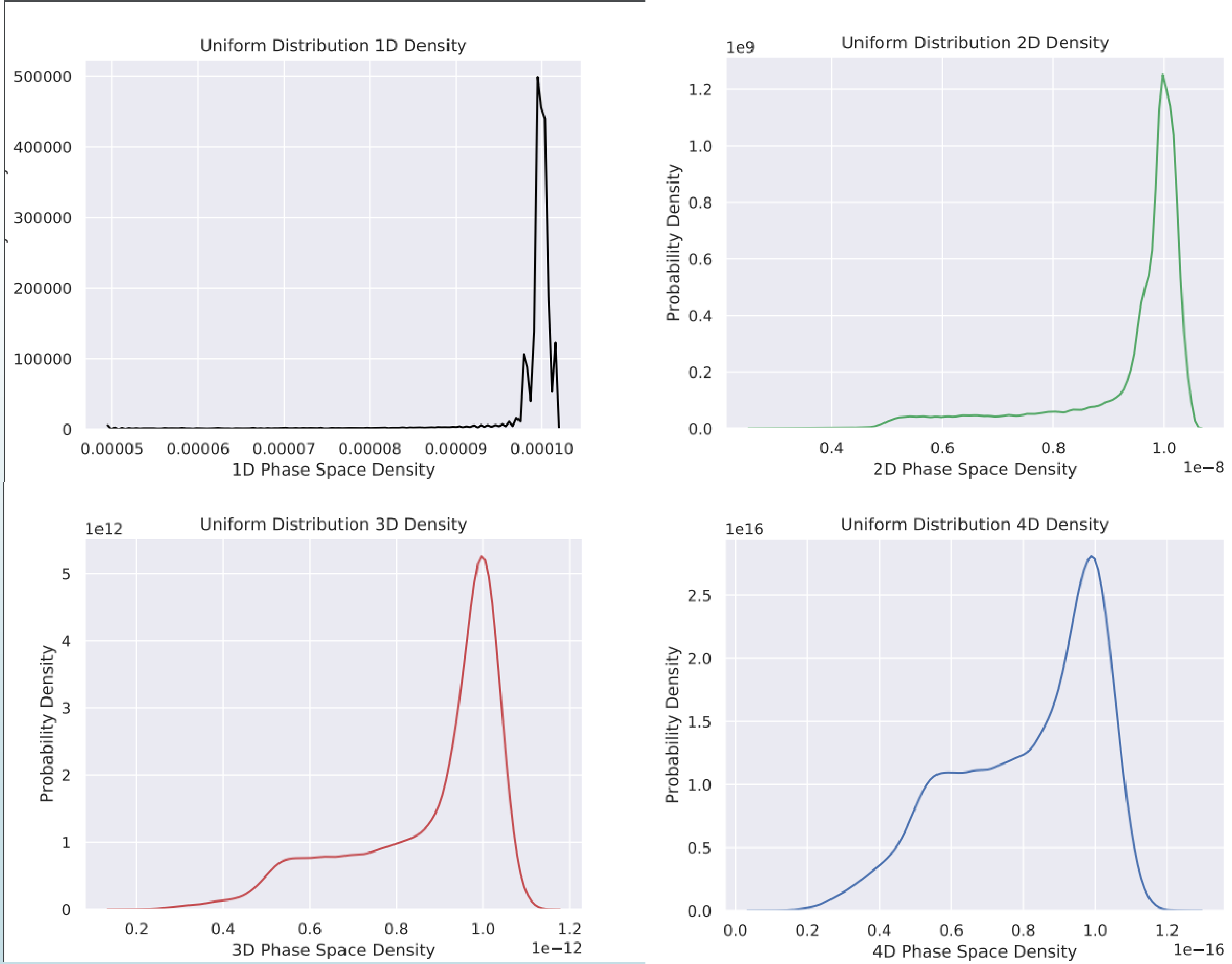
MICE – transmission losses

- Transmission losses leads to survivorship bias
  - We only measure particles downstream which remain within experiment aperture
  - To a certain extent upstream distribution is arbitrary (non-unique) as long as it contains the distribution which makes it downstream
- In MICE so far, the full Upstream sample is compared to the full downstream sample with some corrections made (e.g. normalize for the number of events)
- Does this work? Simple check:
- Split the full upstream sample into the sample which made it downstream and into the sample which didn't make it downstream
- Calculate the density, amplitude and emittance and see how it compares to full upstream sample
- Will show soon, but first what should the distribution look like

Phase-Space Density (Uniform Distribution)

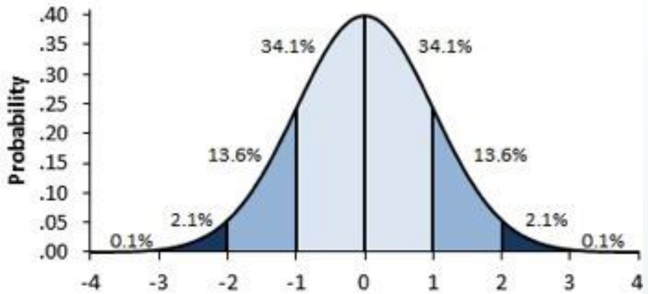
- What should a Phase-Space Density plot look like (sample size = 100,000)?
- For a uniform distribution (e.g. a solid piece of ice):
  - In 1D all particles should have approximately the same peak density, except the edge of the distribution.
  - At the edge of the distribution, the bounding volume the density is calculated over has free space within that volume.
  - In 2D (e.g. a square), for the same sample size, more particles are now at the edge of the distribution, and so fewer particles are found at the peak density.
  - In higher dimensions, more of the particles become edge or bounding particles of the distribution, and thus more particles shift away from the peak density
  - The shape of the phase-space density plot is now determined by both the number of particles (Curse of dimensionality) and shape of your uniform volume (e.g. line -> square -> cube, etc., or line -> circle -> sphere, etc.)


1D
2D
3D
Imagine 4D

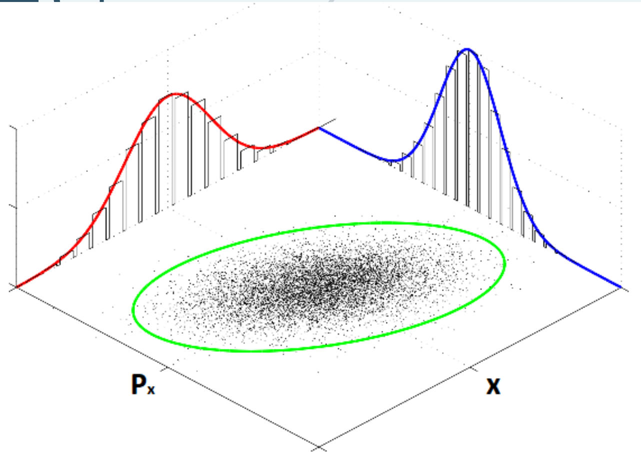

Uniform Distribution 1D Density
500000
400000
300000
200000
100000
0
0.00005
0.00006
0.00007
0.00008
0.00009
0.00010
1D Phase Space Density
Uniform Distribution 2D Density
1e9
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Probability Density
0.4
0.6
0.8
1.0
1e−8
2D Phase Space Density
Uniform Distribution 3D Density
1e12
5
4
3
2
1
0
Probability Density
0.2
0.4
0.6
0.8
1.0
1.2
1e−12
3D Phase Space Density
Uniform Distribution 4D Density
1e16
2.5
2.0
1.5
1.0
0.5
0.0
Probability Density
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1e−16
4D Phase Space Density

Phase-Space Density (Gaussian Distribution)

- What should a Phase-Space Density plot look like (sample size = 100,000)?
- For a Gaussian distribution (e.g. gas leak, density changes further from leak):
  - In 1D, most particles are found at centre of the distribution
  - Away from the centre the particles have to go a further distance to find the same number of neighbours, and so are at a lower density
  - In 2D, each co-ordinate contributes individually (i.e each point in a 1D Gaussian is represented by another 1D Gaussian perpendicular to that point).
  - The combination of the two results in each probability becoming equally likely
  - In the n-th dimensions each point from the (n-1) dimension can again be represented by another Gaussian perpendicular to that point.
  - For n = 3 or higher, it becomes more likely that at least 1 dimension is at a low density, and thus it becomes more and more likely for the particle overall to be found at a low density

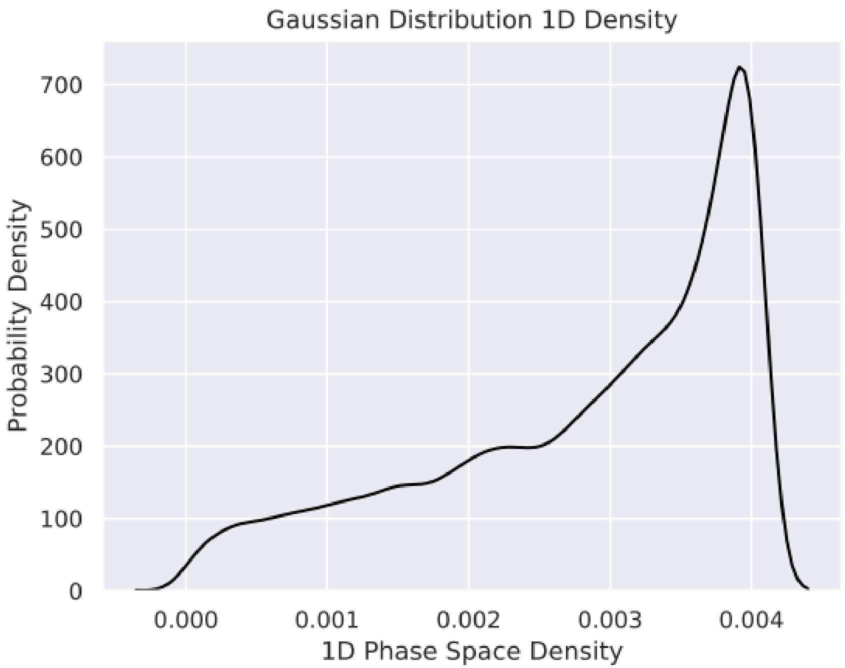


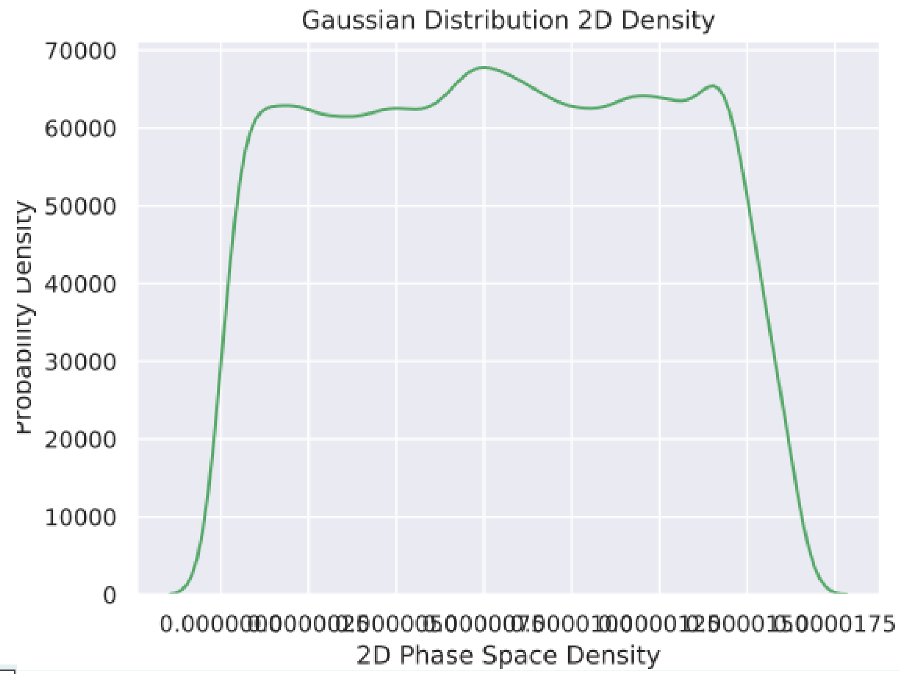
1D Gaussian

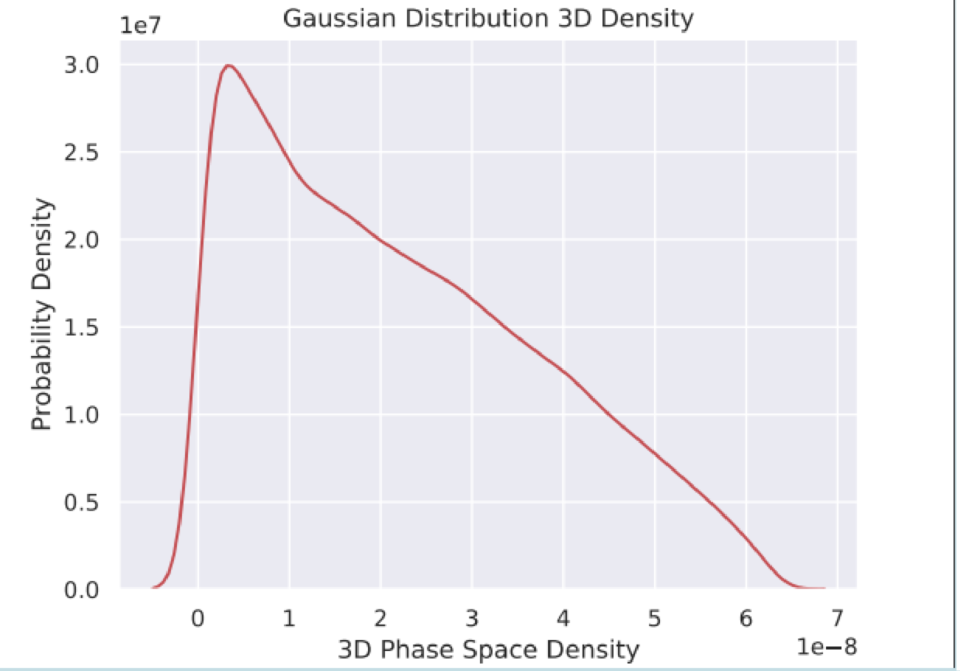


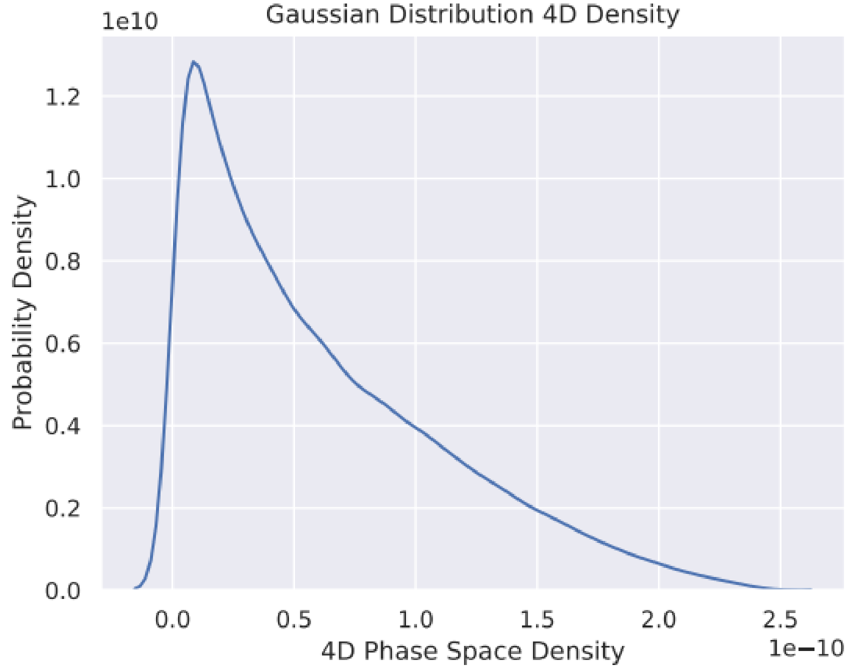
2D Gaussian

Imagine
3D and 4D



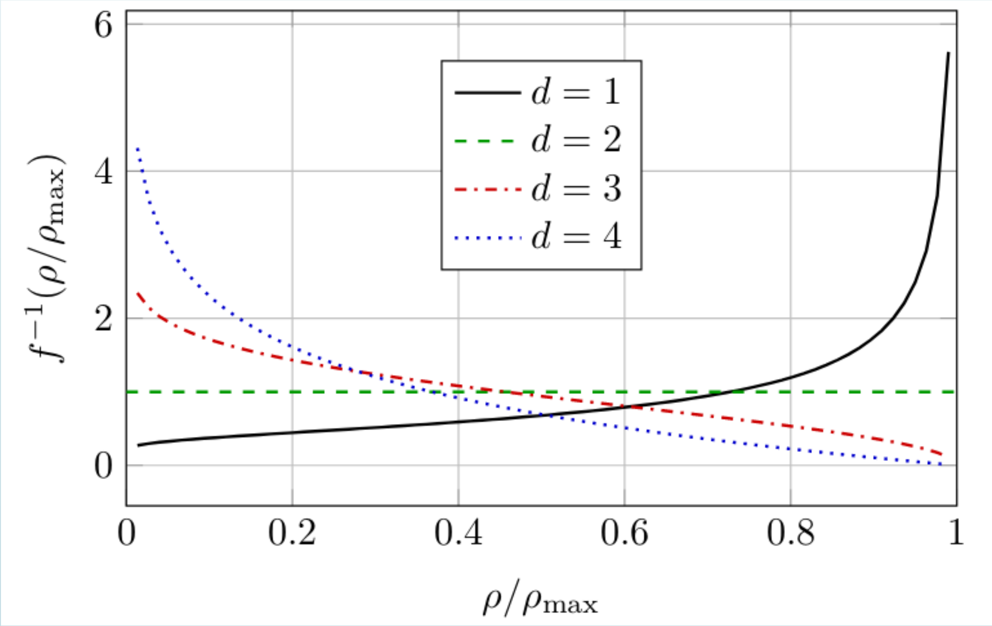






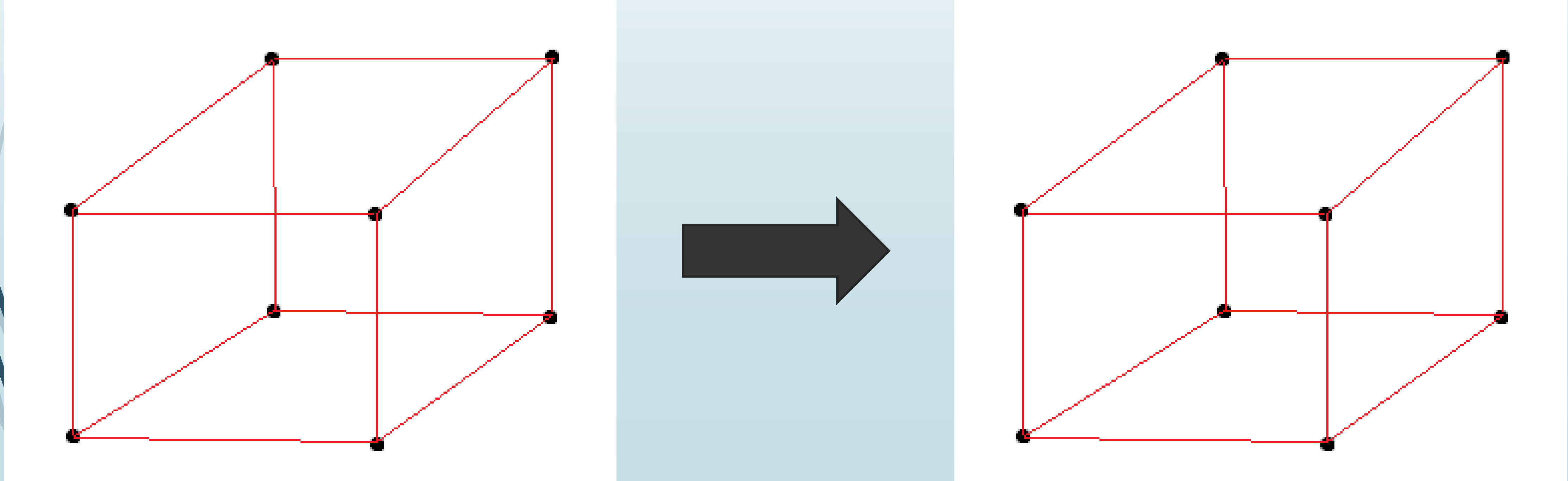
Phase-Space Density (Gaussian Distribution)

- Below, nice plot from Francois in limit n -> infinity
- It shows the expected Density for a Gaussian sample in each dimension normalized to the maximum density. As the dimension increases, particles more likely be found at a low phase space density



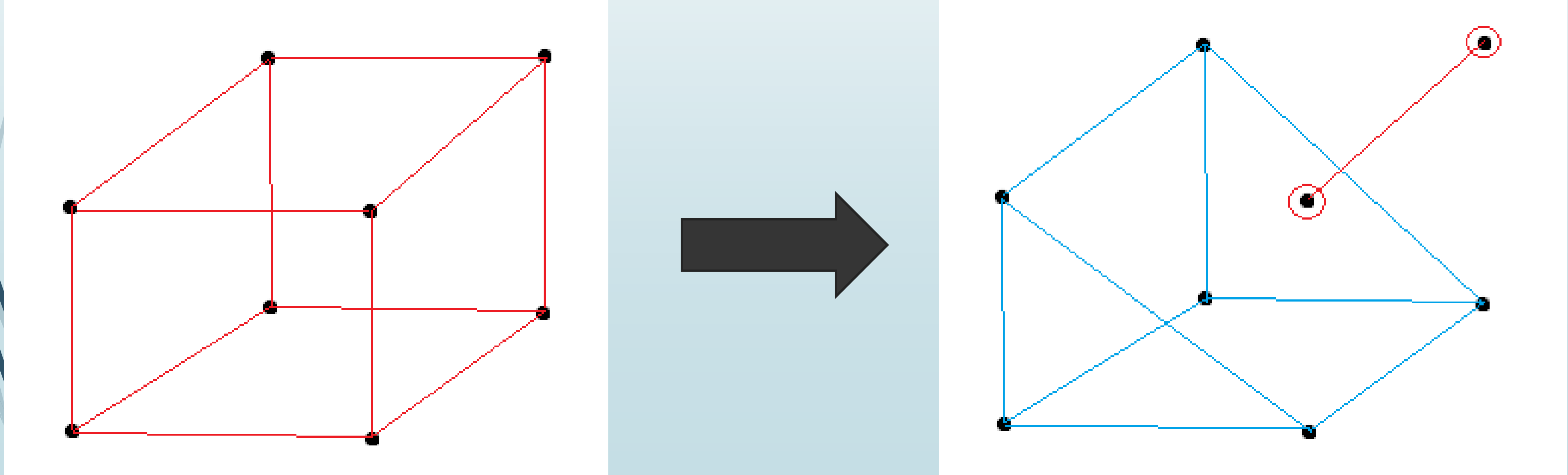
Transmission effects – extreme example

- Imagine phase space distribution given by 8 points arranged in a cube separated by a 1 unit distance, giving a 1 unit volume.
- The system is sent through a magnetic system with no dissipative forces. The points may have changed location, but the 1 unit volume is preserved.



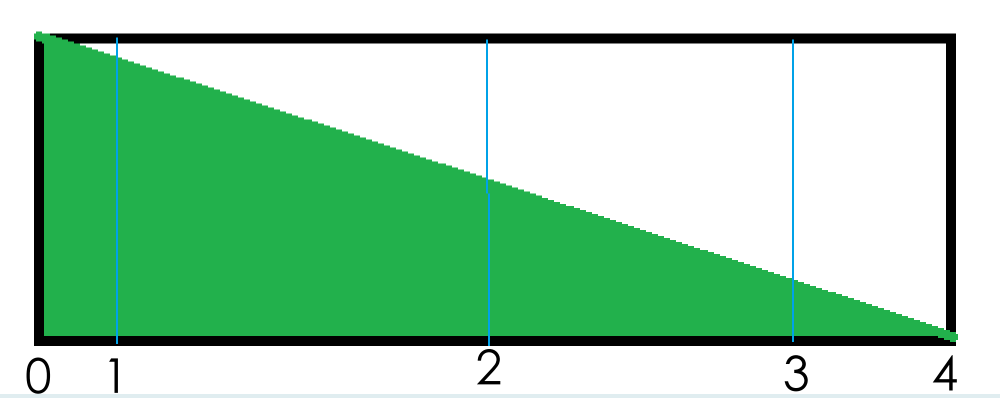
Transmission effects – extreme example

- The eight particles are again put through a magnetic system which has an aperture (acts as a dissipative force), resulting in a loss of two particles.
- The volume of the remaining 6 particles is 0.5 unit volume.
- If one were to normalize the downstream sample by the sample size, one would artificially increase the density (which is wrong). For transmission losses, the change in particle distribution is important.

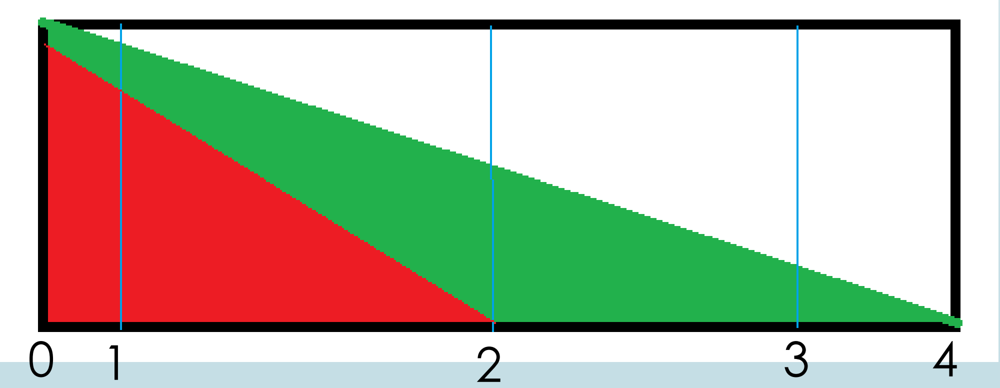


"Just use number of particles at core of beam" - What does Transmission loss mean

- Imagine we now have a non-uniform distribution (e.g. gas leak). It fills the black box and is represented by the green distribution
- The density we measure depends on which volume we look at.
- The density we measure between 0 and 2 will be different from that between 0 and 4



- Our gas leak detection system is not efficient and can only measure the red distribution.
- In our phase-phase calculation our bounding box would only be given by the volume between 0 and 2 (as we don't see particles beyond 2)
- We also can't scale our red distribution by the number of particles as it doesn't represent the parent distribution anymore



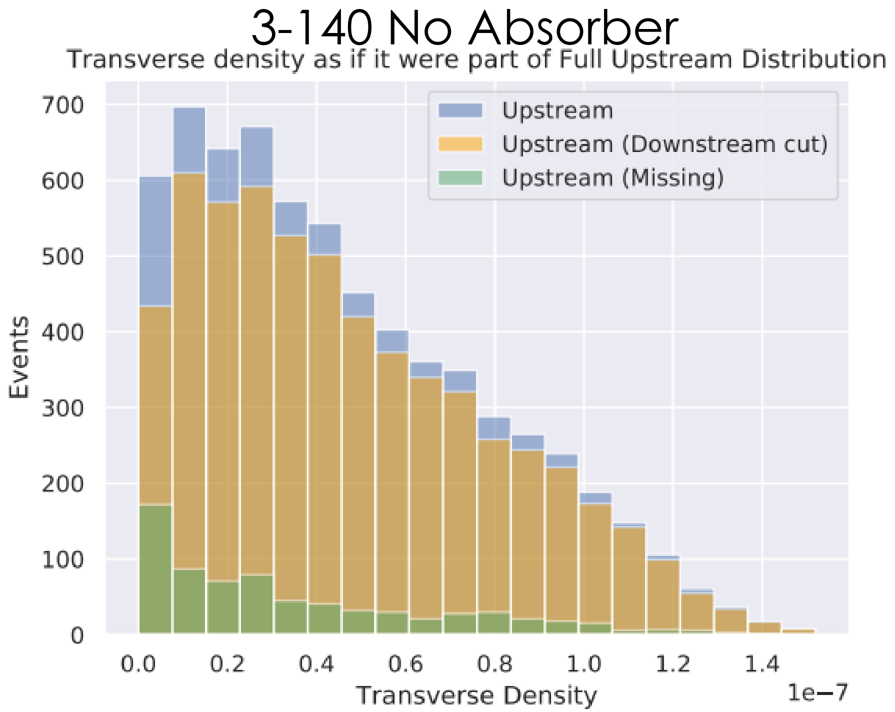
- MICE loses some particles at the core of the beam. The remaining beam core may not share the beam core of the parent distribution – not comparing like with like

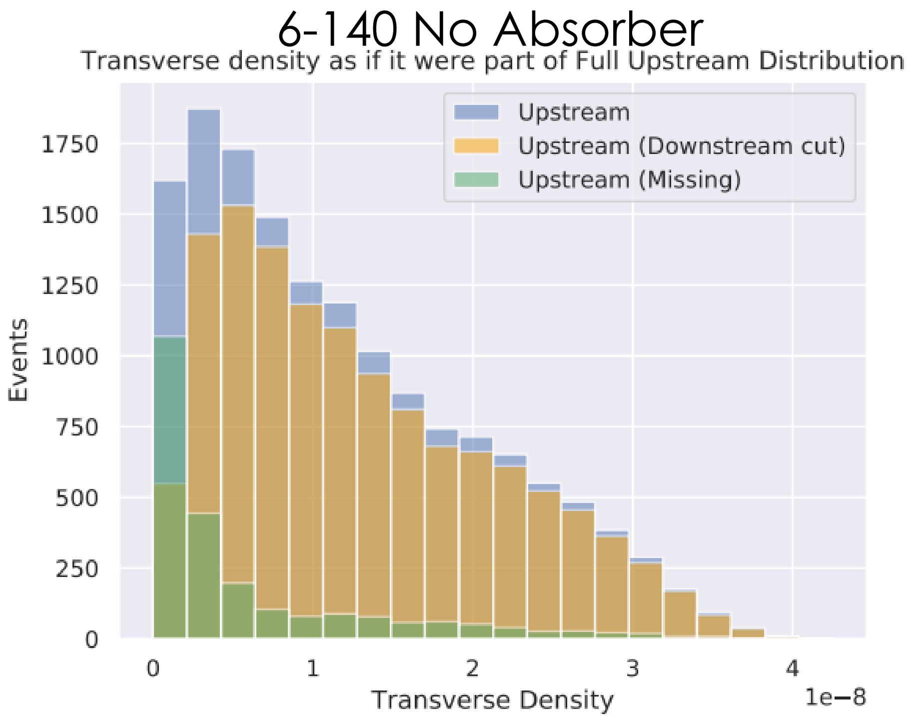
Transverse densities as if they were part of Upstream Distribution making it to TKD or not (i.e. missing) – MC Recon

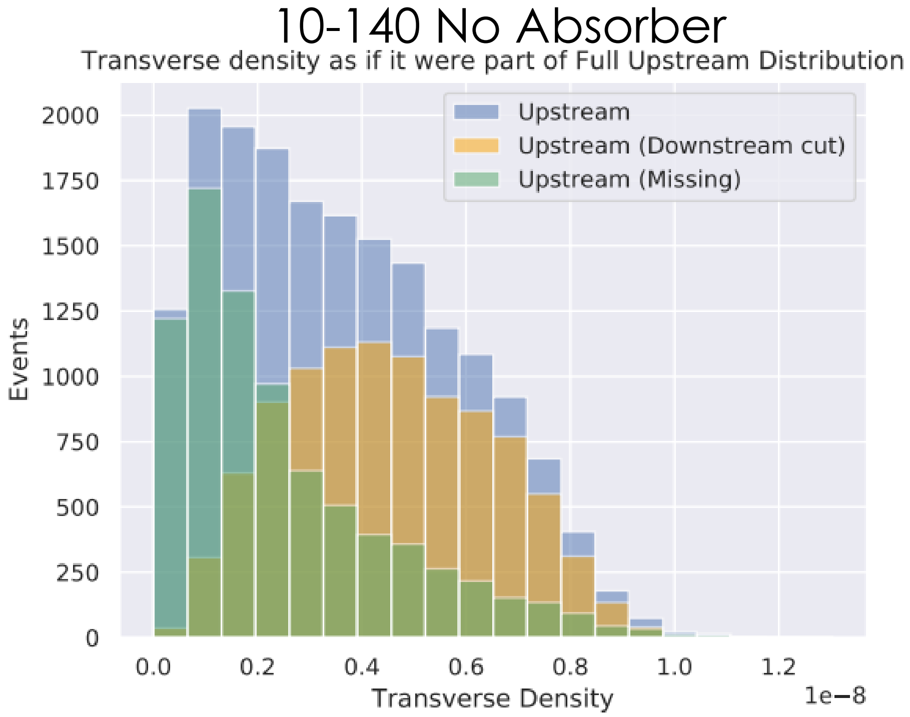
- Top right: 3-140 No Absorber
- Bottom left: 6-140 No Absorber
- Bottom right: 10-140 No Absorber

Small numbers go missing at beam core

Note: beam core is at high density



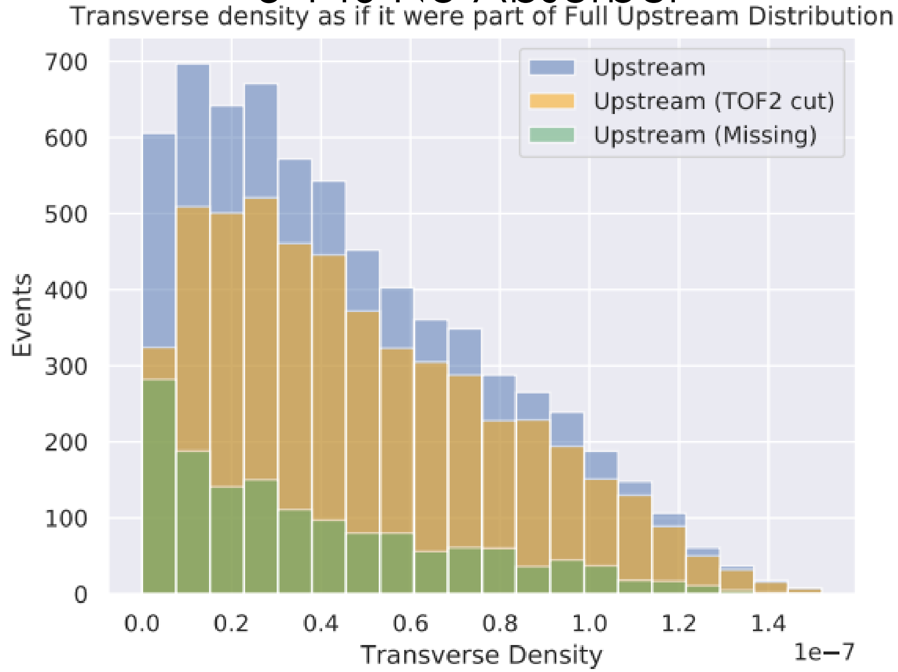




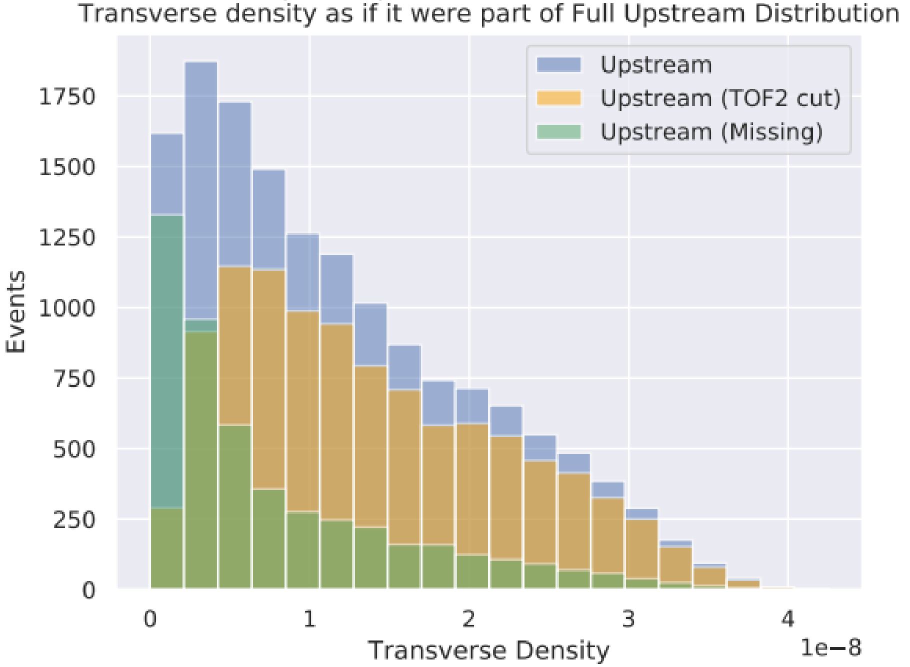
Transverse densities as if they were part of Upstream Distribution making it to TOF2 or not (i.e. missing) – MC Recon

- Wedge Analysis requires TOF2
- TOF2 is even stricter cut than TKD
- More missing particles, even at core

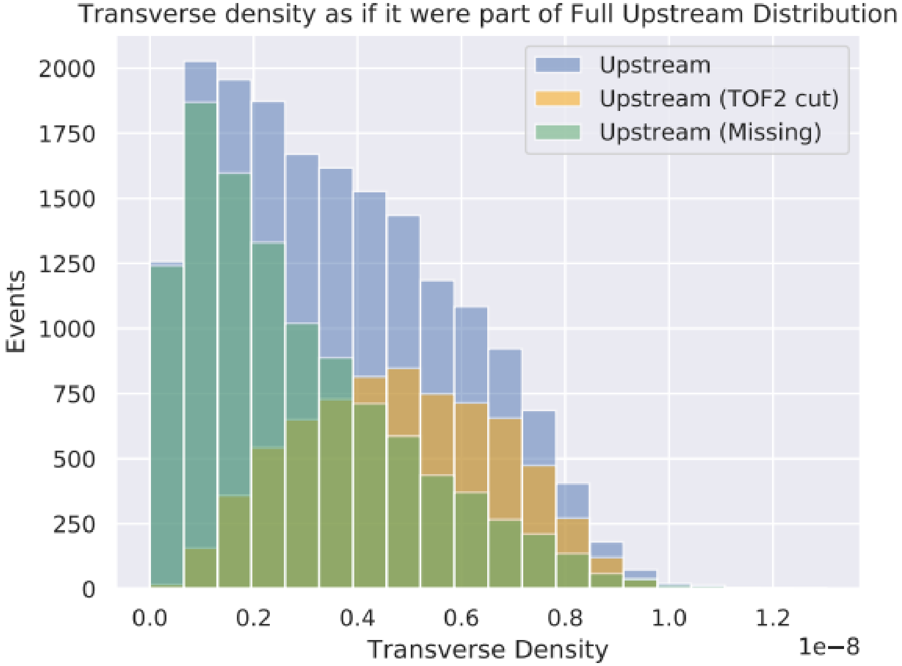
3-140 No Absorber



6-140 No Absorber



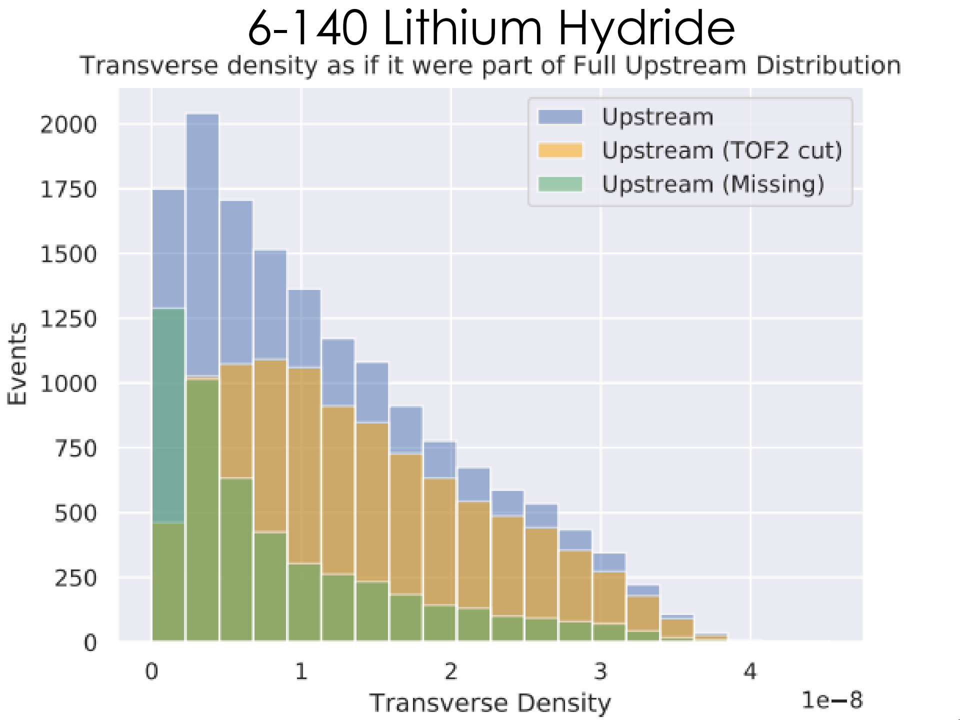
10-140 No Absorber

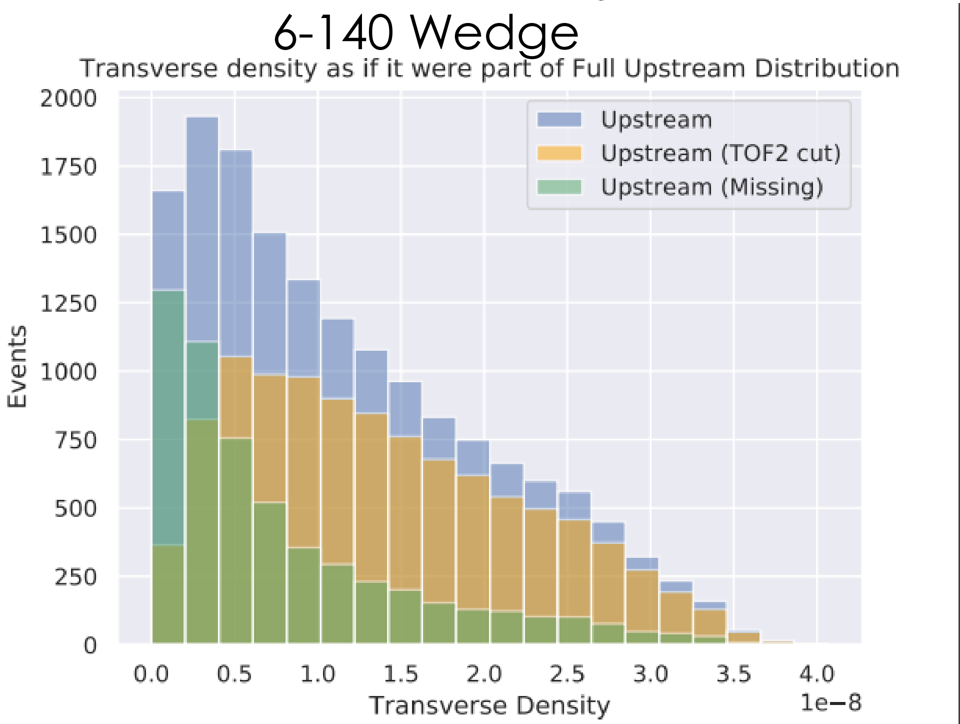


Transverse densities as if they were part of Upstream Distribution making it to TOF2 or not (i.e. missing) – MC Recon

- Further particles will not be detected downstream (i.e. missing from particle distribution function) when the beam encounters an absorber

- Downstream we only have the remaining distribution
- We don’t know what volume the beam would occupy if it had full transmission
- We can only calculate the density for the remaining distribution





Transverse densities as if they were part of Upstream Distribution making it to TOF2 or not (i.e. missing) – MC Recon

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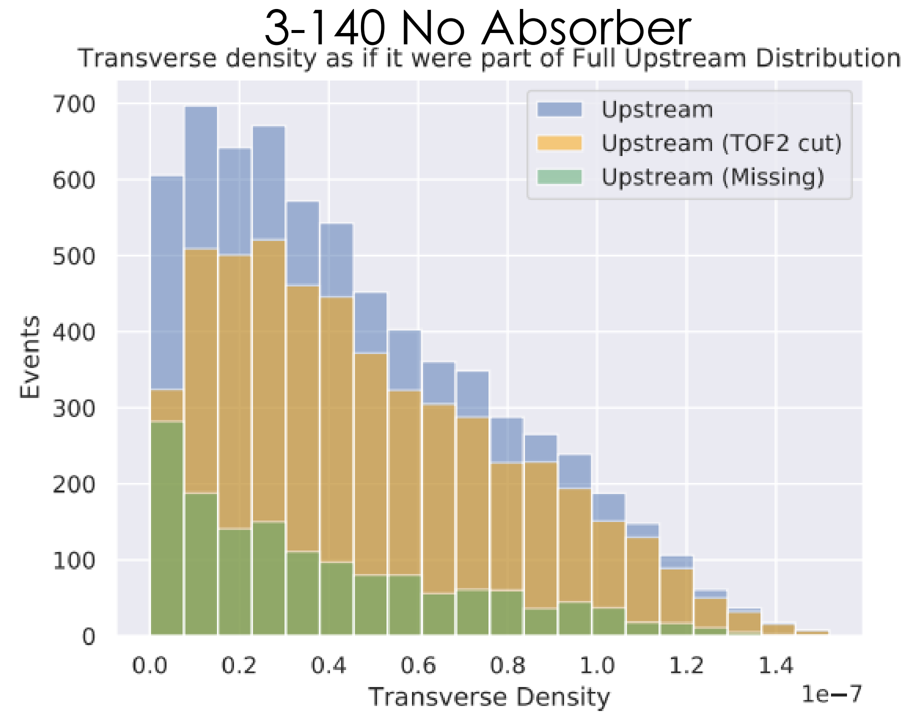
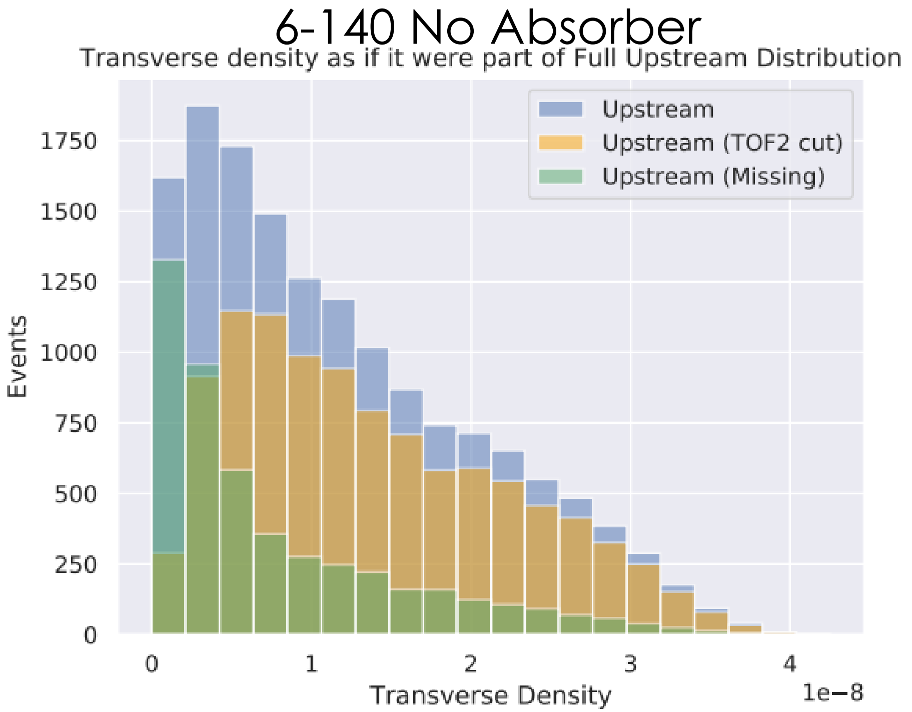
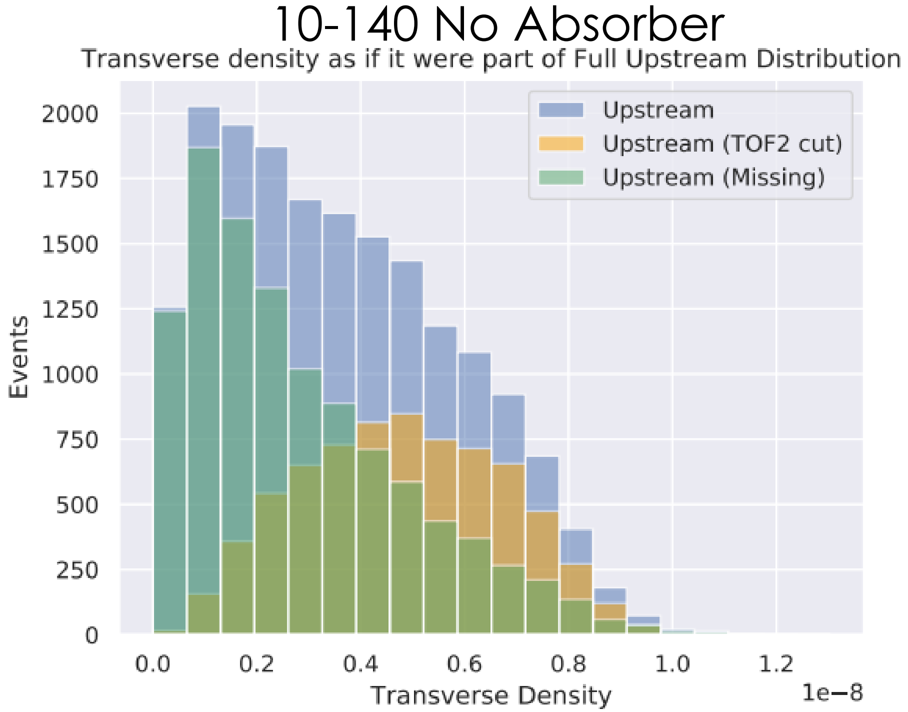
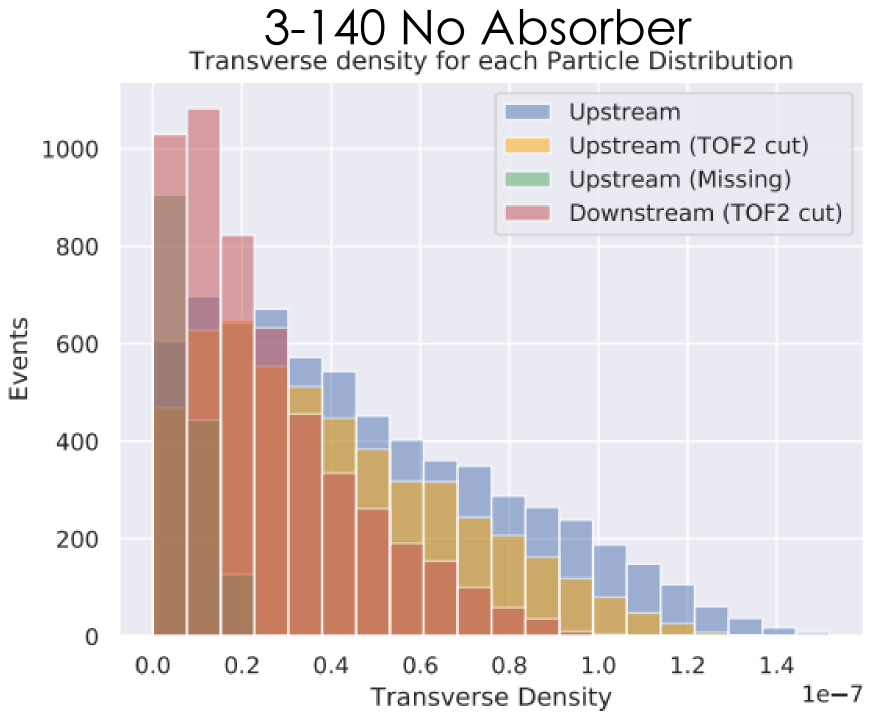


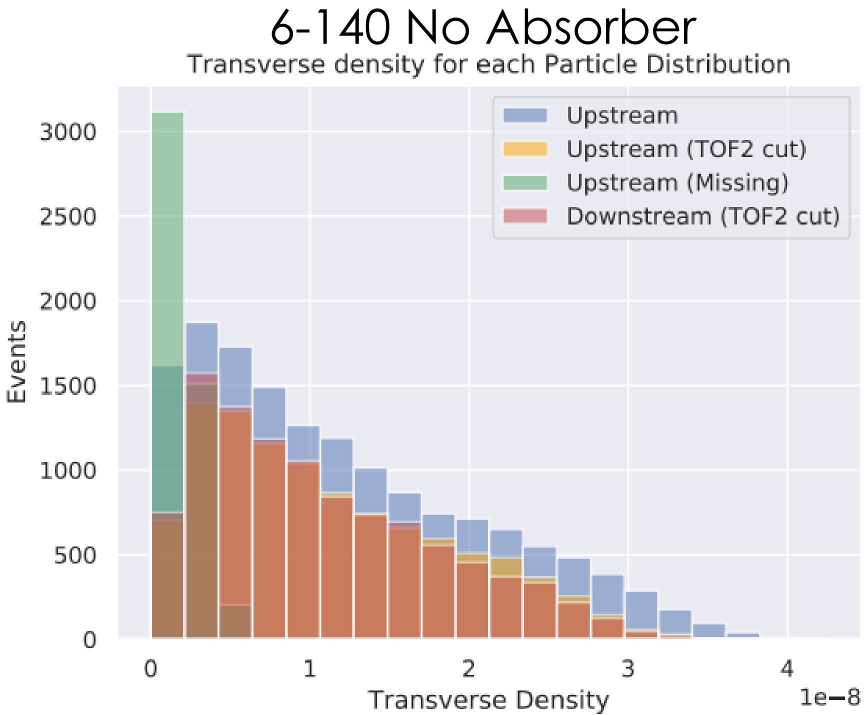


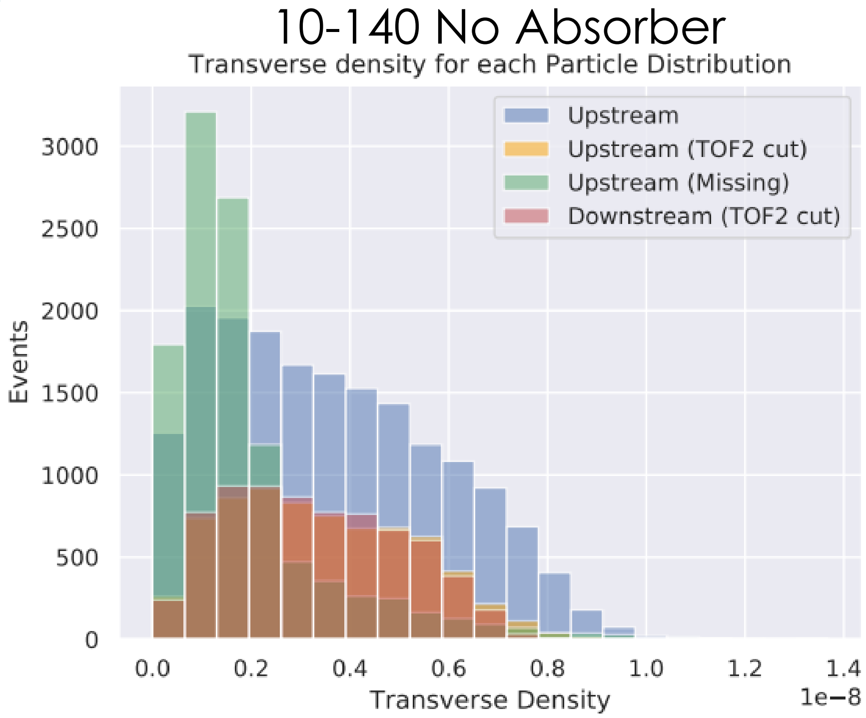


Transvers densities calculated over each measured distribution

- When the Upstream sample (blue) is separated into the sample which makes it downstream (yellow) and the sample which doesn't (green) and the density is calculated for each, their sum doesn't result in the original distribution
- This means there is no simple scaling to compare the full Upstream sample (blue) with the Downstream sample (red)
- We can only compare the yellow and red samples noting it is biased by survivorship, unless we can apply an appropriate correction



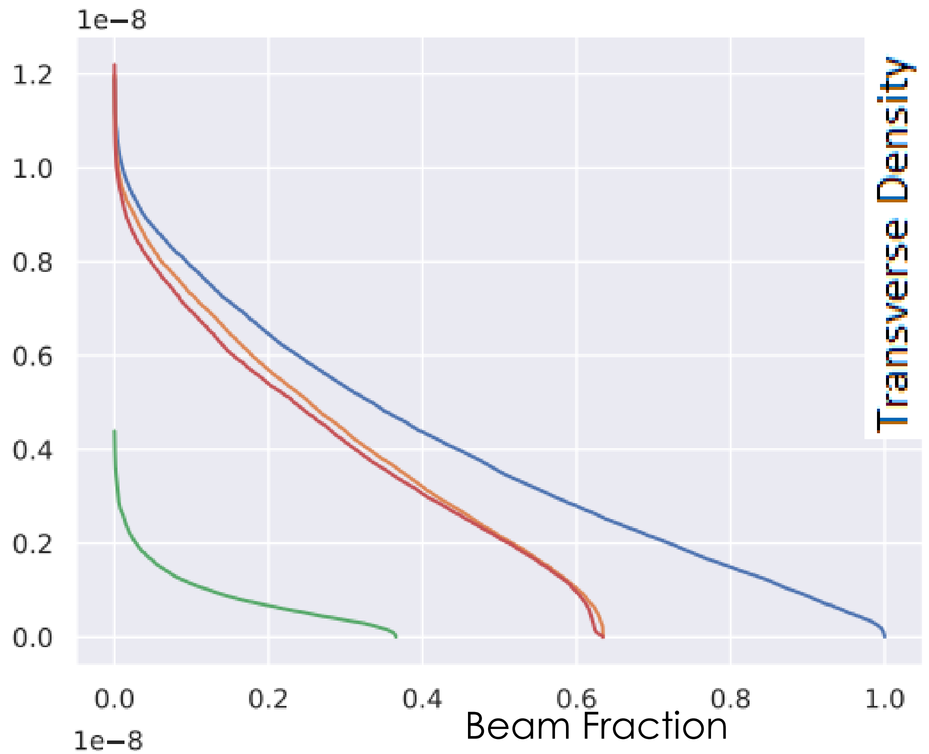


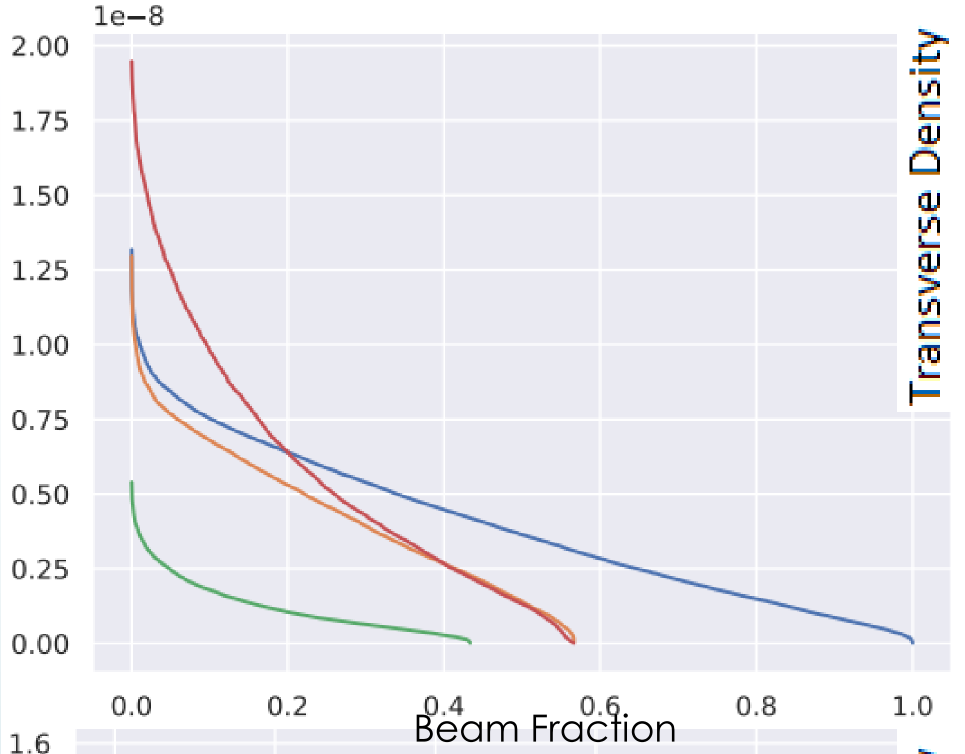


Biased Cooling

- Cooling can be shown in a number of different ways
- One is to through cumulative plots showing what fraction of the beam is above a certain density
- If the downstream line is above the upstream line it shows cooling as the phase space density has increased.
- The opposite is the case for heating
- It is highly affected by transmission losses

Fraction of beam above certain density for 10-140 Data beams



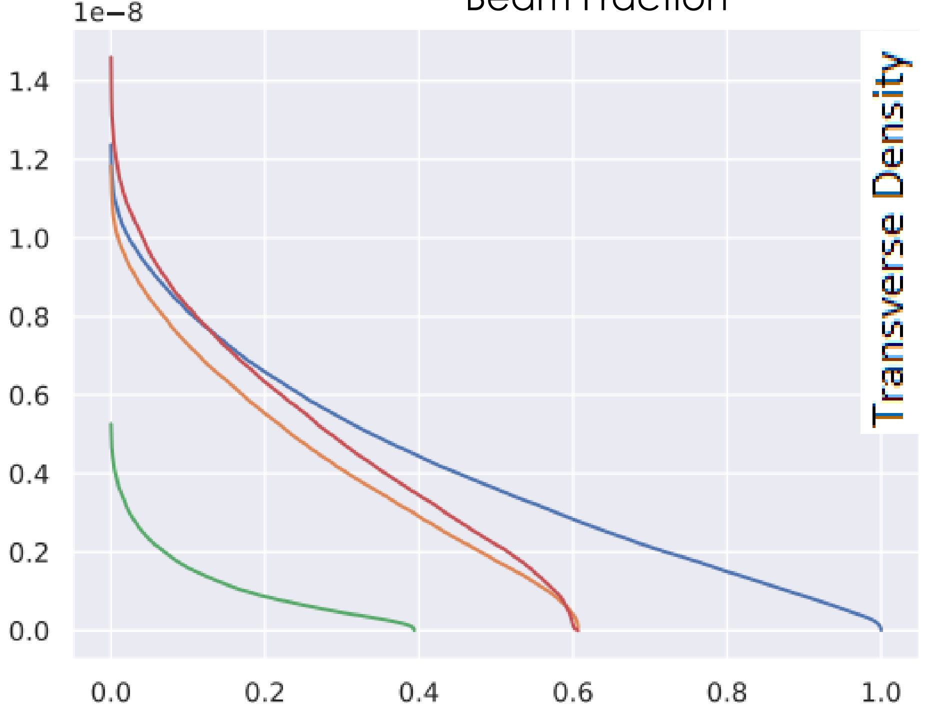


Top Left: No absorber

Top Right: Wedge

Bottom Left: LiH

Bottom Right: LH2

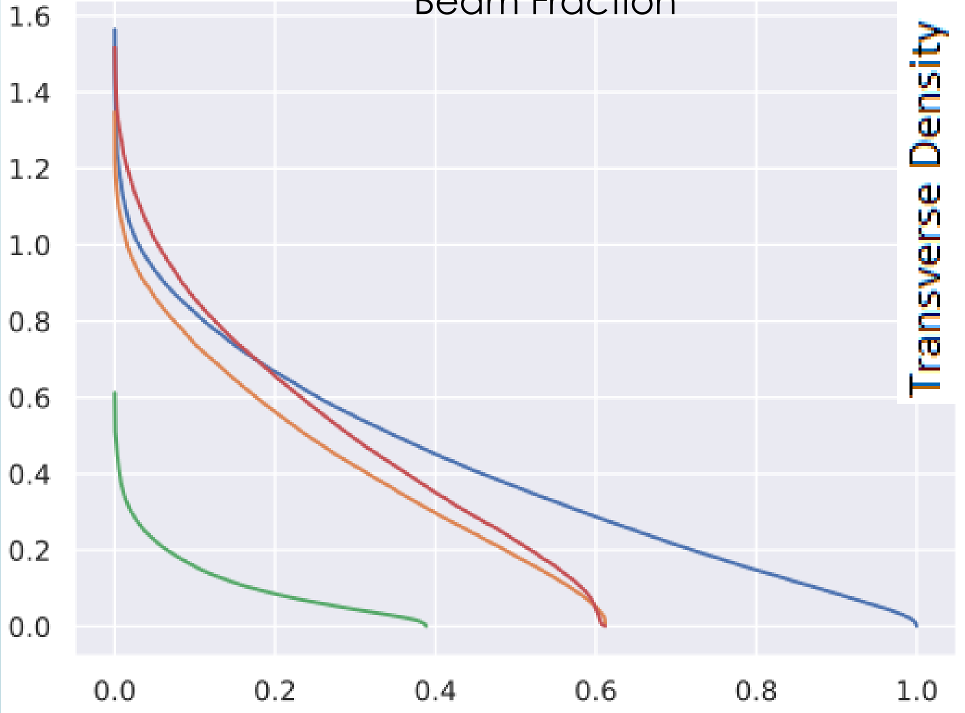


Blue – Full Upstream Sample

Red – Full Downstream Sample

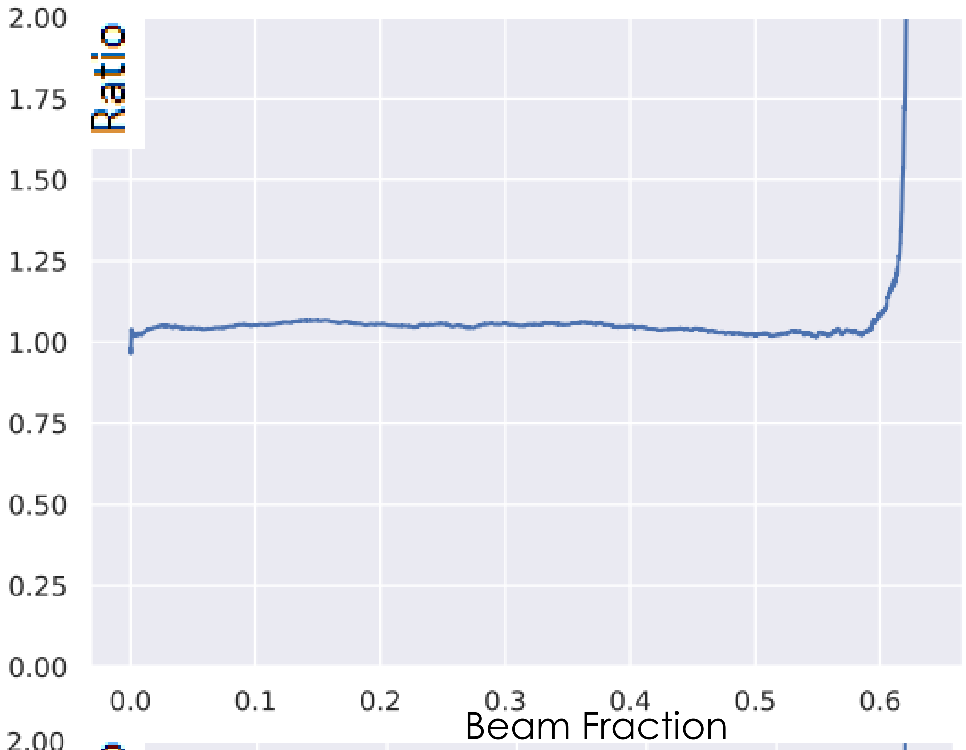
Orange – Upstream Sample which made it Downstream

Green – Upstream Sample which doesn't make it downstream



Biased Cooling

- Another way to show cooling is to take the ratio of the fraction of the upstream sample which makes it downstream to the downstream sample. This should remain constant across the whole fraction of the beam for the symmetric absorber cases. For the wedge the ratio will be proportional to the thickness traversed
- It is biased however as it does not contain the full beam
- Particles are eliminated by the absorber, and some particles make it downstream because of the absorber



Ratio of the Downstream density to the Upstream density which makes it downstream 10-140 Data

Top Left: No absorber

Top Right: Wedge

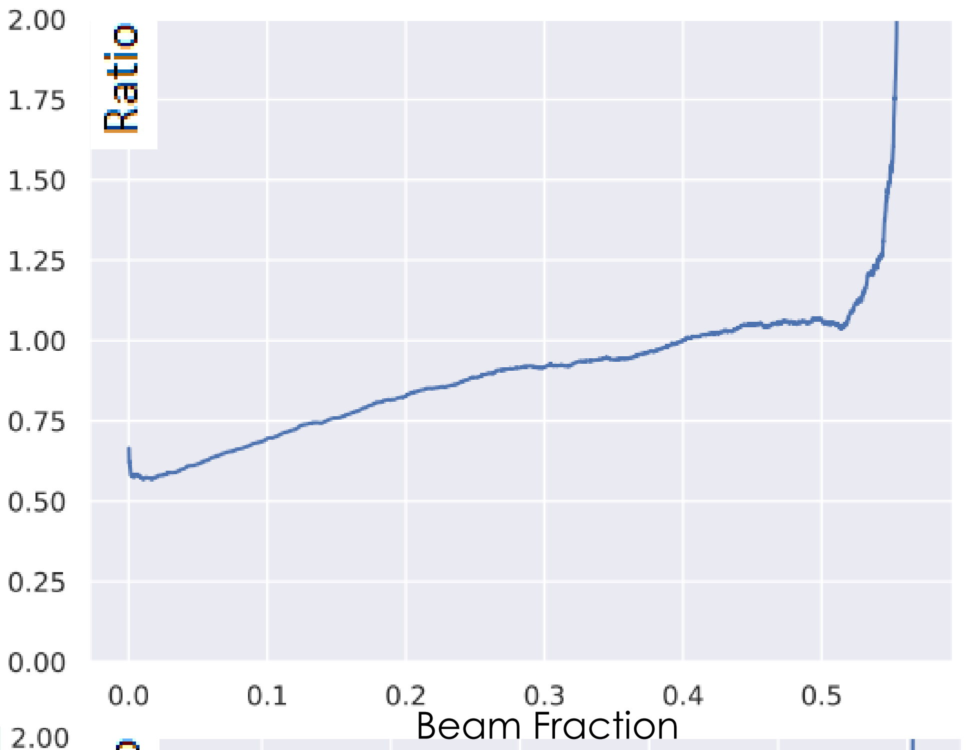
Bottom Left: LiH

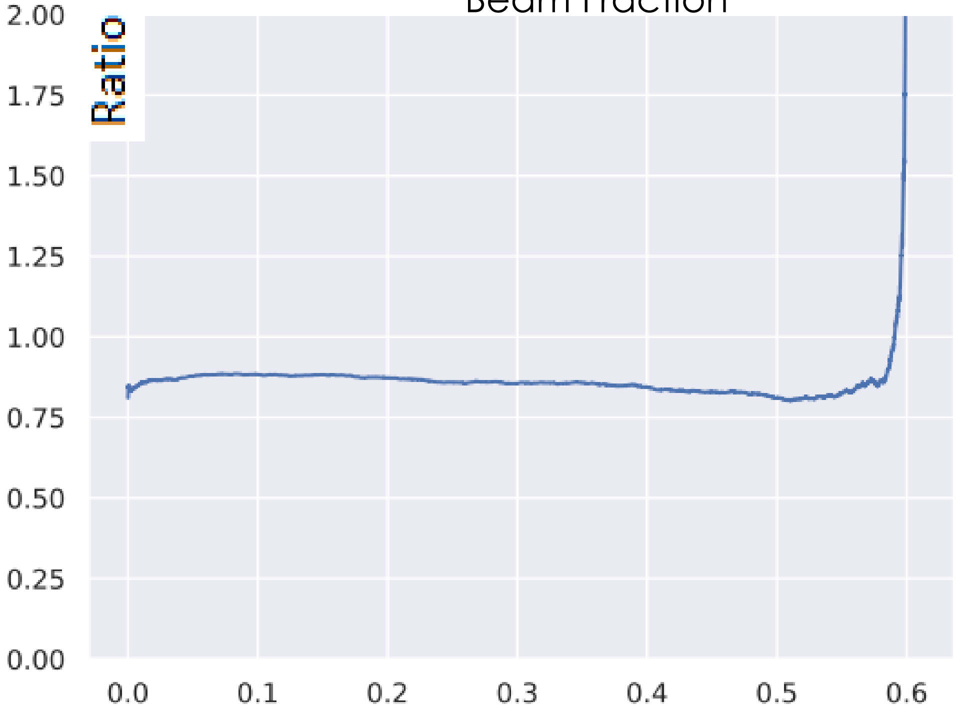
Bottom Right: LH2

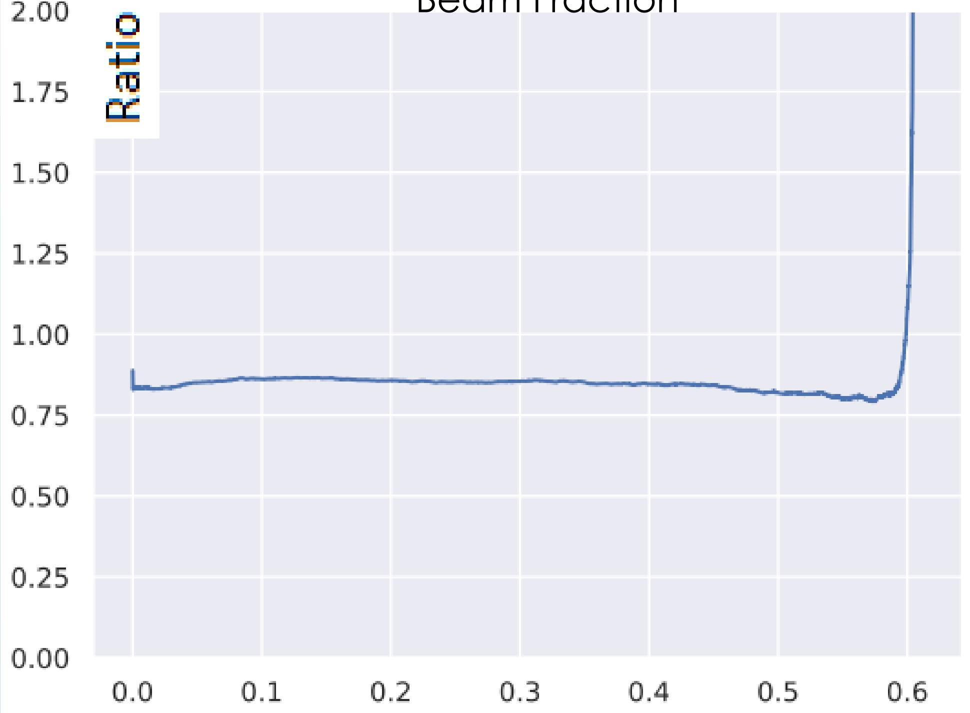
Ratio above one indicates heating while a ratio below one indicates cooling.

Transmission limits the beam to approximately 60% of the full upstream sample.

The min and max are limited by low sample size and scraping respectively







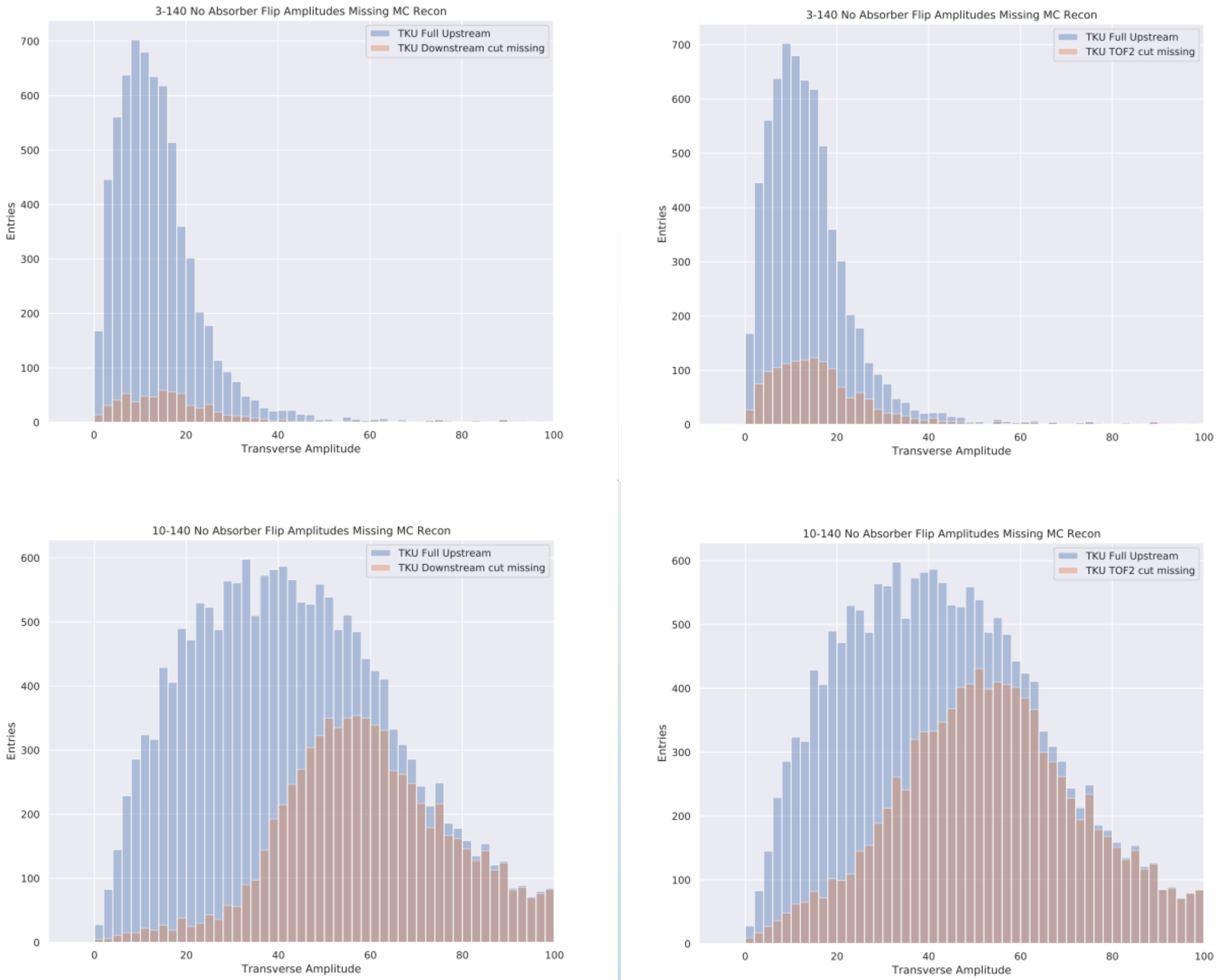
Amplitude

- Seen particles lost at core of beam for Phase-Space Density
- Will show particles at core of beam are also lost for Amplitude

- What to expect:
- MICE beam is approximately Gaussian
- Amplitude distribution should follow Chi-squared distribution with number of degrees of freedom equal to the number of dimensions

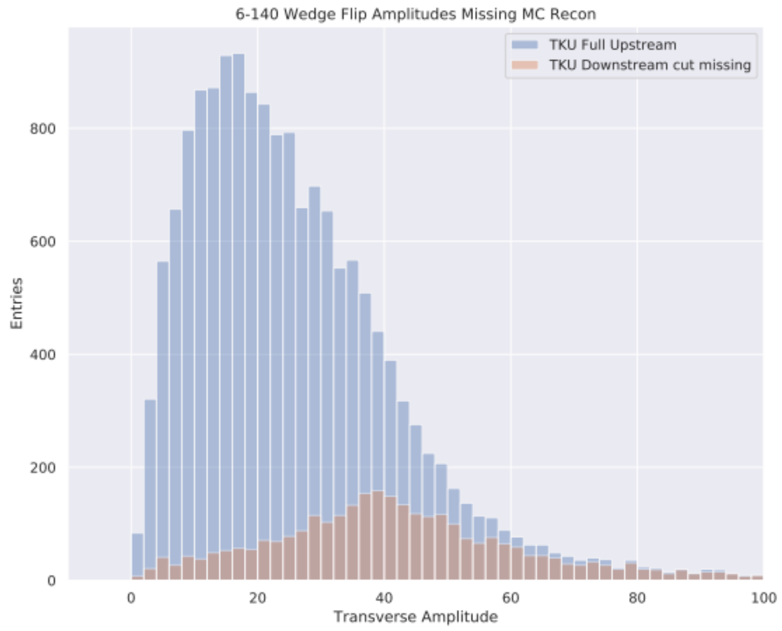
Transverse Amplitude

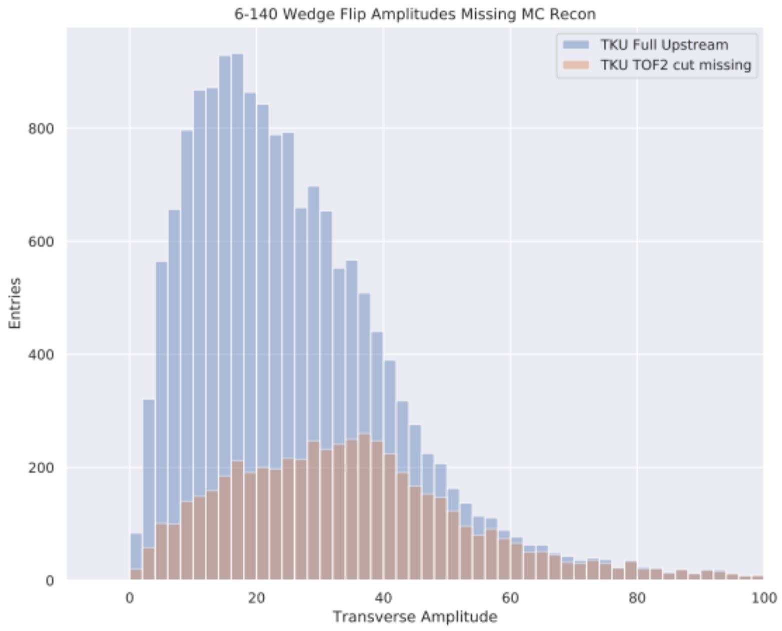
- TOF2 requirement results in more particles being lost
- Particles at beam core are also lost

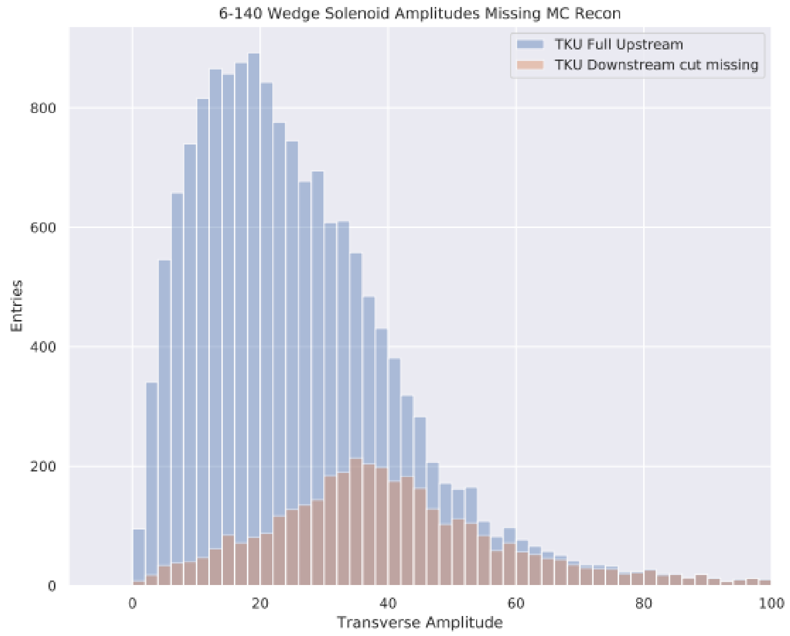


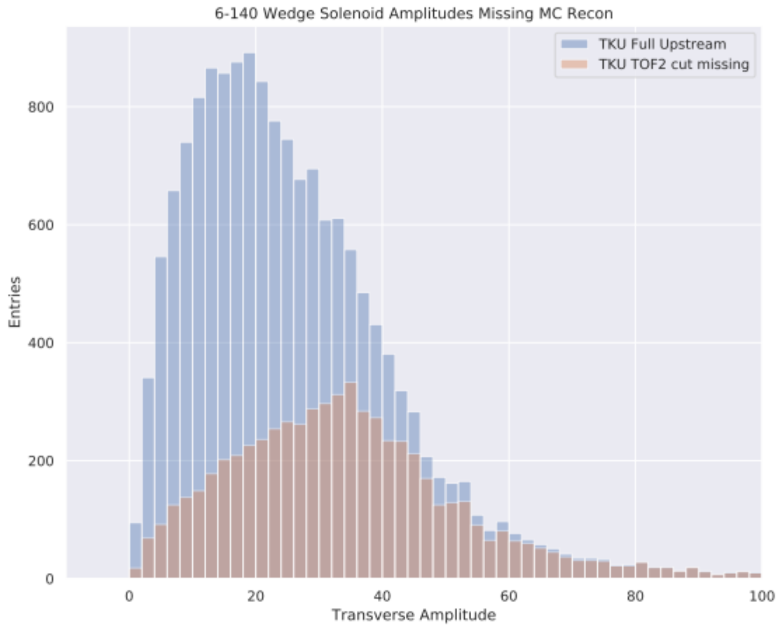
Transverse Amplitude

- Missingness depends both on absorber used and field setting



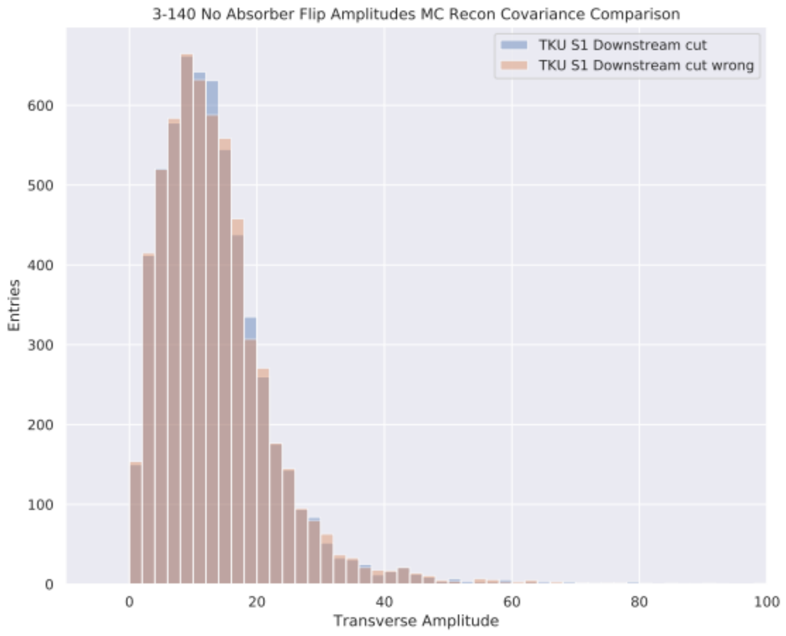


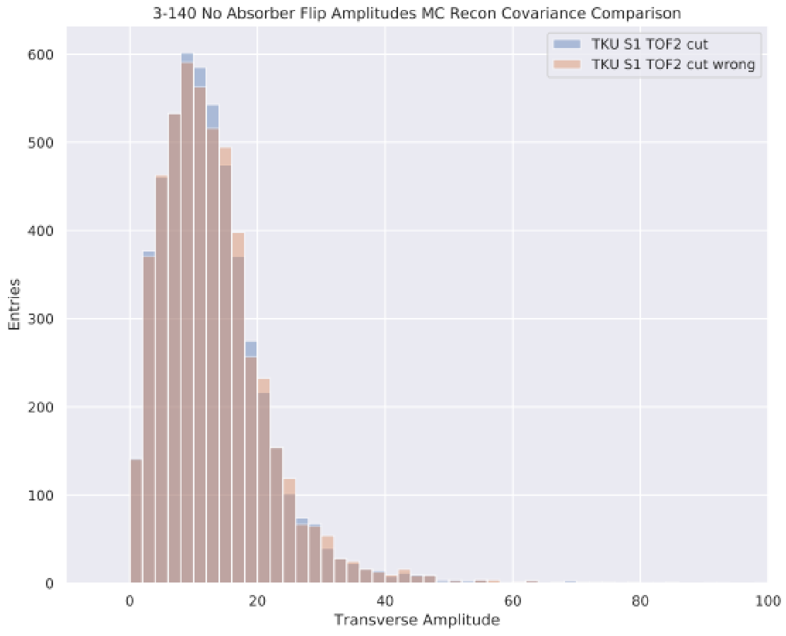


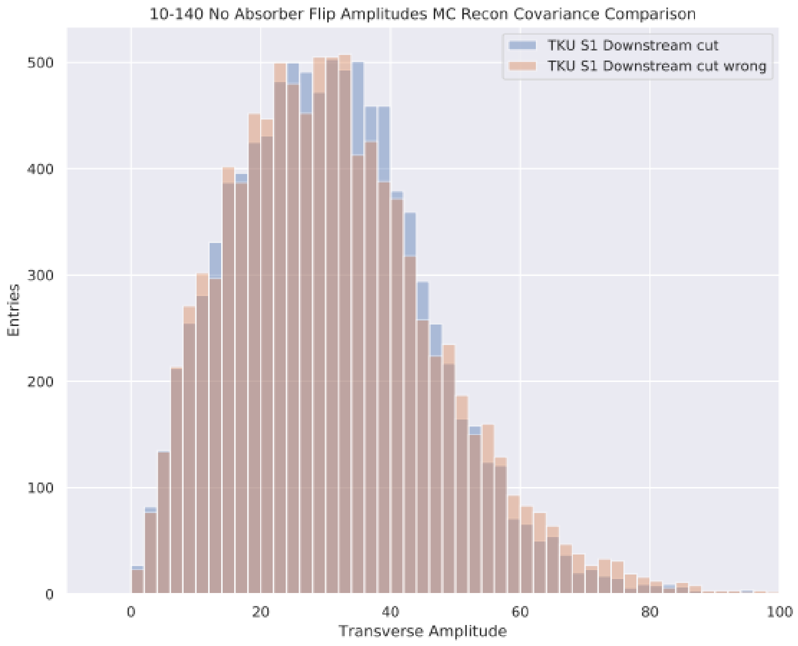


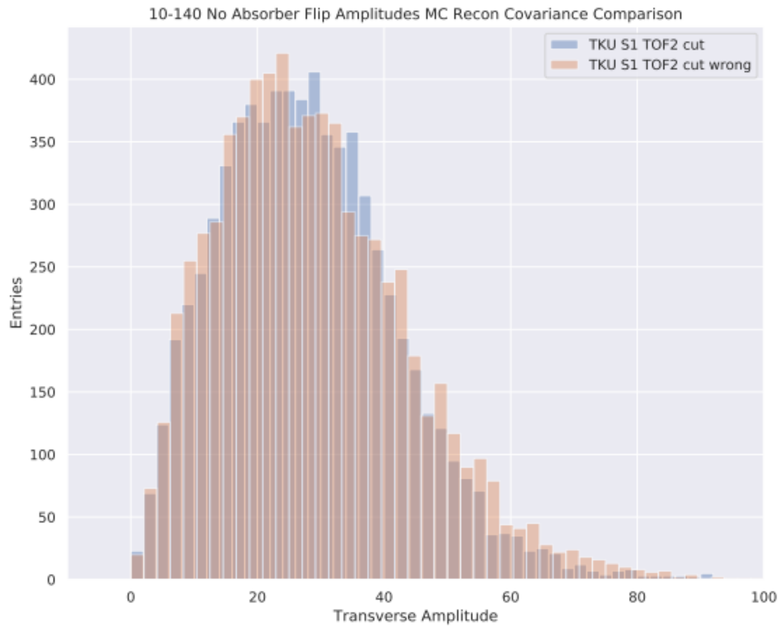
Amplitudes

- Selection bias also affects Amplitude calculation
- Amplitudes at TKU have been calculated using the covariance matrix of the sample which makes it downstream (blue) and using the full Upstream sample (red)
- There is some small variation and thus the amplitude is affected by the input particle distribution



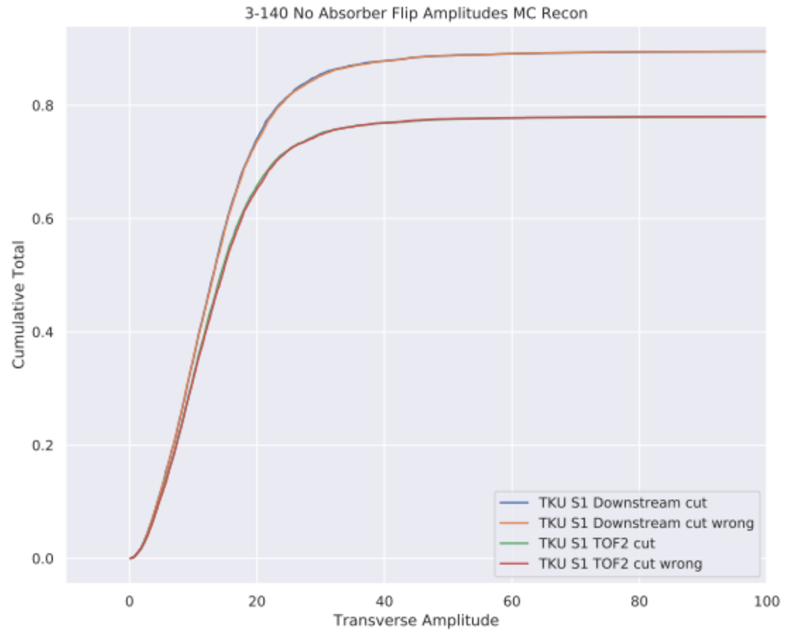


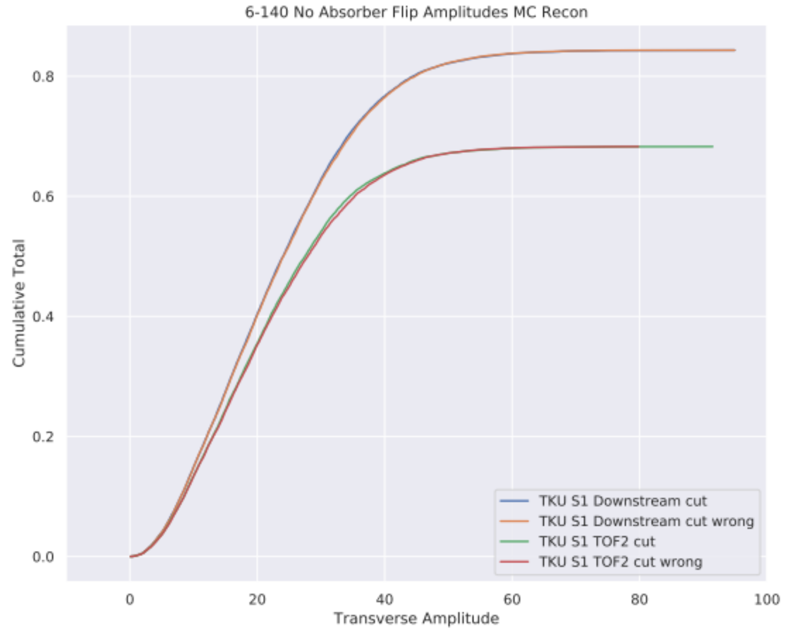


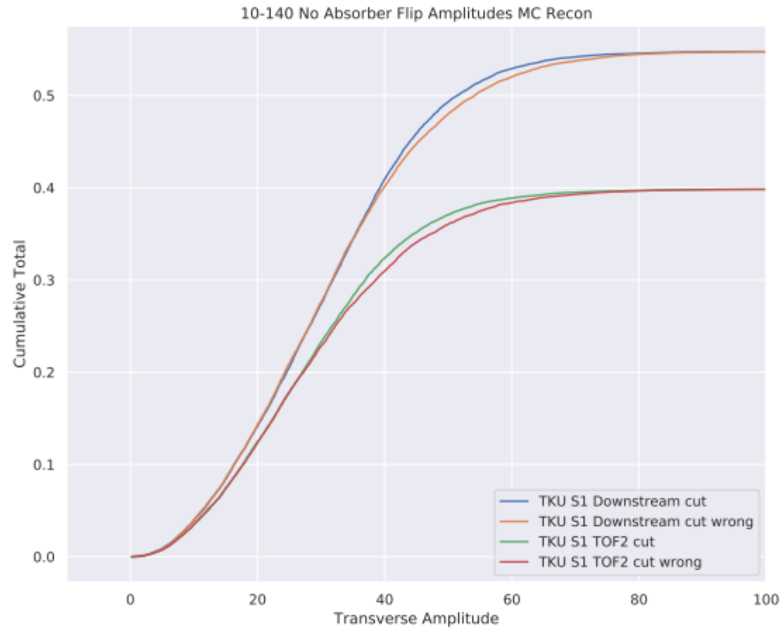


Cumulative Amplitude Plots

- If there are only small variations, does it even matter? Maybe it is just noise
- The cumulative plots also show only some small variation, and mainly at the higher input emittance beams.
- The sampled beam calculation is slightly cooler than that calculated by the full upstream sample
- The cooling performance may be slightly biased (in this case underestimated), although it mainly tells that the input distribution does have some influence

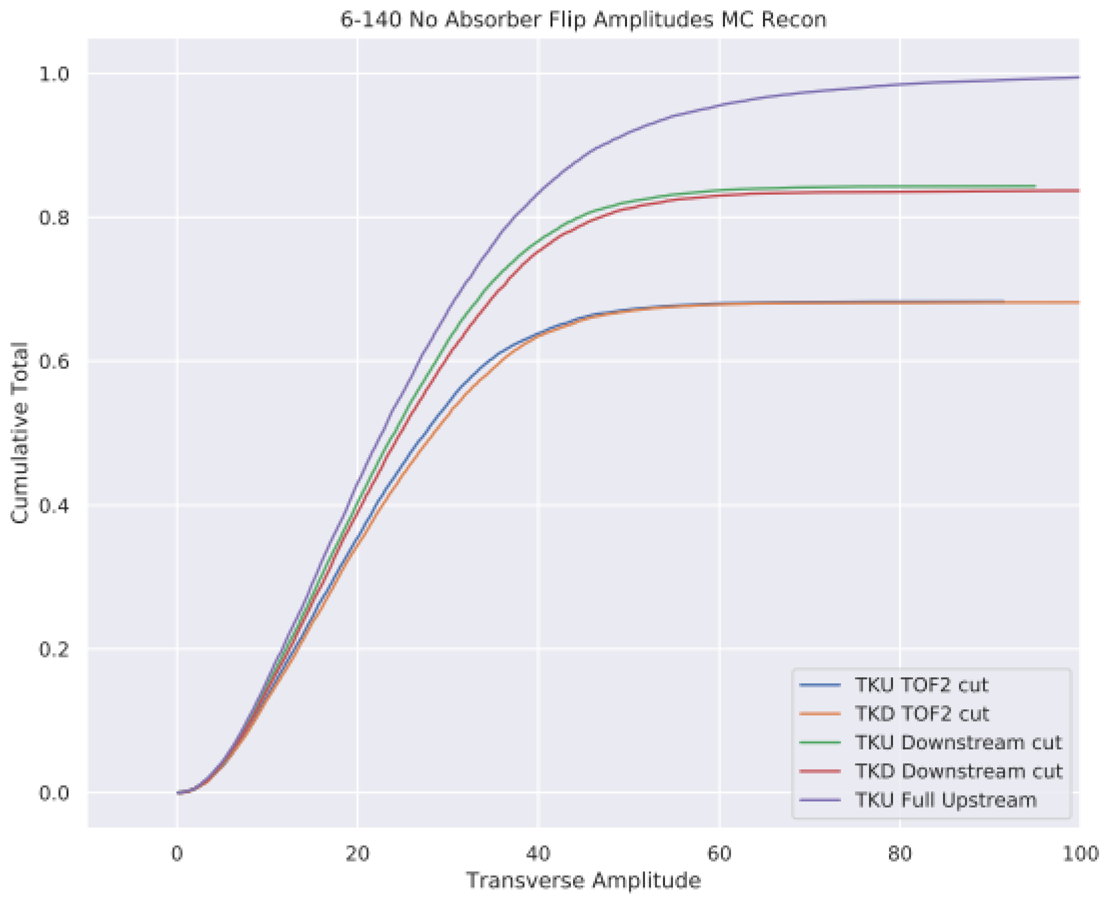


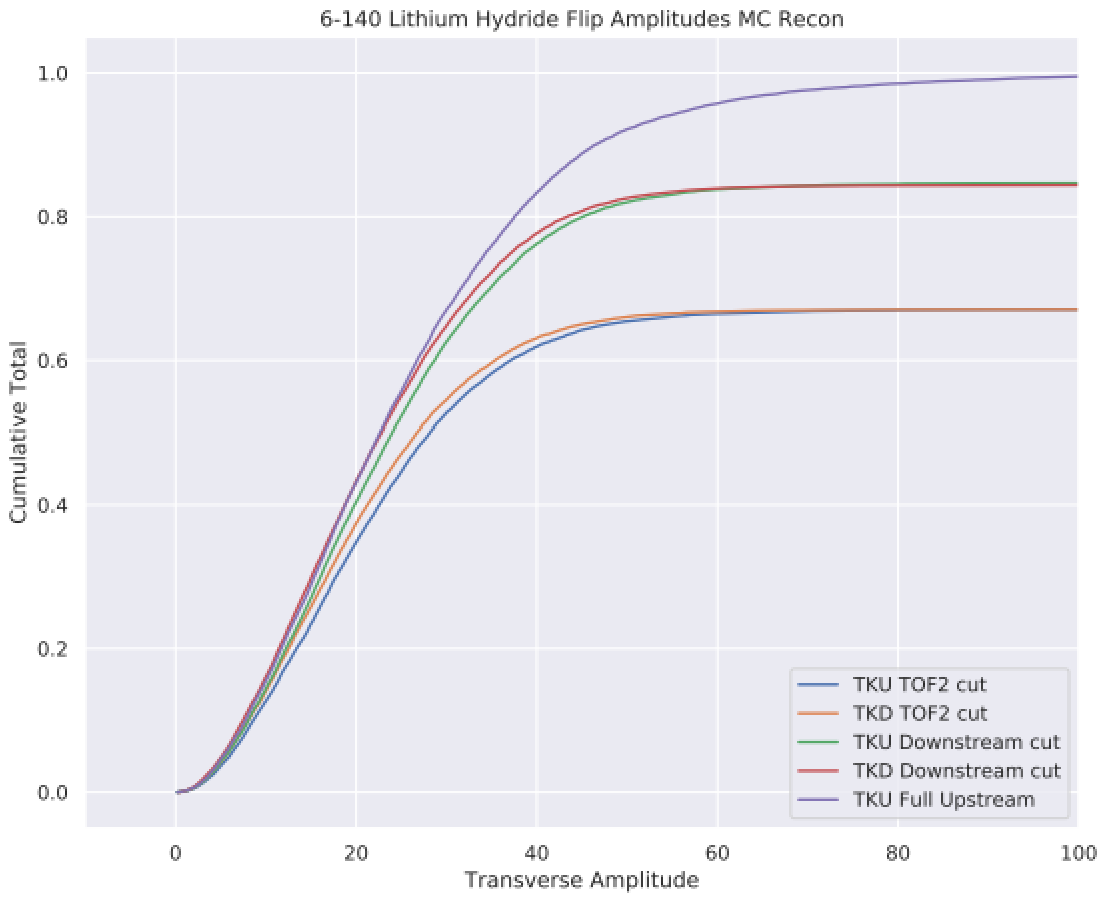




Cumulative Amplitude Plots

- The variation seen for the Upstream sample using the sampled covariance matrix and the full upstream matrix may look small, but then again the expected cooling performance of Lithium Hydride seen in the Cumulative Amplitude plots is also small





Amplitude Correction in MICE

- MICE currently uses an algorithm to correct the amplitudes
- From Chris' note, the algorithm follows:

```
While events in sample {
    Calculate amplitudes
    Remove highest amplitude event
    Update covariance matrix
}
```

- From Francois' thesis this is written as:

$$\langle x_\alpha \rangle_{n-1} = \frac{1}{n-1}\left(n\langle x_\alpha \rangle_n - x_\alpha^n\right)$$

$$\Sigma_{\alpha\beta}^{n-1} = \frac{n-1}{n-2}\Sigma_{\alpha\beta}^{n} - \frac{n}{n-1}\left(x_\alpha^n - \langle x_\alpha \rangle_n\right)\left(x_\beta^n - \langle x_\beta \rangle_n\right)$$

$$with\ \alpha, \beta = x, y, p_x, p_y$$

- The idea of the algorithm is to constantly update the amplitudes so that the cores of the Full Upstream and Downstream sample can be compared

Amplitude Correction in MICE

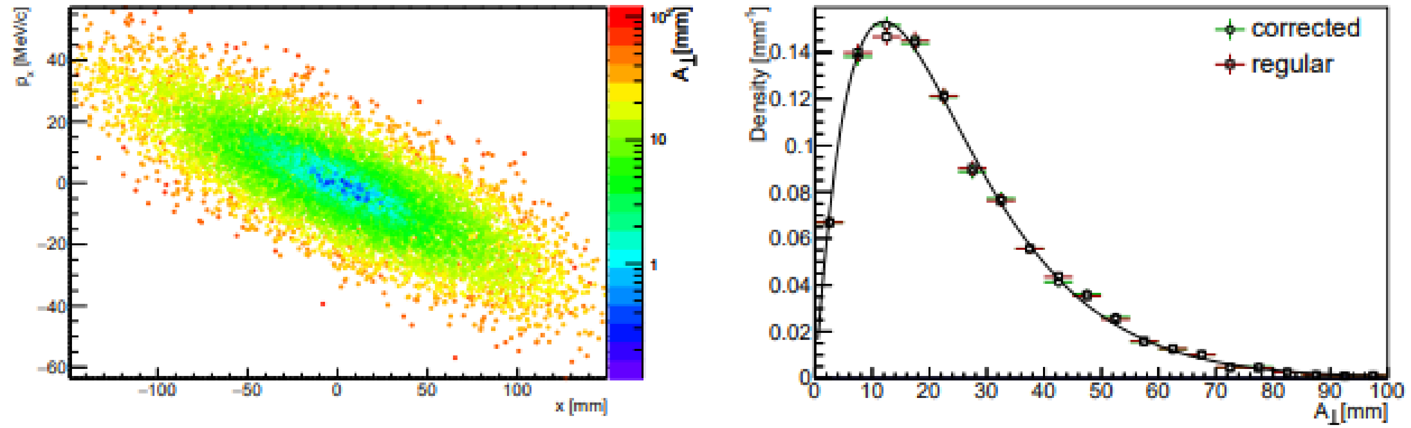


Figure 6.23: (Left) Corrected amplitude scatter plot of a Gaussian beam of $\epsilon_\perp =$ 6 mm. (Right) Regular and corrected amplitude distribution of the Gaussian beam.

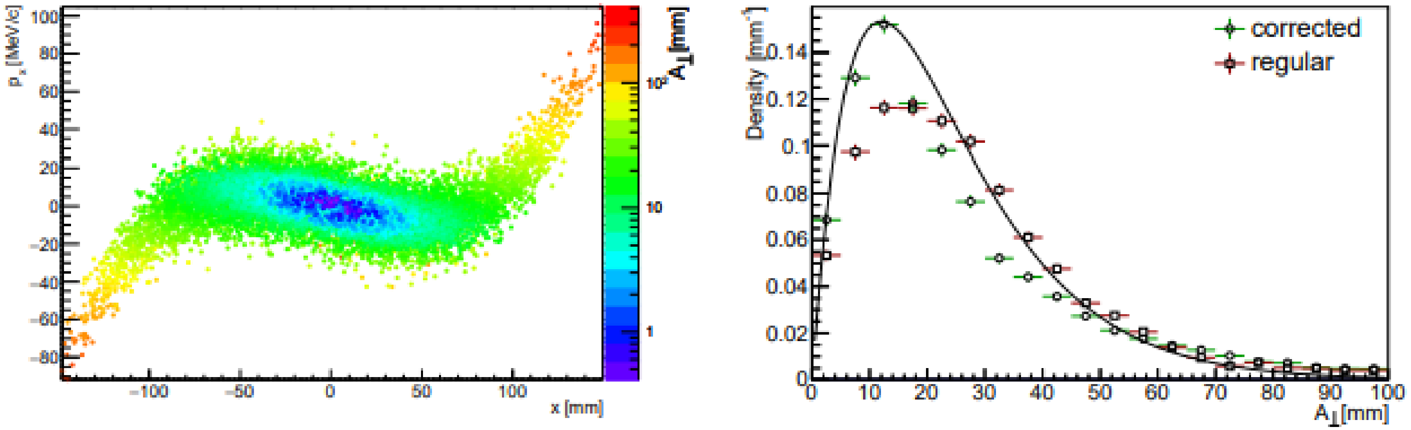


Figure 6.24: (Left) Corrected amplitude scatter plot of a non-Gaussian beam. (Right) Regular and corrected amplitude distribution of the Gaussian beam.

- The idea of the algorithm is to constantly update the amplitudes so that the cores of the Full Upstream and Downstream sample can be compared
- It is an algorithm, which was tested by Francois on the left on two different input distributions.
- However there is no mathematical proof that this is valid (I mention this because of the incorrect MICE normalisation for the phase-space density)

- Concern 1: Each amplitude is calculated using a new covariance matrix. It is effectively a new ruler for each measurement, but each ruler has a slightly different scale
- Concern 2: The idea behind it is that at the core the covariance matrix should remain relatively stable. Therefore there is a limit to the amplitudes which can be compared.
- Concern 3: There is transmission loss. We have already seen that the beam loses particles at the core of the beam, so does the algorithm still remain valid for that scenario

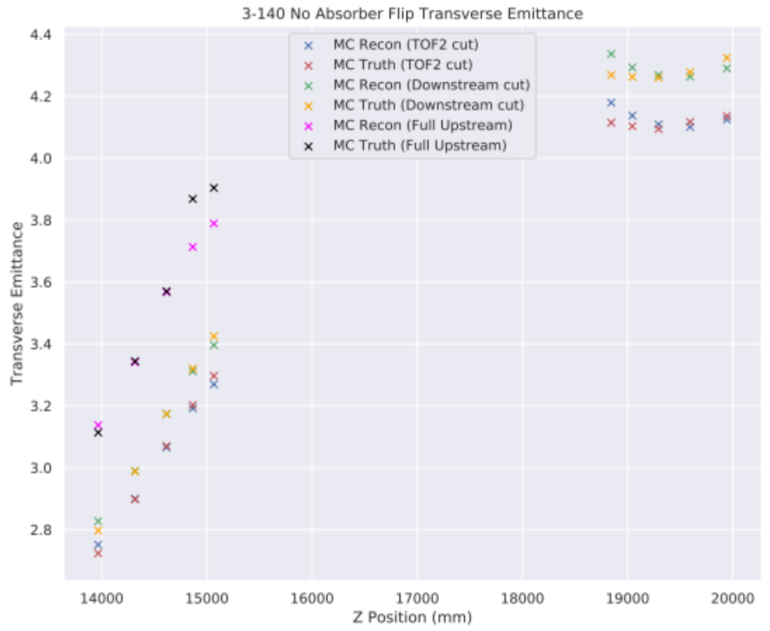
Emittance

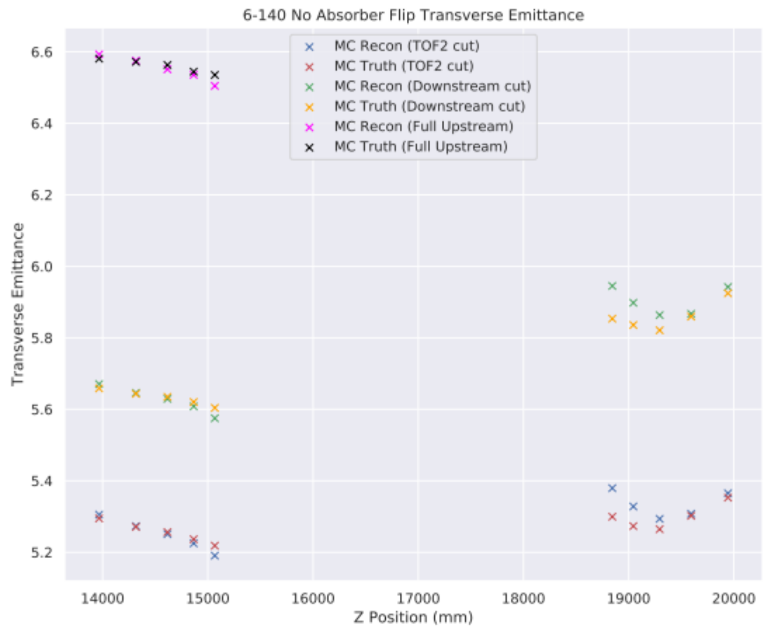
- Emittance like Amplitude is conserved in linear optics
- In MICE emittance growth has been extensively seen

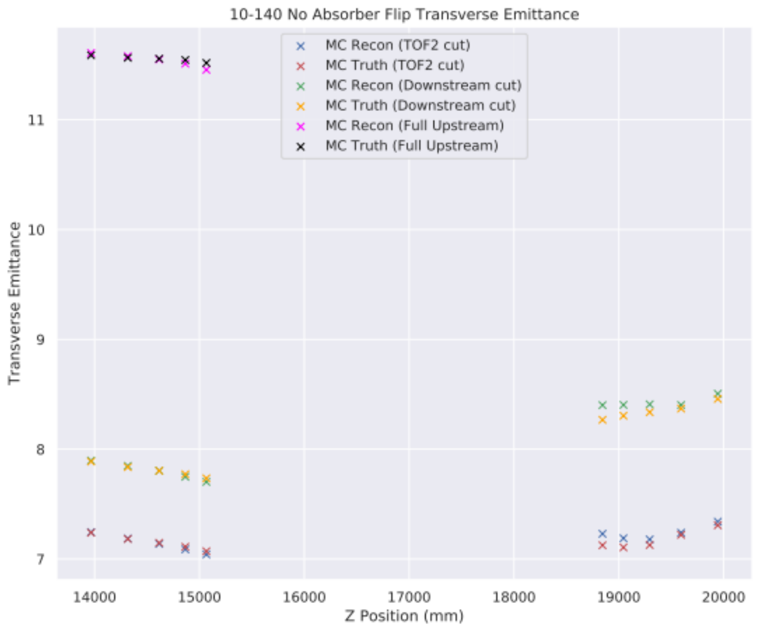
- Emittance "is" the determinant of the covariance matrix
- If Emittance growth is seen it is clear indicator of non-linear effects as the beam becomes more distorted
- Makes it difficult to get a cooling signal using emittance

Emittance

- Upstream Emittance bears no resemblance to Downstream distribution
- Emittance only comparable for a selection
- Emittance growth seen also dependent on particle distribution function. Growth is smaller for a "tighter" selection, i.e. going through less non-linear optics

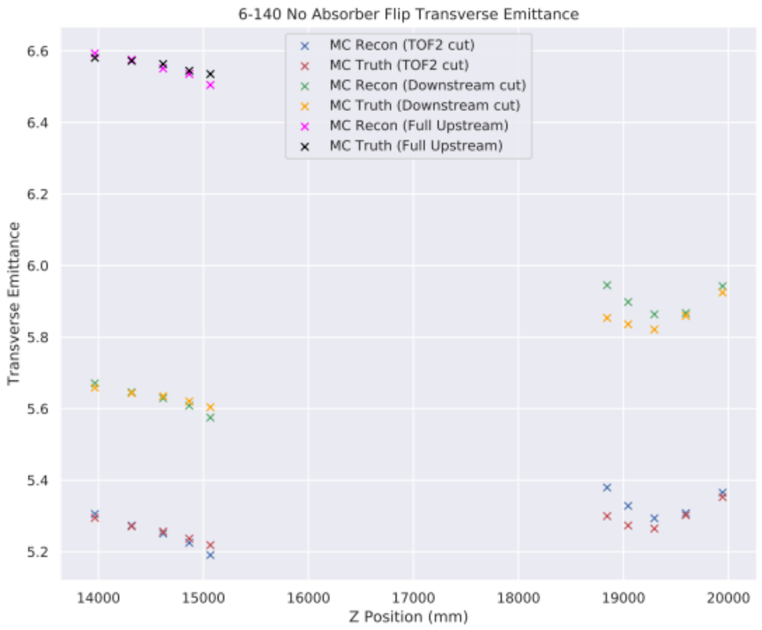


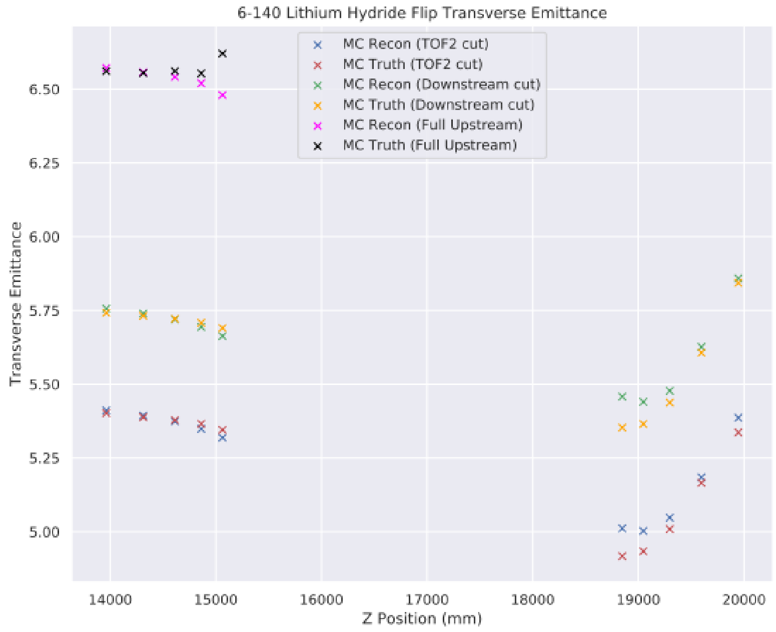


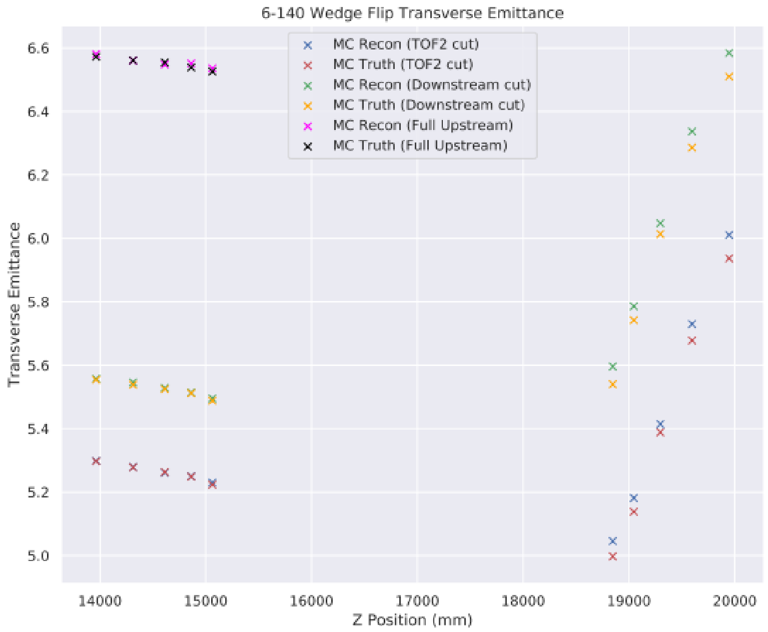


Emittance

- Upstream distributions similar, with similar gradients through Upstream tracker
- In TKD: No Absorber Emittance emittance relatively flat
- For LiH, growth seen within TKD (possibly due to Energy Straggling)
- In Wedge, significant growth in TKD due to dispersion



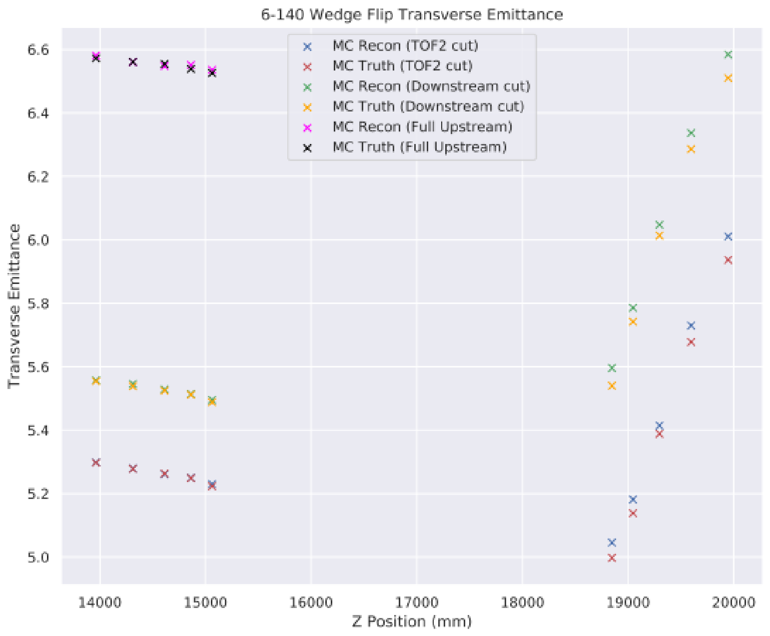


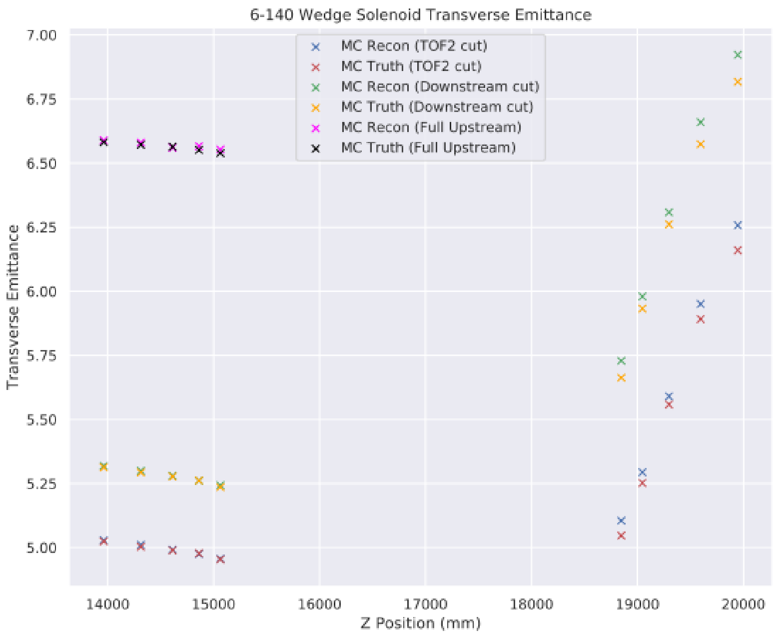


Emittance

- The emittance seen is also dependent on the magnetic field mode i.e. flip or solenoid
- This is because each mode will have different transmission losses and thus affect the downstream survival bias

- For Emittance, Amplitude and Phase-Space Density we can only compare the same particles upstream and downstream, however this is biased by transmission losses
- The full Upstream sample is unbiased
- The challenge is to have an unbiased downstream "cooling" signal
- Emittance, Amplitude and Density all have the Covariance Matrix in common.





MICE – 3 Cooling measurements

- MICE measures cooling using three techniques

- 1. Emittance:

$$\varepsilon_d = \frac{\sqrt[d]{|\Sigma|}}{m_\mu c}$$

- 2. Amplitude:

$$A_d = \varepsilon_d \boldsymbol{x}^T \Sigma^{-1} \boldsymbol{x} = \frac{\sqrt[d]{|\Sigma|} \boldsymbol{x}^T \Sigma^{-1} \boldsymbol{x}}{m_\mu c}$$

- 3. Phase-Space Density:

  - Kernel Density Estimation (KDE)

$$\hat{\rho}(\vec{x}) = \frac{1}{n}\sum_{i=1}^{n} K_{\boldsymbol{H}}(\boldsymbol{x}) = \frac{1}{nh^d}\sum_{i=1}^{n} K\left(\frac{\boldsymbol{x}}{h}\right) = \frac{\sum_{i=1}^{n} exp[-\frac{1}{2}\boldsymbol{x}^T \Sigma^{-1} \boldsymbol{x}]}{n(2\pi)^{d/2}|\Sigma|^{1/2}}$$

  - k-nearest neighbour (KNN)

$$\vec{\rho}(x) = \frac{k}{n\kappa_d R_k^d} = \frac{k\Gamma(\frac{d}{2}+1)}{n\pi^{\frac{d}{2}} R_k^d}$$

What do all 3 share?

Dependence on the Covariance Matrix

How should the Covariance Matrix be altered if there are transmission losses?

Let the full upstream sample be denoted by:

$$\Sigma_1 = \sum_{1}^{N_1} (P_i - \bar{P}_1)(P_i - \bar{P}_1)/N_1$$

where P_i is the Phase Space vector (x, y, z, px, py, pz) and N_1 is the sample size. The Upstream sample which makes it downstream and the sample which doesn't make it downstream are respectively denoted by:

$$\Sigma_2 = \sum_{1}^{N_2} (P_i - \bar{P}_2)(P_i - \bar{P}_2)/N_2\,, \qquad \Sigma_3 = \sum_{1}^{N_3} (P_i - \bar{P}_3)(P_i - \bar{P}_3)/N_3$$

$$with \qquad N_1 = N_2 + N_3 \qquad \text{and} \qquad \bar{P}_1 = \frac{N_2\bar{P}_2 + N_3\bar{P}_3}{N_2 + N_3}$$

Then

$$\Sigma_1 = \sum_{1}^{N_1} \left(P_i - \frac{N_2\bar{P}_2 + N_3\bar{P}_3}{N_2 + N_3}\right)\left(P_i - \frac{N_2\bar{P}_2 + N_3\bar{P}_3}{N_2 + N_3}\right)/(N_2 + N_3)$$

Plan is to write Σ_1 as a combination of Σ_2 and Σ_3.

Separating the Upstream Covariance matrix

$$\Sigma_1 = \sum_1^{N_1}\left(P_i - \frac{N_2\bar{P}_2 + N_3\bar{P}_3}{N_2 + N_3}\right)\left(P_i - \frac{N_2\bar{P}_2 + N_3\bar{P}_3}{N_2 + N_3}\right)/(N_2 + N_3)$$

Multiply above and below by $(N_2 + N_3)^2$

$$= \sum_1^{N_1}((N_2 + N_3)P_i - N_2\bar{P}_2 + N_3\bar{P}_3)((N_2+N_3)P_i - N_2\bar{P}_2 + N_3\bar{P}_3)/(N_2 + N_3)^3$$

$$= \sum_1^{N_1}\Big(N_2^2(P_i - \bar{P}_2)(P_i - \bar{P}_2) + N_2N_3(P_i - \bar{P}_2)(P_i - \bar{P}_3)$$
$$+ N_2N_3(P_i - \bar{P}_3)(P_i - \bar{P}_2) + N_3^2(P_i - \bar{P}_3)(P_i - \bar{P}_3)\Big)/(N_2 + N_3)^3$$

Multiply out and reorder

$$= \sum_1^{N_2}\Big(N_2^2(P_i - \bar{P}_2)(P_i - \bar{P}_2) + N_2N_3(P_i - \bar{P}_2)(P_i - \bar{P}_3)$$
$$+ N_2N_3(P_i - \bar{P}_3)(P_i - \bar{P}_2) + N_3^2(P_i - \bar{P}_3)(P_i - \bar{P}_3)\Big)/(N_2 + N_3)^3$$
$$+ \sum_1^{N_3}\Big(N_2^2(P_i - \bar{P}_2)(P_i - \bar{P}_2) + N_2N_3(P_i - \bar{P}_2)(P_i - \bar{P}_3)$$
$$+ N_2N_3(P_i - \bar{P}_3)(P_i - \bar{P}_2) + N_3^2(P_i - \bar{P}_3)(P_i - \bar{P}_3)\Big)/(N_2 + N_3)^3$$

Separate the sum into sums over N_2 and N_3

Separating the Upstream Covariance matrix

This is the general form partially separating out the covariance matrices

$$\Sigma_1 = \frac{N_2^3}{(N_2+N_3)^3}\Sigma_2 + \frac{N_3^3}{(N_2+N_3)^3}\Sigma_3 + \sum_1^{N_2}\Big(N_2N_3(P_i-\bar{P}_2)(P_i-\bar{P}_3) + N_2N_3(P_i-\bar{P}_3)(P_i-\bar{P}_2) + N_3^2(P_i-\bar{P}_3)(P_i-\bar{P}_3)\Big)/(N_2+N_3)^3 + \sum_1^{N_3}\Big(N_2N_3(P_i-\bar{P}_2)(P_i-\bar{P}_3) + N_2N_3(P_i-\bar{P}_3)(P_i-\bar{P}_2) + N_2^2(P_i-\bar{P}_2)(P_i-\bar{P}_2)\Big)/(N_2+N_3)^3$$

For a symmetric absorber and cooling channel (LiH, lH2, No absorber): $\bar{P}_2 \approx \bar{P}_3$, then

$$\Sigma_1 = \frac{N_2^3}{(N_2+N_3)^3}\Sigma_2 + \frac{N_3^3}{(N_2+N_3)^3}\Sigma_3 + \sum_1^{N_2}\Big(2N_2N_3(P_i-\bar{P}_2)(P_i-\bar{P}_2) + N_3^2(P_i-\bar{P}_2)(P_i-\bar{P}_2)\Big)/(N_2+N_3)^3 + \sum_1^{N_3}\Big(2N_2N_3(P_i-\bar{P}_3)(P_i-\bar{P}_3) + N_2^2(P_i-\bar{P}_3)(P_i-\bar{P}_3)\Big)/(N_2+N_3)^3$$

Substituting for $\bar{P}_2$ and $\bar{P}_3$ in their sums

$$= \Sigma_2\left(\frac{N_2^3 + 2N_2^2N_3 + N_2N_3^2}{(N_2+N_3)^3}\right) + \Sigma_3\left(\frac{N_3^3 + N_2^2N_3 + 2N_2N_3^2}{(N_2+N_3)^3}\right)$$

Separating the Upstream Covariance matrix

$$\Sigma_1 = \Sigma_2\left(\frac{N_2^3 + 2N_2^2N_3 + N_2N_3^2}{(N_2+N_3)^3}\right) + \Sigma_3\left(\frac{N_3^3 + N_2^2N_3 + 2N_2N_3^2}{(N_2+N_3)^3}\right)$$
$$= \Sigma_2\left(\frac{N_2}{N_2+N_3}\right) + \Sigma_3\left(\frac{N_3}{N_2+N_3}\right)$$

Therefore

$$N_1\Sigma_1 = N_2\Sigma_2 + N_3\Sigma_3$$

For a radially symmetric absorber (ignoring dissipative forces) the upstream distribution can be separated into the covariance matrix of the sample which makes it downstream and missing sample weighted by their sample sizes.

No absorber Covariance matrices

Full Upstream Sample (Σ_1)			
2320.82	182.66	56.49	-786.46
182.66	2427.89	675.30	163.87
56.49	675.30	829.47	-60.89
-786.46	163.87	-60.89	773.75

Upstream Sample that made it downstream(Σ_2)			
1518.43	85.98	90.92	-604.66
85.98	1477.41	518.44	73.55
90.92	518.44	711.03	-59.26
-604.66	73.55	-59.26	639.62

Missing Sample (Σ_3)			
3712.64	349.76	-3.34	-1102.37
349.76	4077.46	947.56	320.47
-3.34	947.56	1035.08	-63.73
-1102.37	320.47	-63.73	1006.53

Recombined Upstream Sample ($\Sigma_2\left(\frac{N_2}{N_1}\right)+\Sigma_3\left(\frac{N_3}{N_1}\right)$)			
2320.42	182.39	56.46	-786.58
182.39	2427.71	675.28	163.80
56.46	675.28	829.47	-60.90
-786.58	163.80	-60.90	773.72

The determinant of a matrix

- The determinant of a matrix can be separated into parts using:

$$|\Sigma_1| = \sum_{i=0}^{n} \Gamma_n^i \left| {}^{\Sigma_2}/_{\Sigma_3{}^i} \right| = |\Sigma_2| + |\Sigma_3| + \sum_{i=1}^{n-1} \Gamma_n^i \left| {}^{\Sigma_2}/_{\Sigma_3{}^i} \right|$$

Where Γ_n^i represents substituting all combinations of i^{th} lines from Σ_2 by the same lines in Σ_3 and taking the subsequent determinant of the new matrix

- For the symmetric case (LiH, LH2 and no absorber) the previous and above substitutions could be made to compare the upstream and downstream densities. Due to the asymmetry this cannot be done for the wedge and requires further derivation for the asymmetric case.

Potential next step

- The missing data downstream is inaccessible, however the upstream sample which makes it downstream can be compared to the downstream sample
- The transport, M, of a covariance matrix from upstream to downstream can be given by:

$$\Sigma_{down} = \langle X_{down}\tilde{X}_{down}\rangle = \langle MX_{up}\tilde{M}\tilde{X}_{up}\rangle = M\langle X_{up}\tilde{X}_{up}\rangle\tilde{M} = M\Sigma_{up}\tilde{M}$$

- The determinant is given by:

$$|\Sigma_{down}| = |M\Sigma_{up}\tilde{M}| = |M|^2|\Sigma_{up}| = |\Sigma_{up}|$$

- The question is can a matrix M be found such that the Matrix is not dependent on its input parameters, i.e. no position or momentum dependence
- We know there are non-linear effects, therefore a linear transfer map won't work, i.e. will require a higher order transfer map
- Recon values are based on a extended linear Kalman filter – Leads to inherent biases. These biases would be fed as inputs into the higher order transfer map
- => Need to understand Recon biases

Higher order transfer map

- A normal linear transfer map looks like:

$$\boldsymbol{x}_d = M\boldsymbol{x}_u$$

$$\begin{pmatrix} x_d \\ x'_d \\ y_d \\ y'_d \\ z'_d \end{pmatrix} = \begin{pmatrix} M_{00} & M_{01} & M_{02} & M_{03} & M_{04} \\ M_{10} & M_{11} & M_{12} & M_{13} & M_{14} \\ M_{20} & M_{21} & M_{22} & M_{23} & M_{24} \\ M_{30} & M_{31} & M_{32} & M_{33} & M_{34} \\ M_{40} & M_{41} & M_{42} & M_{43} & M_{44} \end{pmatrix} \begin{pmatrix} x_u \\ x'_u \\ y_u \\ y'_u \\ z'_u \end{pmatrix} + \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix}$$

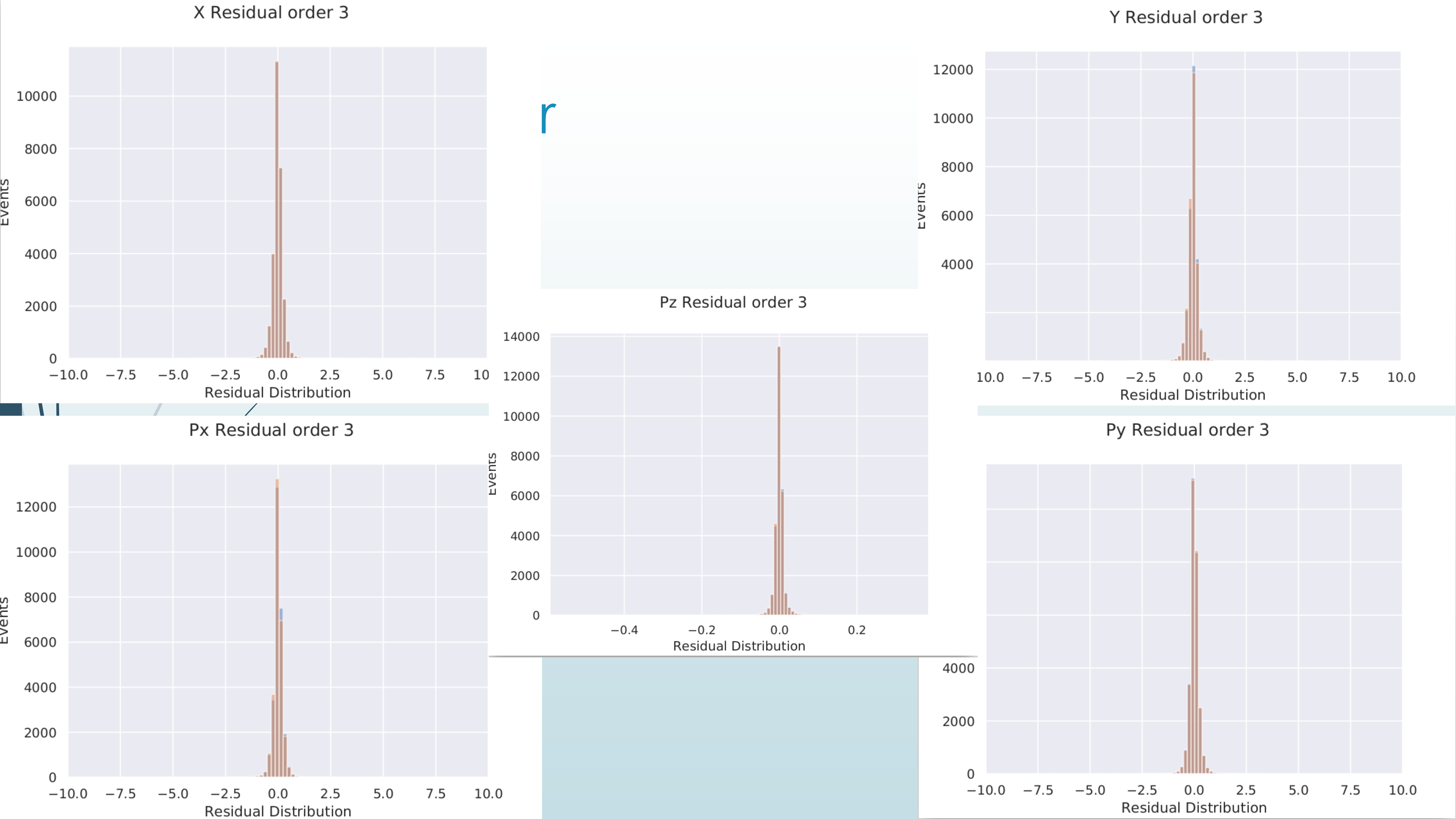
- The a terms are to account for misalignments

- For higher orders, the matrix is expanded to include the likes of x^3, $x^2x', xx'y$, etc.

Higher Order Transfer Map

- Example scenario - Transfer map was calculated between TKU S2 and TKU S1
- The Residuals for a run were then calculated between the measurement at TKU S1 and the track propagated to TKU S1 from TKU S2 by the transfer map
- The Residuals decreased going to higher orders but didn't improve beyond 3rd order.

- A separate second run (independent sample) was then looked at
- The same scenario was then repeated however using a 3rd order transfer map from the first run
- The residuals are shown on next slide


X Residual order 3
Events
Residual Distribution
Y Residual order 3
Events
Residual Distribution
Pz Residual order 3
Events
Residual Distribution
Px Residual order 3
Events
Residual Distribution
Py Residual order 3
Residual Distribution

Higher Order Transfer Map

- Residuals for independent run look encouraging, variance seems to be at level of what the fibre width can measure
- In future, need to apply transfer matrix approach between TKU and TKD
- Need to also check that transfer matrix isn't affected by different Particle Distribution functions

- TKU S2 to TKU S1 will be highly correlated with Kalman filter (i.e. particle pulls). Expect the transfer map to then produce small residuals. It also bakes in the Recon bias into the transfer map. Need to check transfer map approach in MC Truth as well.
- Need to check how Recon bias affects the transfer map.
- Want to understand Recon bias

What would transfer map mean?

- Full Upstream distribution is unbiased
- Downstream distribution has survival bias. This effects the Covariance Matrix of the remaining sample and thus the calculated emittance, amplitude and density
- Want to calculate the downstream emittance, density and amplitude as if there were no transmission losses (no bias then).
- Use the transfer map to transport the full Upstream sample Covariance matrix. Calculate the downstream emittance, amplitude and density using the transported covariance matrix.

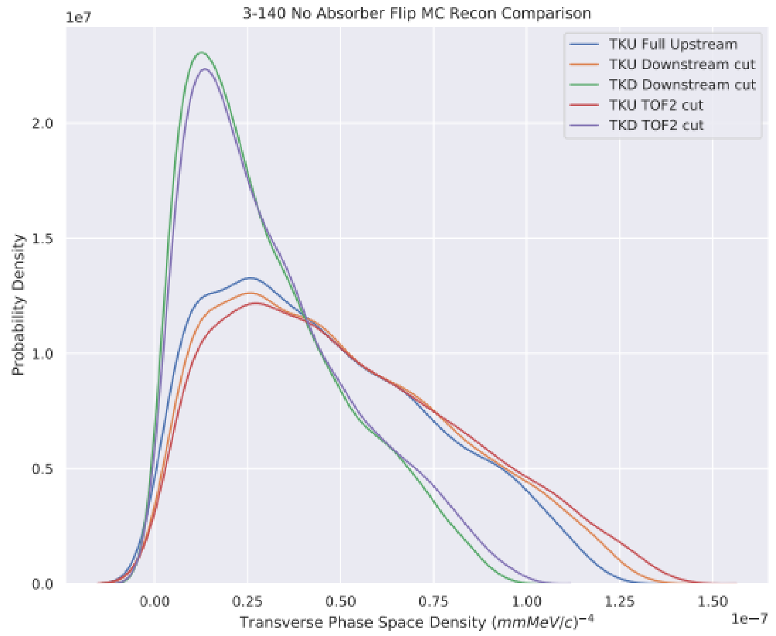
$$\Sigma_{down} = \langle X_{down}\tilde{X}_{down}\rangle = \langle MX_{up}\tilde{M}\tilde{X}_{up}\rangle = M\langle X_{up}\tilde{X}_{up}\rangle\tilde{M} = M\Sigma_{up}\tilde{M}$$

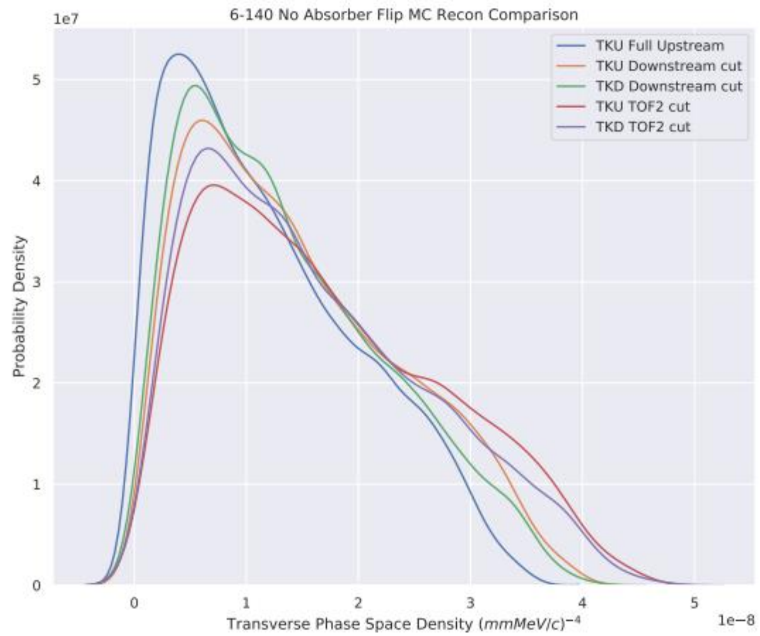
$$|\Sigma_{down}| = \left|M\Sigma_{up}\tilde{M}\right| = |M|^2\left|\Sigma_{up}\right| = \left|\Sigma_{up}\right|$$

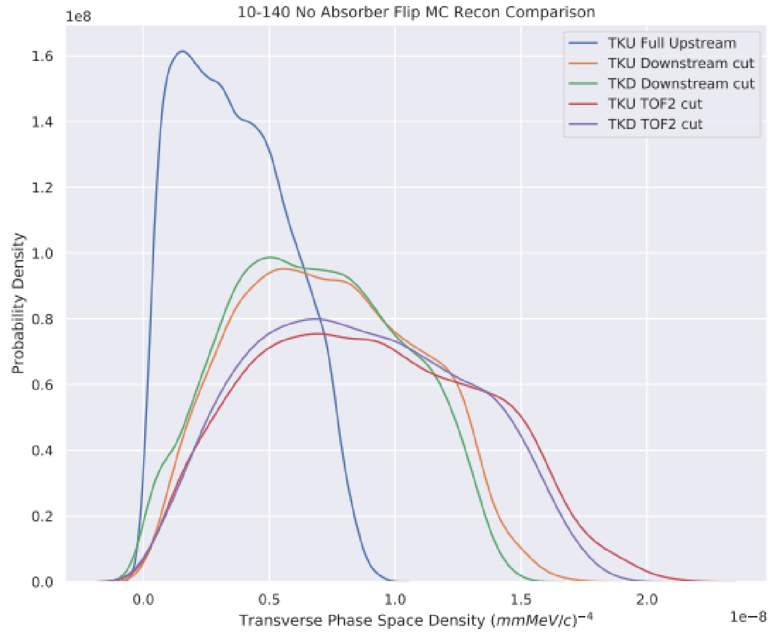
- Can now compare the remaining downstream emittance, amplitude and density with the full upstream sample, having removed bias due to selection

What do the transverse density and Amplitude plots look like

- Density depends on selection can only compare like with like Upstream and Downstream (i.e. same cut)
- It also depends if one is looking at MC Truth or MC Recon as there are some Recon effects
- 3-140 case is difficult to explain in density, amplitude and emittance. Perhaps transverse momentum is too low with TKU measurement residual too large

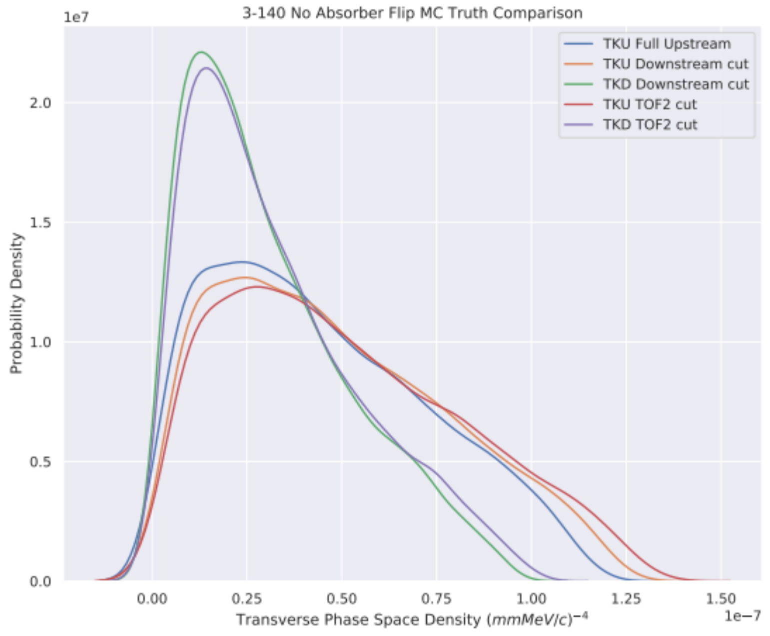


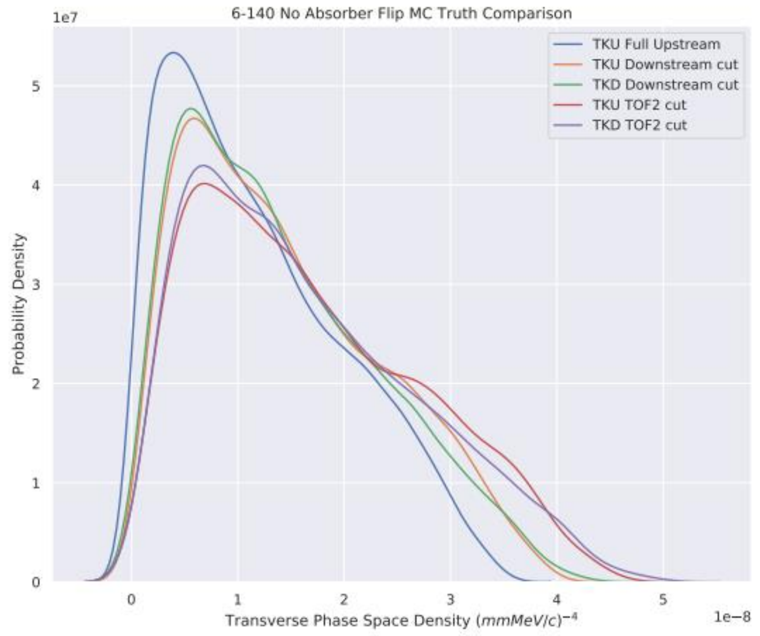


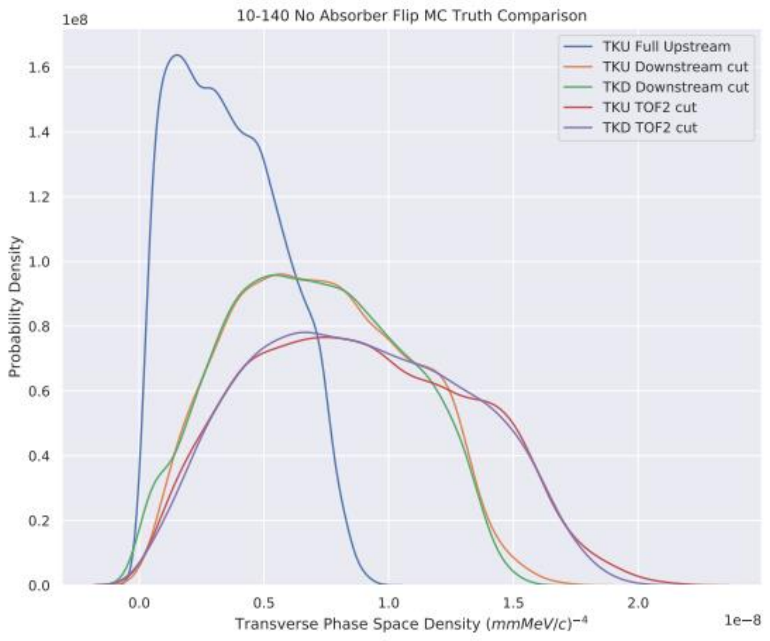


What do the transverse density and Amplitude plots look like

- In MC Truth the change in Phase Space density looks better between upstream and downstream, i.e. less growth
- Can be instructive to look at growth between stations

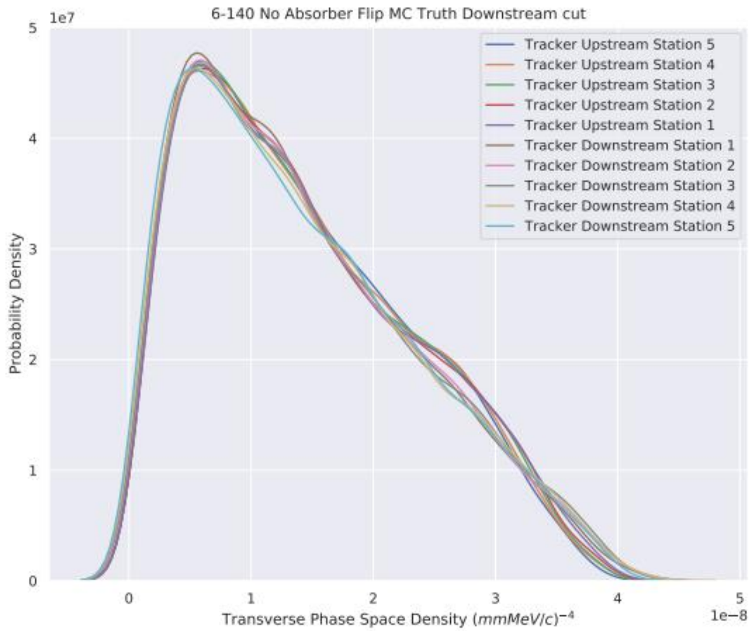


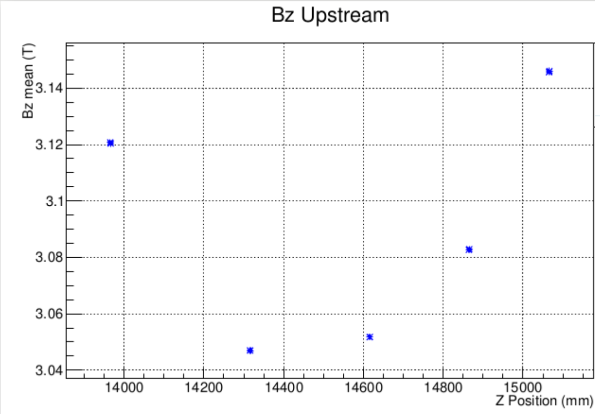


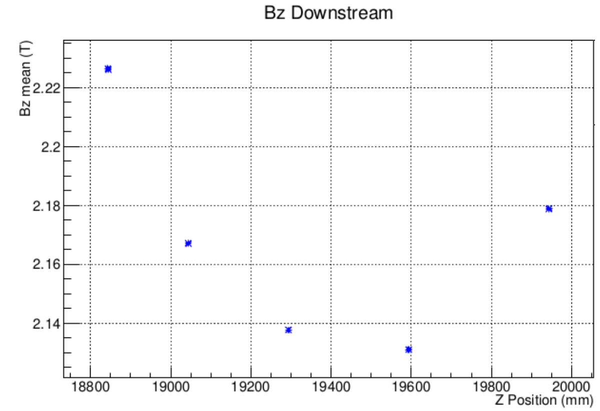


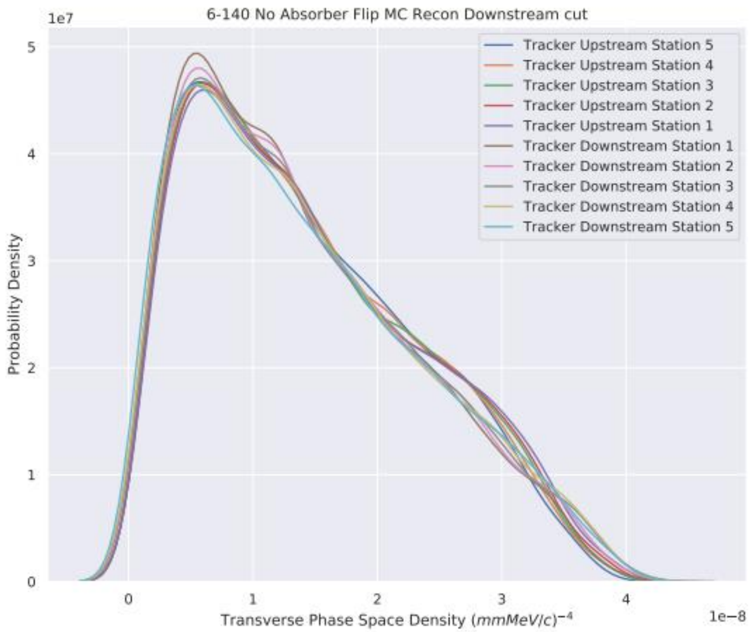
What do the transverse density and Amplitude plots look like

- Why does the Recon density move more?
- One hint is the Transverse Momentum Reconstruction
- It varies between stations and between Trackers
- The magnetic field is not uniform in the trackers. The “U” shaped residuals in the trackers are similar to the shape of the Bz field (more on that later)



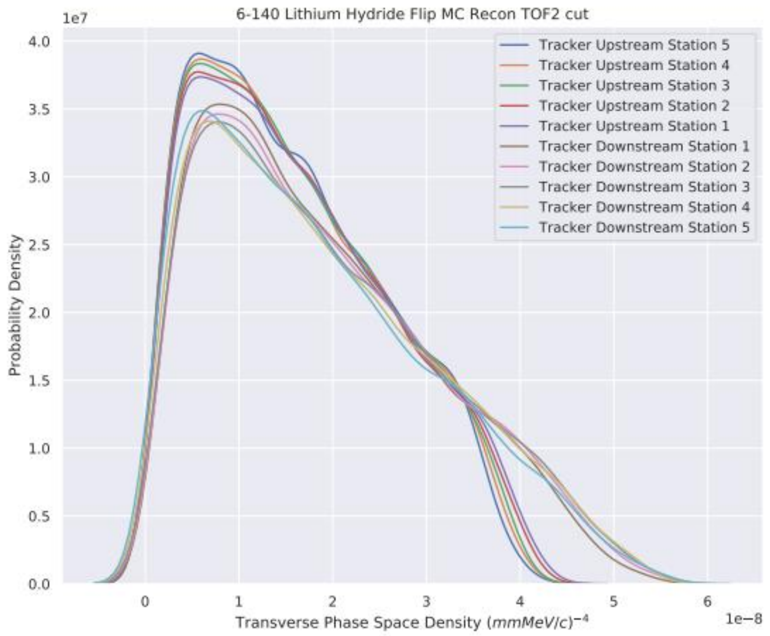


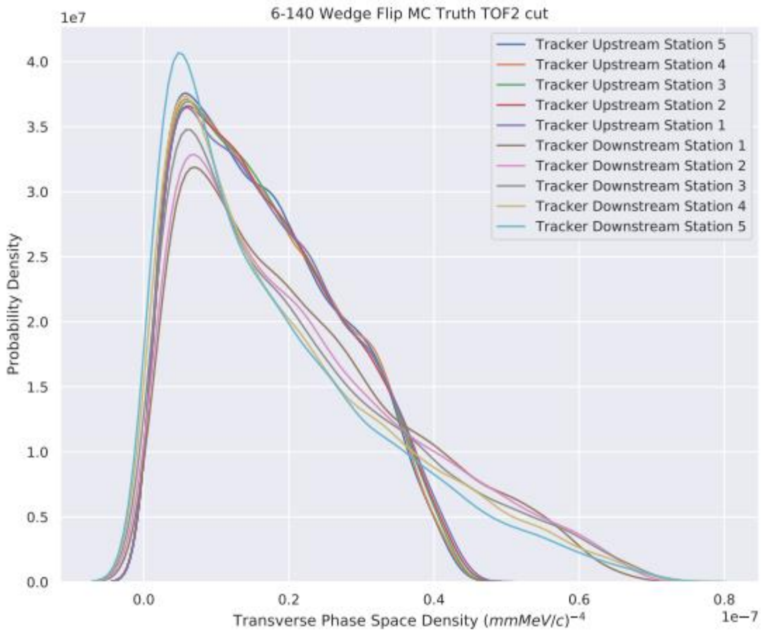




What do the transverse density and Amplitude plots look like

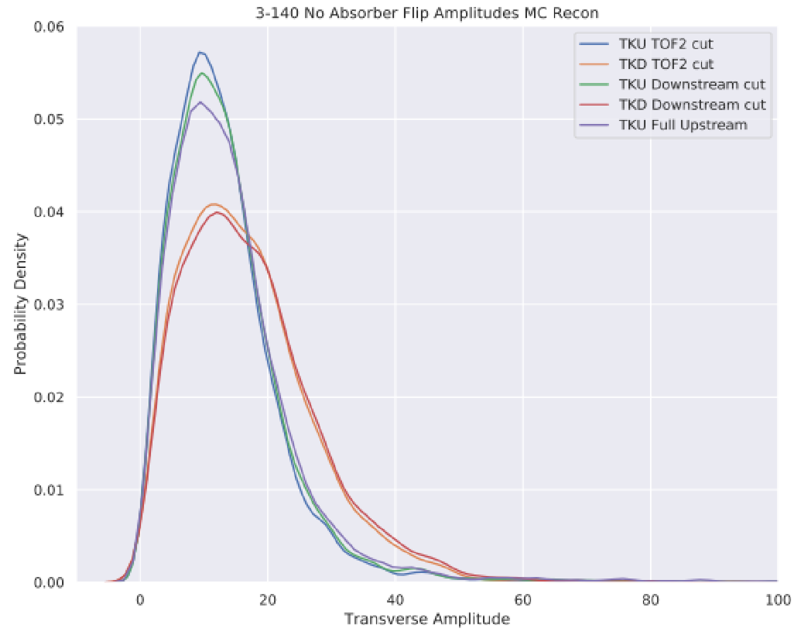
- When an absorber is present, the downstream densities vary more between planes due to both Energy straggling and/or dispersion
- The approximation of a "momentum bite" is no longer valid
- Particles with different Pz will have different phase advances
- The non-homogenous magnetic field also blurs the line where separation of transverse and longitudinal components remains valid

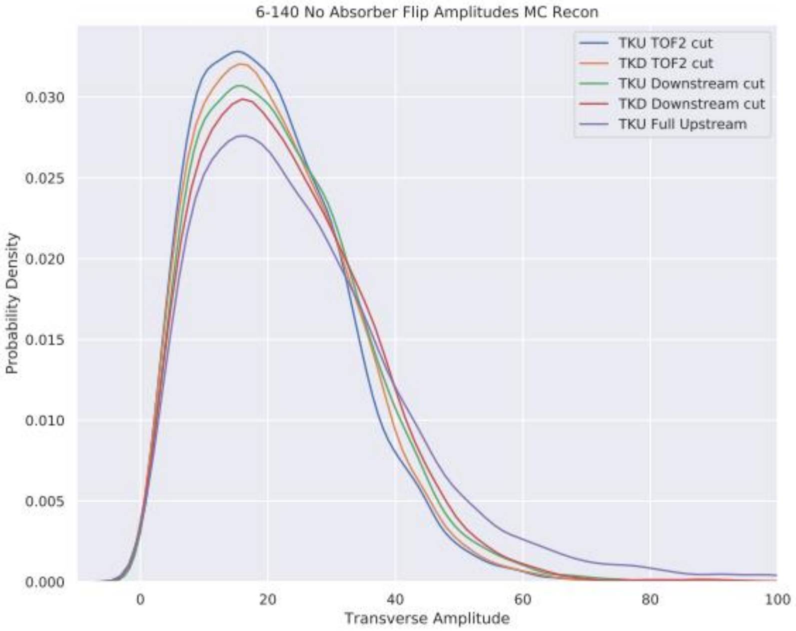


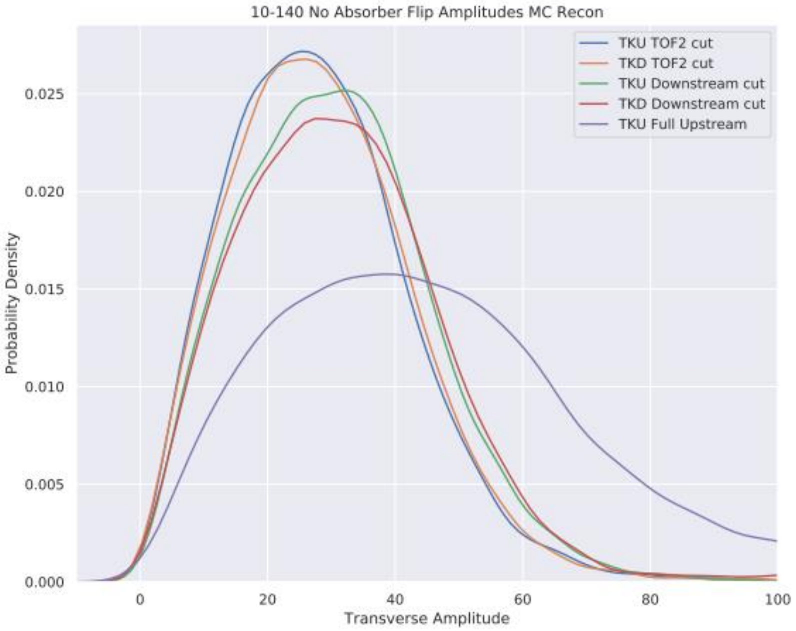


What do the transverse density and Amplitude plots look like

- Amplitude plots look similar to density plots
- Same difference between MC Recon and MC Truth is seen

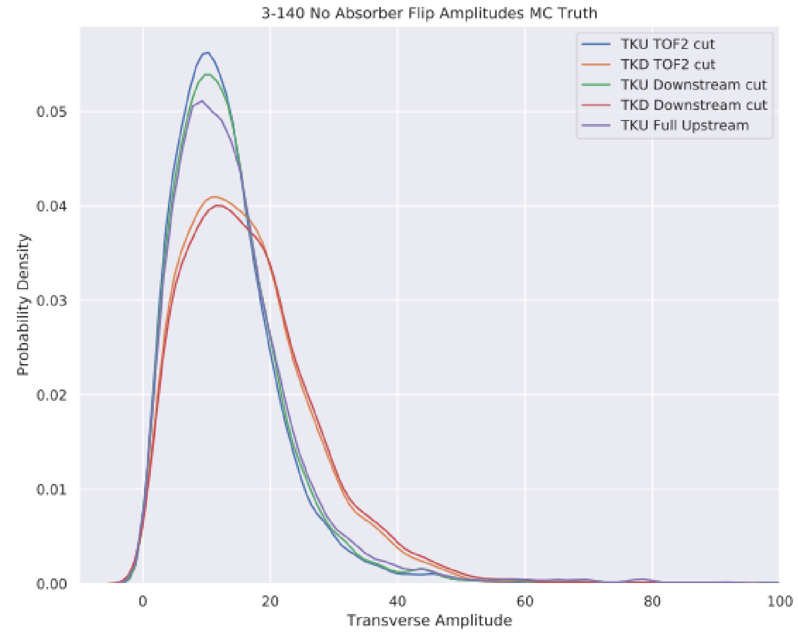


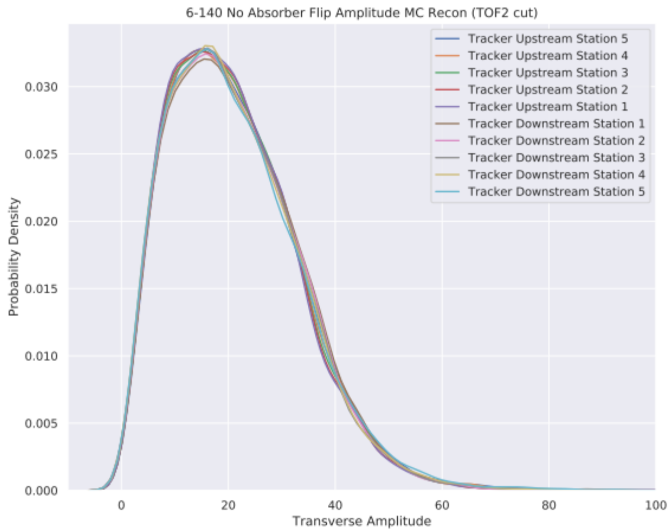


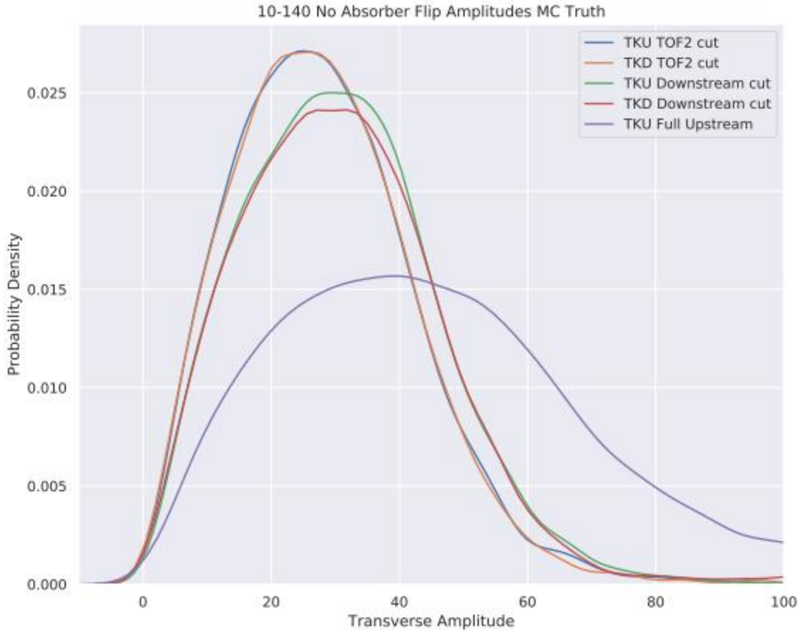


What do the transverse density and Amplitude plots look like

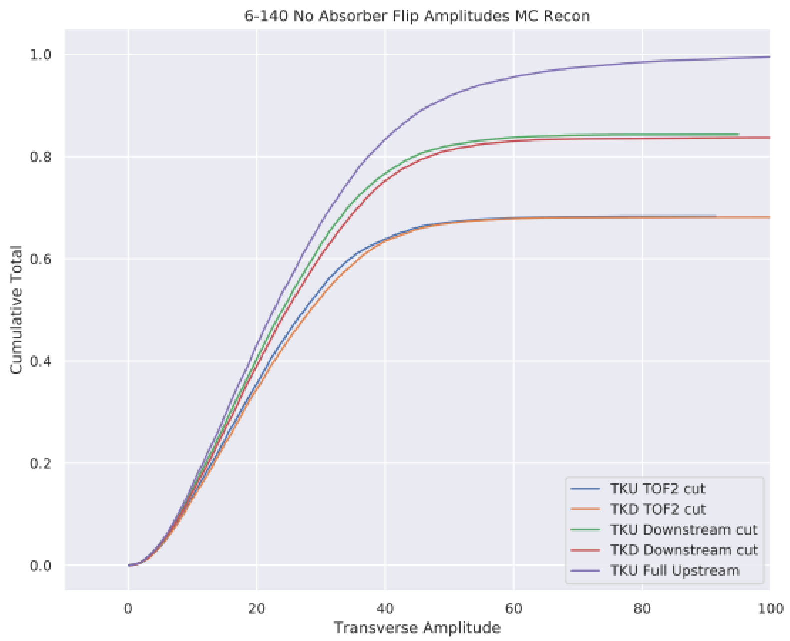
- Amplitude plots look similar to density plots
- Same difference between MC Recon and MC Truth is seen
- Evolution through tracker is also similar
- Cumulative plots are more interesting

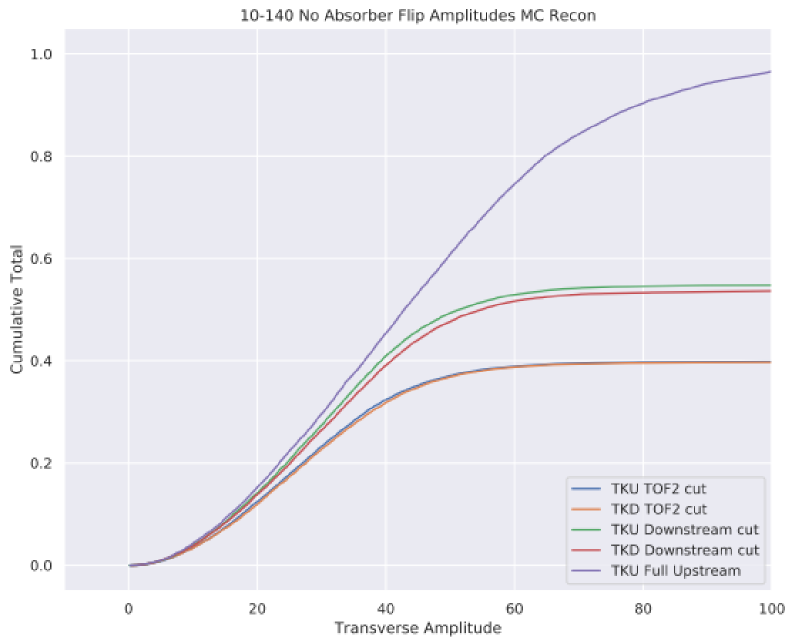


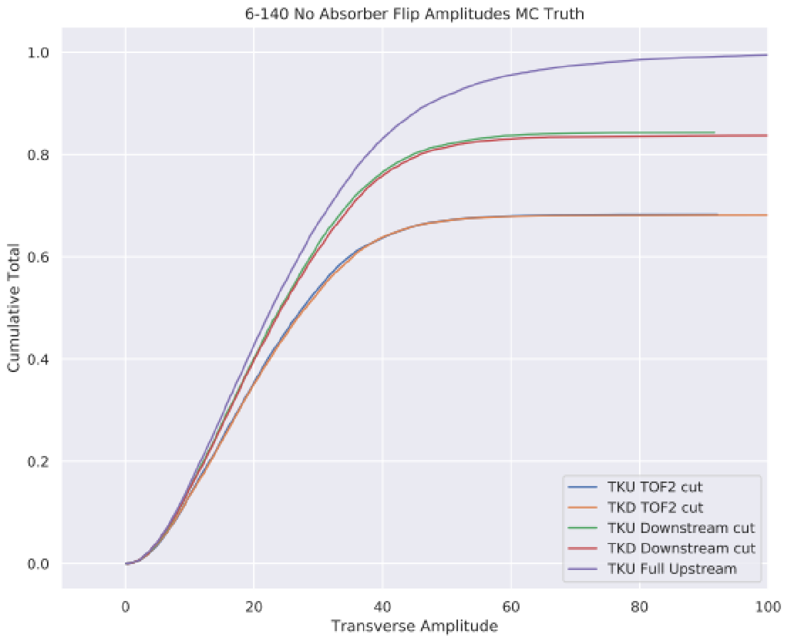




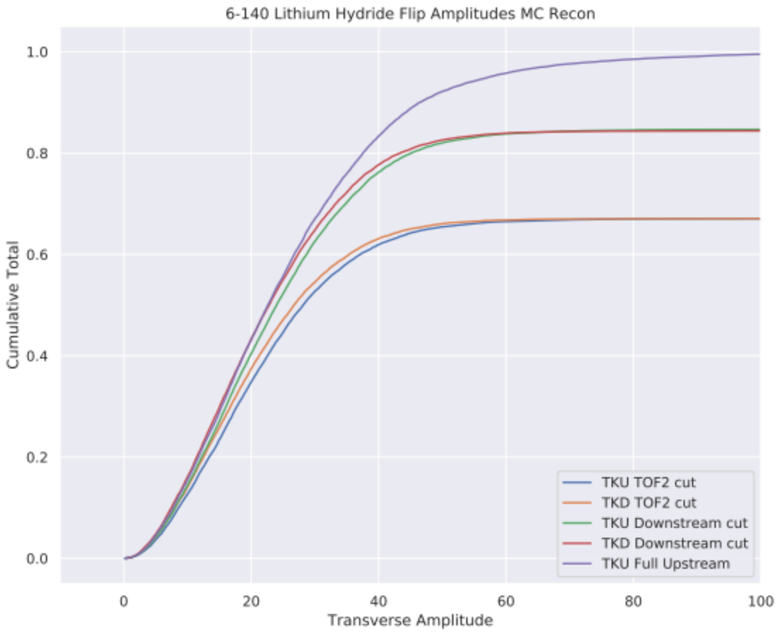
- Truth shows little change between TKU and TKD
- Recon shows some separation, as if some heating has taken place

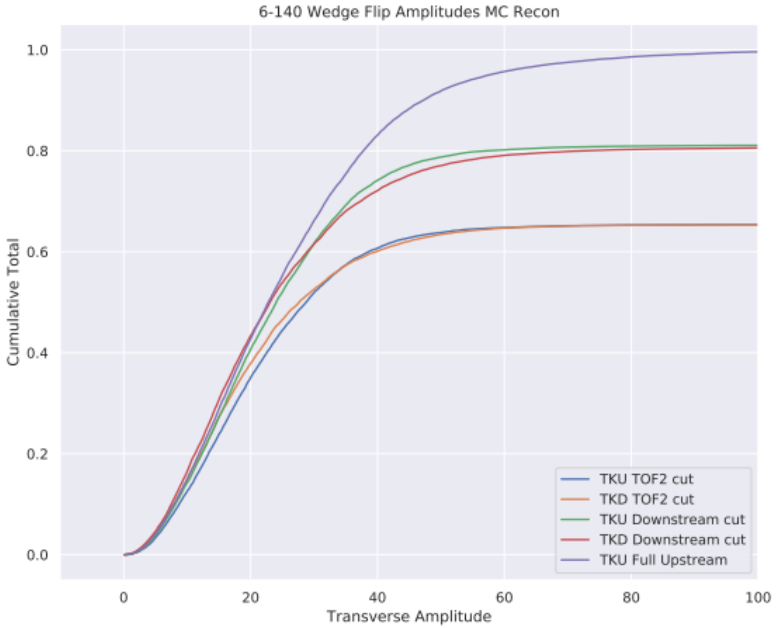


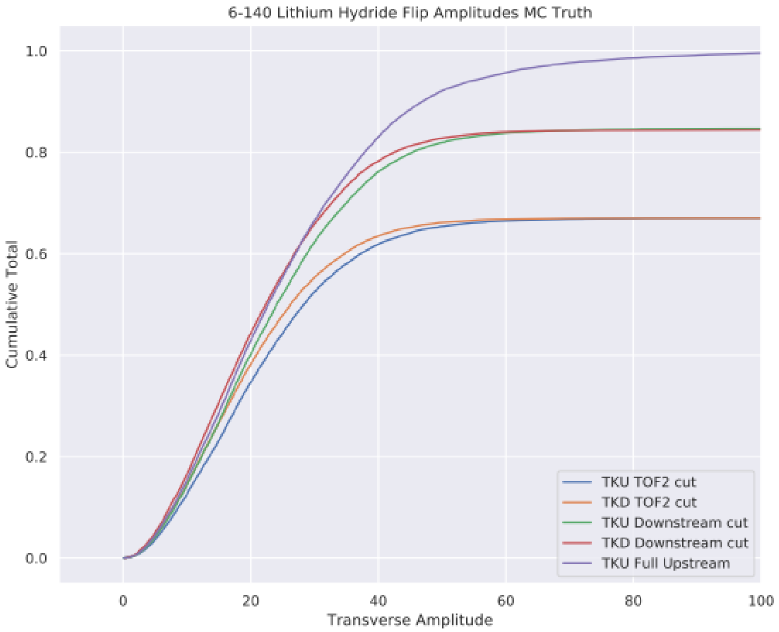


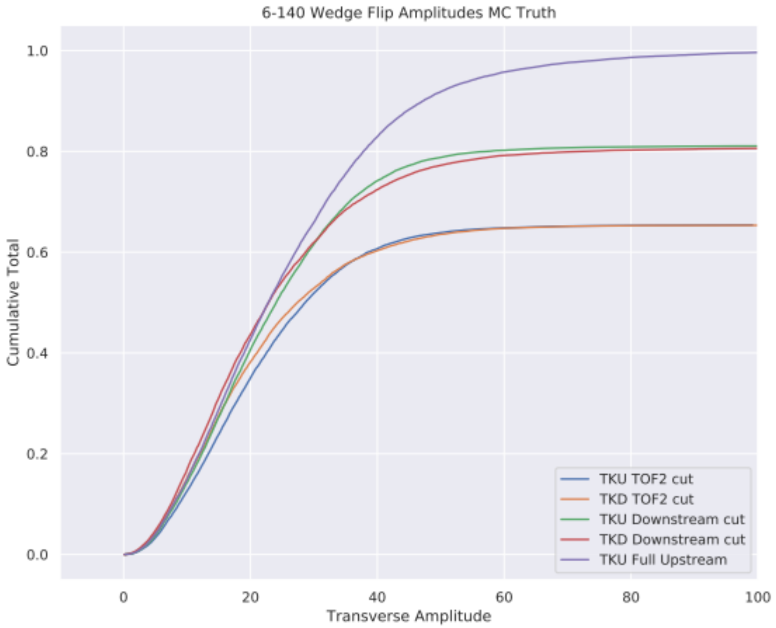


- LiH shows cooling for both Truth and Recon, however Recon slightly underestimates cooling
- Compared to the parent distribution no noticeable effect is seen
- Ratio plots will help, but caution due to the lower number of events
- Wedge case is more interesting with a longitudinal component
- -> Introduce a time Component









Liouville's theorem

- The complete state of a particle can be given by its coordinates and moments
- A particle beam can be described by the distribution of the particles in the beam also known as the phase space density $\rho(x, y, z, p_x, p_y, p_z)$.
- Liouville's theorem states that the density of particles in phase space is a constant i.e. ${}^{d\rho}/_{dt} = 0$ (providing there are no dissipative forces)
- The number of particles in a phase-space volume is then given by:

$$N = \int \rho(x, y, z, p_x, p_y, p_z) dx dy dz dp_x dp_y dp_z = \int \rho dV$$

- The phase-space density is directly related to the phase space volume
- We know x, y, z, p_x, p_y, p_z at the longitudinal z-planes. The particles in the beam will have different arrival times.
- The plan will be to create a spread in z-space such that all particles are measured at the same time and thus we retain the constant volume of the beam

Introducing a Time Coordinate

- Choose start time to be at TOF1 plane and end time to be at the TOF2 plane
- Calculate time between 2 stations for each particle based on the distance between those 2 stations and the measured pz and Energy of each particle at each station, c is speed of light:

$$\Delta\, t_{stations} = \frac{\Delta z_{stations} E}{c \times p_z}$$

- Upstream time at each station based on adding the time between each station/TOF1 to the TOF1 start time (i.e. 0 sec)
- Downstream time at each station based on subtracting the time between stations/TOF2 from the TOF2 time
- It means Downstream tracks have the additional TOF2 cut
- Each particle at each station now has a time coordinate, the time it takes from travelling from TOF1
- Particle follows helical path, so some error likely

Calculating Longitudinal Coordinate

- The mean arrival time is found at each station. The delta between the mean arrival time and the actual arrival time is used to calculate a z-coordinate for that particle.
- The z-coordinate is based on a Δz, the z distance the particle would travel in the time delta between mean arrival time and the actual arrival time

- Calculating Δz in terms of the time coordinate:
- $v = \beta c = \frac{\Delta z}{\Delta t} = c\sqrt{1 - \frac{1}{\gamma^2}}$
- $\Delta z = \mathrm{c}\Delta\mathrm{t}\sqrt{1 - \frac{1}{\gamma^2}} = c(t_{measured})\sqrt{1 - \frac{1}{\gamma_{measured}{}^2}} - c(t_{mean})\sqrt{1 - \frac{1}{\gamma_{mean}{}^2}}$

- An assumption of a small Pz momentum bite is used, such that no corrections are made to the transverse components i.e. the time bite of the beam is so small that the difference in rotation of the particles in the beam through field gradients are negligible. This won't be the case for larger Pz distributions and still needs to be looked into.

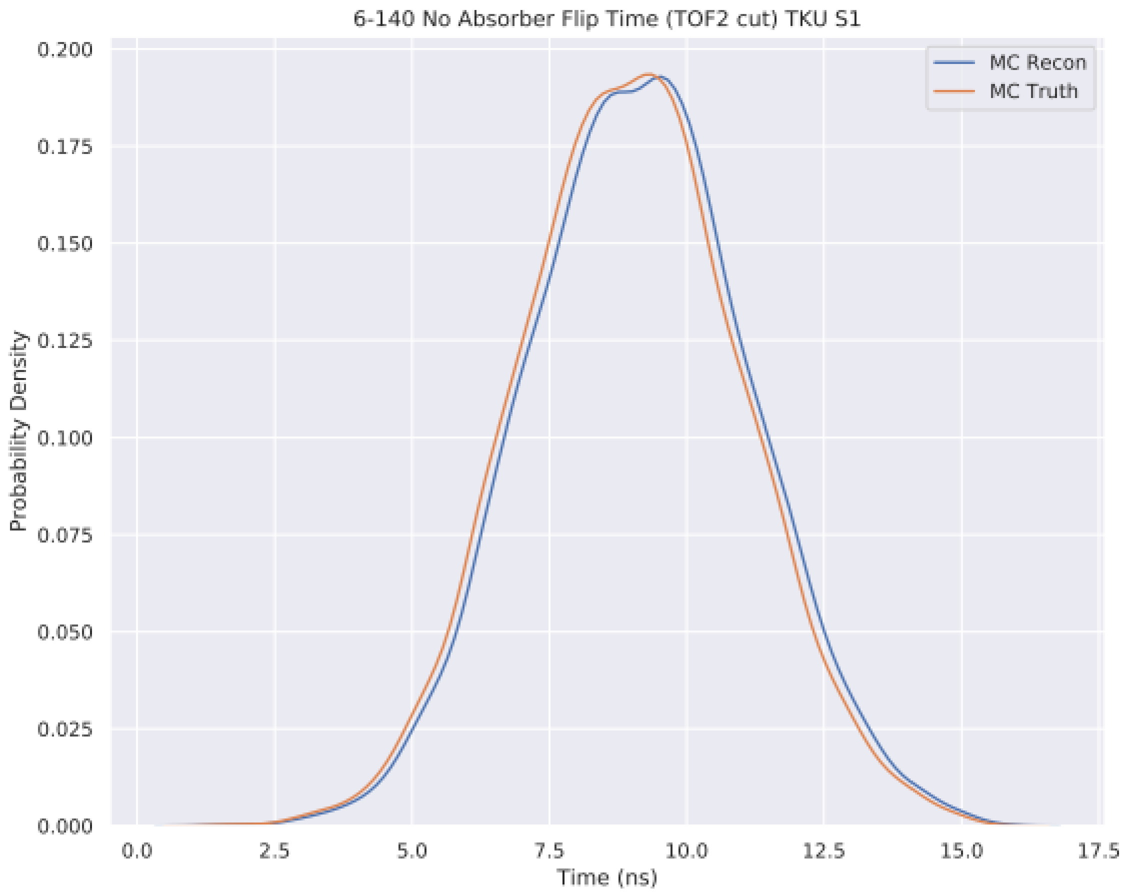
Introduce Time off-set

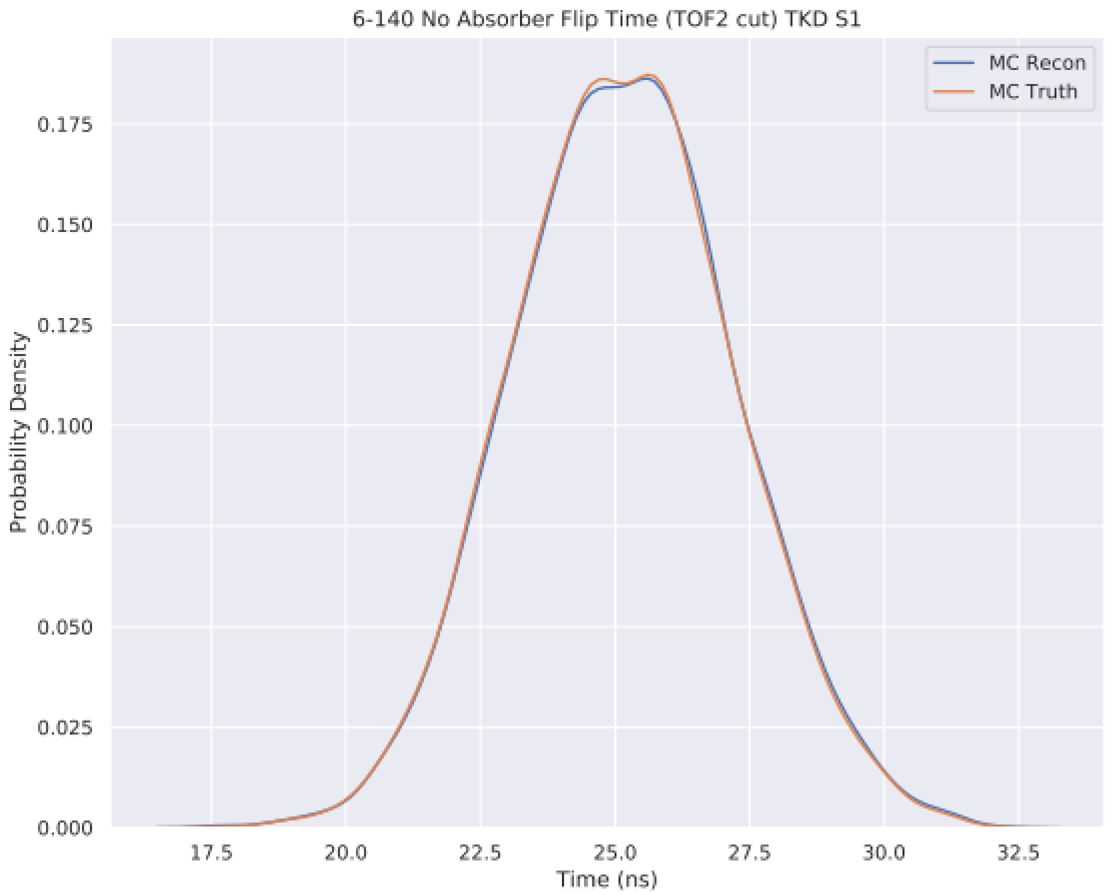
- All Particles leaving TOF0 at the same time is not a realistic beam.
- The starting beam has been smeared by a Gaussian Time Distribution. Each particle at the subsequent stations is offset by the same amount
- The starting beam has a standard deviation of 2 nanoseconds, to be on a similar scale with a beam exciting a 200 MHz RF cavity

- Note:
- Energy Loss through the diffuser still needs to be added. Will create only a tiny spread in the beam, but provide a correction for the mean offset.
- Still not sure on Energy Loss model – Want to use something similar as Model used for tracker stations.
- Interestingly the TOF/Tracker combined uses Globals to account for the Energy loss in the diffuser.
- Preference is for a full Reconstruction, rather than a Hybrid which uses some simulation

Time Coordinates at Reference Planes

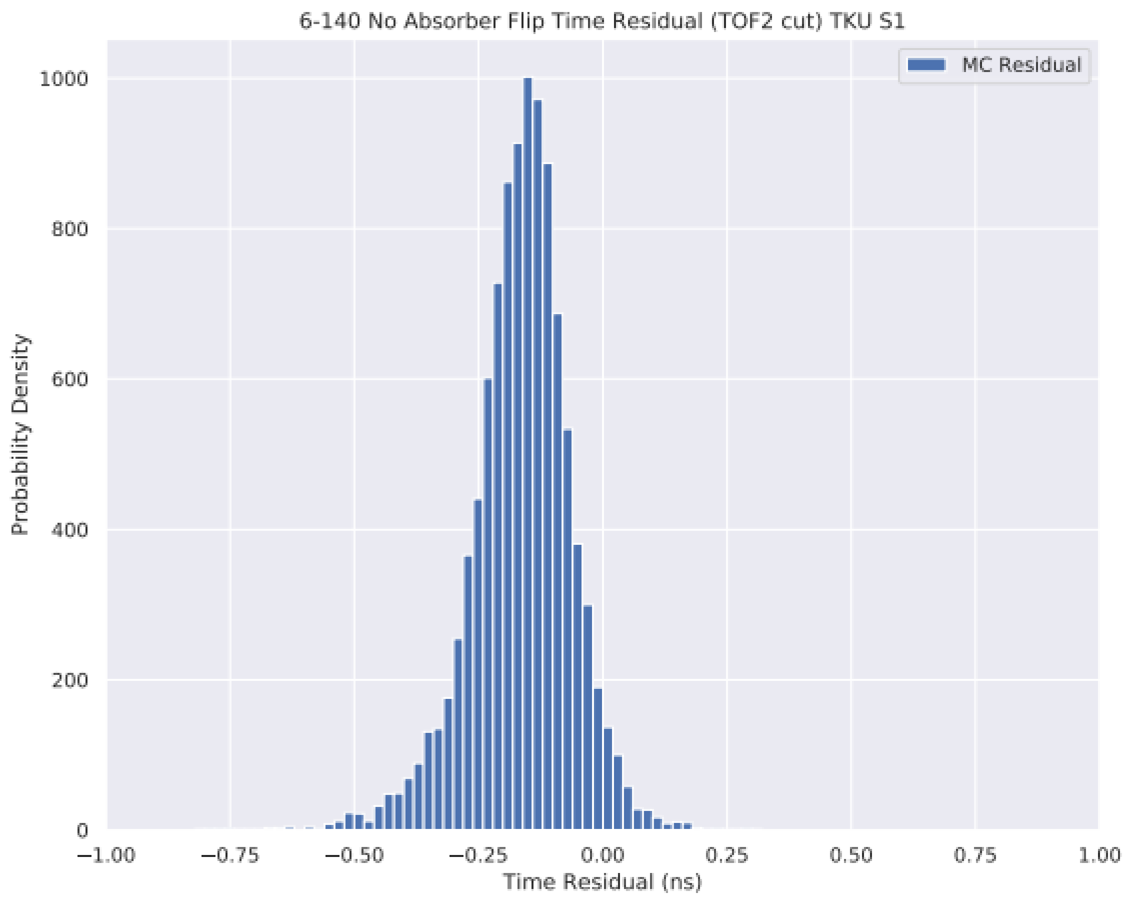
- Truth and Recon distributions are similar, slight offset in TKU as Energy loss for diffuser still needs to be incorporated

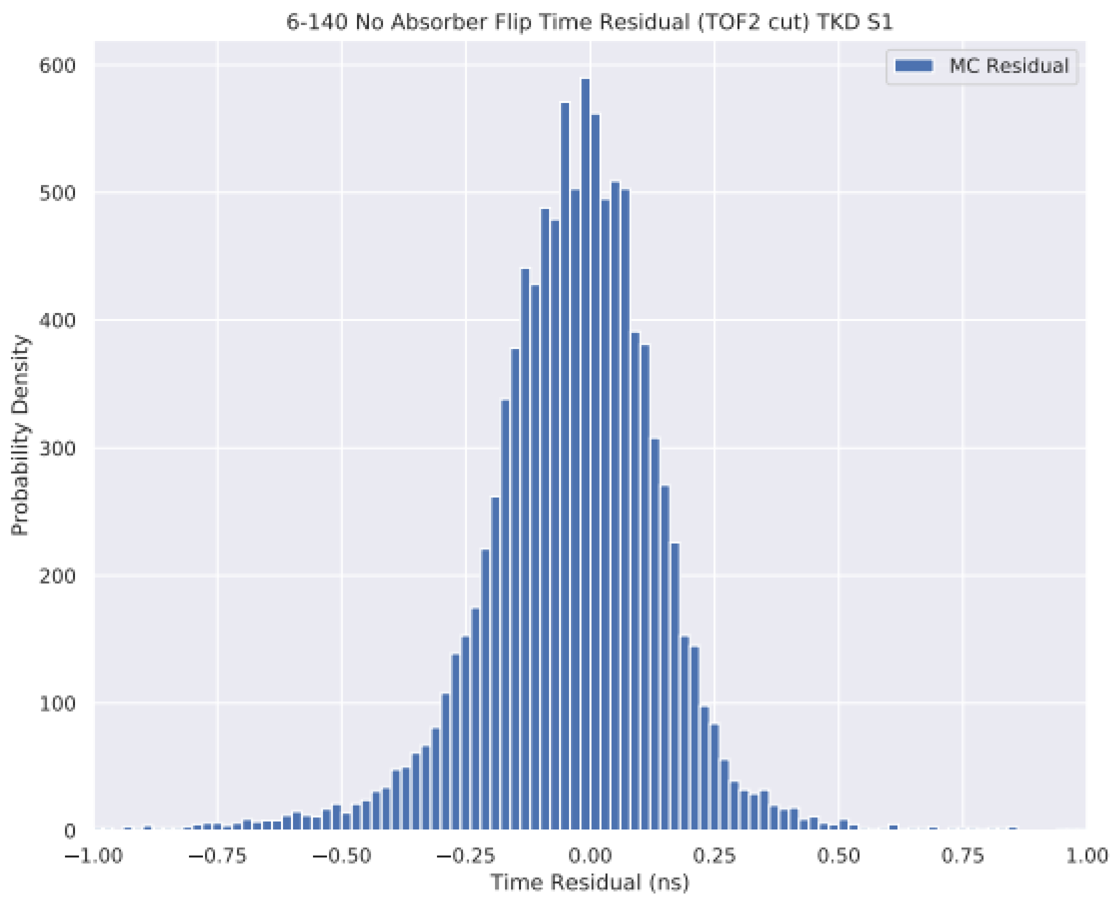




Time Coordinate Residuals at Reference Planes

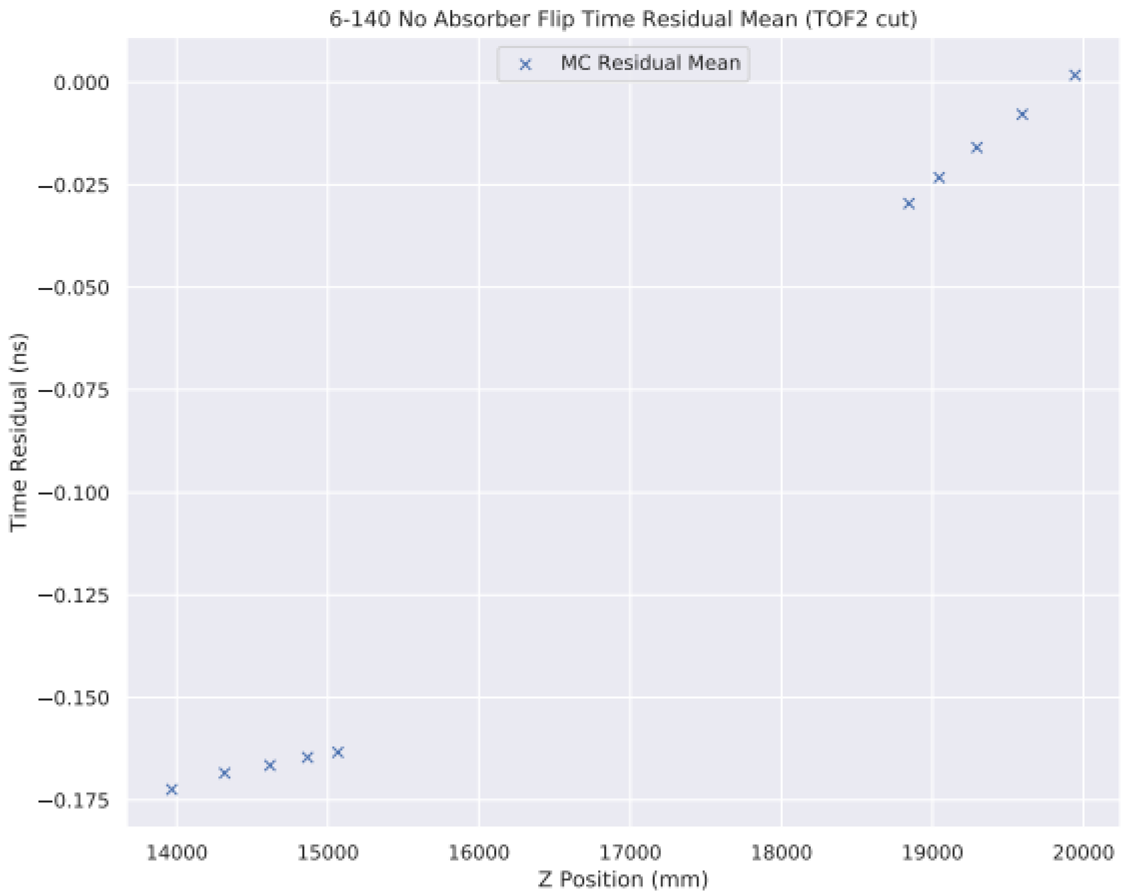
- RMS is slightly larger for downstream distribution due to measurement error as TOF1 is taken to have zero measurement error as the input distribution

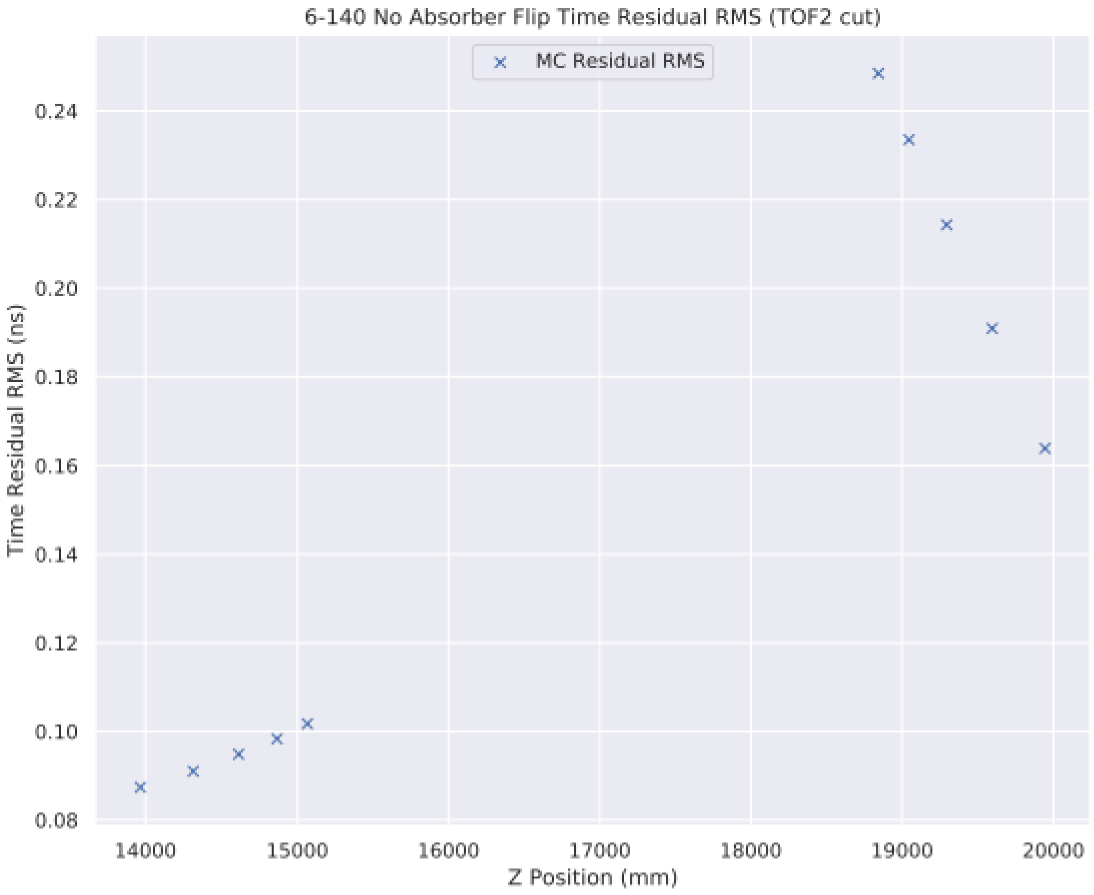




Time Residuals – Mean and RMS

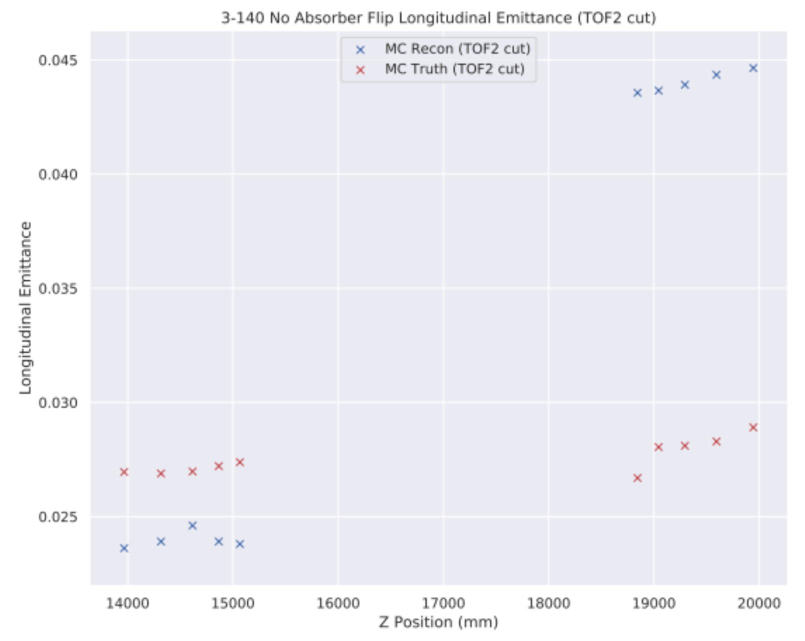
- TKU shows little variance while TKD shows the time is on average of by 25 ps
- The RMS in both trackers grows the further away from TOF the particle is

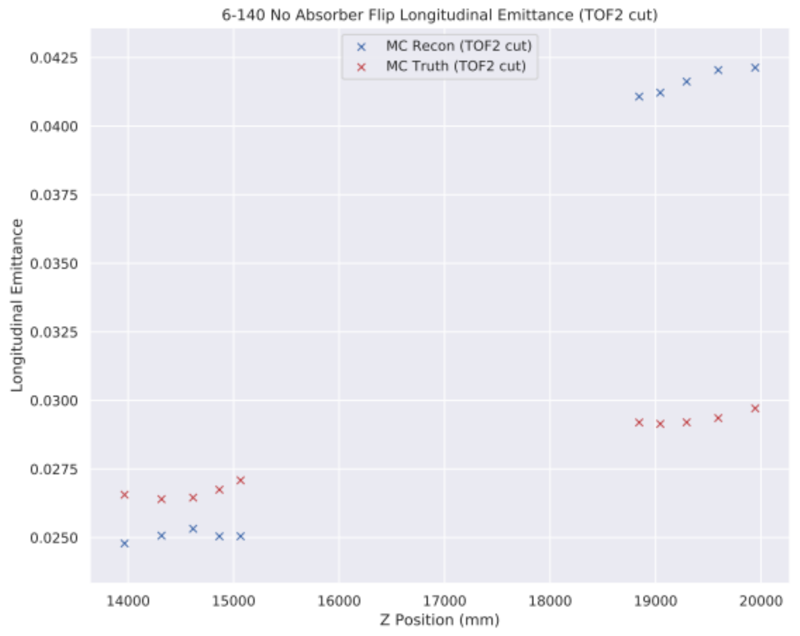


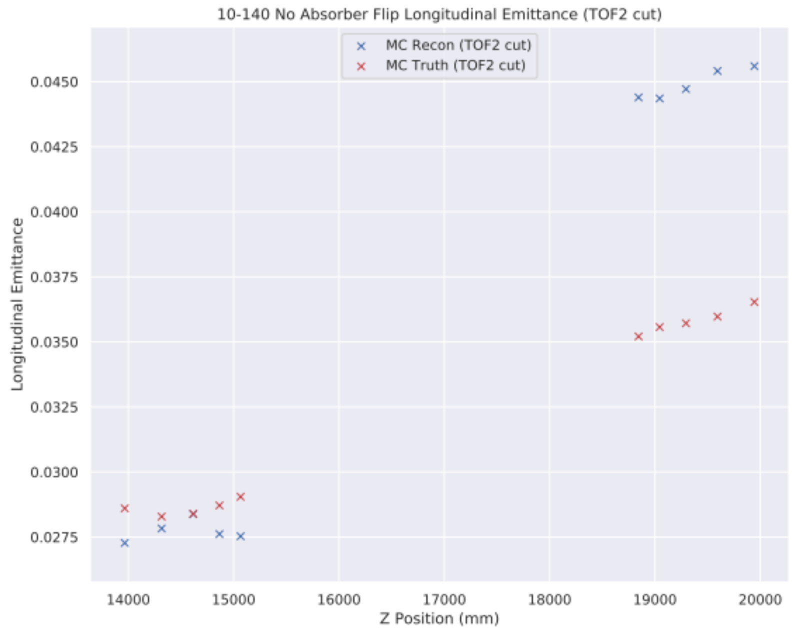


Longitudinal Emittance, Amplitude and Density

- The longitudinal Reconstruction looks terrible
- Its dependent on 2 components: z and pz
- Will redo the Time coordinate Reconstruction but will use the virtual Pz coordinate instead of the Recon Pz

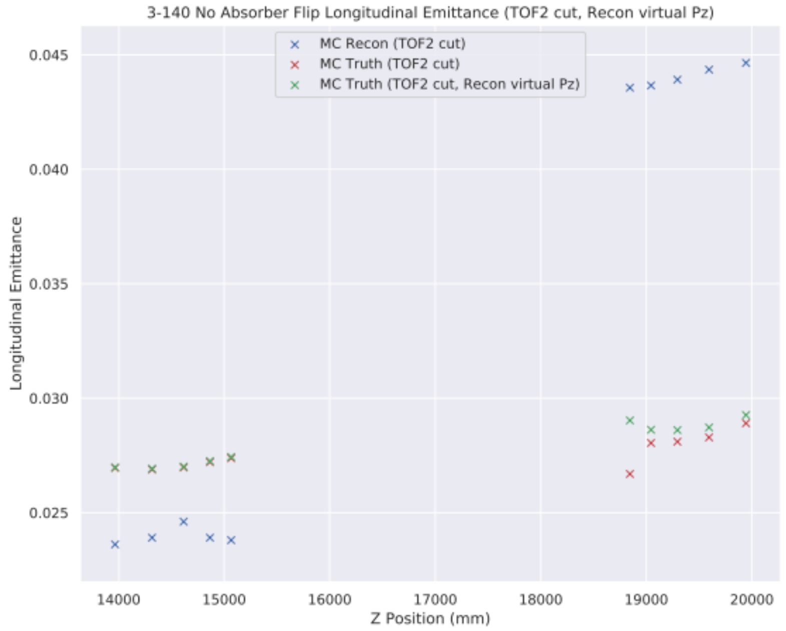


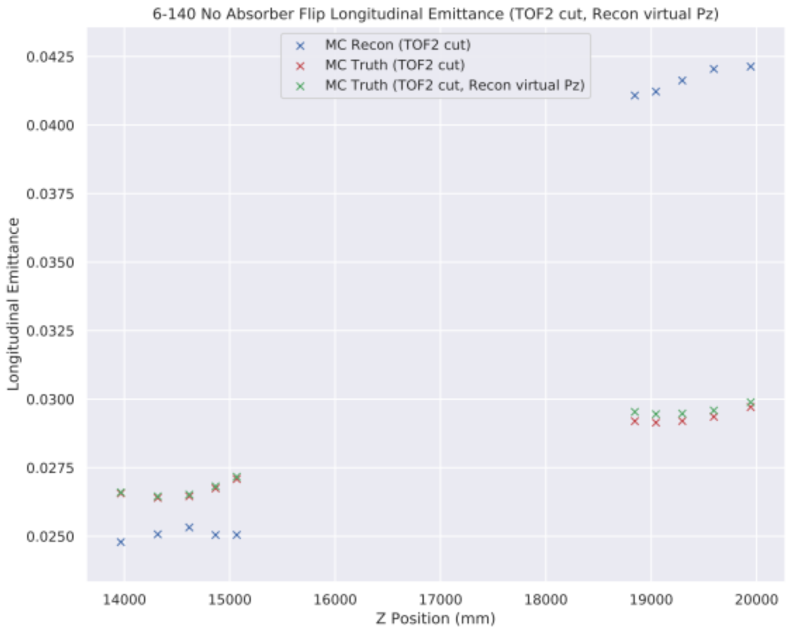




Longitudinal Emittance, Amplitude and Density

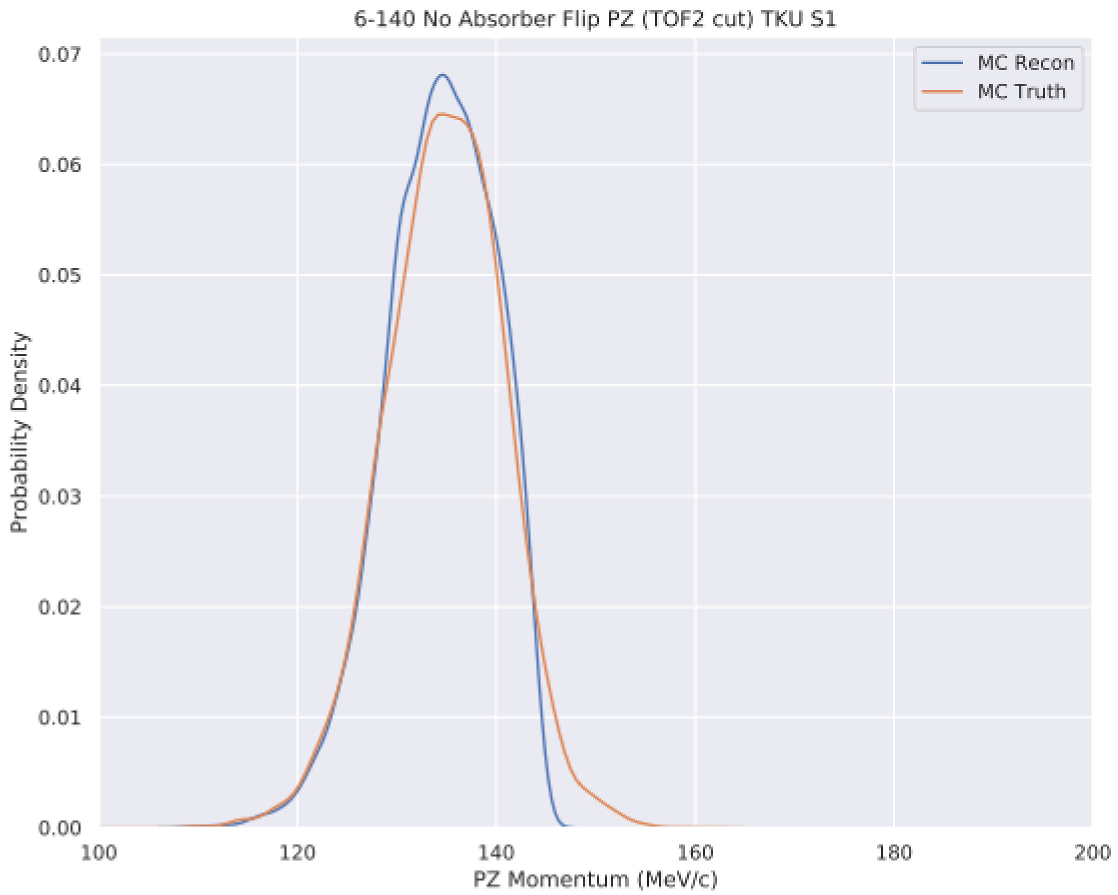
- Recon and Truth look much better in that case
- The problem in the longitudinal Reconstruction must be Pz then
- This can be seen in the following slides

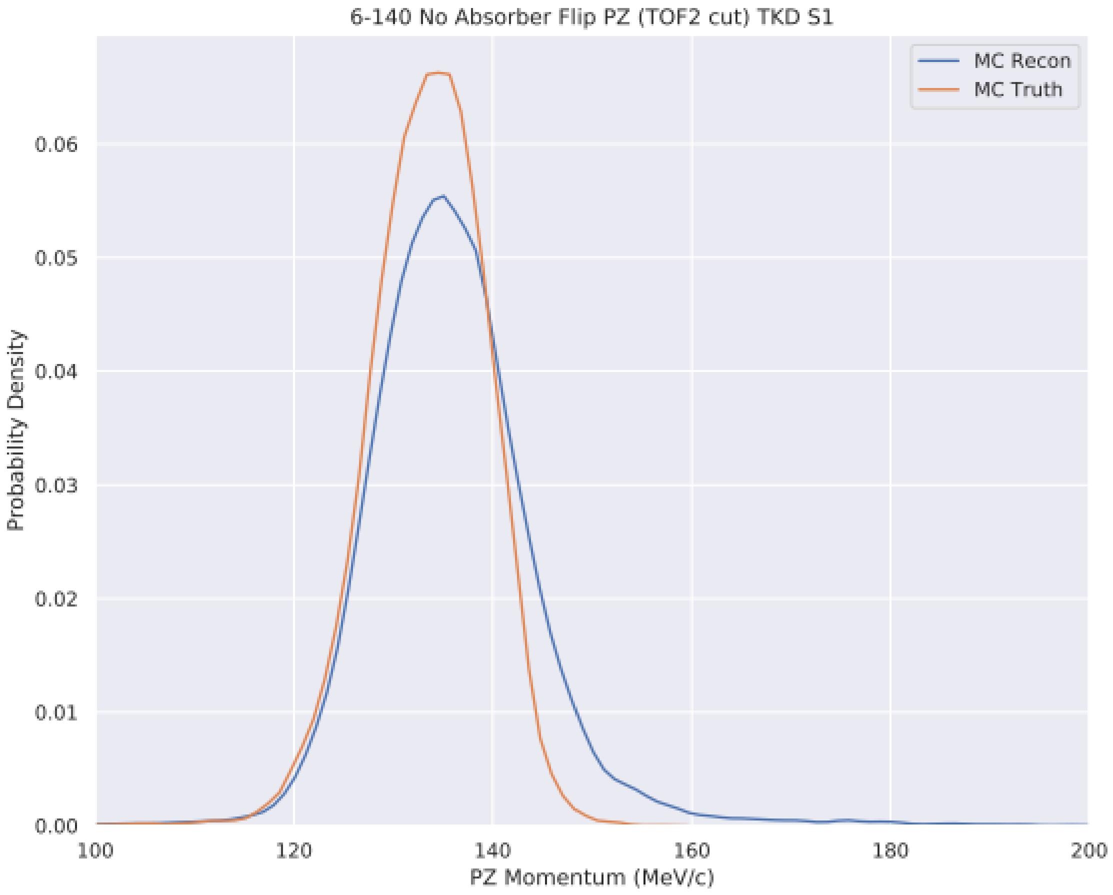




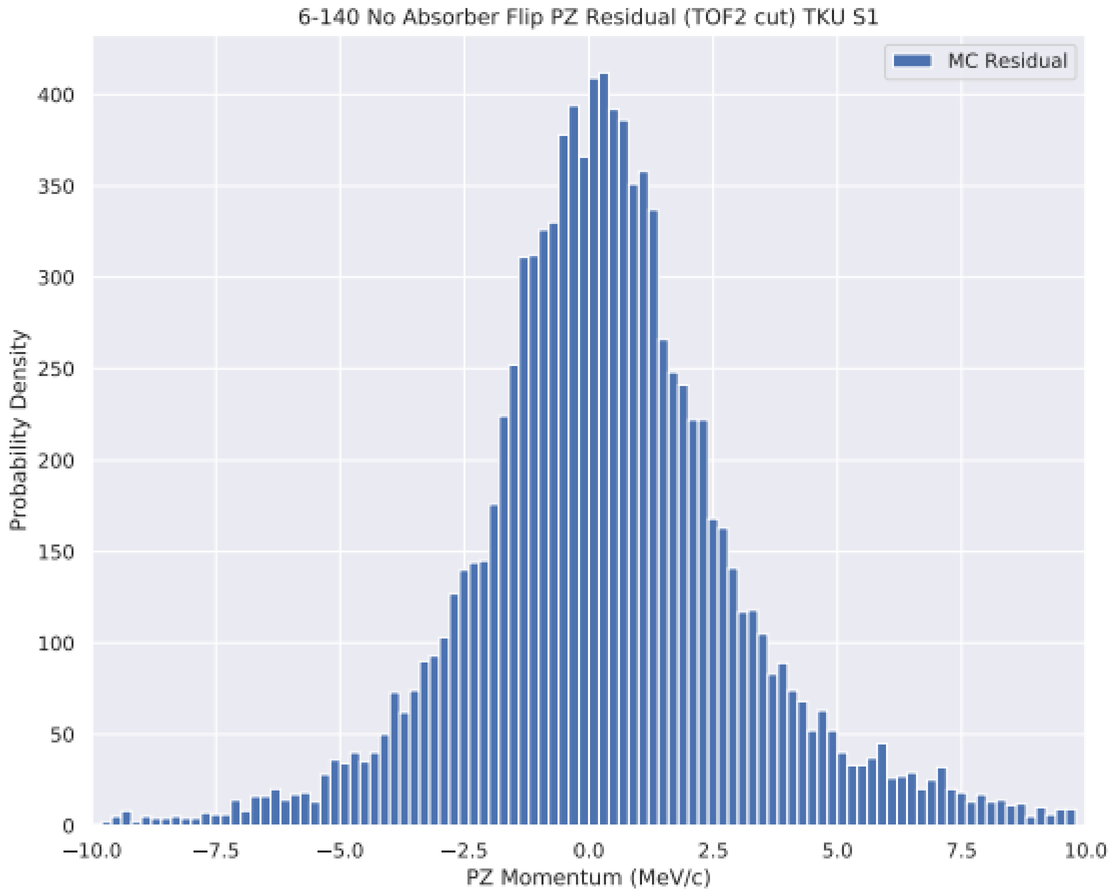
Pz Distribution at Reference Planes

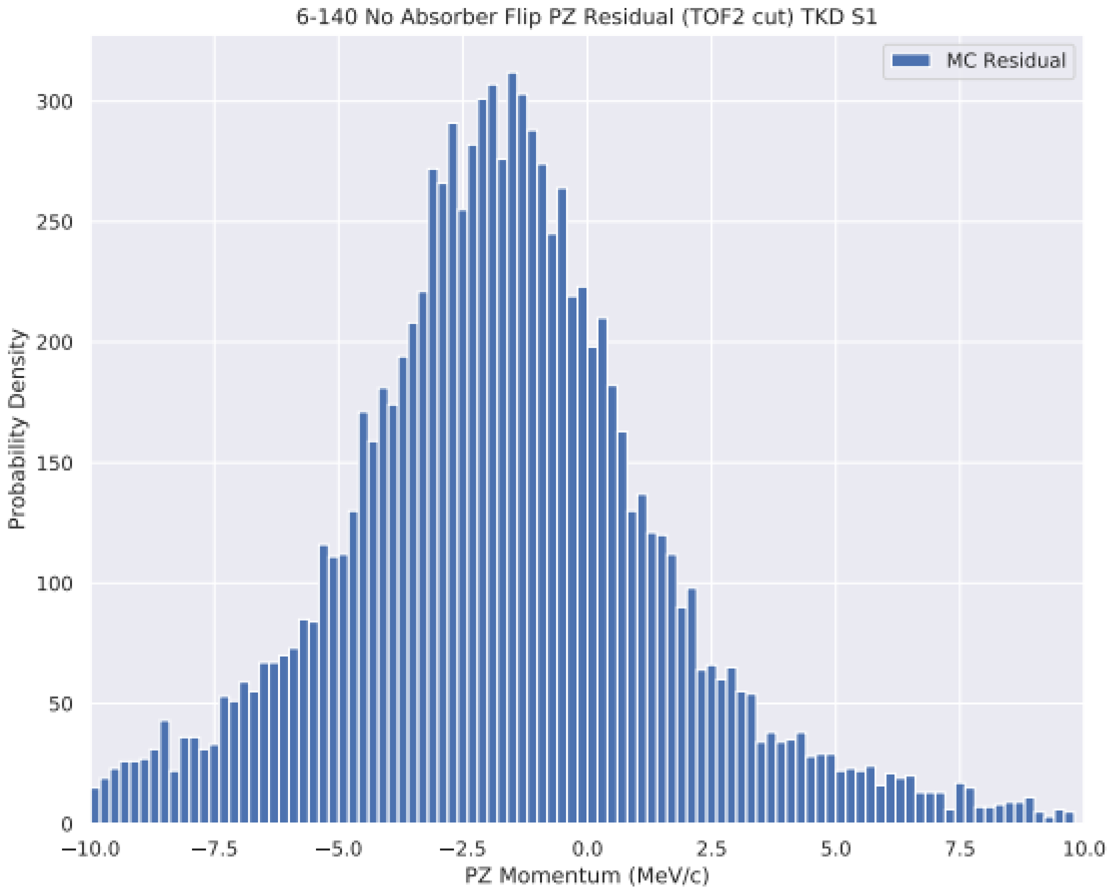
- TKU: Recon Probability Distribution is narrower and taller
- TKD: Recon Probability Distribution is broader and smaller
- Trackers are not identical in their reconstruction -> systematic bias





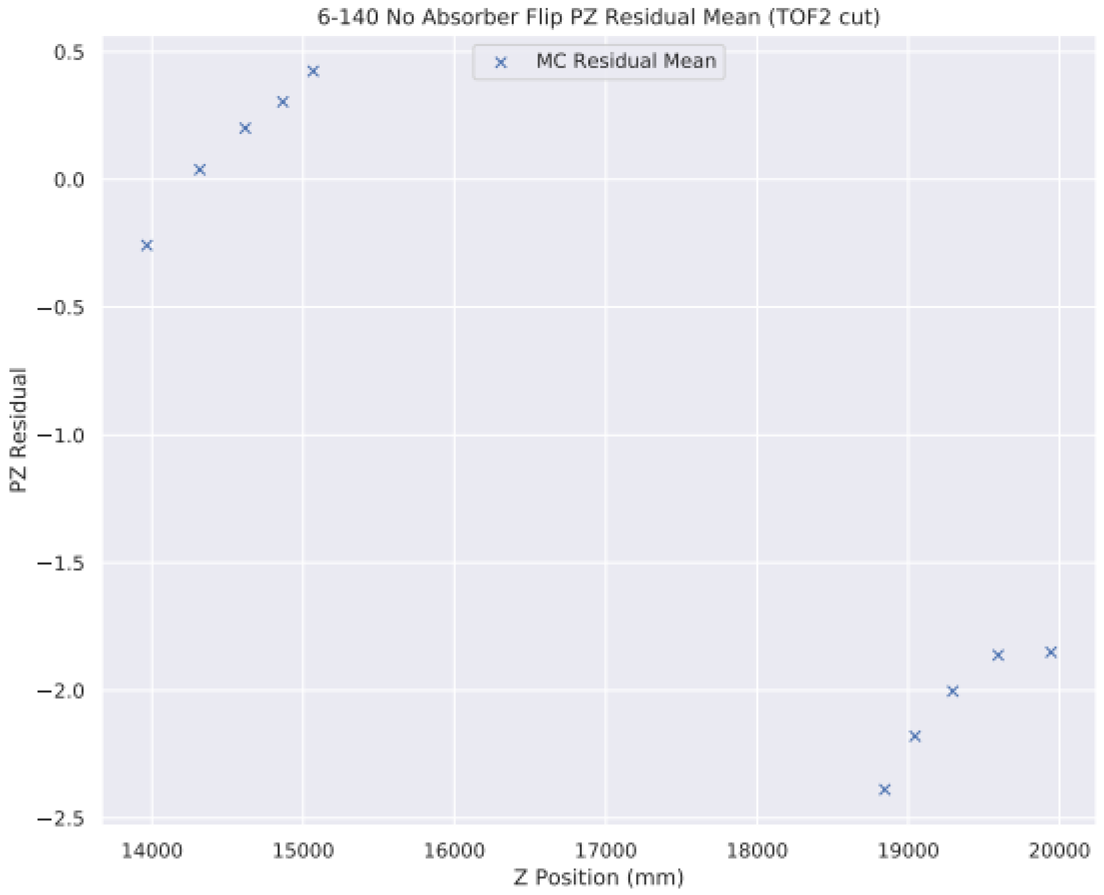
Pz Residuals at Reference Planes

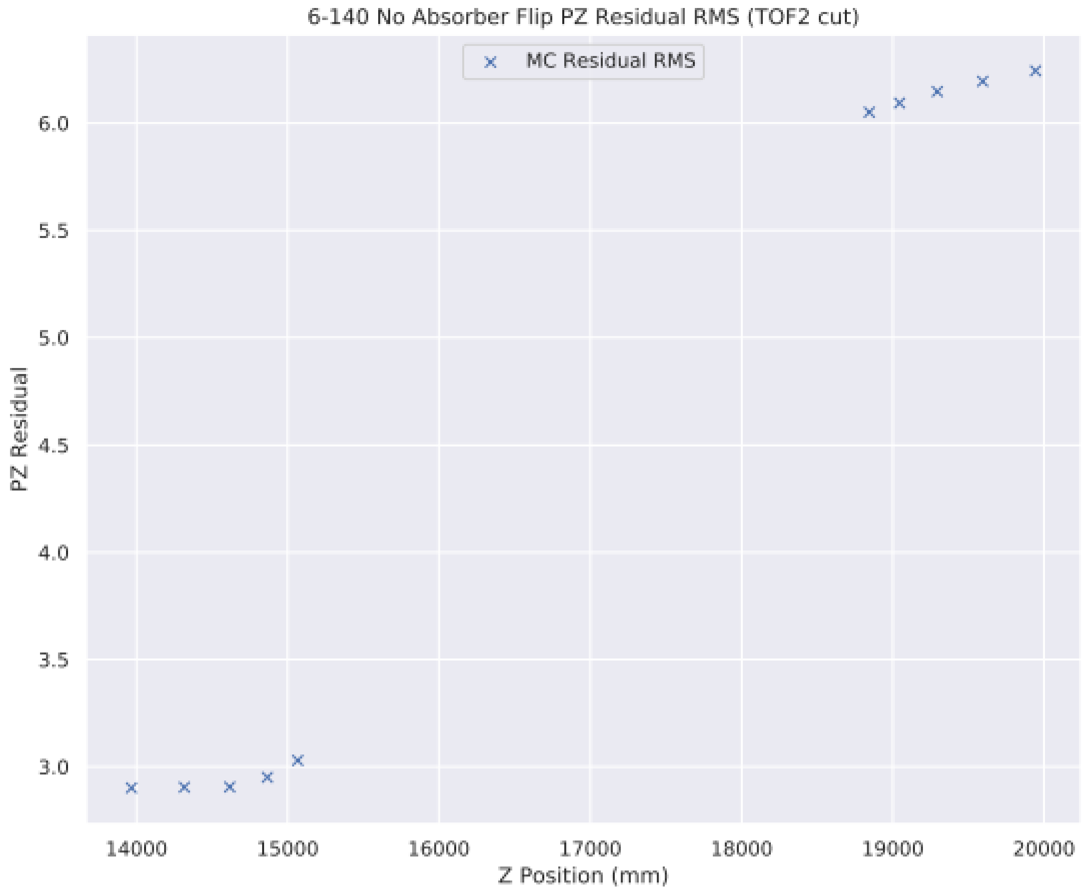




Pz Mean and RMS

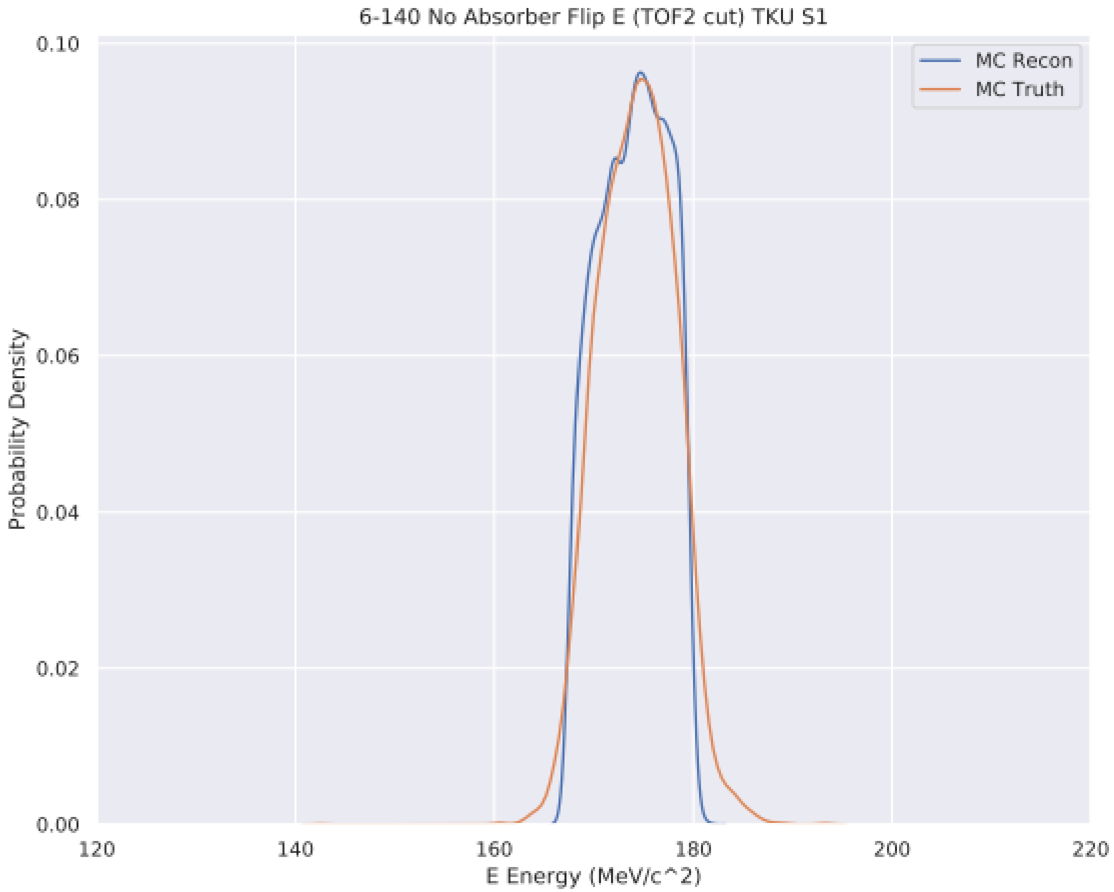
- Recon adds more Pz in TKD

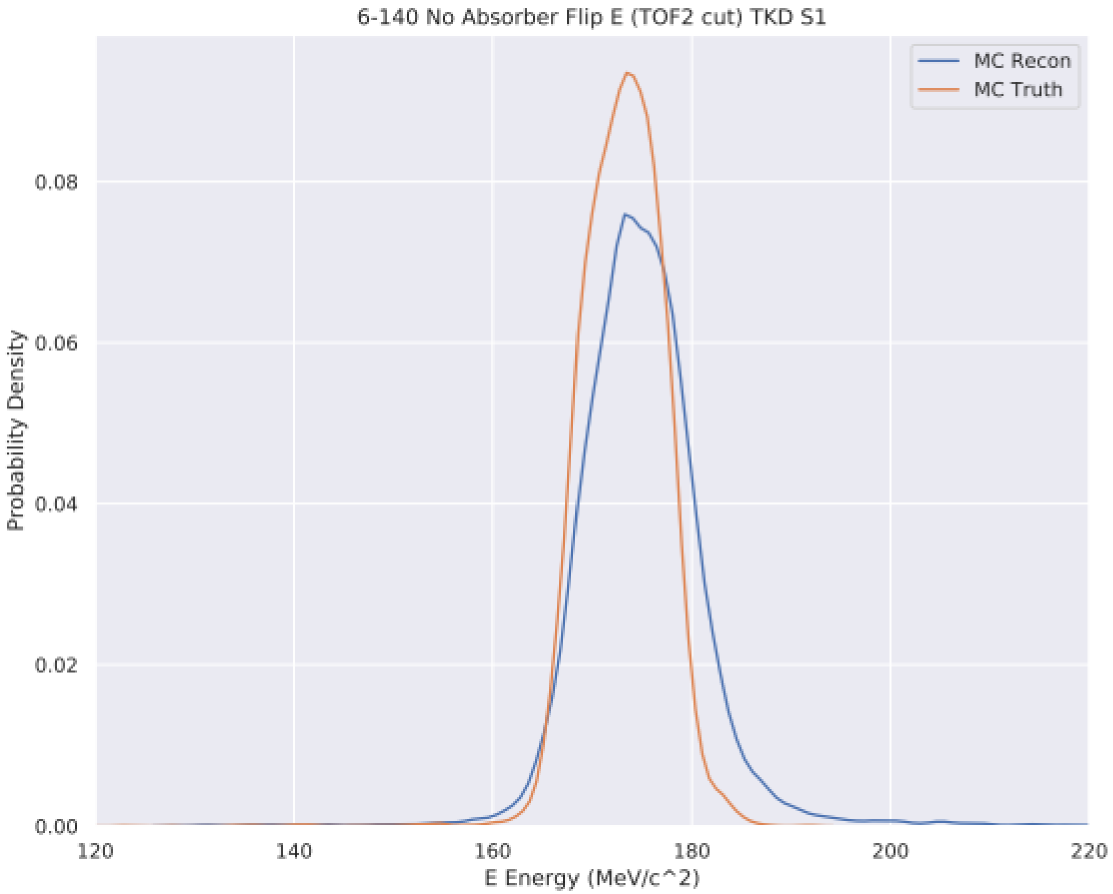




Energy distribution at Reference Planes

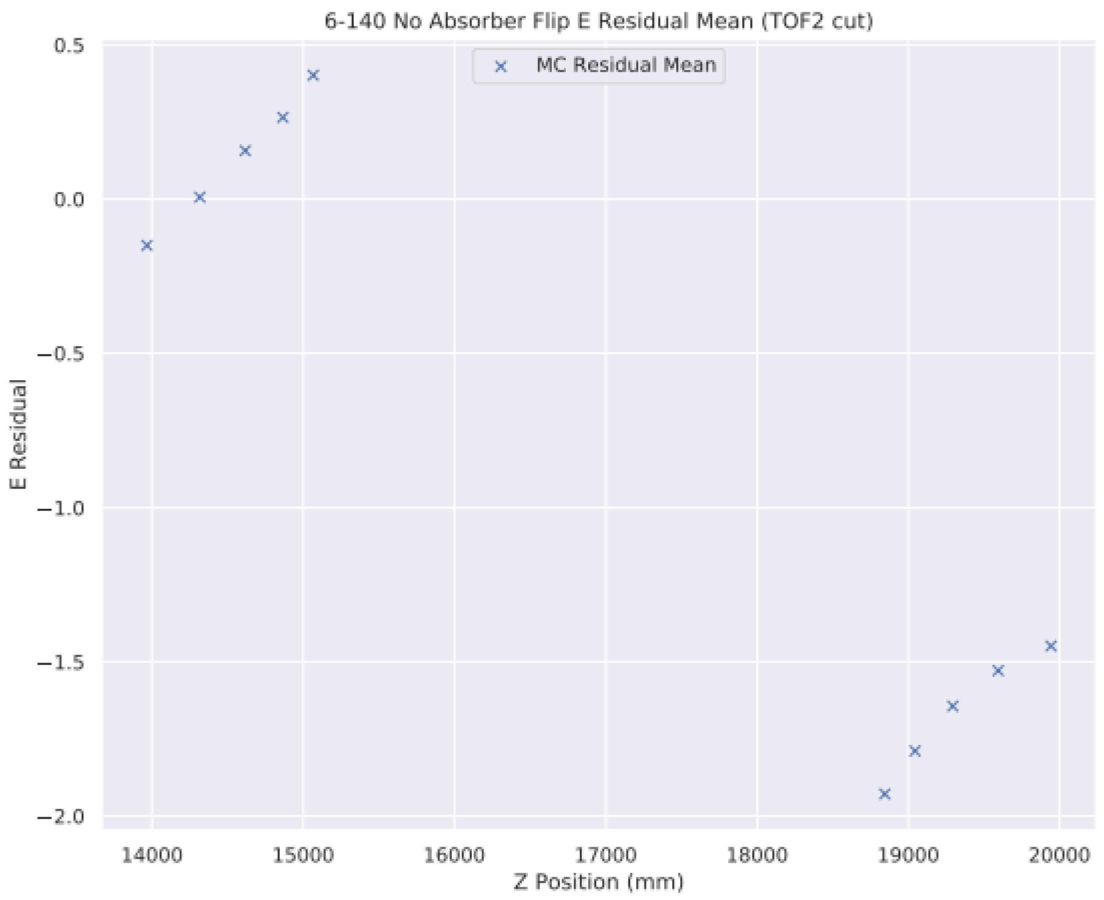
- Similar to Pz

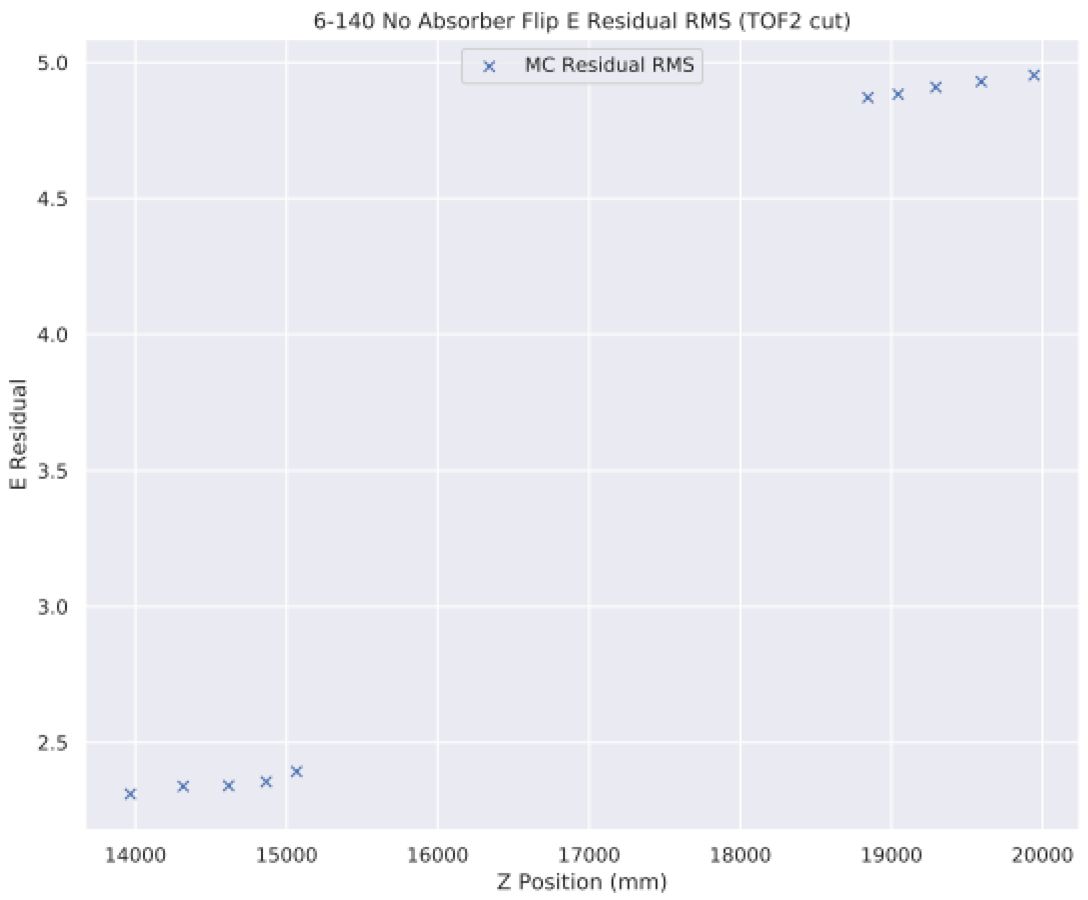




Energy Residual Mean and RMS

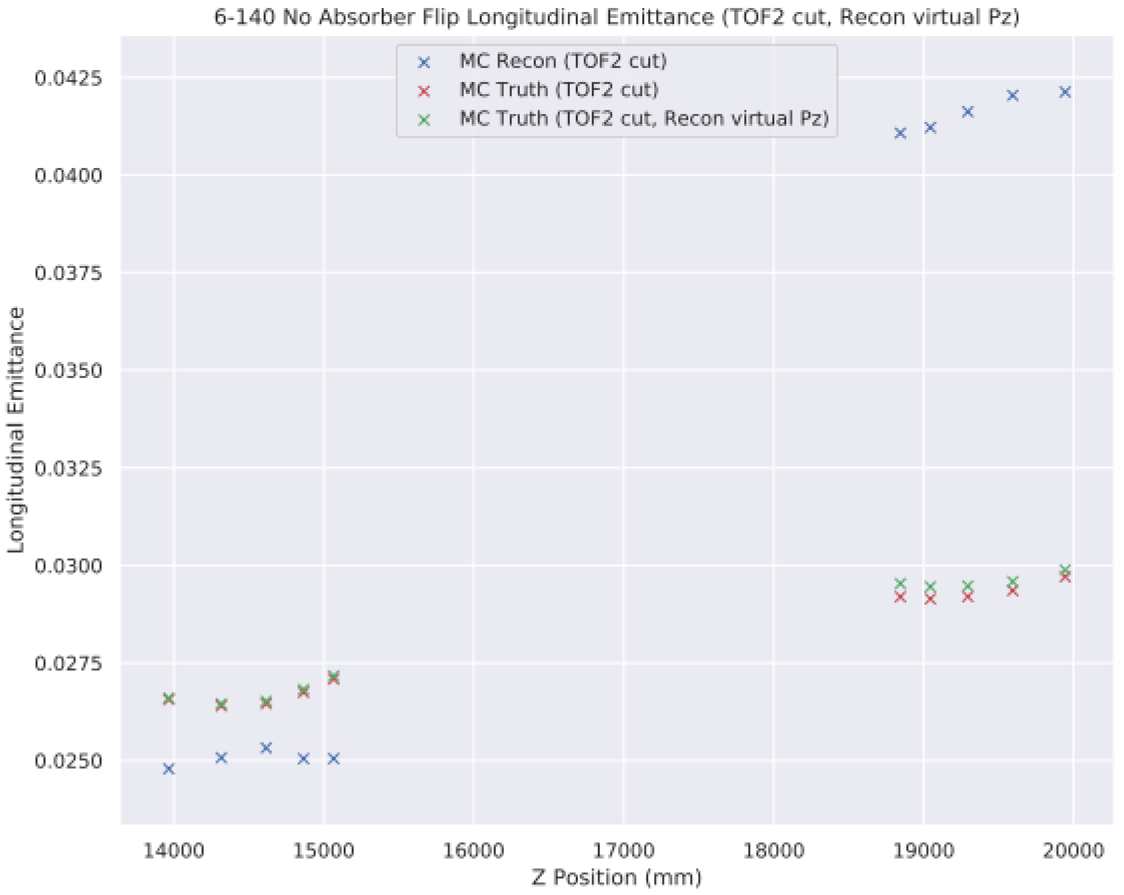
- Recon adds Energy to System – we no longer have a conserved system
- This has implications for Longitudinal/6D Emittance, Amplitude and Density

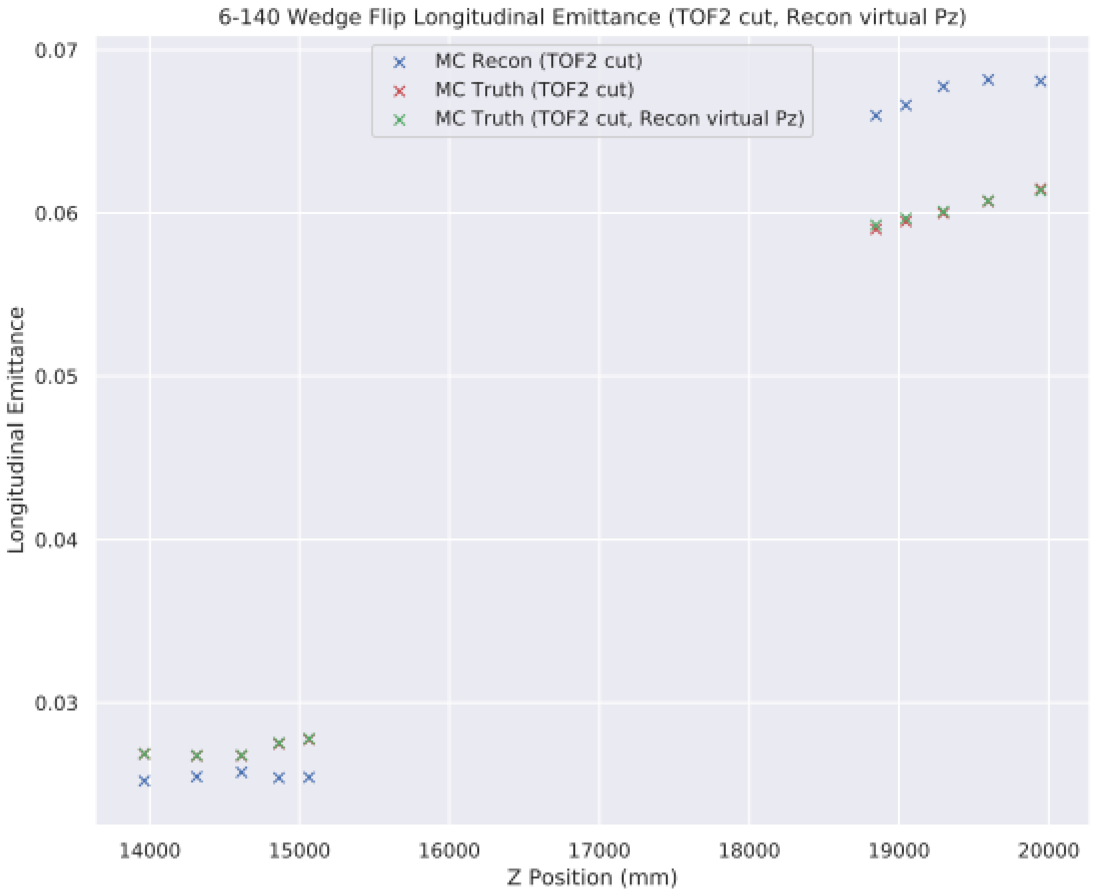




Longitudinal Emittance

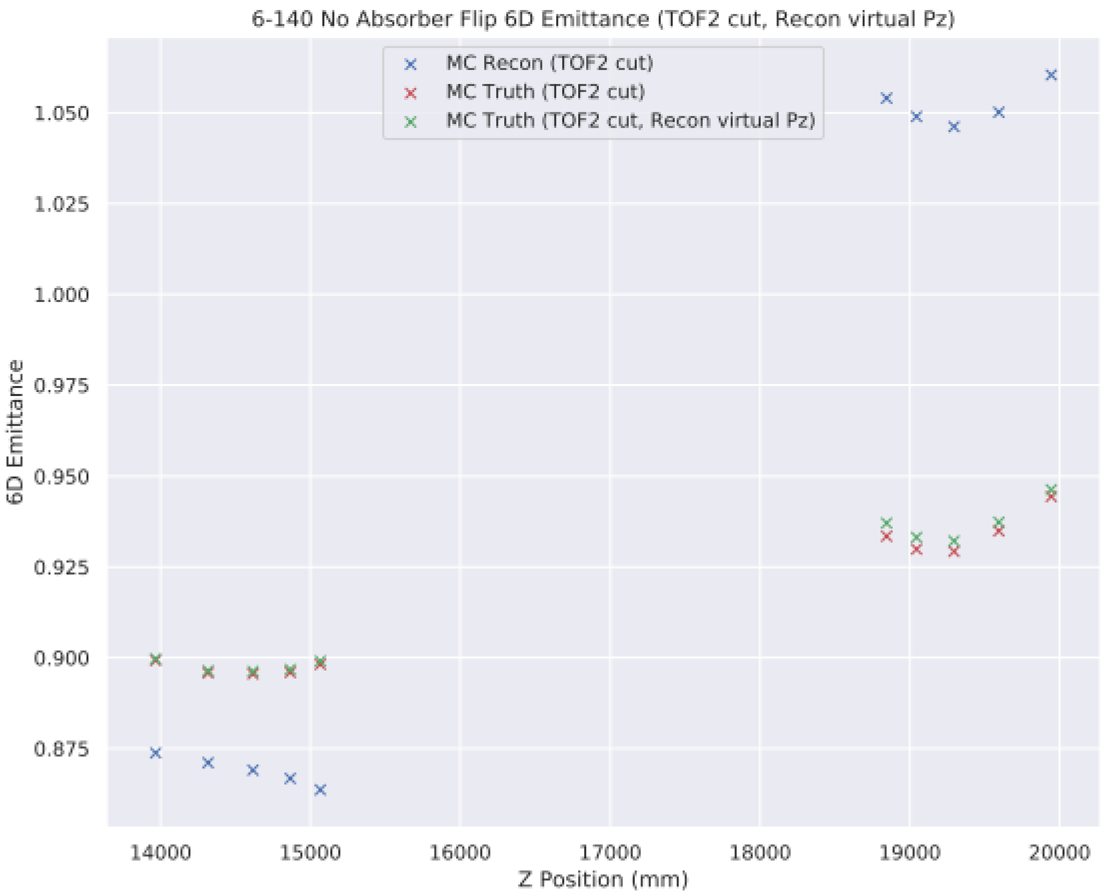
- Green is same as Truth except Time has been reconstructed using Truth Pz
- Recon shows larger discrepancies (Resolution effect)
- No Absorber Longitudinal Emittance shows small change, but Wedge Longitudinal Emittance doubles between TKU and TKD

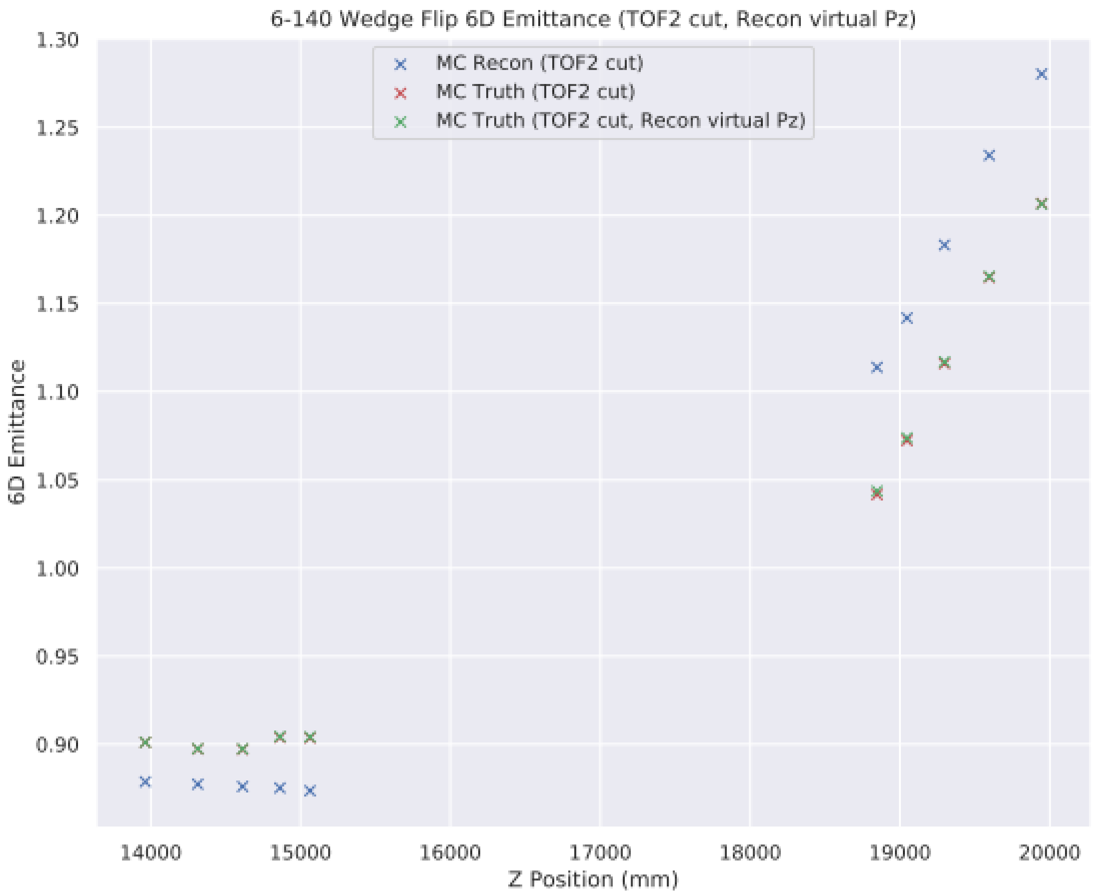




6D emittance

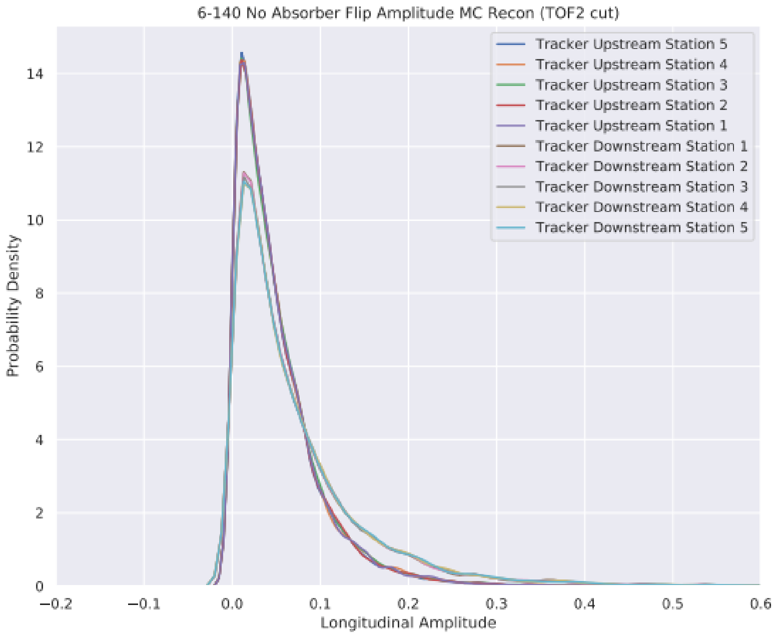
- Green is same as Truth except Time has been reconstructed using Truth Pz
- Recon shows larger discrepancies (Resolution effect)
- Could take larger momentum bite, but would then need to correct transverse components. Probably need to in Wedge case due to dispersion downstream

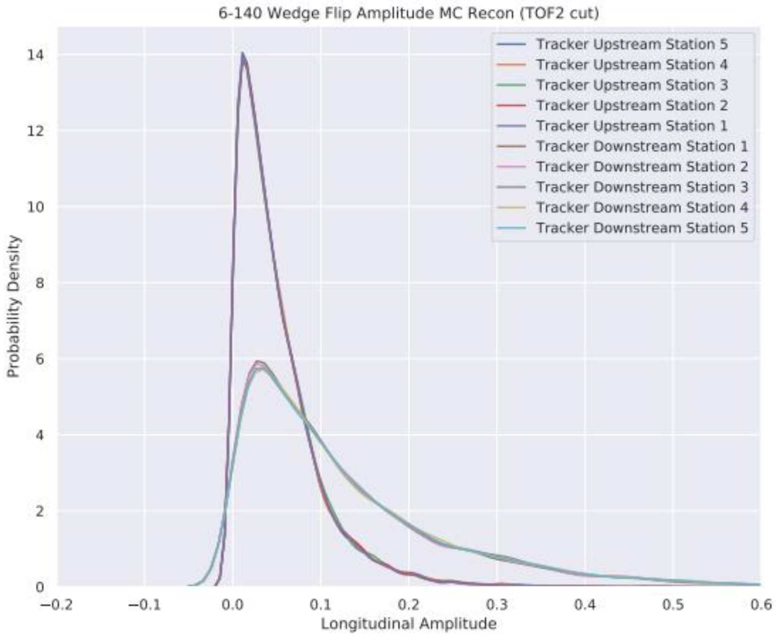


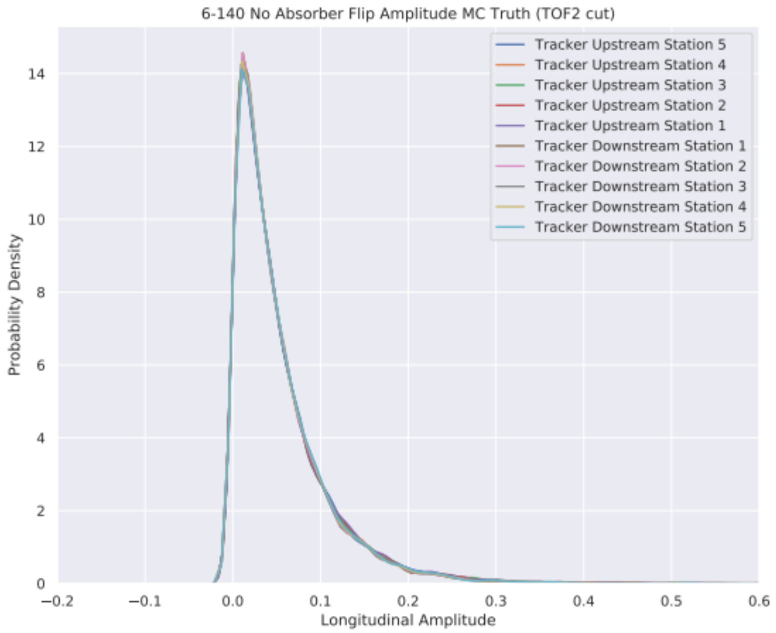


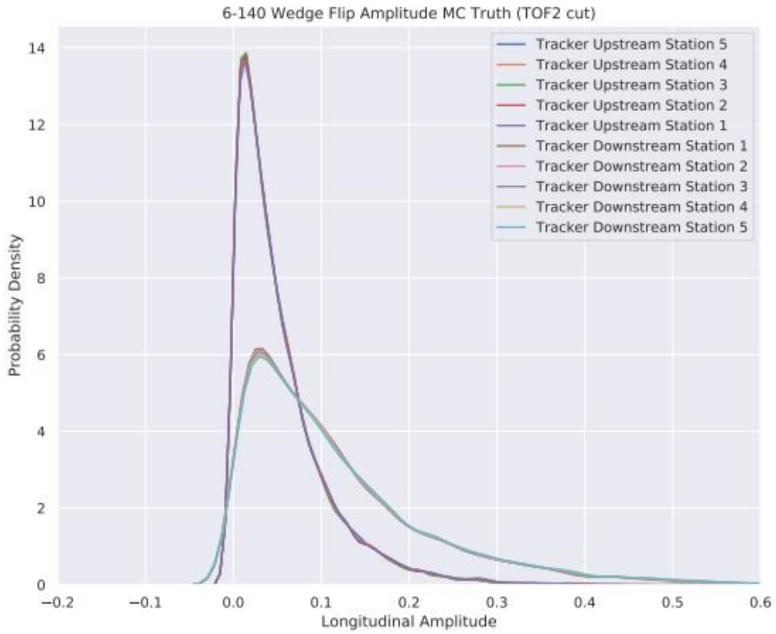
Longitudinal Amplitude

- MC Truth shows conservation between TKU and TKD for No Absorber
- Growth for Wedge
- Recon is off due to the Pz being reconstructed differently in TKU and TKD



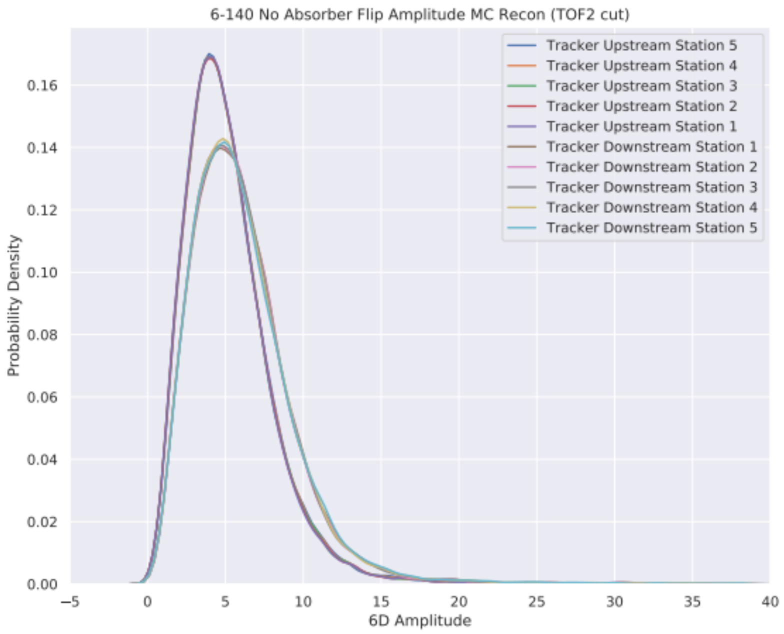


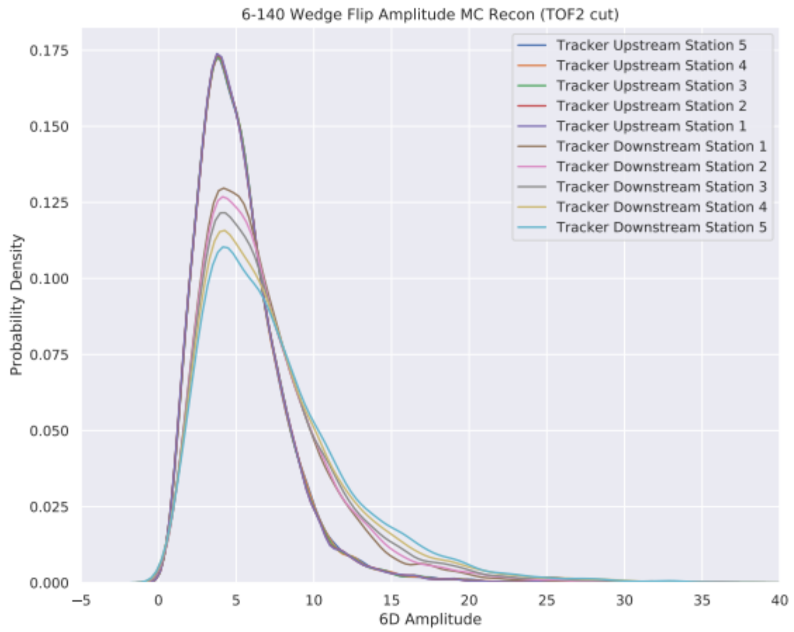


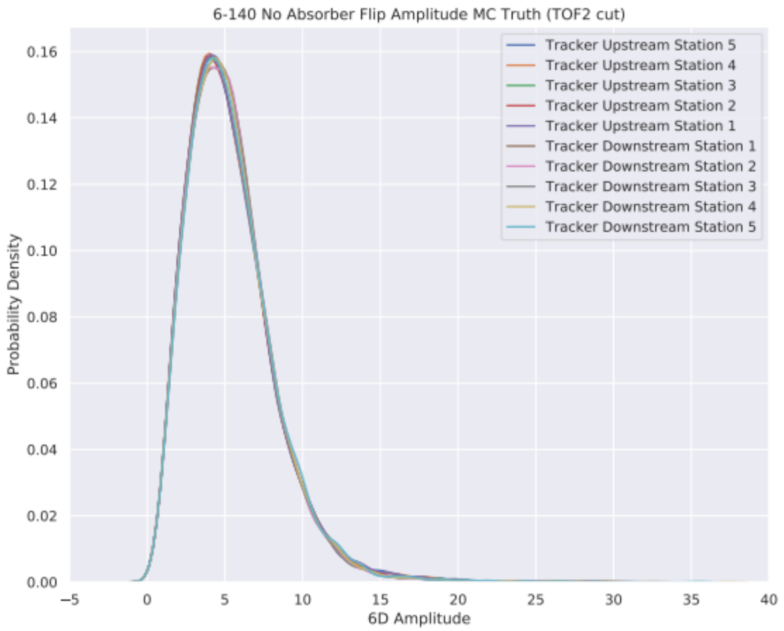


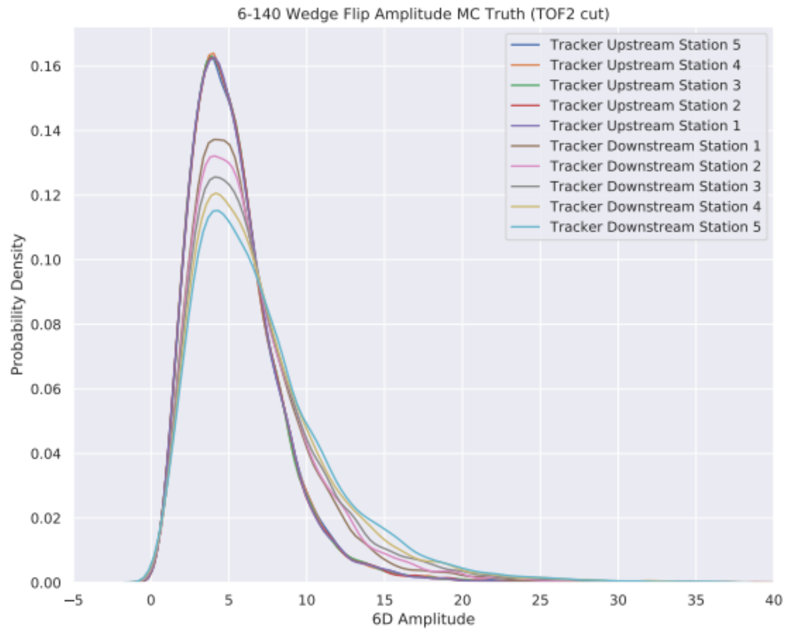
6D Amplitude

- Truth shows 6D conservation for No Absorber case
- Wedge shows change between TKU and TKD and within TKD due to dispersion
- Likely need to correct Transverse components for extra rotation
- Makes separation of 6D into Transverse and Longitudinal components tricky



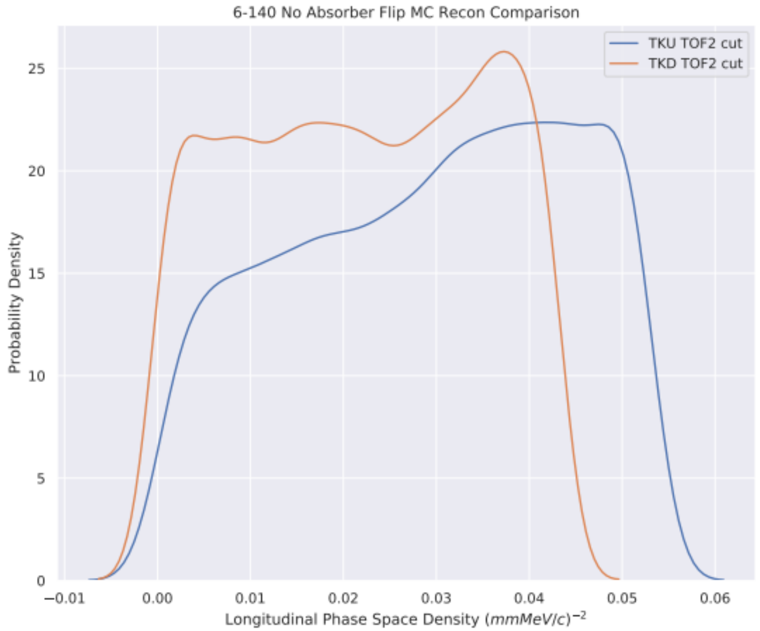


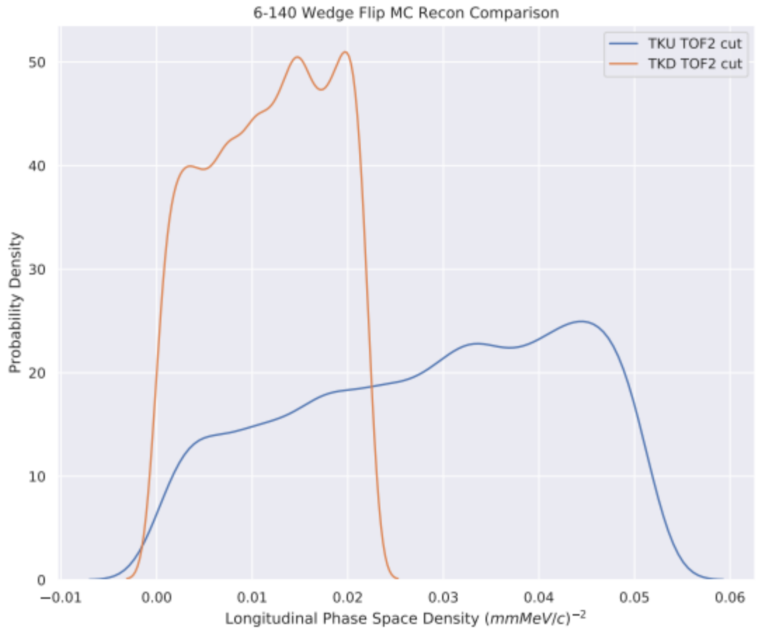


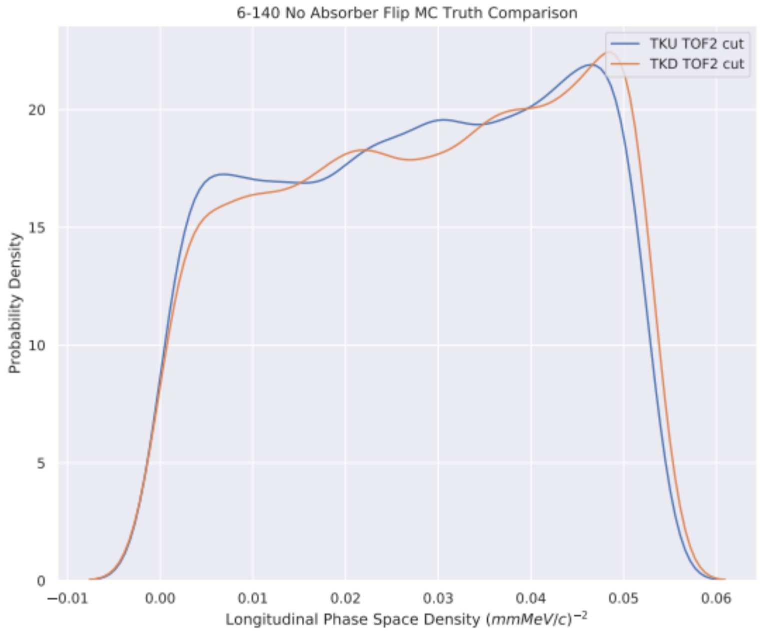


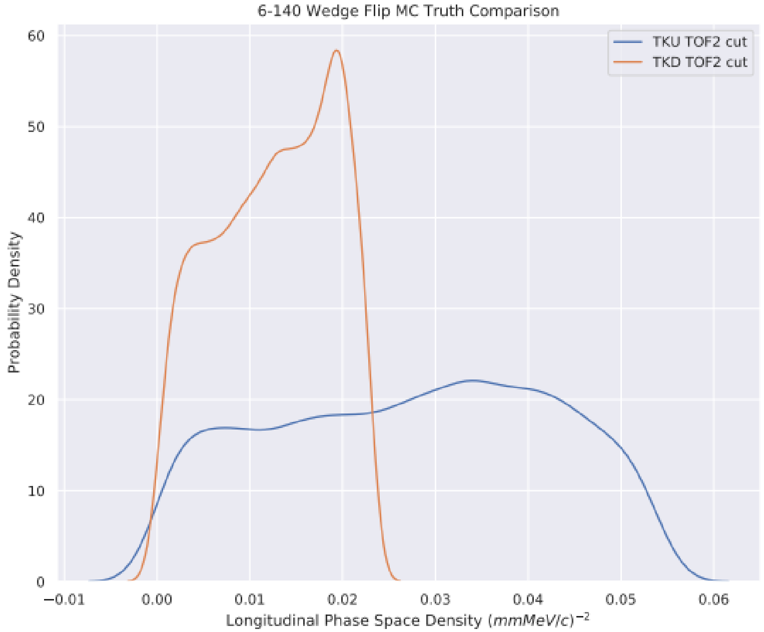
Longitudinal Density

- See similar effects for longitudinal density as for Amplitude
- Truth conserves No Absorber density and halves Wedge density of beam.
- Wedge density shape change indicates some edge effect



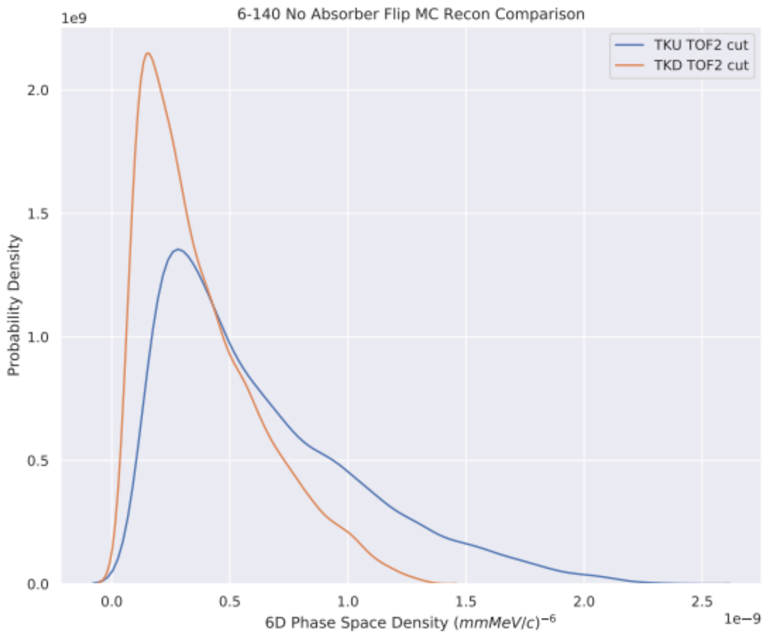


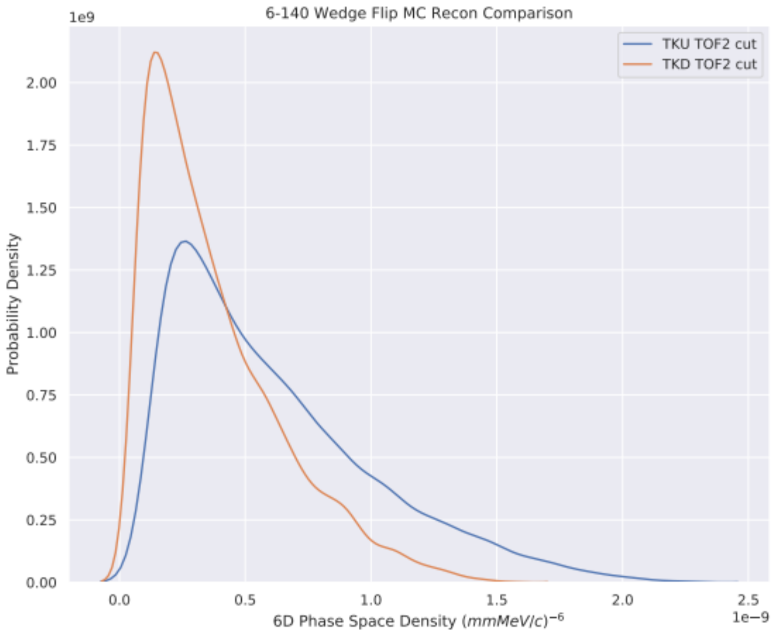


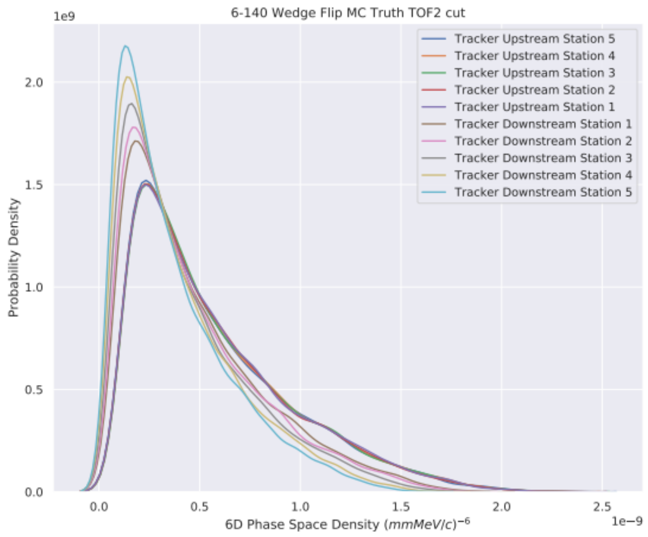


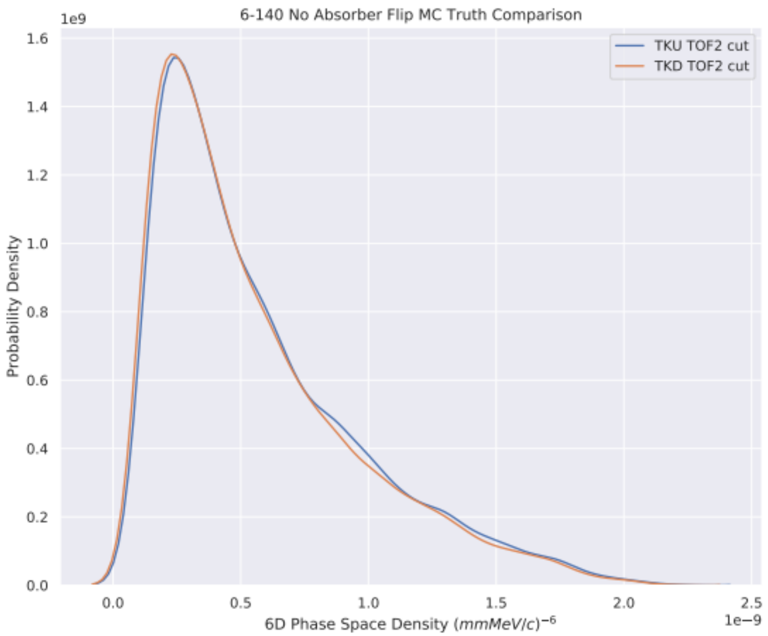
6D Density

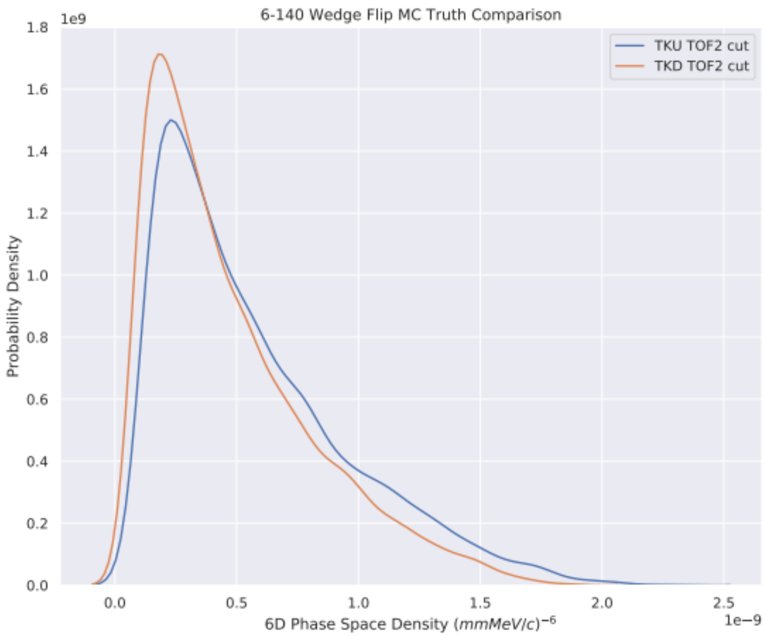
- 6D density is also similar to 6D Amplitude
- It also has same effect through TKD -> Transverse Components need correction











Emittance, Amplitude and Density

- Caution, these plots are affected by survivorship bias
- Emittance and Amplitude conserved in linear optics, not necessarily when higher order effects apply
- The choice of vector potential in the non-linear case becomes important

$$K = -\left(1+\frac{x}{\rho}\right)\left[p^2-(p_x-eA_x)^2-(p_z-eA_z)^2\right]^{\frac{1}{2}}-eA_s$$

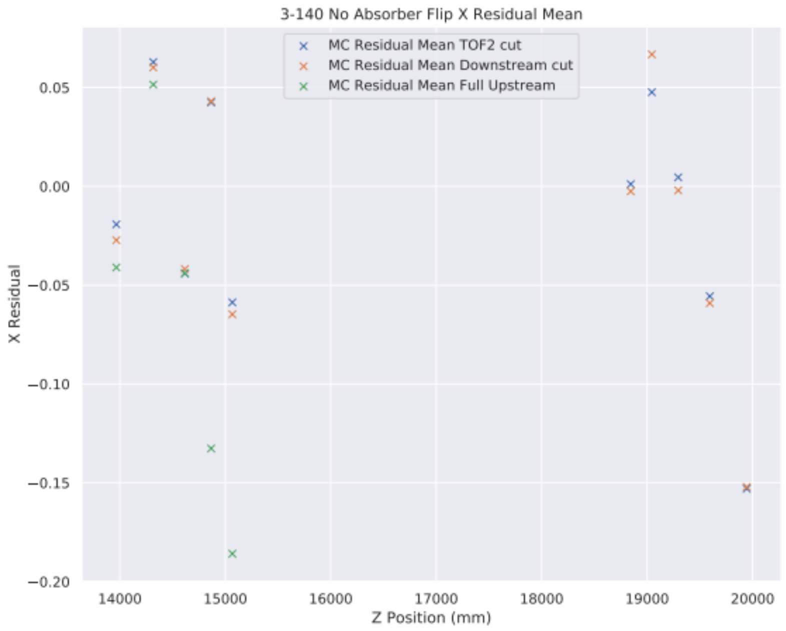
- As well as where approximations are then made in the expansion

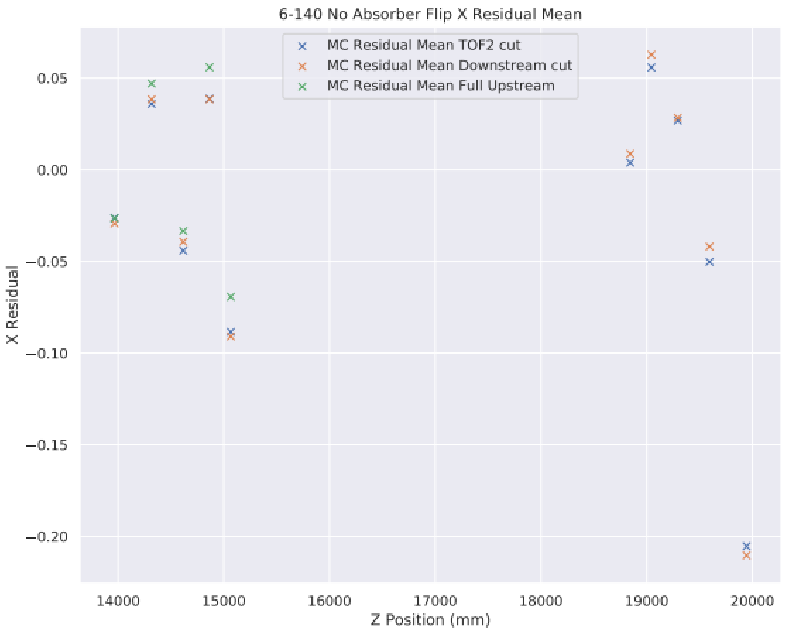
$$\tilde{H} = \frac{K}{p_0} = -\left(1+\frac{x}{\rho}\right)\frac{1}{p_0}\sum_{l=0}^{\frac{1}{2}}\frac{\frac{1}{2}!}{l!\left(\frac{1}{2}-l\right)!}\left(p^2\right)^{\frac{1}{2}-l}$$
$$\times\left[-\left(p_x-ez\sum_{k=0}^{\infty}\frac{b_{2k}}{2(k+1)}\left(x^2+z^2\right)^k\right)^2 - \left(p_z+ex\sum_{k=0}^{\infty}\frac{b_{2k}}{2(k+1)}\left(x^2+z^2\right)^k\right)^2\right]^l$$

- Currently MICE Recon uses linear optics i.e. only l=0 and l=1 terms in the expanded Hamiltonian (on right)
- Solenoid is finite length, stations near edge may deviate further from approximation
- Density remains a constant (only affected by dissipative forces), but it can be difficult to find a constant volume element

Recon bias

- Is it a problem?
- Depends, if it is a simple offset then can correct for it
- If not, then need to figure out what each part of the Recon does to see how it introduces biases


3-140 No Absorber Flip X Residual Mean
MC Residual Mean TOF2 cut
MC Residual Mean Downstream cut
MC Residual Mean Full Upstream
X Residual
Z Position (mm)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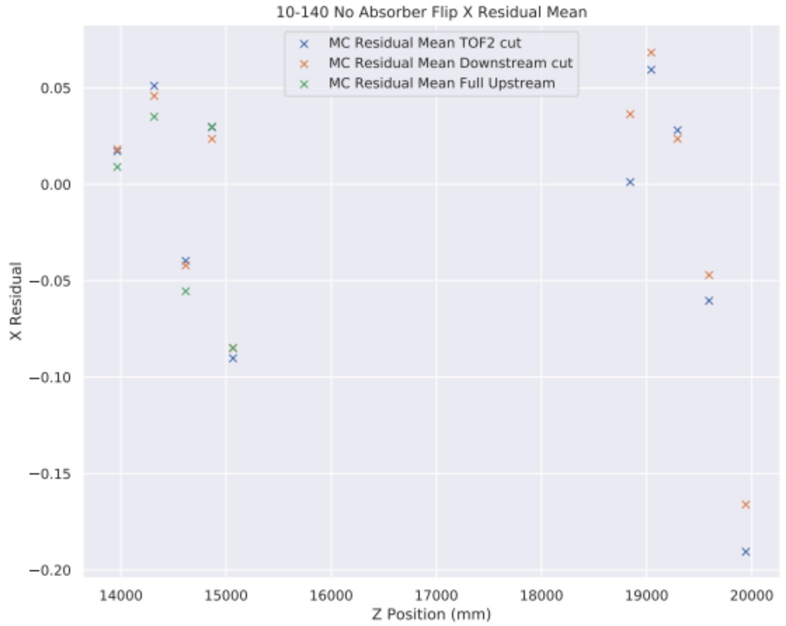
MC Residual Mean TOF2 cut
MC Residual Mean Downstream cut
MC Residual Mean Full Upstream
X Residual
Z Position (mm)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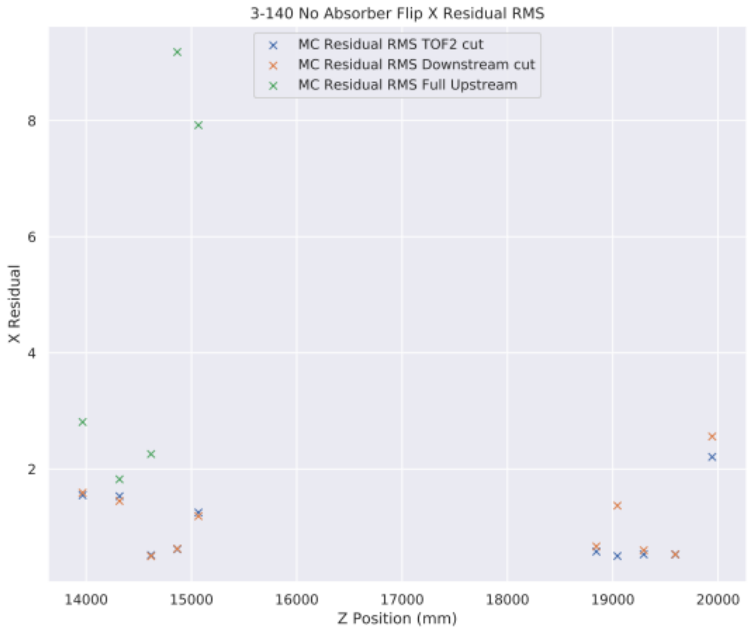
MC Residual Mean Downstream cut
MC Residual Mean Full Upstream
X Residual
Z Position (mm)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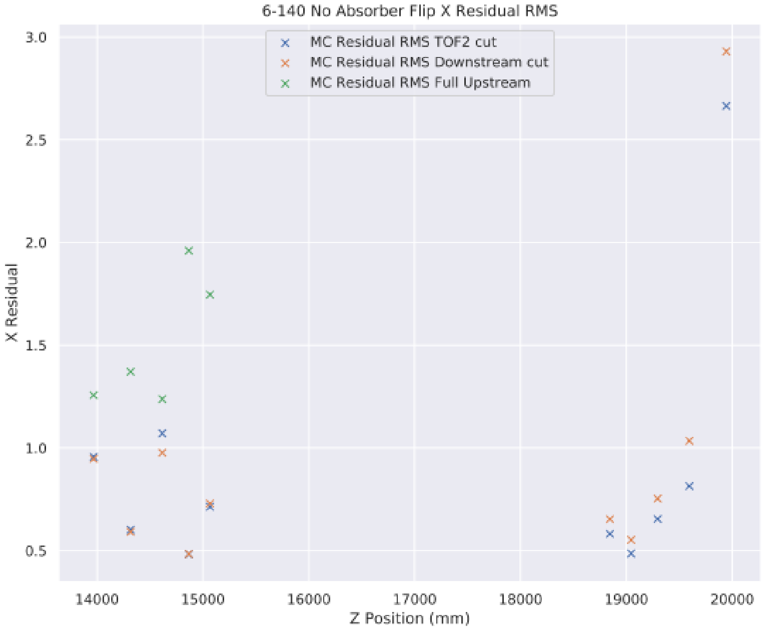
MC Residual RMS TOF2 cut
MC Residual RMS Downstream cut
MC Residual RMS Full Upstream
X Residual
Z Position (mm)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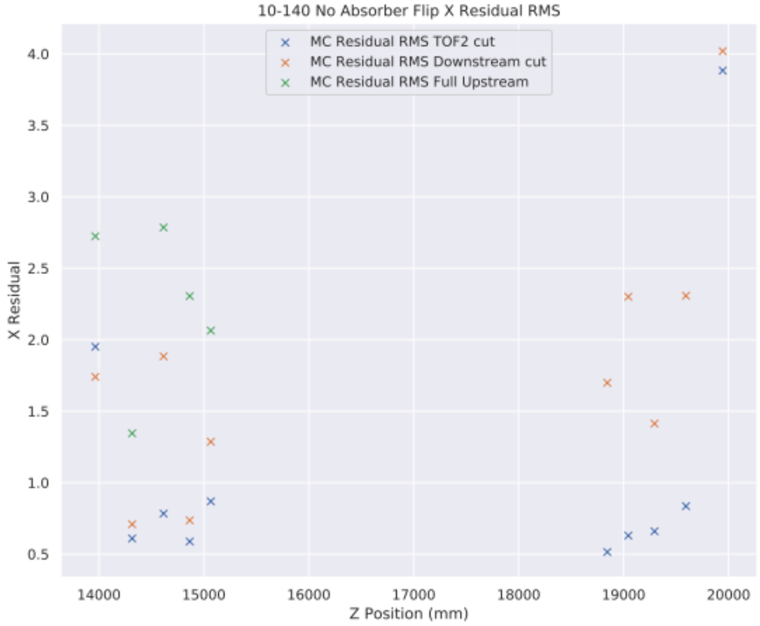
MC Residual RMS TOF2 cut
MC Residual RMS Downstream cut
MC Residual RMS Full Upstream
X Residual
Z Position (mm)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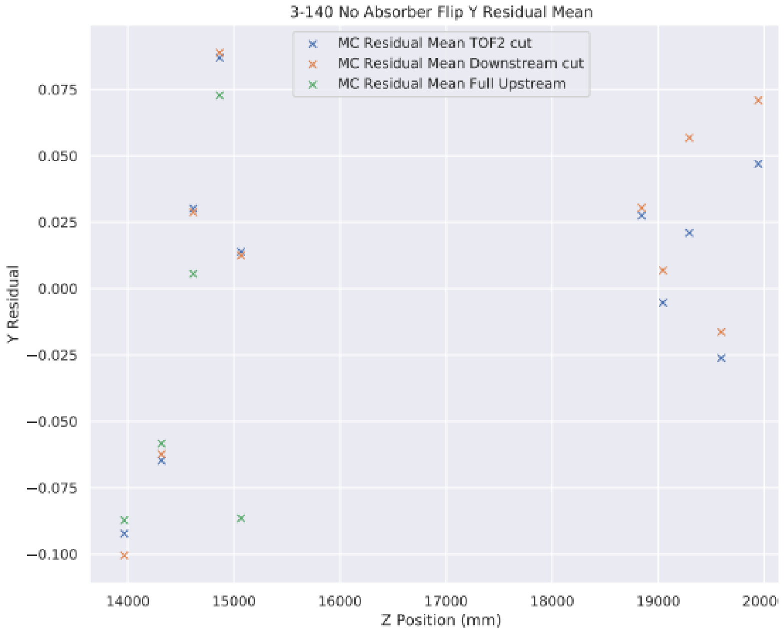
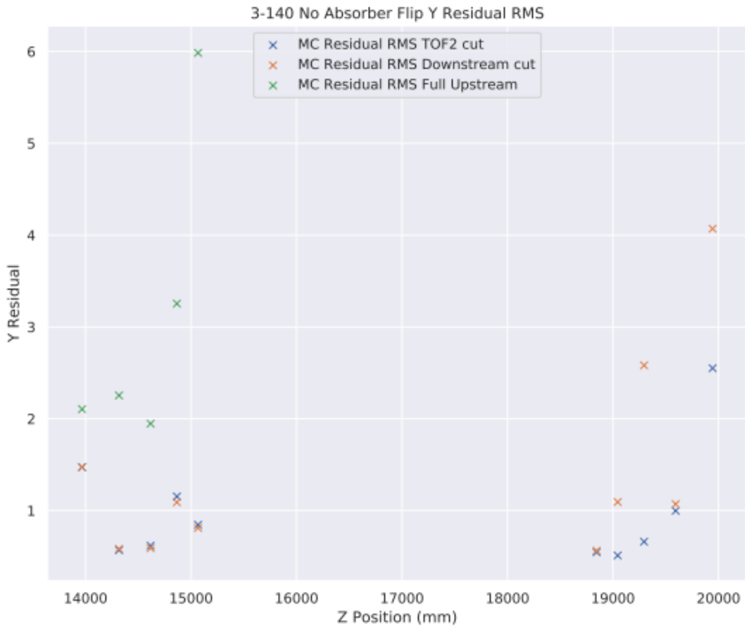
MC Residual RMS TOF2 cut
MC Residual RMS Downstream cut
MC Residual RMS Full Upstream
X Residual
Z Position (mm)


3-140 No Absorber Flip Y Residual Mean
MC Residual Mean TOF2 cut
MC Residual Mean Downstream cut
MC Residual Mean Full Upstream
Y Residual
Z Position (mm)


6-140 No Absorber Flip Y Residual Mean
MC Residual Mean TOF2 cut
MC Residual Mean Downstream cut
MC Residual Mean Full Upstream
Y Residual
Z Position (mm)


10-140 No Absorber Flip Y Residual Mean
MC Residual Mean TOF2 cut
MC Residual Mean Downstream cut
MC Residual Mean Full Upstream
Y Residual
Z Position (mm)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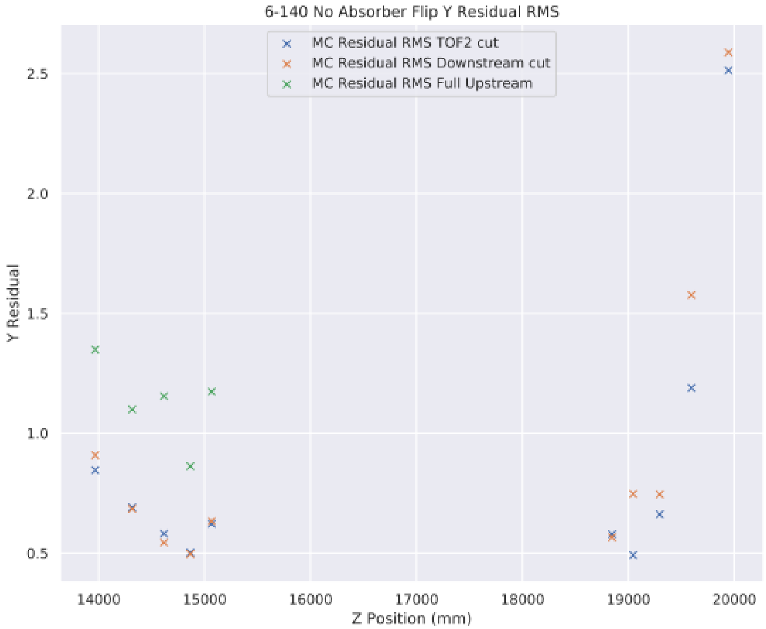
MC Residual RMS Full Upstream
Y Residual
Z Position (mm)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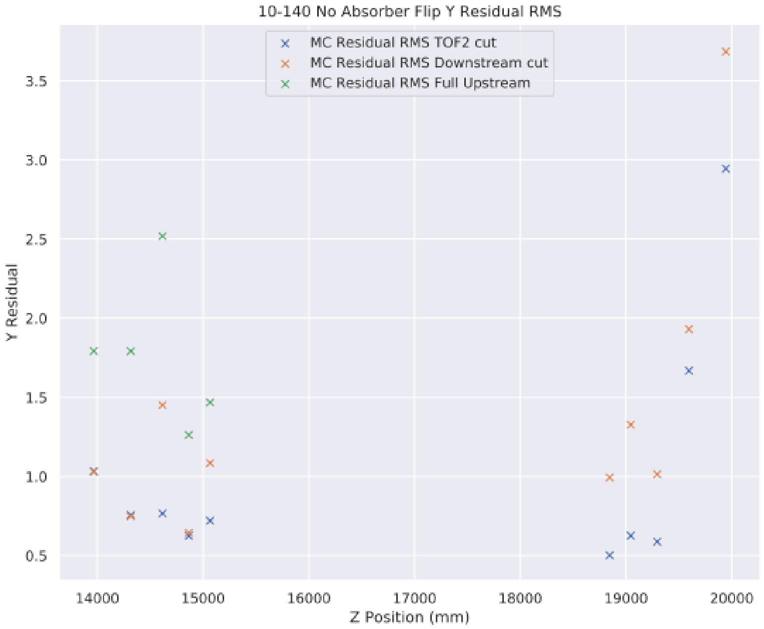
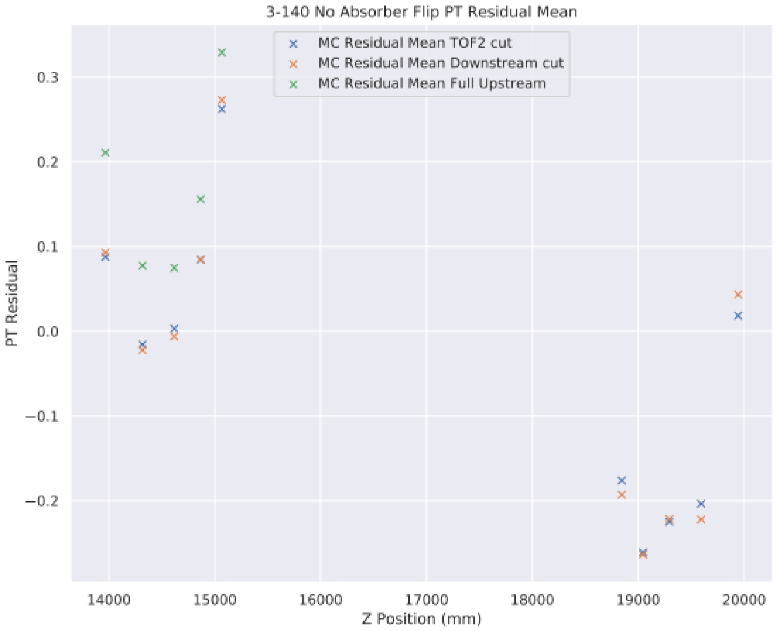
MC Residual RMS Full Upstream
Y Residual
Z Position (mm)


10-140 No Absorber Flip Y Residual RMS
MC Residual RMS TOF2 cut
MC Residual RMS Downstream cut
MC Residual RMS Full Upstream
Y Residual
Z Position (mm)

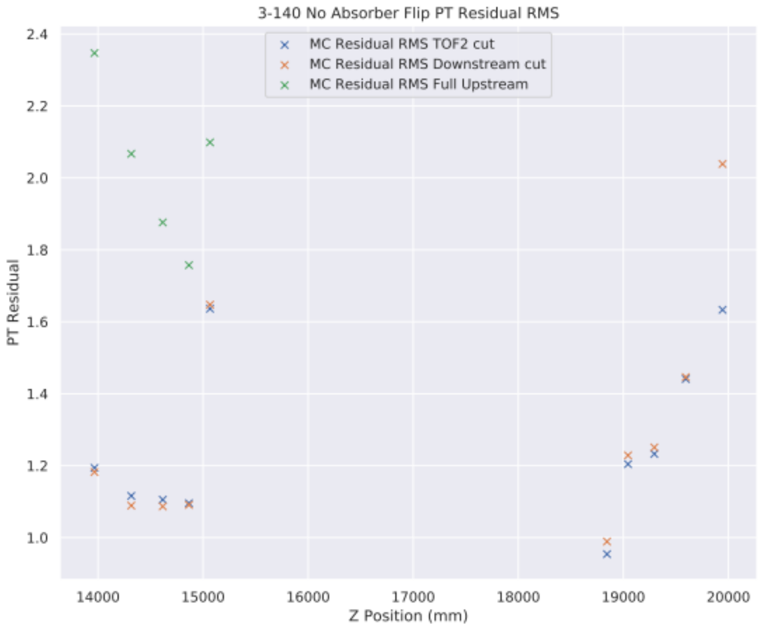
X + Y Components

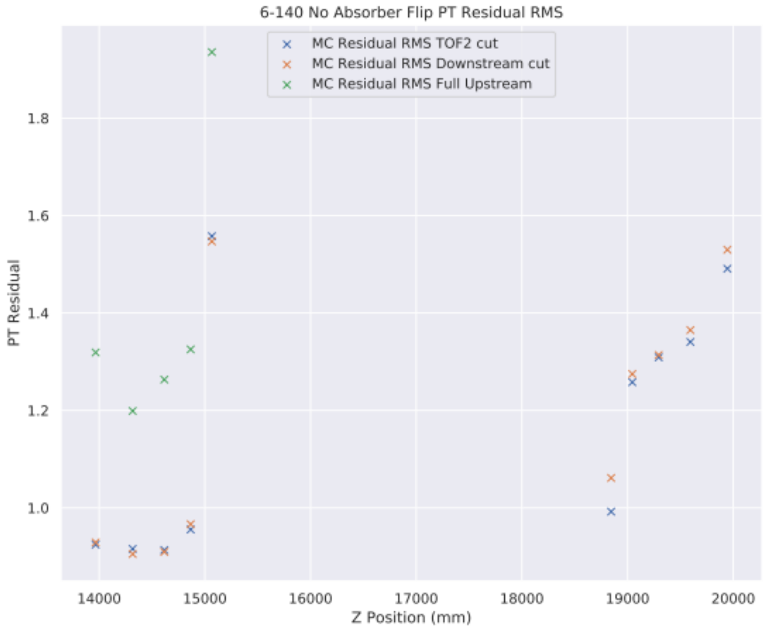
- Average Pulls of up to 0.1 mm from the centre of the fibre may not seem that concerning
- The fibres themselves however are very small, and as a percentage that is quite large
- Pt and Pz may give better hints

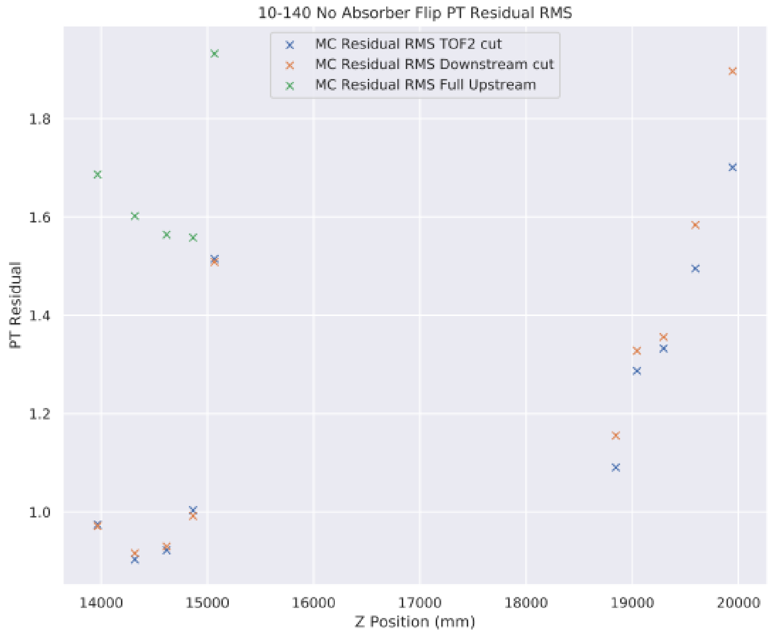

3-140 No Absorber Flip PT Residual Mean
MC Residual Mean TOF2 cut
MC Residual Mean Downstream cut
MC Residual Mean Full Upstream
PT Residual
Z Position (mm)

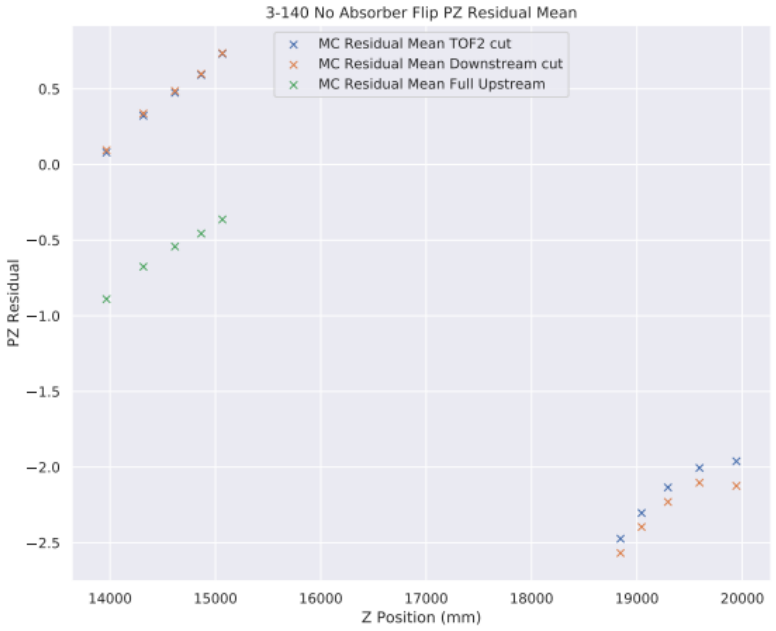

6-140 No Absorber Flip PT Residual Mean
MC Residual Mean TOF2 cut
MC Residual Mean Downstream cut
MC Residual Mean Full Upstream
PT Residual
Z Position (mm)


10-140 No Absorber Flip PT Residual Mean
MC Residual Mean TOF2 cut
MC Residual Mean Downstream cut
MC Residual Mean Full Upstream
PT Residual
Z Position (mm)

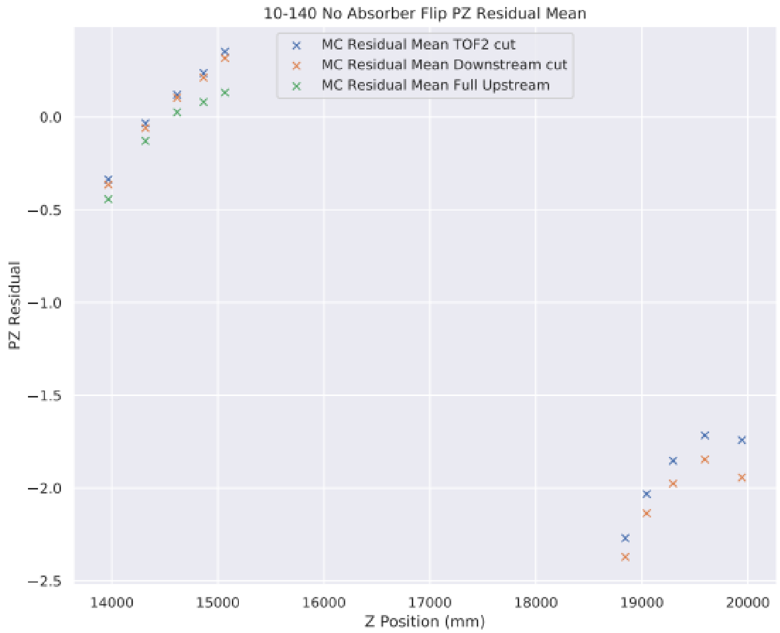

3-140 No Absorber Flip PT Residual RMS
MC Residual RMS TOF2 cut
MC Residual RMS Downstream cut
MC Residual RMS Full Upstream
PT Residual
Z Position (mm)


6-140 No Absorber Flip PT Residual RMS
MC Residual RMS TOF2 cut
MC Residual RMS Downstream cut
MC Residual RMS Full Upstream
PT Residual
Z Position (mm)


10-140 No Absorber Flip PT Residual RMS
MC Residual RMS TOF2 cut
MC Residual RMS Downstream cut
MC Residual RMS Full Upstream
PT Residual
Z Position (mm)


3-140 No Absorber Flip PZ Residual Mean
MC Residual Mean TOF2 cut
MC Residual Mean Downstream cut
MC Residual Mean Full Upstream
PZ Residual
Z Position (mm)


6-140 No Absorber Flip PZ Residual Mean
MC Residual Mean TOF2 cut
MC Residual Mean Downstream cut
MC Residual Mean Full Upstream
PZ Residual
Z Position (mm)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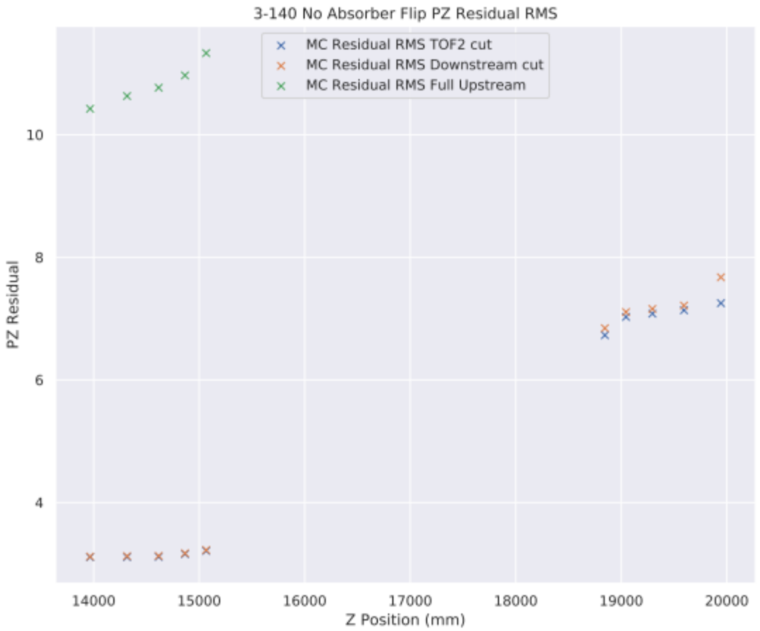
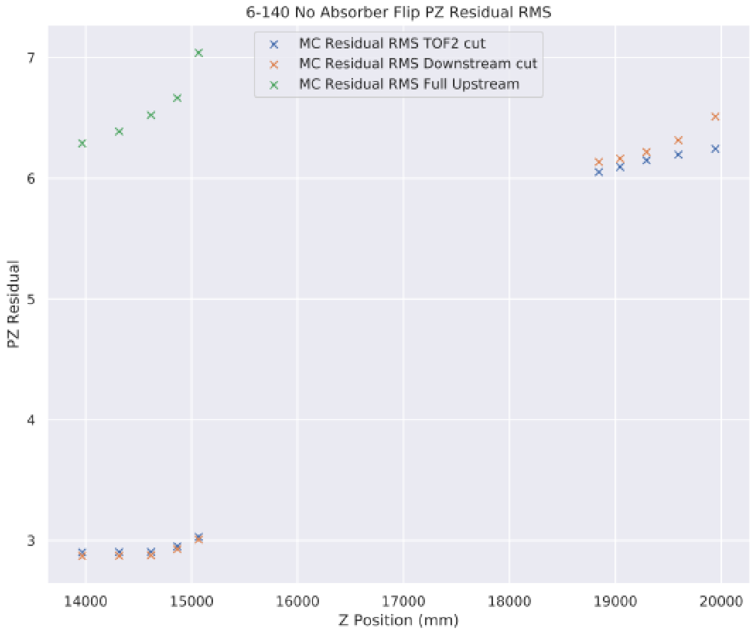
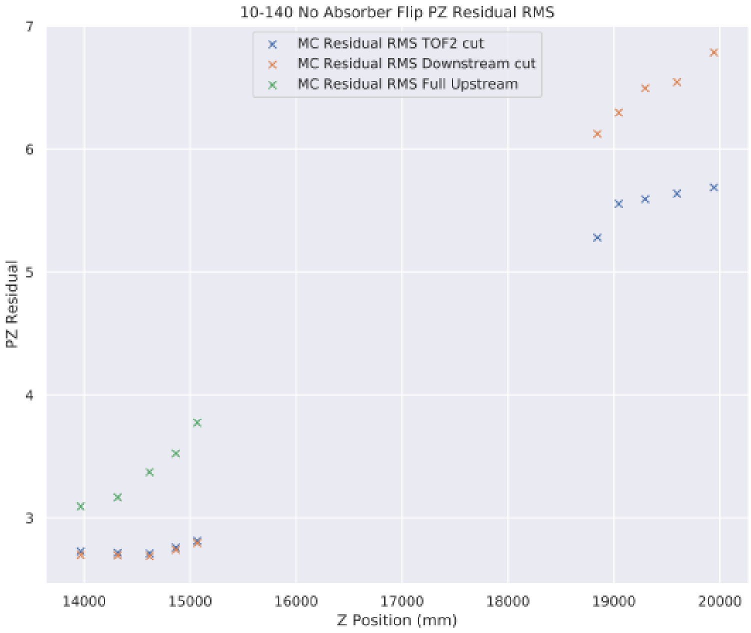
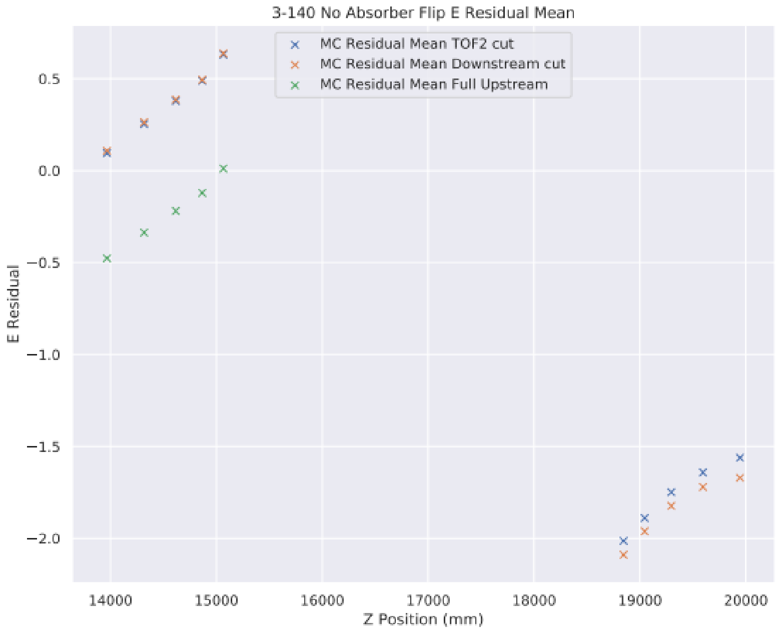
MC Residual Mean TOF2 cut
MC Residual Mean Downstream cut
MC Residual Mean Full Upstream
PZ Residual
Z Position (mm)


3-140 No Absorber Flip PZ Residual RMS
MC Residual RMS TOF2 cut
MC Residual RMS Downstream cut
MC Residual RMS Full Upstream
PZ Residual
Z Position (mm)


6-140 No Absorber Flip PZ Residual RMS
MC Residual RMS TOF2 cut
MC Residual RMS Downstream cut
MC Residual RMS Full Upstream
PZ Residual
Z Position (mm)


10-140 No Absorber Flip PZ Residual RMS
MC Residual RMS TOF2 cut
MC Residual RMS Downstream cut
MC Residual RMS Full Upstream
PZ Residual
Z Position (mm)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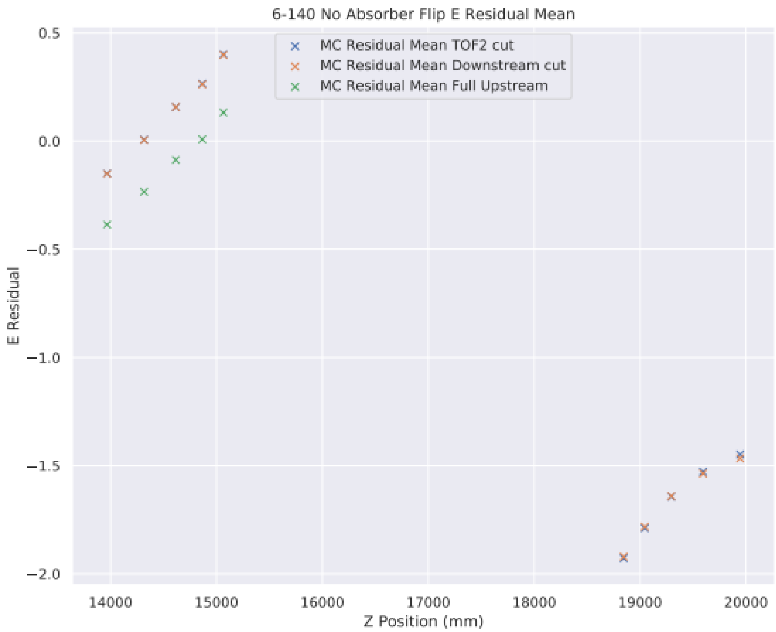
MC Residual Mean TOF2 cut
MC Residual Mean Downstream cut
MC Residual Mean Full Upstream
E Residual
Z Position (mm)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MC Residual Mean TOF2 cut
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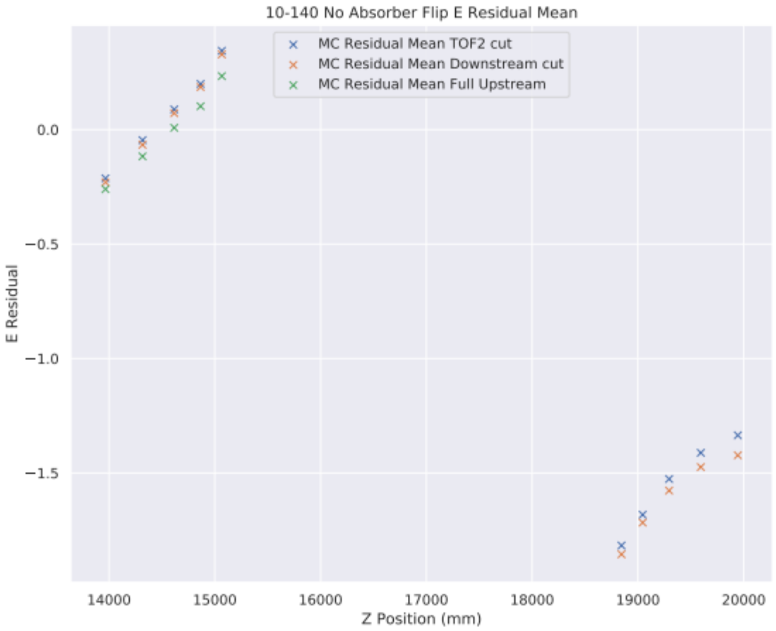
E Residual
Z Position (mm)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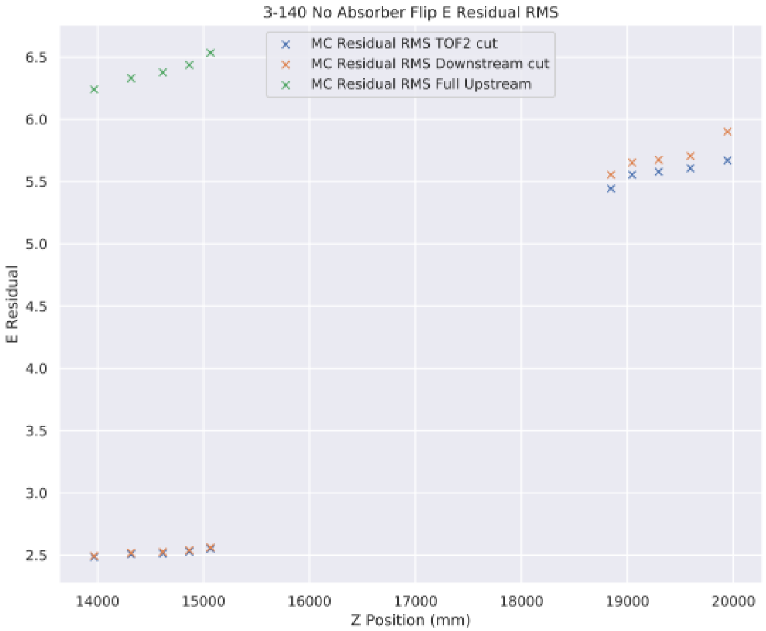
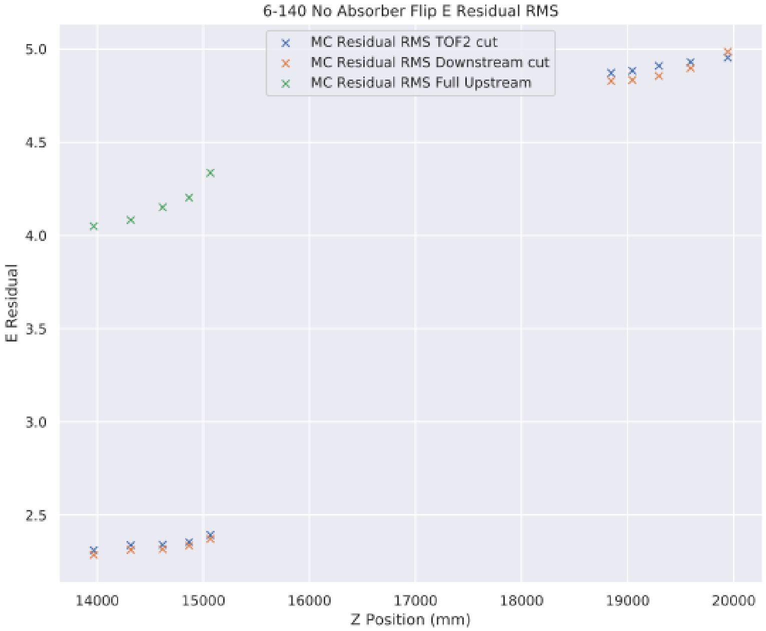
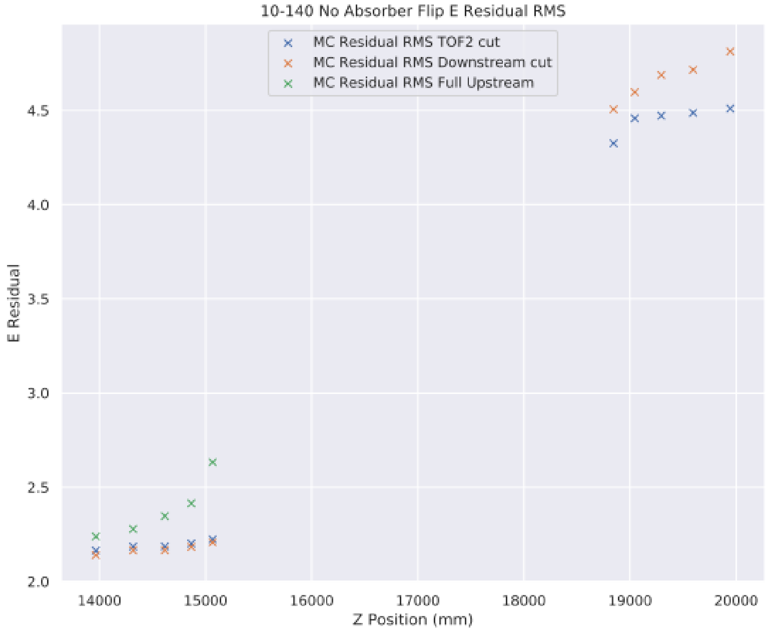
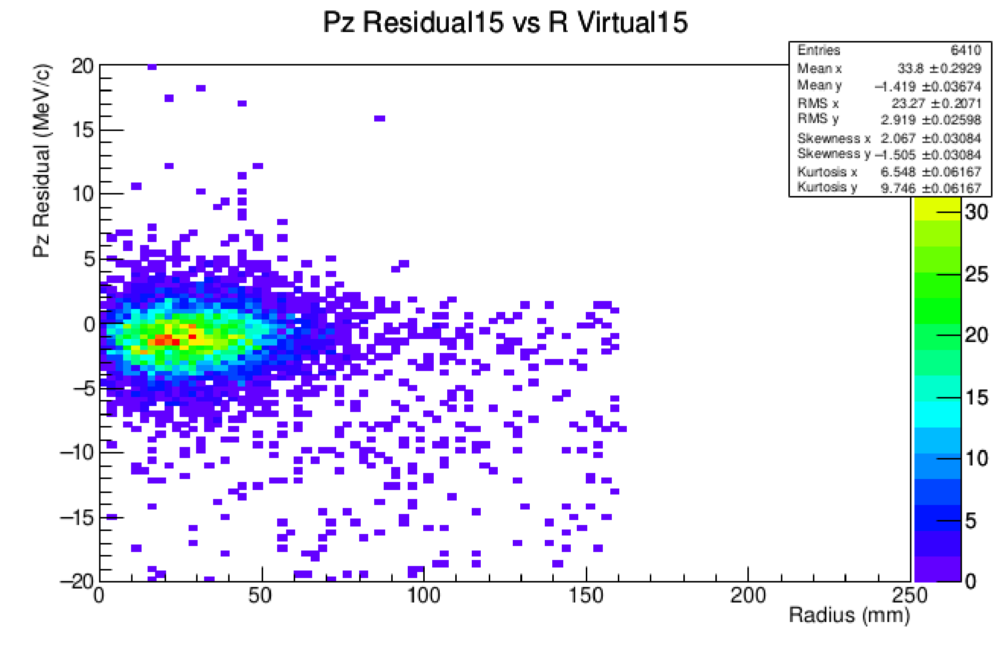
MC Residual Mean Downstream cut
MC Residual Mean Full Upstream
E Residual
Z Position (mm)

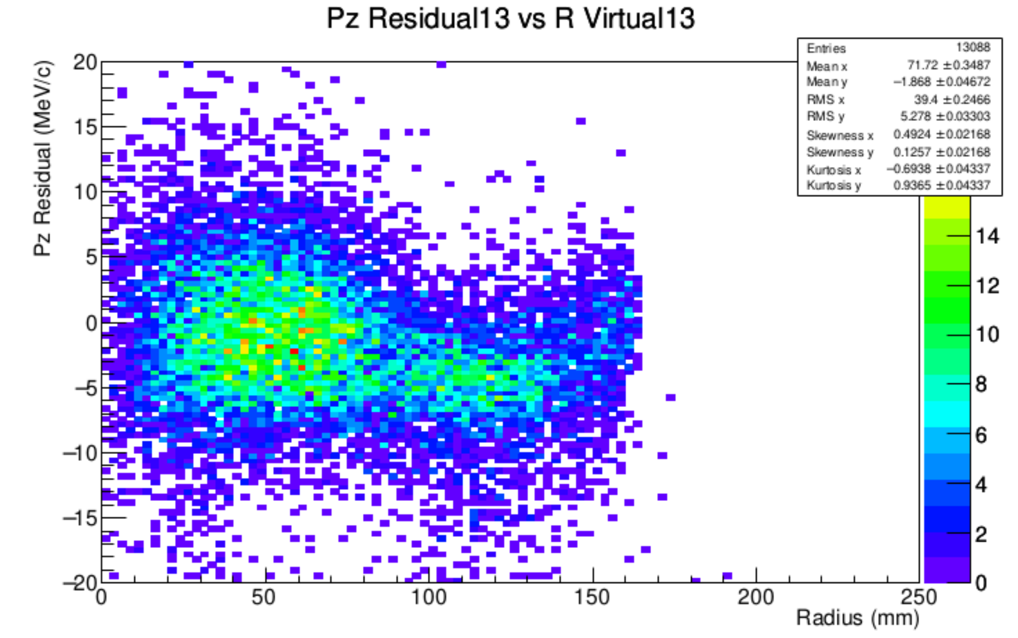

3-140 No Absorber Flip E Residual RMS
MC Residual RMS TOF2 cut
MC Residual RMS Downstream cut
MC Residual RMS Full Upstream
E Residual
Z Position (mm)


6-140 No Absorber Flip E Residual RMS
MC Residual RMS TOF2 cut
MC Residual RMS Downstream cut
MC Residual RMS Full Upstream
E Residual
Z Position (mm)

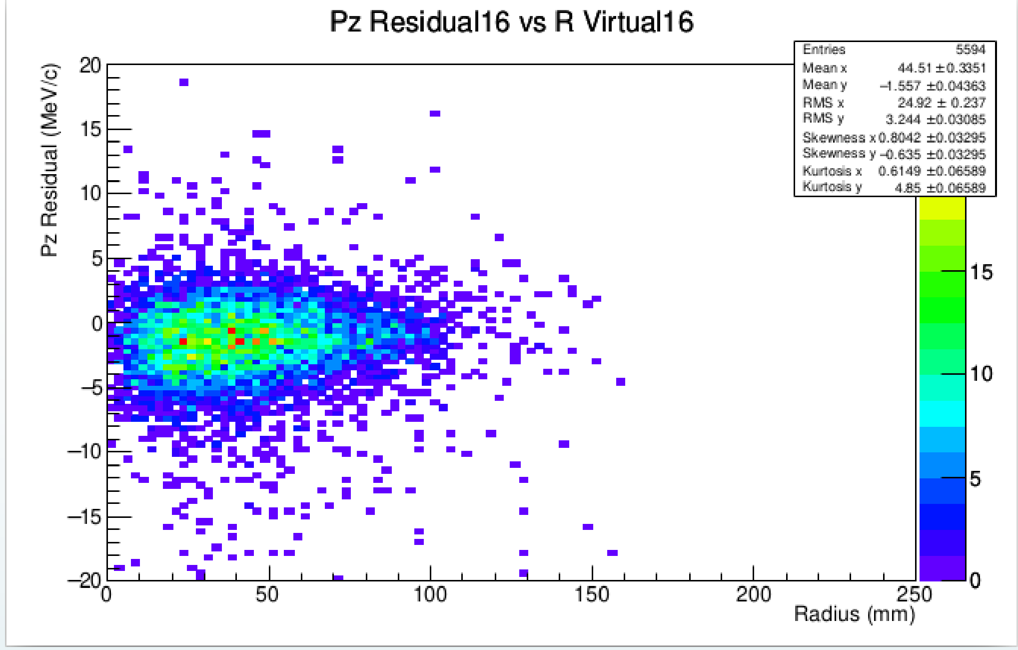

10-140 No Absorber Flip E Residual RMS
MC Residual RMS TOF2 cut
MC Residual RMS Downstream cut
MC Residual RMS Full Upstream
E Residual
Z Position (mm)



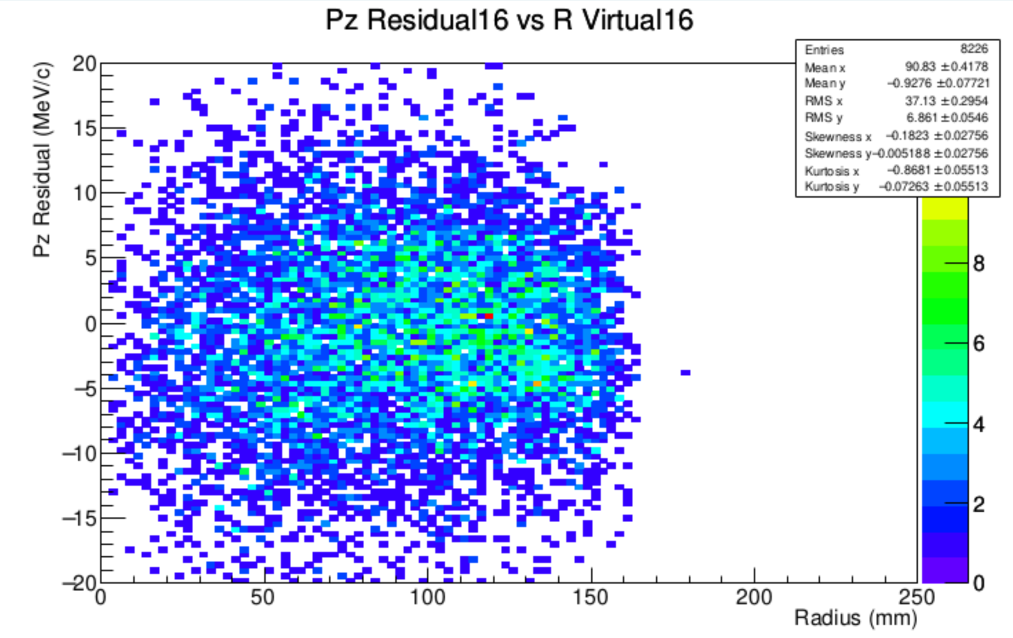
TKU Reference Plane: 3-140 above
TKU Reference Plane: 3-240 Below

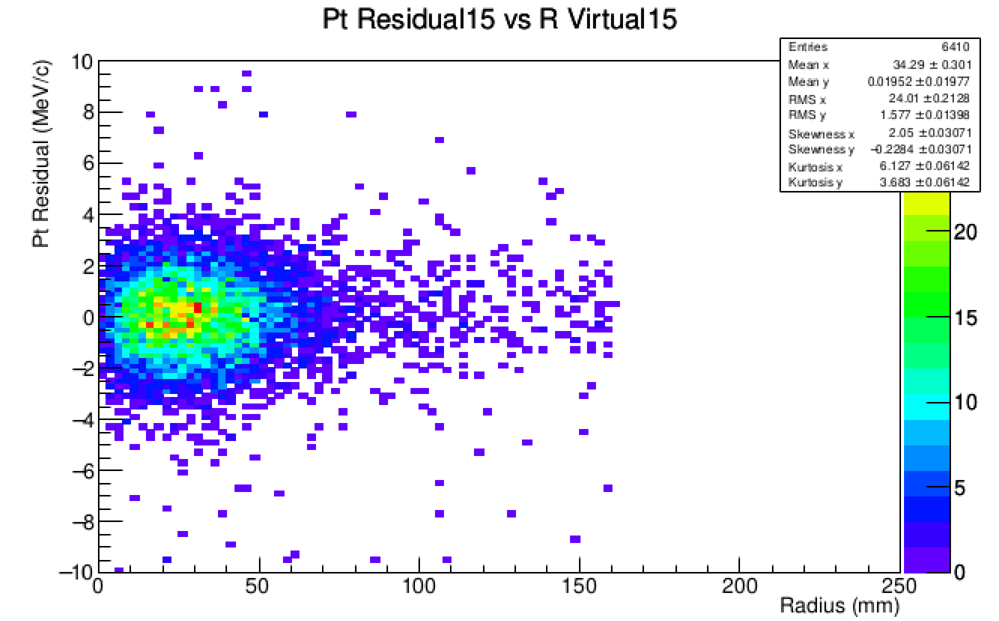


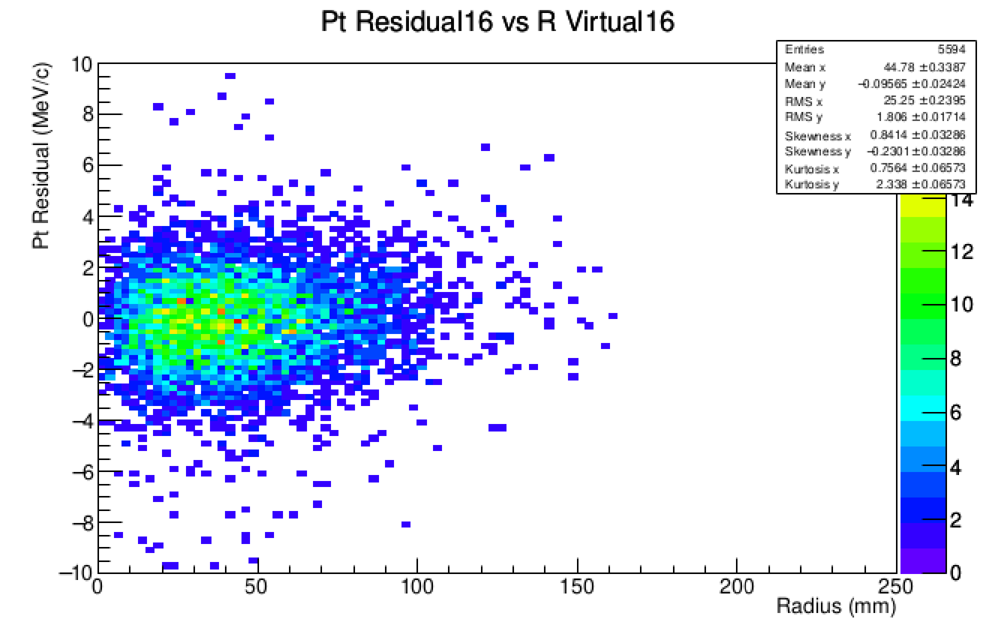
- Pz Residual vs Radial Position at Station
- The Radial Position can affect the Residual, as well as the parent momentum
- It can also be different between trackers



TKD Reference Plane: 3-140 above
TKD Reference Plane: 3-240 Below



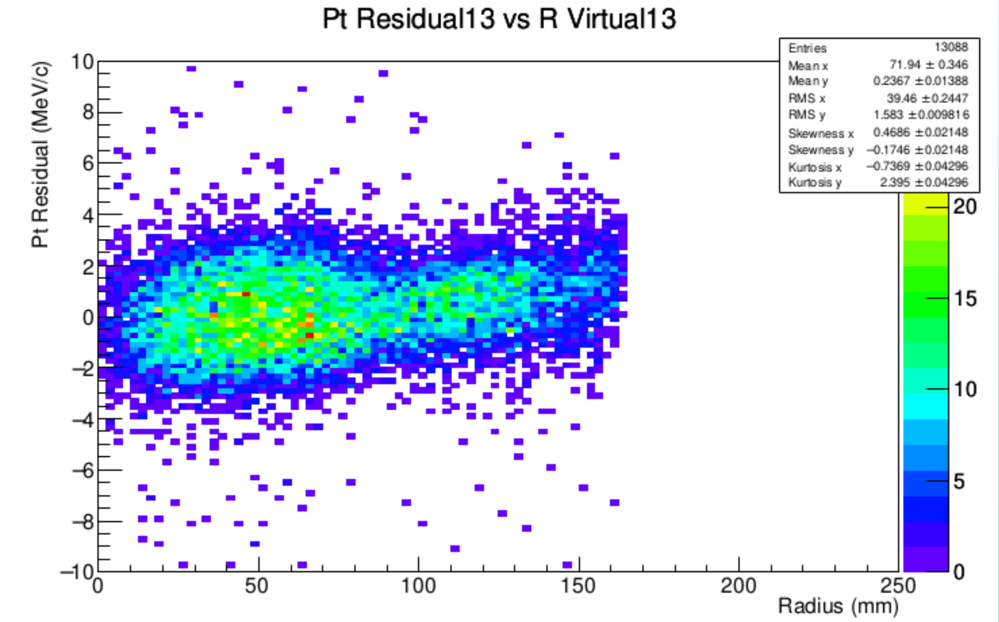


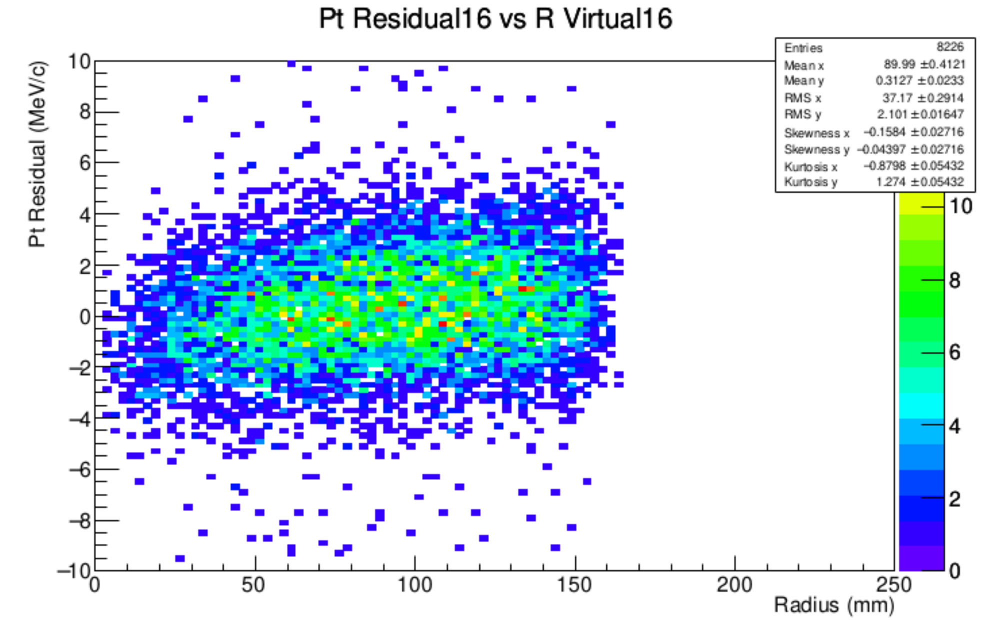


- Pt Residual vs Radial Position at Station
- The Radial Position can affect the Residual, as well as the parent momentum
- It can also be different between trackers

TKU Reference Plane: 3-140 above
TKU Reference Plane: 3-240 Below

TKD Reference Plane: 3-140 above
TKD Reference Plane: 3-240 Below



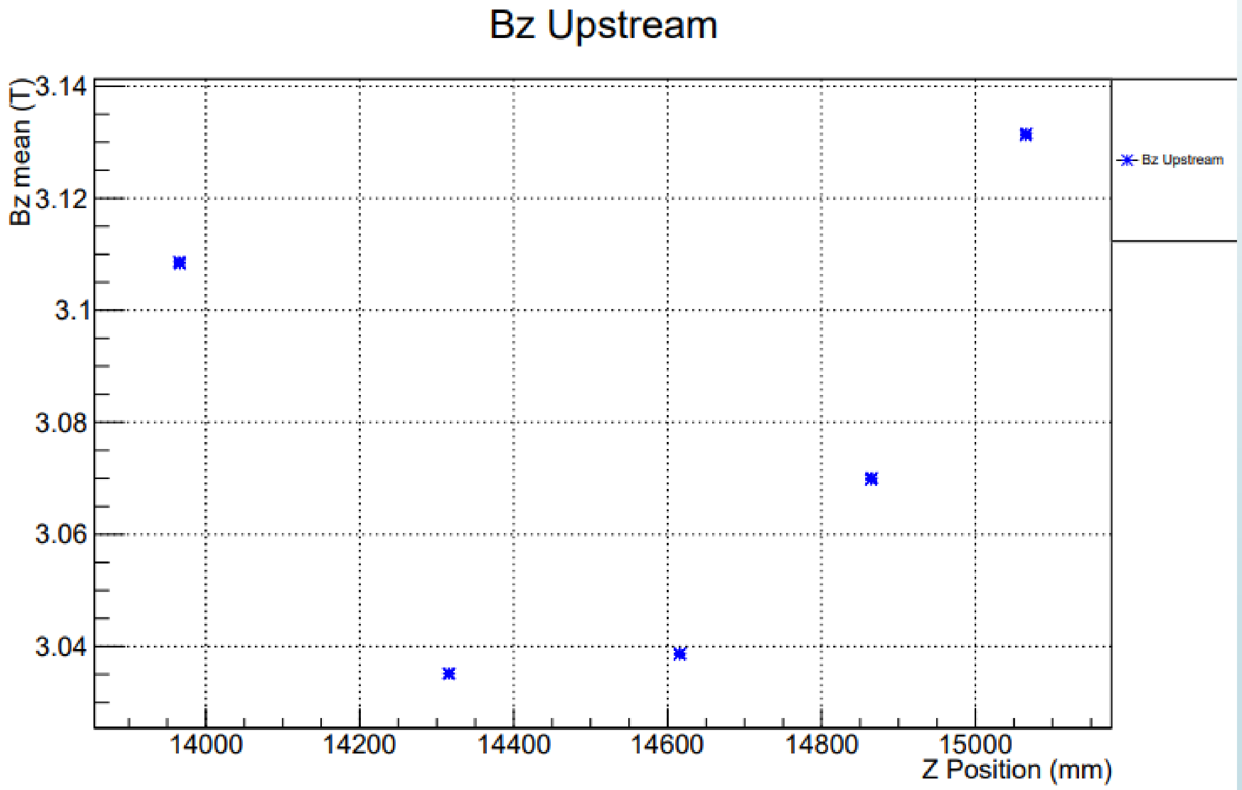


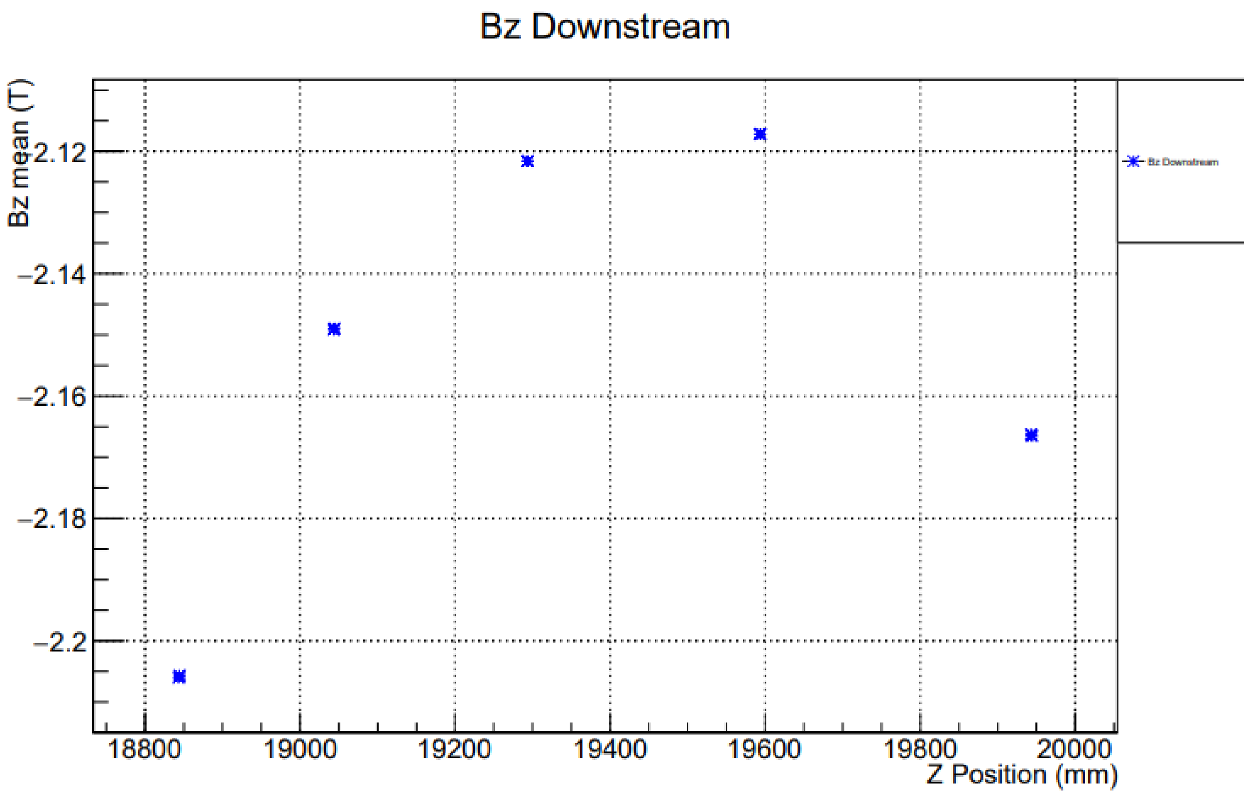
Pt, Pz and E

- They show Tracker dependence
- They show Plane Dependence
- They show cut selection dependence
- Recon shows momentum dependence
- Input Emittance Beam shows relatively small effect

- Recon assumes magnetic field homogeneity of +/- 1%
- Earlier showed Pt Residual followed similar shape to Bz field
- What is the Bz Residual

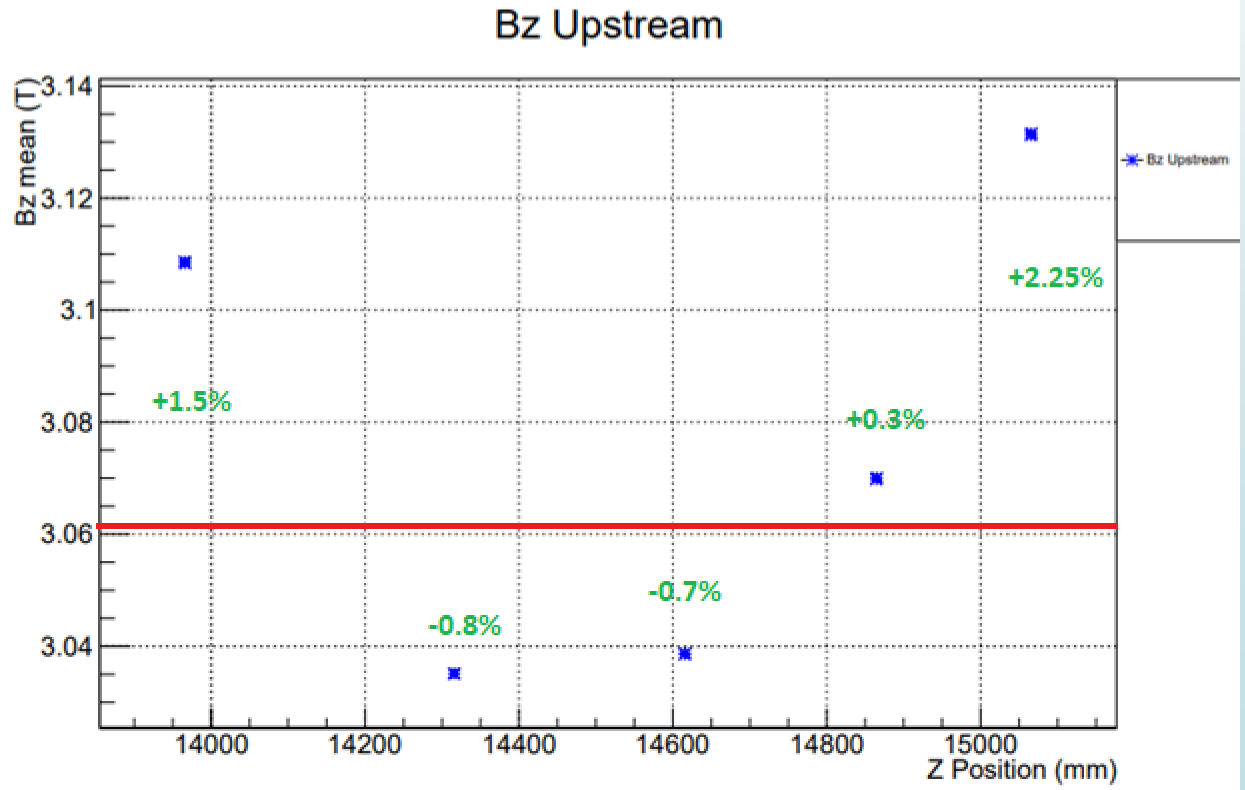
Bz Fields

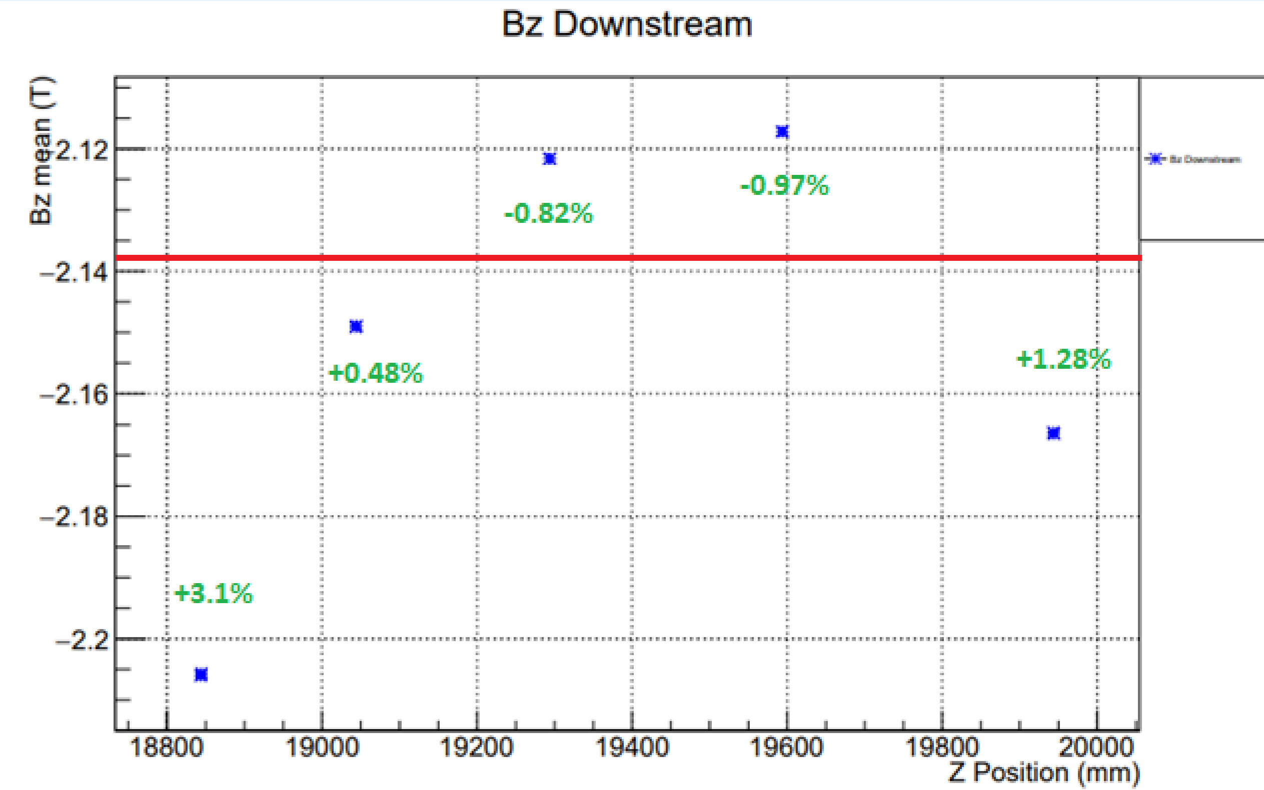

Bz Upstream
Bz mean (T)
3.14
3.12
3.1
3.08
3.06
3.04
14000
14200
14400
14600
14800
15000
Z Position (mm)
Bz Upstream

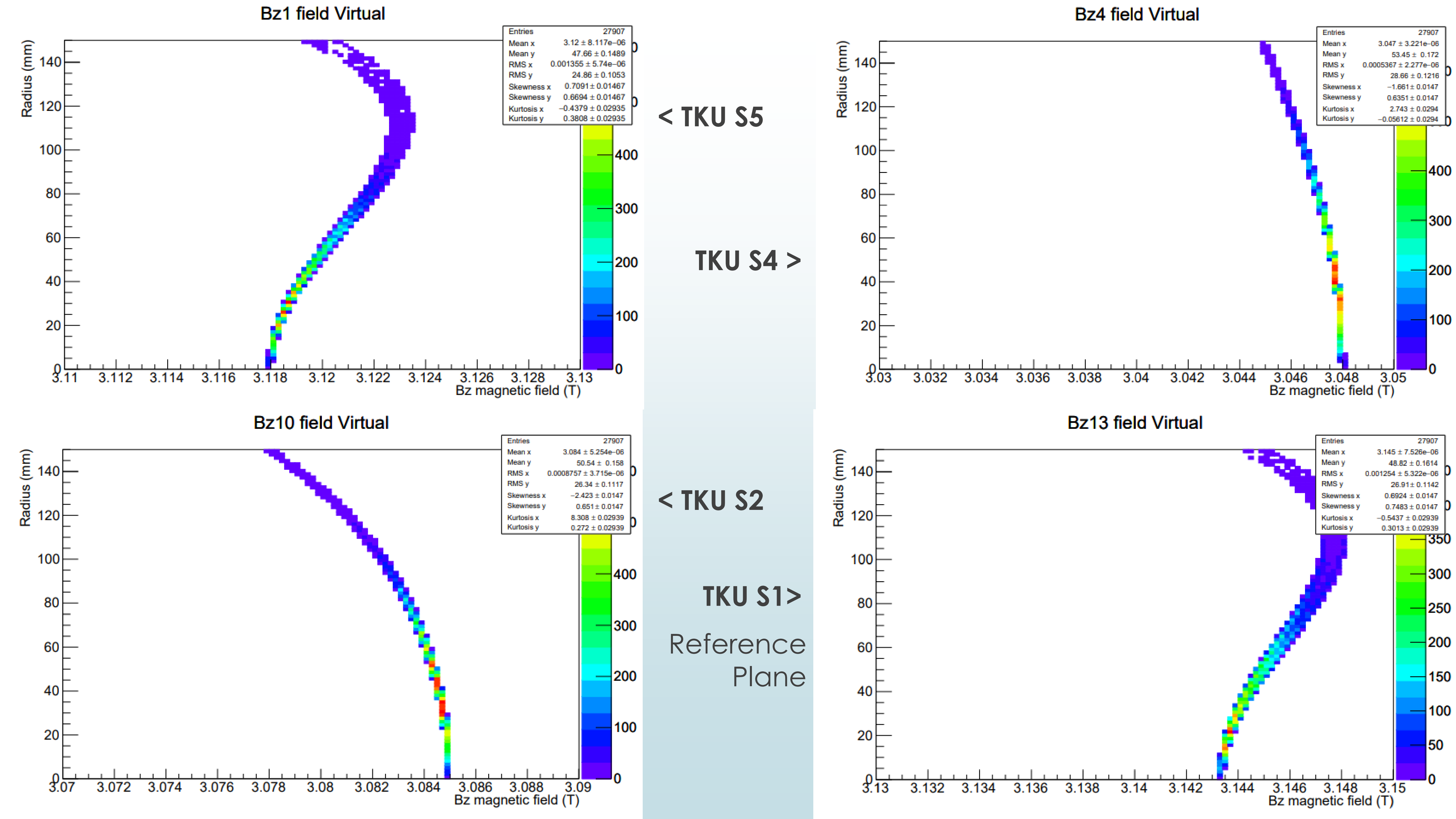

Bz Downstream
Bz mean (T)
−2.12
−2.14
−2.16
−2.18
−2.2
18800
19000
19200
19400
19600
19800
20000
Z Position (mm)
Bz Downstream

Bz fields

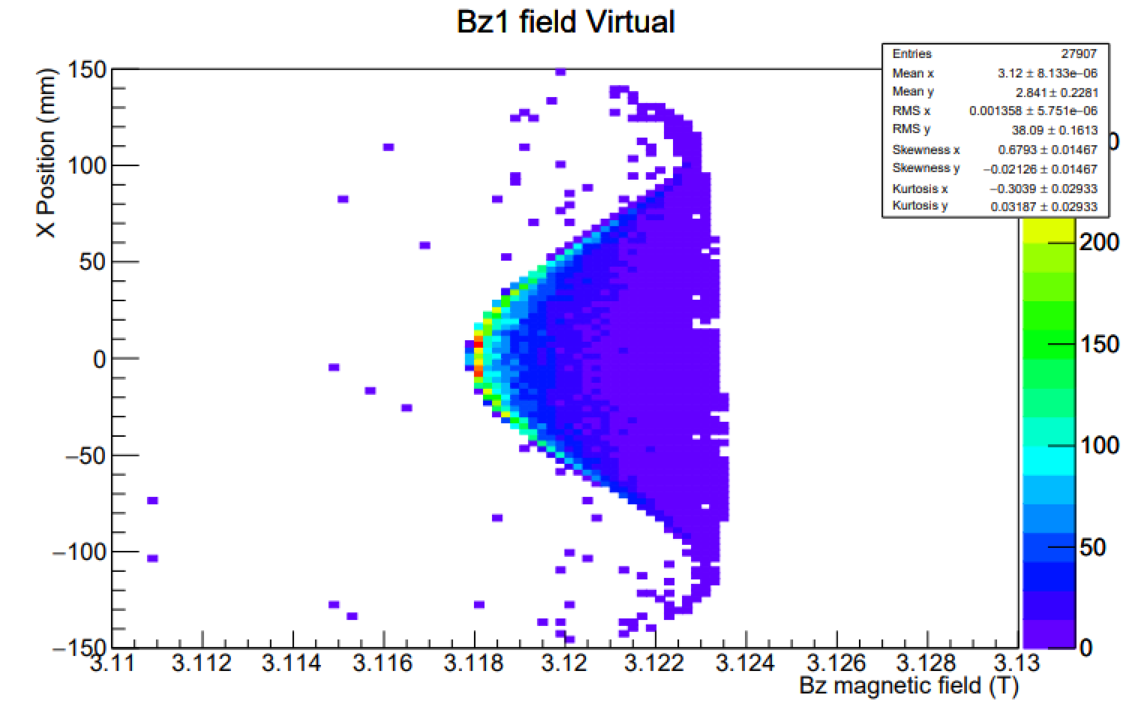
- Recon assumes mean magnetic field through Tracker
- However field varies significantly and is worst at reference planes (2-3%)
- It is also different between upstream and downstream due to different field strengths
- Next Slide: Field is not even uniform within station (Apologies: from different run)



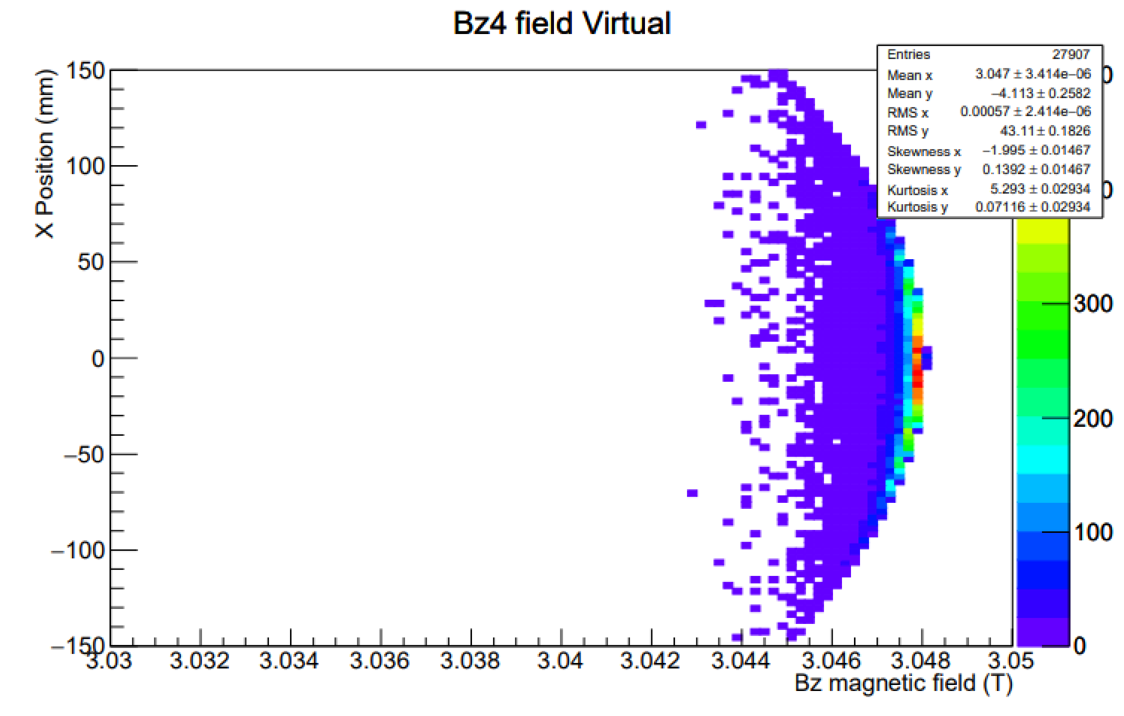




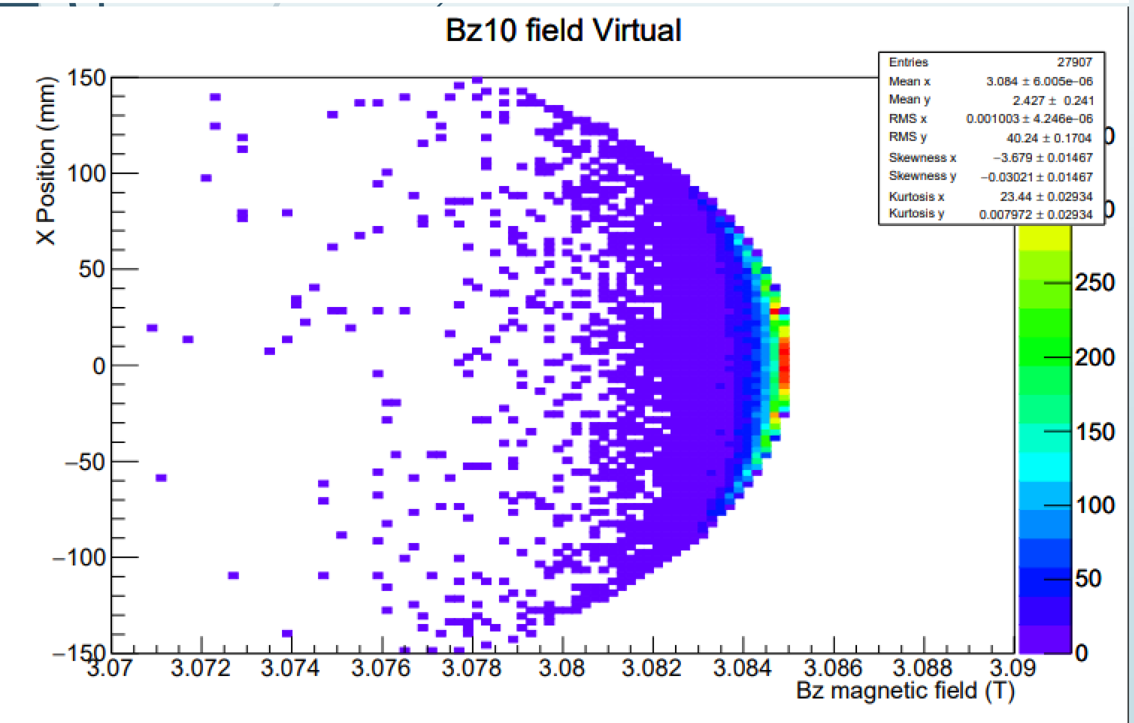
Bz1 field Virtual
Entries 27907
Mean x 3.12 ± 8.117e−06
Mean y 47.66 ± 0.1489
RMS x 0.001355 ± 5.74e−06
RMS y 24.86 ± 0.1053
Skewness x 0.7091 ± 0.01467
Skewness y 0.6694 ± 0.01467
Kurtosis x −0.4379 ± 0.02935
Kurtosis y 0.3808 ± 0.02935
Radius (mm)
Bz magnetic field (T)
Bz4 field Virtual
Entries 27907
Mean x 3.047 ± 3.221e−06
Mean y 53.45 ± 0.172
RMS x 0.0005367 ± 2.277e−06
RMS y 28.66 ± 0.1216
Skewness x −1.661 ± 0.0147
Skewness y 0.6351 ± 0.0147
Kurtosis x 2.743 ± 0.0294
Kurtosis y −0.05612 ± 0.0294
Radius (mm)
Bz magnetic field (T)
< TKU S5
TKU S4 >
< TKU S2
TKU S1>
Reference Plane
Bz10 field Virtual
Entries 27907
Mean x 3.084 ± 5.254e−06
Mean y 50.54 ± 0.158
RMS x 0.0008757 ± 3.715e−06
RMS y 26.34 ± 0.1117
Skewness x −2.423 ± 0.0147
Skewness y 0.651 ± 0.0147
Kurtosis x 8.308 ± 0.02939
Kurtosis y 0.272 ± 0.02939
Radius (mm)
Bz magnetic field (T)
Bz13 field Virtual
Entries 27907
Mean x 3.145 ± 7.526e−06
Mean y 48.82 ± 0.1614
RMS x 0.001254 ± 5.322e−06
RMS y 26.91 ± 0.1142
Skewness x 0.6924 ± 0.0147
Skewness y 0.7483 ± 0.0147
Kurtosis x −0.5437 ± 0.02939
Kurtosis y 0.3013 ± 0.02939
Radius (mm)
Bz magnetic field (T)



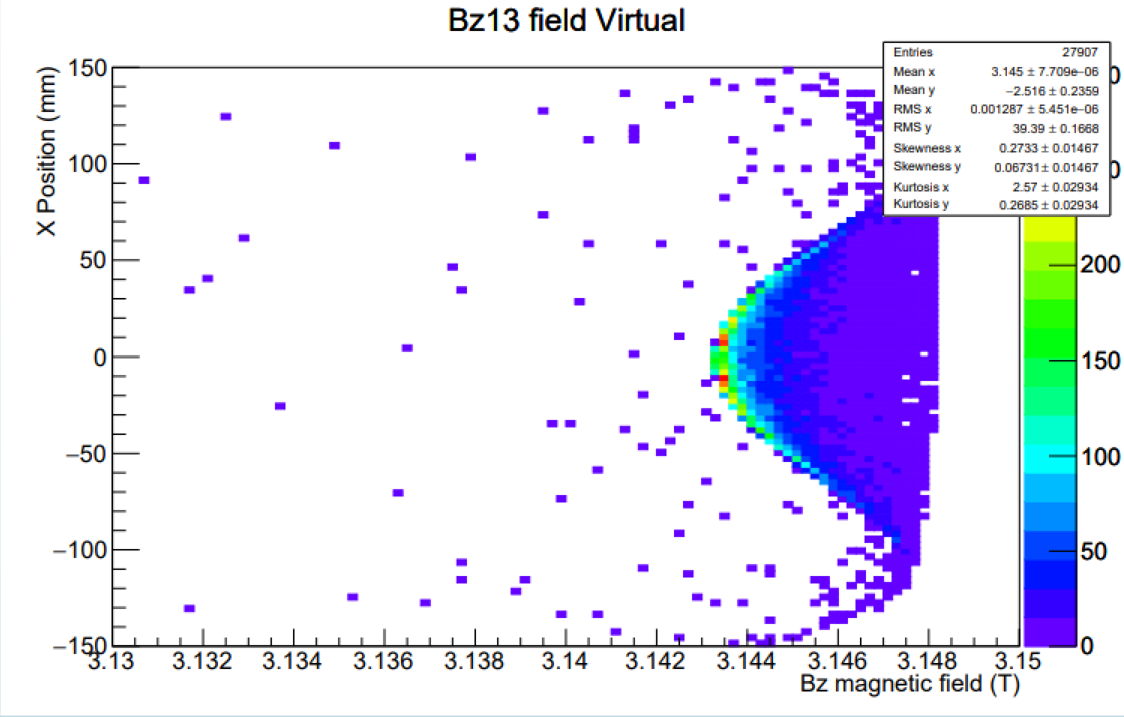
< TKU S5



TKU S4 >



< TKU S2



TKU S1>

Reference Plane

Bz field

- Field is not uniform
- Recon will underestimate/overestimate at certain station
- Not the only effect

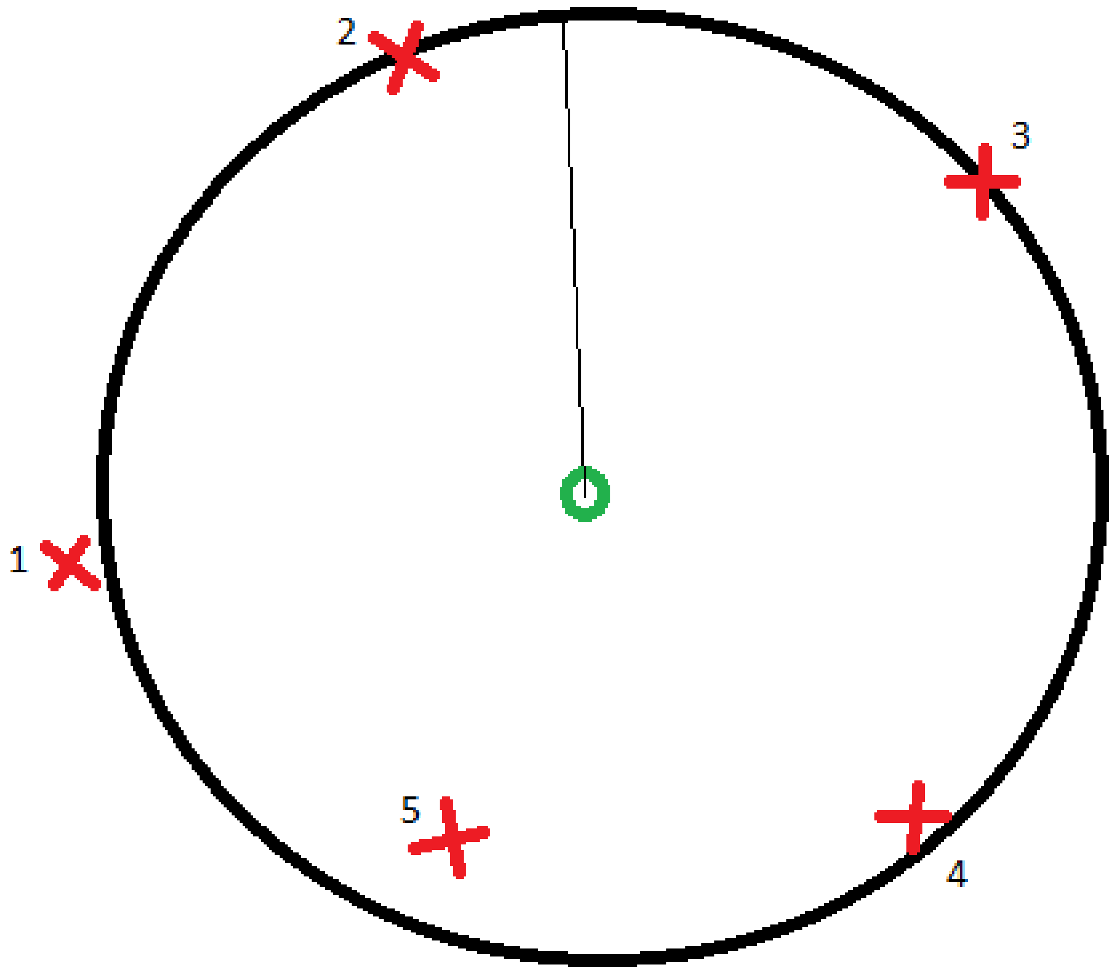
- Next: Energy loss

Path of particle in ideal solenoid

- If there is no Energy Loss, then the particle will follow a constant radius path
- If there is a constant Energy Loss with no scattering, then the particle will spiral towards a centre with radius $r = a\varphi$, where φ is the turning angle and a is angle of the polar slope (between tangent and polar circle, dictates expansion of spiral).
- dE/dx is fairly constant through the stations as the Energy Loss is small (or as implemented by MAUS)
- In MICE we have 5 stations per tracker. Between stations the particles follow a helical path (with no Energy Loss, assume perfect vacuum) and are deviated at the station.
- At the station, Energy Loss occurs, and the particle is deviated to a lower radius path but remains tangential to the circle centre unless scattered.
- This in turn creates a new circle centre along the radial path. The radius change is proportional to the Energy Loss.

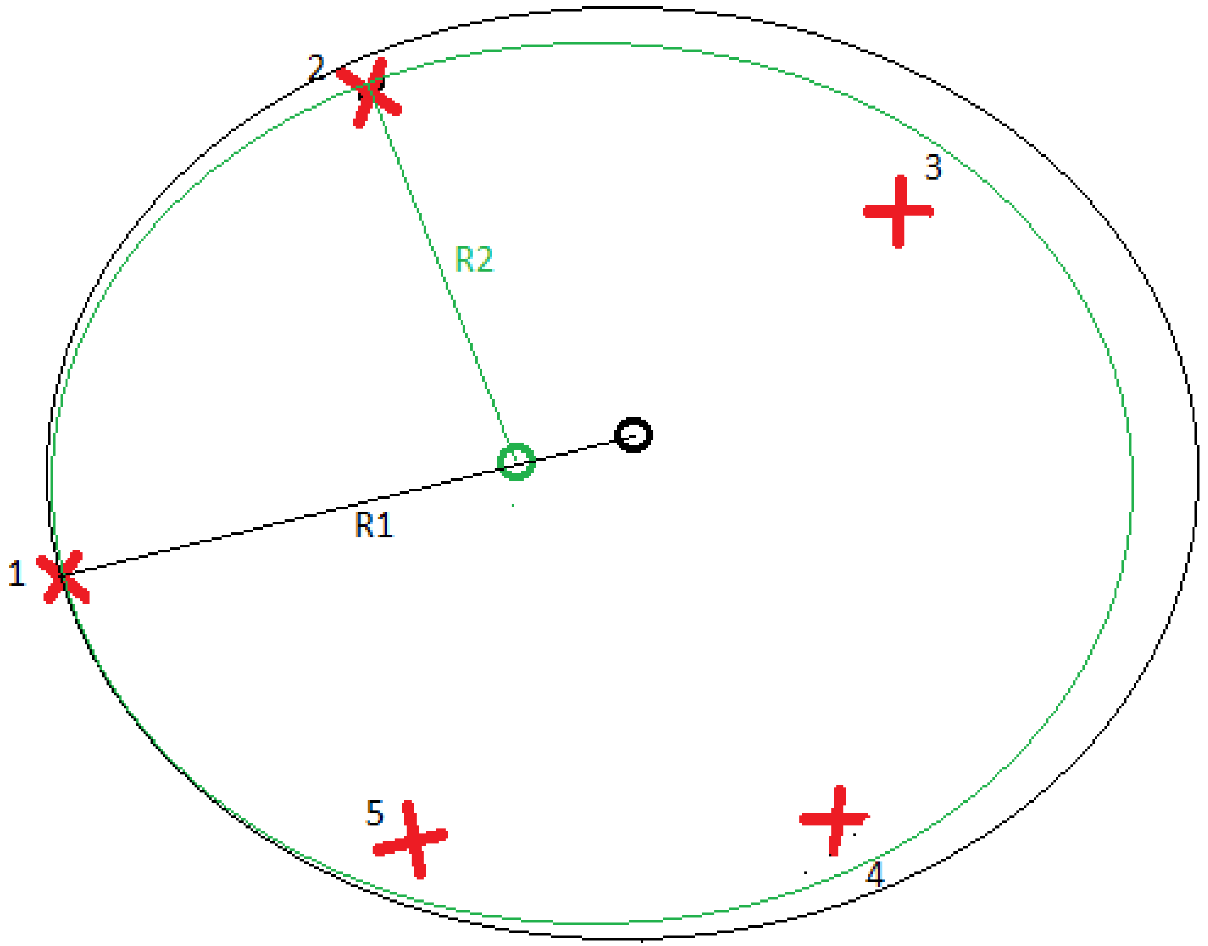
Exaggerated case – not to any scale

Circle fit radius of five stations

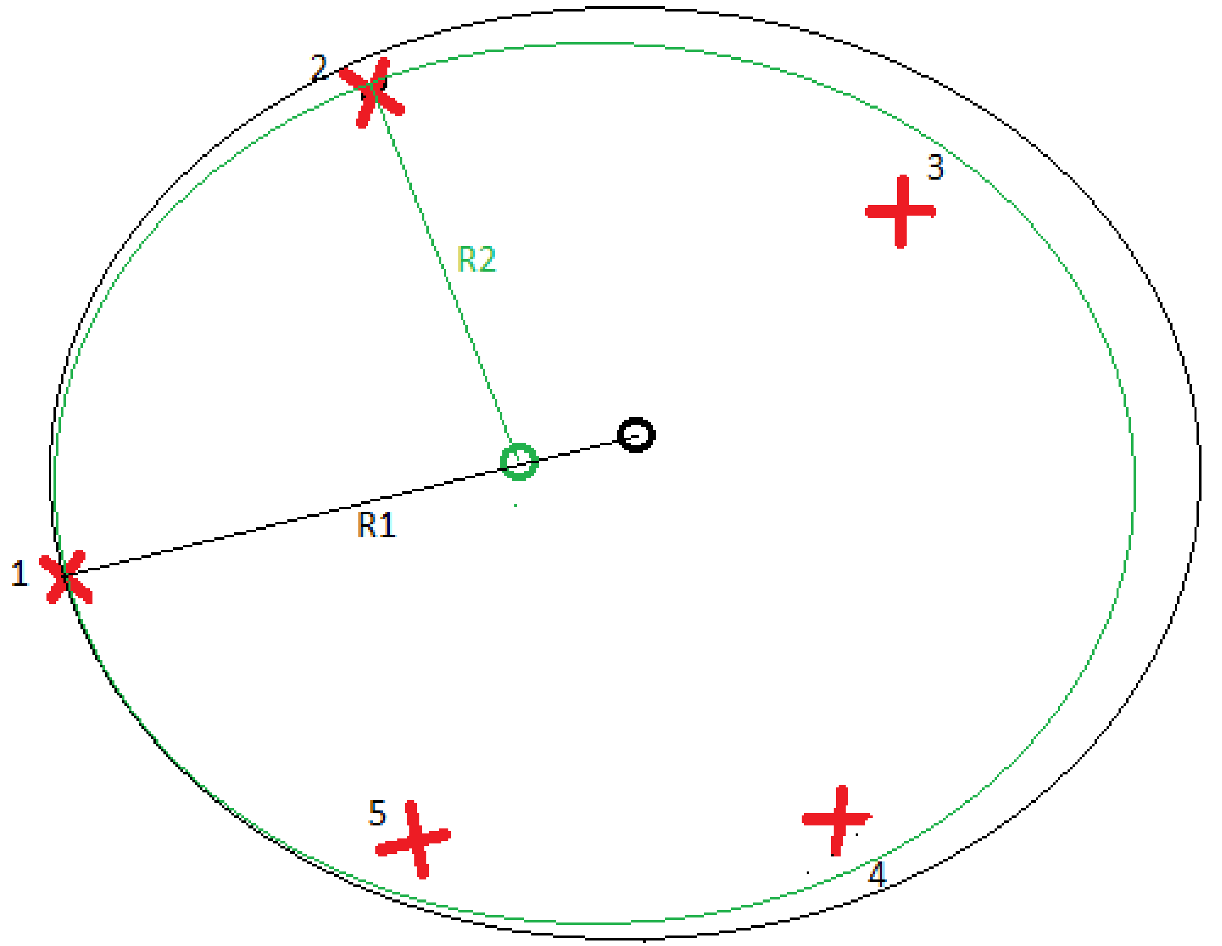


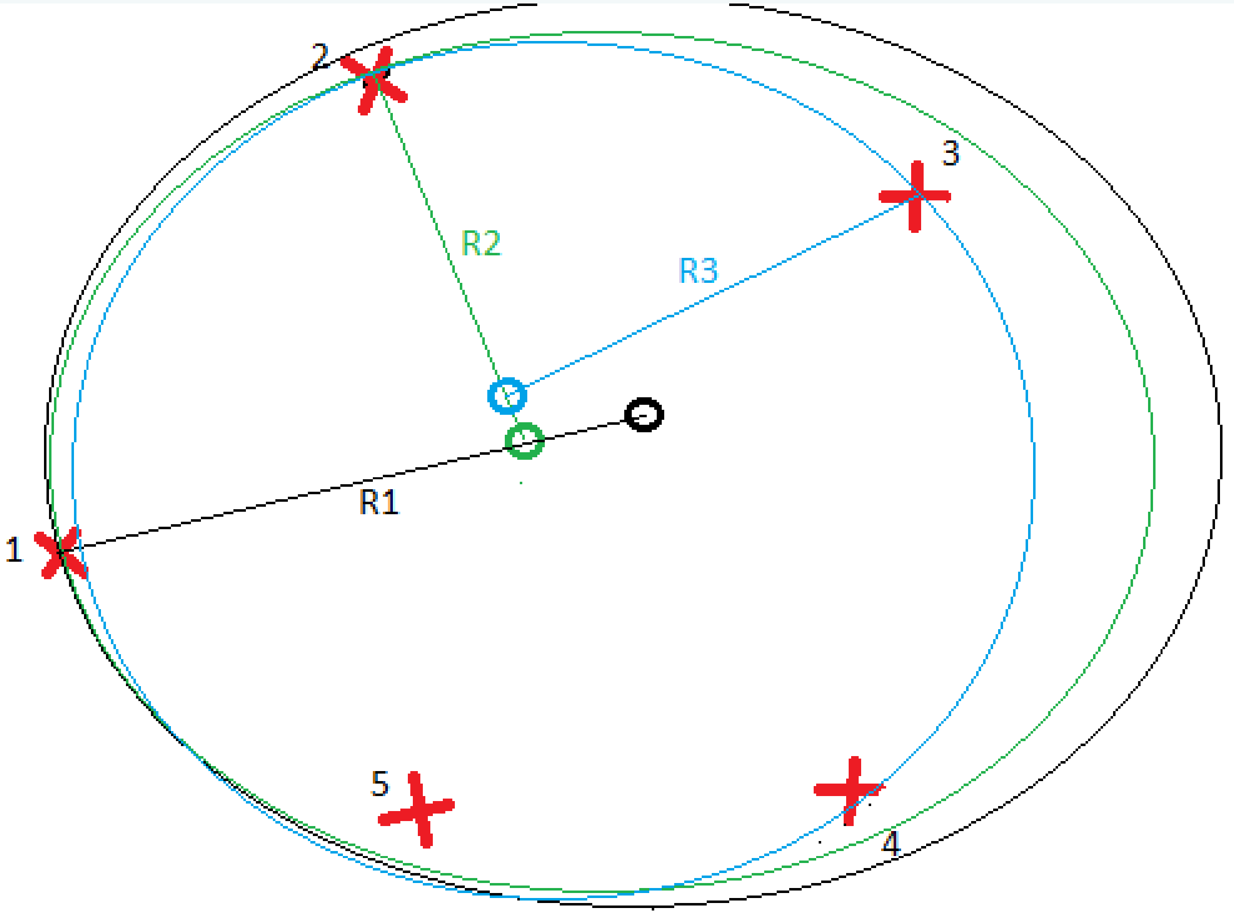
R1 true radius of initial particle

R2 true radius of particle after Energy Loss through 1st station, with new centre

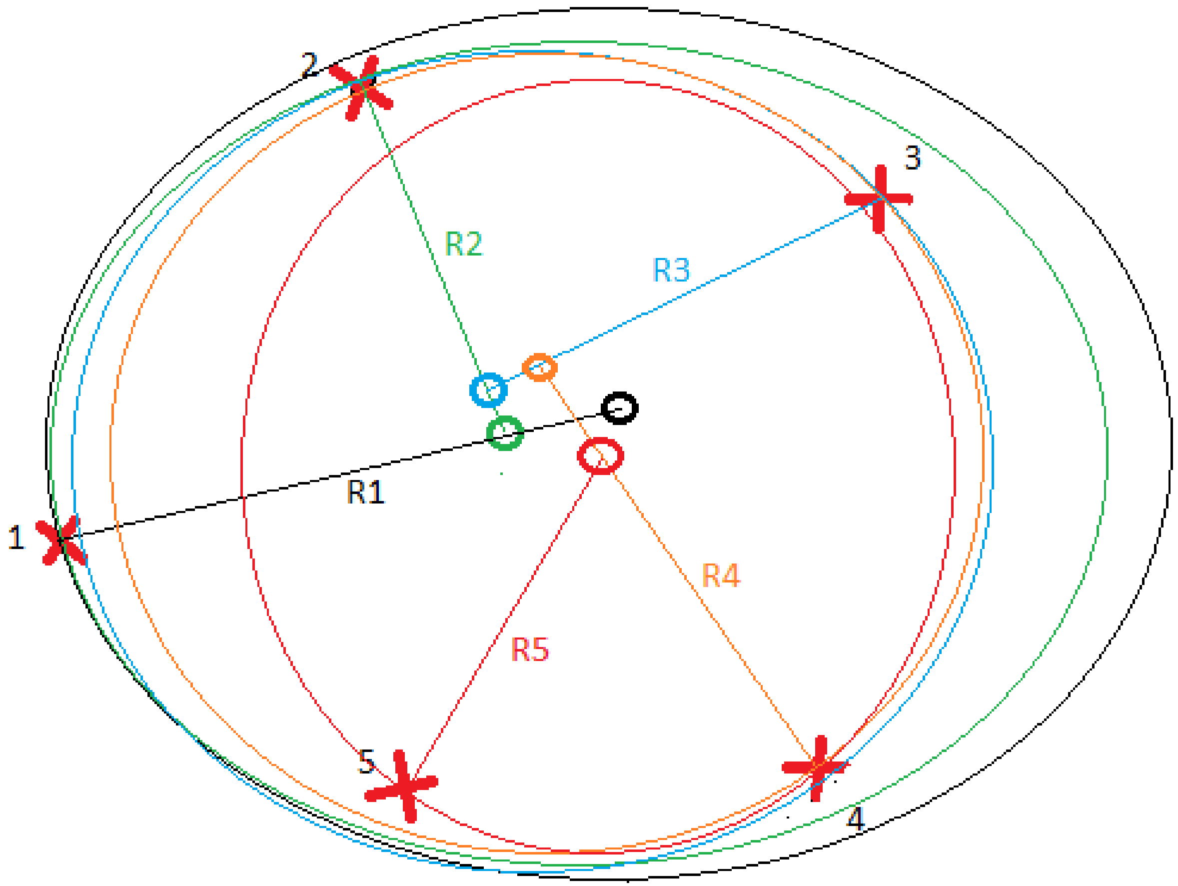


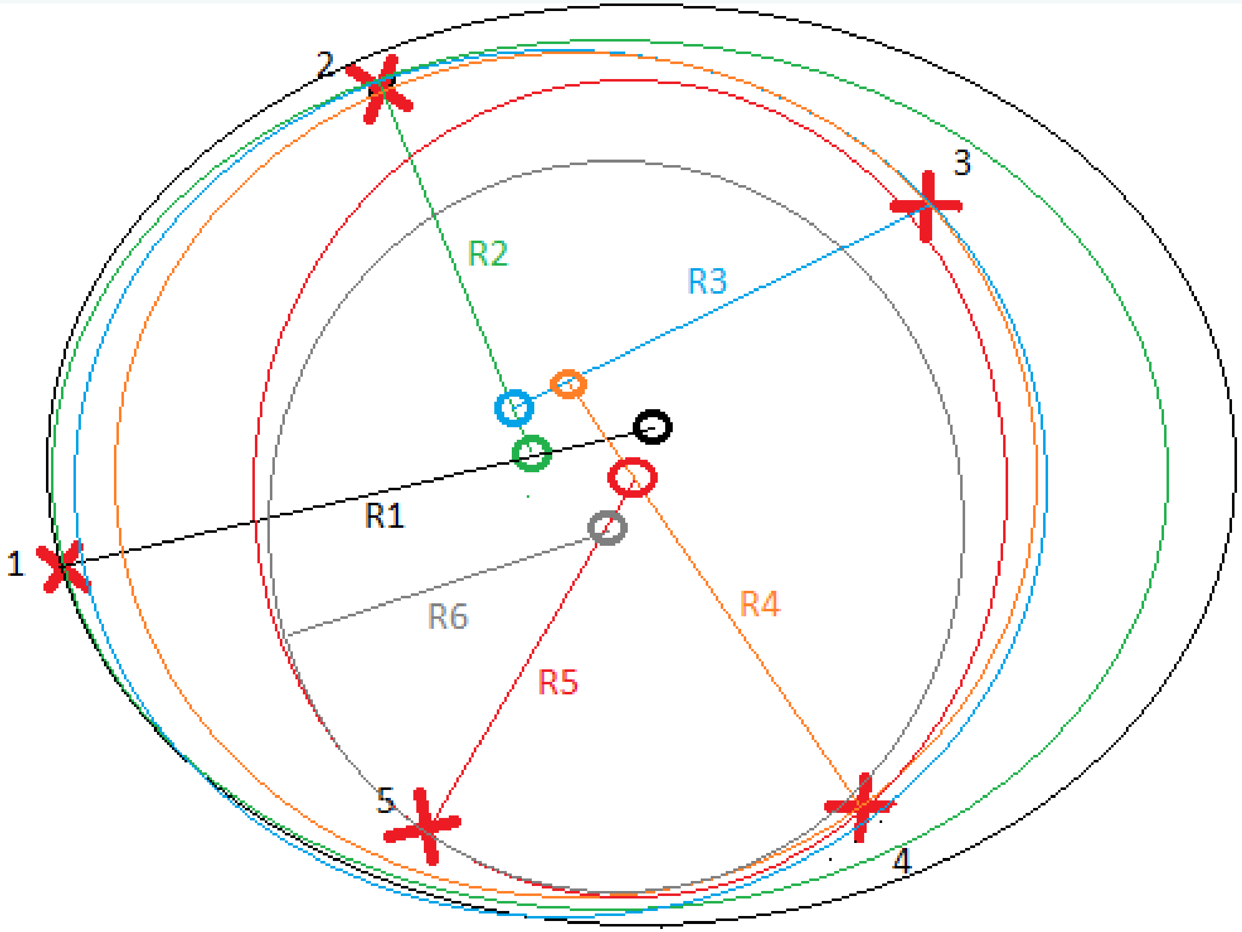
Before Station 2 to after Station 2





Before Station 5 to after Station 5





What affect does it have on Pt and Pz

- $p_t = cBQR$
- c, B and Q are constant (should be), so transverse momentum changes by radius loss
- A particle loses approximately 0.6 MeV per station, so ~ 3 MeV per tracker, which for a 140 MeV particle is ~2%
- Therefore the radius from start to finish reduces by 2%
- For a high radius particle, e.g. 100mm, this radius reduction would be more than a few widths of fibres, leading to a poor qui-squared value for the circle fit and thus being excluded

What effect does it have on Pt and Pz

- z-s plane
- Another qui-squared cut is made in the z-s plane, if the fit in the z-s plane fits a straight line.
- $z = \frac{dz}{ds} s - s_0 \; with \; s = R\varphi$, however if the radius is not constant, or not the appropriate radius (wrong circle centre), then the phase advance will be wrong.
- Should have straight line between stations in s-z plane, however a small deviation at each station. That deviation should be similar at each station (i.e. angle change)
- A too strict straight line qui-squared cut may exclude valid particles, but more importantly:

$$p_z / p_t = \Delta z / R\Delta\varphi$$

- The p_t to R ratio should be fairly constant and thus p_z is heavily influenced by the phase advance.
- If the movement of circle centre isn't accounted for, then will have the wrong phase advance angle

Energy Loss and varying Magnetic field - Effect on Particle Motion

- This is a back of the envelope calculation to a certain extent – MC to come in future
- 100 column shows what radius at that station is with real field as percentage for comparable Mean field radius of 100% through the 10 stations
- 140 column shows what pz = 140 using mean field would equate to with field at station
- No Energy loss added
- There is variance on order of up to 4%, and also difference between TKU and TKD

					R				
Mean Field	Bx	By	Bz	3.01436	1	100	20		140
3.01436	-0.0103	-0.0033	3.06		0.985084967	98.50849673	19.70169935		142.1197203
	-0.0028	-0.00189	2.988		1.008821954	100.8821954	20.17643909		138.7757269
	0.0008	-0.0018	2.9929		1.007170303	100.7170303	20.14340606		139.0033042
	0.007	-0.0024	3.0247		0.996581479	99.65814792	19.93162958		140.4802346
	0.0081	-0.001	3.0873		0.976374178	97.63741781	19.52748356		143.3876511
				2.1169					
2.1169	0.0216	-0.0069	2.1877		0.967637245	96.76372446	19.35274489		144.6823185
	0.0112	-0.0122	2.1246		0.996375788	99.63757884	19.92751577		140.5092352
	0.00018	-0.00415	2.1004		1.007855647	100.7855647	20.15711293		138.9087817
	-0.0024	-0.00176	2.0928		1.011515673	101.1515673	20.23031346		138.40616
	-0.0062	0.007	2.1413		0.988605053	98.8605053	19.77210106		141.6136804

Energy Loss and varying Magnetic field - Effect on Particle Motion

- Now, add in rough 0.6 MeV Energy Loss per station to see affect on radius
- Comparing then solely affect of change in radius on PZ
- Last column then shows combined effect
- One can see between reference planes approximately 2.5 MeV has been added

	IN	OUT	140				
100			0.6			140	
98.50849673	98.50849673	98.08631746	139.4	0.995714286		142.1197203	141.5106358
100.8821954	100.4498432	100.0174909	138.8	0.995695839		138.7757269	140.9015512
100.7170303	99.85374147	99.42209706	138.2	0.995677233		139.0033042	140.2924667
99.65814792	98.37682887	97.94972252	137.6	0.995658466		140.4802346	139.6833822
97.63741781	95.9636335	95.54518743	137	0.995639535		143.3876511	139.0742977
96.76372446	96.76372446	96.33994172	136.4	0.995620438		144.6823185	141.4972981
99.63757884	99.20120988	98.76484092	135.8	0.995601173		140.5092352	140.874876
100.7855647	99.90277139	99.46137475	135.2	0.995581738		138.9087817	140.2524539
101.1515673	99.82256858	99.37956902	134.6	0.99556213		138.40616	139.6300317
98.8605053	97.12864243	96.69567672	134	0.995542348		141.6136804	139.0076096

TKU to TKD adds ~ 2.5 MeV to Pz

Energy Loss and varying Magnetic field - Effect on Particle Motion

- Will be running MC to test effect of each scenario

- Uniform vs non-uniform field
- Affect of Energy Loss vs No Energy Loss at Trackers on Recon (for varying fields)
- Combination will be different in TKU and TKD, due to when Energy loss takes place. Delta Z between trackers are opposite and thus Energy Loss contribution comes mainly in latter half of TKU and earlier half of TKD
- Flip vs solenoid – Stations are near edges of field. Radial components will have different push/pull effects depending on flip or solenoid mode in TKD

- Try to combine effects to make meaningful correction to Recon

Magnetic field concerns

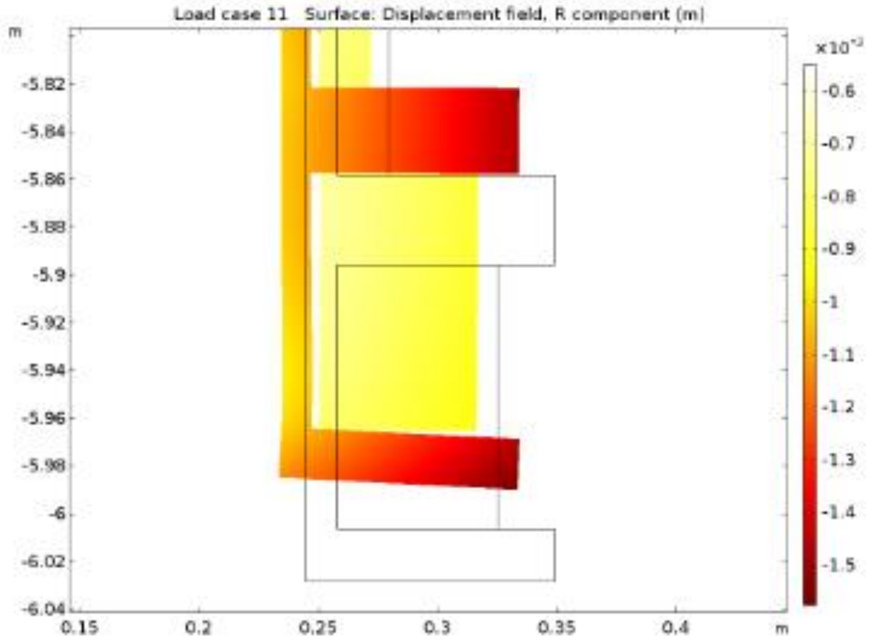


Fig. 3. Deformation plot of the E2 coil and the bobbin (deformation ten times amplified). The colour indicates the radial displacement.

SSU	M1 [mm]	M2 [mm]	E1 [mm]	CC [mm]	E2 [mm]
z	3449.22	3013.39	2613.39	1863.39	1113.39
z'	16106.55	15670.49	15270.29	14519.90	13769.50

Table 3.7 Table of centre positions in z and z' for all of SSU's coils. Coloured cells are those that are calculated from the method described above.

- Talk to long already, will talk more about this at next meeting
- Fields in MAUS 3.3.2 has scaled TKU field by 2% and TKD field by 1.8%
- Is this sensible?

- MAUS tracker and field rotate by different amounts depending on run configuration. Why? No answer yet
- Cobb/Blackmore – there are misalignments, worse in TKD, will affect separation of Longitudinal and Transverse components
- MAUS field doesn't include PRY, Langlands showed some variation
- MAUS uses warm magnet dimensions, not cold dimensions. There are also forces between coils – Witte, Langlands and Blackmore showed it is not insignificant
- Also to note Mandrel and coils have different cooling coefficients, factor of two difference
- Further investigations to come

Conclusion

- Covariance Matrix has significant effect on Emittance, Density and Amplitude calculations
- Transmission losses leads to survivorship bias which can skew or calculated cooling performance
- Method developed to separate the Covariance Matrix into a sample contribution and a missing contribution
- Method developed for a higher order Transfer Matrix. Needs to be tested between TKU and TKD
- Will lead to an unbiased determination of cooling
- Introduced Time-Coordinate to allow Wedge analysis, showed negligible bias
- Longitudinal emittance, amplitude and density measurements have problems due to Recon
- Recon introduces biases
- Biases appear to be due to assumptions used. Uniform field and Helix motion. Back of envelope calculation of Pz bias due to assumptions on par with Pz bias seen in Recon
- Non-uniform field is not insignificant
- Energy Loss distorts motion of particle from Helix to more of a spiral
- Magnetic Field concerns
- More MC to follow, which can hopefully aid in Pz Bias

THE END