# Weak gravity conjecture in an accelerating Universe

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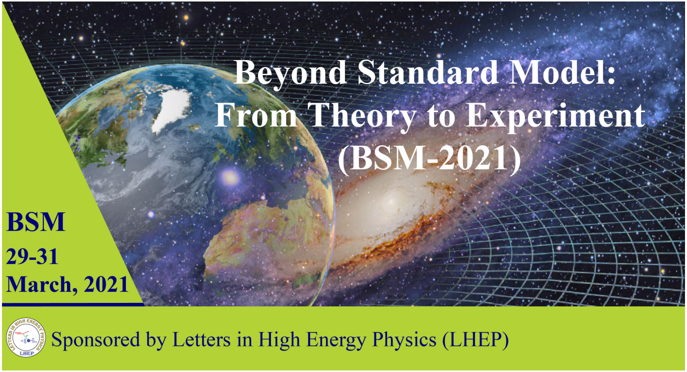


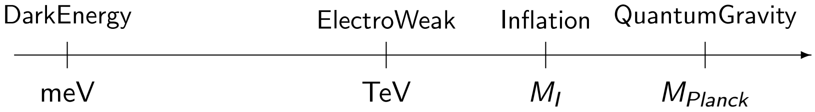
# Universe evolution: based on positive cosmological constant

- Dark Energy

  simplest case: infinitesimal (tuneable) +ve cosmological constant

- Inflation (approximate de Sitter)

  describe possible accelerated expanding phase of our universe



# de Sitter spacetime

vacuum solution of Einstein equations with +ve cosmological constant

and maximal symmetry: 10 isometries like flat space

$SO(4,1)$ vs Poincaré $E_4$

hyperboloid from 5 dimensions: $-y_0^2 + \vec{y}^2 = \frac{1}{H^2}$

$$R_{\mu\nu\lambda\rho} = H^2(g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda}) \qquad R = 12H^2 = 4\Lambda$$

Flat slicing: $ds^2 = -dt^2 + e^{2Ht}d\vec{x}^2$ exponential expansion

FRW with flat 3-space and scale factor $a(t) = e^{Ht}$

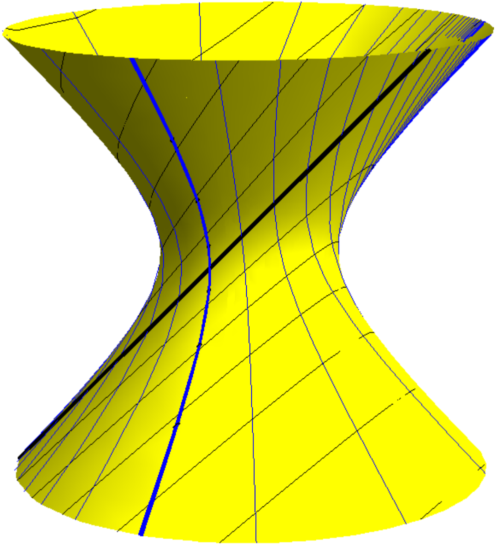
isometries: 3 space translations, 3 rotations, 1 scale, 3 special conformal

e.g. scale: $\vec{x} \to \omega^2\vec{x}$ and $t \to t - \omega/H$

Closed slicing: $ds^2 = -dt^2 + \frac{1}{H^2}ch^2Ht\, d\Omega_3^2 \leftarrow$ unit sphere $S^3$

Open slicing: $ds^2 = -dt^2 + \frac{1}{H^2}sh^2Ht\, dH_3^2 \leftarrow$ unit hyperbolic $H^3$

# de Sitter spacetime



# de Sitter spacetime: static coordinates

$$ds^2 = -(1 - H^2 r^2)dt^2 + \frac{dr^2}{1 - H^2 r^2} + r^2 d\Omega_2^2 \quad \leftarrow \text{ unit sphere } S^2$$

describes 1/4 of the spacetime

similarity with a black hole metric:

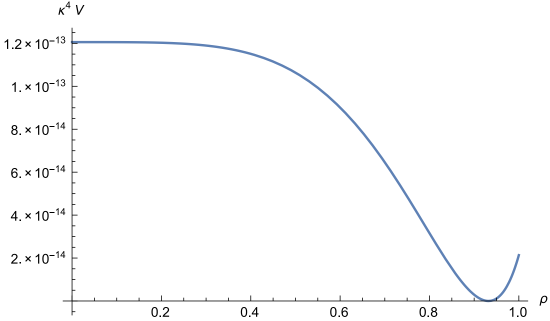
no singularity but cosmological horizon at $r = H^{-1} \equiv r_C$ [9] [11]

# Inflation:
## theoretical paradigm consistent with cosmological observations

a small region of space becomes fast exponentially large $\Rrightarrow$

explains homogeneity, isotropy and flatness problems

Inflaton potential




slow-roll region with $V', V''$ small compared to the de Sitter curvature

# Swampland de Sitter conjecture

String theory: vacuum energy and inflation models

related to the moduli stabilisation problem

Difficulties to find dS vacua led to a conjecture:

$$\frac{|\nabla V|}{V} \geq c \quad \text{or} \quad \min(\nabla_i \nabla_j V) \leq -c' \quad \text{in Planck units}$$

with $c, c'$ positive order 1 constants    Ooguri-Palti-Shiu-Vafa '18

Dark energy: forbid dS minima but allow maxima

Inflation: forbid standard slow-roll conditions

Assumptions: heuristic arguments, no quantum corrections

$\longrightarrow$ ongoing debate...

# Swampland Program

Vafa '05, Ooguri-Vafa '06

Not all effective field theories can consistently coupled to gravity

-anomaly cancellation is not sufficient

- consistent ultraviolet completion can bring non-trivial constraints

those which do not, form the 'swampland'

criteria $\Rrightarrow$ conjectures

supported by arguments based on string theory and black-hole physics

The first and most established example is the Weak Gravity Conjecture:

gravity is the weakest force implying a minimal non-trivial charge

$$q \geq m/\sqrt{2} \quad \text{in Planck units } 8\pi G = \kappa^2 = 1$$

Arkani-Hamed, Motl, Nicolis, Vafa '06

# Reissner-Nordstøm black hole

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \qquad M = \frac{m}{8\pi},\ Q^2 = \frac{q^2}{32\pi^2}$$

$Q^2$: repulsive electric energy, while $-2M$: attractive gravity force [5]

Two horizons at $r = r_\pm$ satisfying $f(r) = 0$: $r_\pm = M\left(1 \pm \sqrt{1 - \frac{Q^2}{M^2}}\right)$

- $Q^2 < M^2$: two real roots with $0 < r_-$ (inner) $< r_+$ (outer horizon)

  $r_-$ hides the singularity at $r = 0$, while between horizons $t$ is space like

- $Q^2 = M^2$: $r_- = r_+ \Rightarrow$ extremal BH

  electric and gravity forces are balanced

- $Q^2 > M^2$: complex roots, no horizon $\Rightarrow$ naked singularity at $r = 0$

  the repulsive force is stronger than gravity and forbids BH horizons

# Weak Gravity conjecture

Existence of states with $Q^2 > M^2$ minimal non-trivial charge

$\Rightarrow$ Charged black holes can decay

no BH remnants

since naked singularities are forbidden by the Weak Cosmic Censorship

Next: generalisation to de Sitter space using similar arguments

I.A.-Benakli '20

# Reissner-Nordstøm black hole in de Sitter space [5]

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3}r^2 \qquad M = \frac{m}{8\pi},\ Q = \frac{q^2}{32\pi^2},\ \Lambda = \frac{3}{l^2} = 3H^2$$

$f(r) = 0 \Rightarrow$ 4 roots: one -ve (unphysical), one +ve, two +ve or complex

Define $P(r) \equiv -r^2 f(r) = l^{-2}r^4 - r^2 + 2Mr - Q^2$

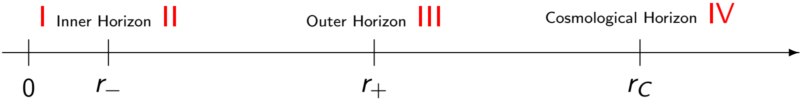
$\Rightarrow$ its discriminant $\Delta \propto -\frac{27}{l^2}(Ml)^4 + (l^2 + 36Q^2)(Ml)^2 - Q^2(l^2 + 4Q^2)^2$

- $\Delta > 0 \Rightarrow$ 3 positive roots: $0 < r_- < r_+ < r_C$

  $r_C$: cosmological horizon ($\to \infty$ when $\Lambda \to 0$)
- $\Delta = 0 \Rightarrow r_- = r_+ < r_C$, or $r_- < r_+ = r_C$
- $\Delta < 0 \Rightarrow r_\pm$ complex and $r_C > 0$, or $r_- > 0$ and $r_+, r_C$ complex

# Reissner-Nordstøm black hole in de Sitter space

$\Delta > 0 \Rightarrow$ 3 Horizons    4 Regions




[14]

$\Delta$ is quadratic polynomial of $M^2 l^2$ with roots

$$M_{\pm}^2(l, Q^2) = \frac{1}{54l}\left[l(l^2+36Q^2) \pm \left(l^2 - 12Q^2\right)^{3/2}\right]$$

$\Delta < 0$ outside the roots (for $l^2 \geq 12Q^2$), or for $l^2 \leq 12Q^2$

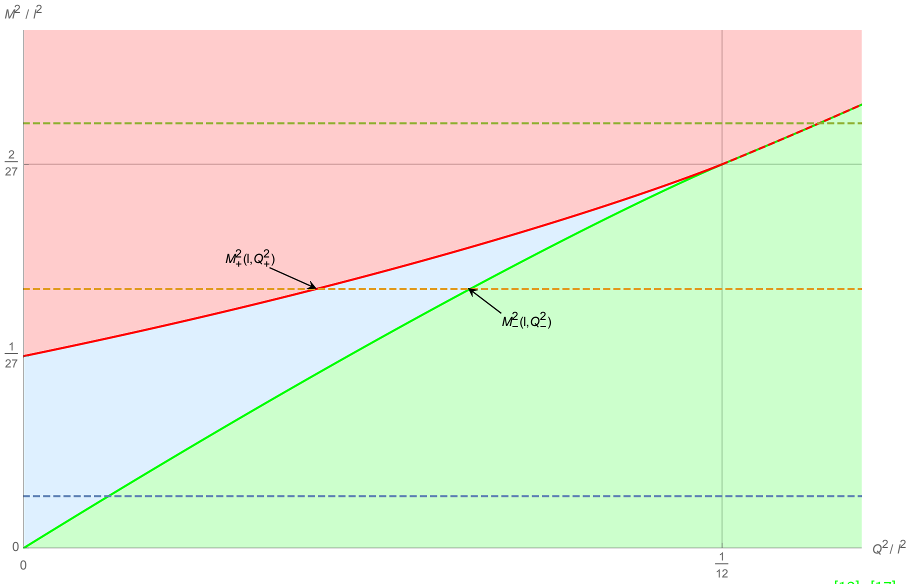
For $\Delta > 0 \Rightarrow$ four regions: $0 < r_- < r_+ < r_C$

- **Region IV:** $r > r_C$

  $t$ space-like, the cosmological constant dominant over all forces
- **Region III:** $r_+ \leq r \leq r_C$ $\quad f(r) \sim 1$ constant
- **Region II:** $r_- \leq r \leq r_+$ BH interior

  $t$ space-like, dominance of gravitational attraction
- **Region I:** $0 < r \leq r_-$ $\quad$ dominance of electromagnetic repulsion

Define $Q_{\pm}$: $M_{\pm}^2(l, Q_{\pm}^2) = M^2$ $\quad Q_+ \leq Q_-$




[12] [17]

# Comparison of forces

1. $M^2 < \frac{l^2}{27}$: $Q_+$ does not exist

   As $Q \nearrow$, $Q < Q_-$ and $M > M_-(l, Q^2) \Rightarrow r_- \nearrow$, $r_+ \searrow$, $r_C \nearrow$

   Region II shrinks with $r_+ \to r_-$

   As $Q > Q_-$ and $M^2 < M_-^2(l, Q^2) \Rightarrow \Delta < 0$ and Region II disappears

   The repulsive electric force is stronger and forbids BH horizons

2. $\frac{l^2}{27} \leq M^2 \leq \frac{2l^2}{27}$: 3 horizons $\Rightarrow Q \in [Q_+, Q-], M \in [M_-, M_+]$

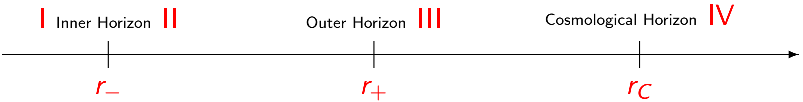
   As $Q \searrow$ towards $Q_+ \Rightarrow r_- \searrow$, $r_+ \nearrow$ and $r_C \searrow$ Region III shrinks

   For $Q < Q_+$ Region III disappears and dS space is 'eaten' by the BH

   As $Q \nearrow$ towards $Q_- \Rightarrow r_- \nearrow$, $r_+ \searrow$ and $r_C \nearrow$ Region II disappears

   For $Q > Q_-$ the electric force is strong and forbids again BH horizons

# Comparison of forces




3. $M^2 = \frac{2l^2}{27} \Rightarrow Q_+ = Q_- = l/\sqrt{12}$

   at $Q = Q_\pm$ the 3 horizons coincide $r_- \to r_+ \to r_C \to l/\sqrt{6}$

4. $M^2 > \frac{2l^2}{27}$: there is only one horizon defined at $\delta M = \delta Q^2/l$

   in the parametrization $M = \sqrt{\frac{2}{27}}\, l + \delta M,\ Q^2 = \frac{l^2}{12} + \sqrt{\frac{2}{3}}\delta Q^2$

   $\delta M > \delta Q^2/l$: dS 'eaten' by the BH

   $\delta M < \delta Q^2/l$: electric repulsion forbids BH horizons

Weak Gravity conjecture in dS space: minimal non-trivial charge $q_{\rm min}(m, l)$

defined in the green region of the figure [14] I.A.-Benakli '20

# Weak gravity conjecture in dS space [14]

- **Small charge:** $Q^2 \leq \frac{l^2}{12} \left( q^2 \leq \frac{\pi}{\Lambda G} \right)$:

  $M^2 < M_-^2(l, Q^2) = \frac{1}{54l} \left[ l(l^2 + 36Q^2) - \left( l^2 - 12Q^2 \right)^{3/2} \right]$

  $\Rightarrow$ flat space limit: $Q^2 > M^2 + \frac{M^4}{l^2} + \mathcal{O}(1/l^4)$

- **Large charge:** $Q^2 \geq \frac{l^2}{12} \left( q^2 \geq \frac{\pi l^2}{3G} \right)$: $M^2 < \frac{3}{2} \frac{1}{l^2} \left( Q^2 + \frac{5}{36} l^2 \right)^2$

  $\Rightarrow$ strong curvature limit ($l \to 0$): $Q^2 > \sqrt{\frac{2}{3}} lM - \frac{5}{36} l^2$

  independent of the Newton constant: $q > \left( \frac{32\pi^2}{3} \right)^{1/4} \sqrt{lm}$

# Conclusions

Weak gravity conjecture in an accelerating Universe:

- existence of a state with charge larger than a minimal value

  generalising the flat space result $Q^2 > M^2$ in Planck units

  minimal charge depends on the mass and the Hubble constant

- small cosmological constant $H < M$ (also $H < \frac{M_P}{\sqrt{12}Q}$) $\Rightarrow$

  power corrections to the flat result $Q^2 > M^2 + M^4 H^2$

- large cosmological constant $\Rightarrow$

  minimal charge$^2$ linear in mass $Q^2_{\rm min} \sim M/H$

  constraints for particle physics models of inflation