WEAK GRAVITY CONJECTURE(S)

Karim Benakli

CNRS, Sorbonne Université

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Based on works with Carlo Branchina + Gaëtan Lafforgue-Marmet

Looking for firm grounds in the Swampland:

Among the consistent QFTs, which ones can descend from a quantum theory of gravity?

The briefest glance at the

Weak Gravity Conjecture

The weak gravity conjecture is easy to state in an asymptotically flat space-time. In fact, there are several ways to formulate it:

- The simplest is the following: The attractive gravitational interaction (think: Newton's force) is weaker than the repulsive gauge interaction (think: Coulomb's force).

  But there are also other formulations. For example:

- Extremal black holes decay completely, leaving no remnants.
- A super-extremal state exists.

Arkani-Hamed, Motl, Nicolis, Vafa '06

- All these formulations lead to the following practical statement about the charge/mass ratio: Arkani-Hamed, Motl, Nicolis, Vafa '06

In flat space-time, for any theory with $U(1)$ gauge symmetry with coupling g, a state of charge $q(>0)$ and mass m satisfying the inequality

$$\sqrt{2}gq \geq \frac{m}{M_{Pl}}$$

must exist.

All the above conditions are the same in flat space-time

The WGC implies the non-existence of global symmetries in quantum gravity.

I would like to make a couple of points about this conjecture:

- First: not only is it easy to state but it is the most verified conjecture of the Swampland. It has been shown to be valid in many vacua of string theory.
- Second, on closer inspection, there is something troubling. All these conditions are equivalent when we are in an asymptotically flat space-time. The question then arises as to what the real fundamental criterion is.

I will not discuss cosmology, where we are not in a flat space-time, and where we can study a form like the one presented by Ignatios for de Sitter space.

Instead I can examine how this criterion is applied by a phenomenologist, beyond the fact, well known since decades, that there is no global symmetry.

Let us look at some examples.

Let's put numbers to see how it impacts phenomenology

- For electromagnetism: $U(1)_{em}$. Mass of the electron $m_e = 511$ keV. Coupling $g_{em} \sim 0.3$.

$$0.3 \gtrsim 10^{-21}$$

- For a TeV scale new state $(q = 1)$:

$$g \gtrsim 10^{-15}$$

- For a meV scale new state $(q = 1)$:

$$g \gtrsim 10^{-30}$$

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In flat space-time: not the most useful constraint ...

The WGC applied for the corresponding magnetic monopole leads to:

In flat space-time, for any theory with $U(1)$ gauge symmetry with coupling g, there is an ultraviolet scale

$$\Lambda_{UV} \sim gM_{Pl}$$

which sets the cut-off of the Effective Field Theory (EFT).

- This is the Magnetic Weak Gravity Conjecture.
- It obviously implies the non-existence of global symmetries (forbids $g \to 0$).

Thus, if we have a model with very small g, Λ_{UV} will fall within the range of energies accessible to experiments. New states are necessarily present in this model at this scale.

I will illustrate this on a simple example.

The XENON1T example:

K. B., Branchina, Lafforgue-Marmet '20 - Anchordoqui, Antoniadis, K.B., Lust '20

- The example is provided by an attempt to fit the Xenon1T anomaly.
- One possibility that could explain the Xenon1T anomaly is the presence of a dark photon at the keV scale that couples very weakly to electrons.
- This coupling can be either direct or through kinetic mixing. Then, the mixing of the dark photon X and SM photon γ needs to be very small

$$\epsilon_{\gamma X} \sim \frac{e g_X}{16\pi^2} C_{\mathrm{Log}} \lll 1 \quad \rightarrow \quad g_X \lll 1$$

- In any case, the coupling must be tiny an the "model" should have a UV cut-off:

$$g_X \lesssim \mathcal{O}(10^{-13} - 10^{-12}) \qquad \Rightarrow \qquad \Lambda_{UV} \lesssim 10 - 100 \text{ TeV}$$

- What kind of stuff can appear at the Λ_{UV}?

The XENON1T example: K. B., Branchina, Lafforgue-Marmet '20 - Anchordoqui, Antoniadis, K.B., Lust '20

- A way to get a tiny coupling in string theory is by using Large Extra-Dimensions or Low String Scale:

$$g_X = \sqrt{\frac{16\ \pi}{g_s}}\ \frac{M_s}{M_{\rm Pl}} \sim 4 \times 10^{-14} \left(\frac{0.2}{g_s}\right)^{1/2} \left(\frac{M_s}{10\ \rm TeV}\right),$$

* The coupling needed to fit XENON1T is the *weakest one can get* this way.

It corresponds to extra-dimensions or a string scale of order $10 - 100$ TeV

$\Rightarrow$ A case for a 100 TeV collider.

The WGC + (massless) scalars:

The Repulsive Force Conjecture (RFC): taken far apart, two copies of the RFC state feel a repulsive force between them.

Palti '17 - Heidenreich, Reece and Rudelius '19

→ RFC avoids gravitational bound states.

→ RFC involves forces from: gravitons + $U(1)$ gauge bosons + massless scalars

- What about cases with only scalar bosons i.e. without gauge bosons?

- Problem 1: Moduli exchange lead to attractive forces.
- Problem 2: Dealing with scalars having non-vanishing potentials.

A very brief overview of the

Scalar Weak Gravity Conjecture

Many attempts for a Scalar Weak Gravity Conjecture (SWGC). In particular:

[1] Palti '17 $\rightarrow$ attempts to generalize the RFC to scalars $\rightarrow$ the SWGC

[2] Gonzalo, Ibanez '19 $\rightarrow$ self-interacting scalars $\rightarrow$ some relation between the derivatives of the scalar potential.

[3] Freivogel, Gasenzer, Hebecker and Leonhardt '19 $\rightarrow$ criticize [2] $\rightarrow$ size of gravitational bound state (No functional relation).

[4] K. B., Branchina, Lafforgue-Marmet '20 $\rightarrow$ Gravity is the weakest interaction.

[5] Gonzalo, Ibanez '20 $\rightarrow$ 2 to 2 processes $\rightarrow$ recovers [4] in their explicit example.

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We try to formulate the weakness of gravity as:

For any scalar Φ, there exist at least one state X such that the leading interaction between X and Φ is stronger than the gravitational one.

K. B., Branchina, Lafforgue-Marmet

Case $X = \phi$:
We consider a real scalar ϕ with the potential:

$$V(\phi) = \frac{1}{2} m_0^2 \phi^2 + \frac{\mu}{3!} \phi^3 + \frac{\lambda}{4!} \phi^4.$$

At energy scales $E \sim m_0$, the theory is *non-relativistic*. We study fluctuations around $\phi = 0$ and make the field redefinition:

$$\phi(x) = \frac{1}{\sqrt{2m_0}} \left(\psi(\mathbf{x}, t) e^{-im_0 t} + \psi^*(\mathbf{x}, t) e^{im_0 t} \right)$$

The potential for the non-relativistic field ψ should be of the form

$$V_{eff}(\psi\psi^*) = m_0 \psi\psi^* + \frac{\tilde{\lambda}}{16 m_0^2} (\psi\psi^*)^2 .$$

where:

$$\tilde{\lambda} = \lambda - \frac{5}{3} \frac{\mu^2}{m_0^2}.$$

The resulting sign of $\tilde{\lambda}$ tells us about the attractive (< 0) or repulsive (> 0) nature of the effective interaction and, in the case where they are in competition, which one of the two terms dominate at energies $E \sim m_0$.

- It is essential in the comparison to fix the energy scale, and naturally it is given by the mass of the scalar particle, and consider the gravitational scattering in the s-channel at $s \sim 4m_0^2$.

Requiring that gravity is the weakest force at low energy amounts then to impose:

$$\left|\tilde{\lambda}\right| = \left|\lambda - \frac{5}{3}\frac{\mu^2}{m_0^2}\right| \geq \frac{m_0^2}{M_{Pl}^2}.$$

We have put an absolute value on the left hand side so that it holds independently of the sign of the self-interaction.

- The quantity $\sqrt{|\tilde{\lambda}|M_{Pl}}$, could be interpreted as an ultra-violet cut-off scale.

In more generic field case, we will impose a stronger condition

$$4m_0^2\left|\frac{\partial^4 V_{eff}}{\partial^2\psi\partial^2\psi^*}\right|_{\psi=0} \geq \frac{\tilde{c}}{M_{Pl}^2}\left|\frac{\partial^2 V_{eff}}{\partial\psi\partial\psi^*}\right|^2_{\psi=0}$$

and take the order one constant $\tilde{c}$.

- We consider the quartic scalar potential

$$V(\phi,\bar{\phi}) = -m^2\bar{\phi}\phi + \lambda(\bar{\phi}\phi)^2.$$

with $\lambda > 0$, insuring stability, and $m^2 > 0$.

- It is convenient to use the parametrization $\phi(x) = \frac{1}{\sqrt{2}}\rho(x)e^{i\pi(x)}$. The gravitational interaction is subdominant for:

$$\frac{m^2}{3\lambda} < \rho^2 \leqslant \frac{14}{3}\tilde{M}_{Pl}^2 + \frac{17}{21}\frac{m^2}{\lambda} + \mathcal{O}(\tilde{M}_{Pl}^{-2})$$

- At the minimum, where $\rho^2 = \frac{m^2}{\lambda} \equiv v$, we get $\tilde{\lambda} = -24\lambda$, and the conjecture is then verified in the case:

$$\lambda \geq \frac{1}{12}\frac{m^2}{\tilde{M}_{Pl}^2} \sim 10^{-17}$$

If we take m to be the electroweak scale.

$$v^2 \leq 12\tilde{M}_{Pl}^2 \sim 10^{37} GeV^2$$

Multiple Scalar and Moduli Fields

Consider the case of a massive complex scalar X:

$$\mathcal{L}_{int} = \mu\phi|X|^2 + \cdots$$

where the dots stand for sub-leading higher order terms. We can write the potential as:

$$V(X,\phi) = m_X^2(\phi)\,|X|^2, \qquad \mu = \partial_\phi m_X^2$$

The preeminence of scalar interactions must be taken at the mass scale $\sim 2m_X$ and reads then:

$$|\partial_\phi m_X| \geq \frac{m_X}{\tilde{M}_{Pl}}$$

Newton vs Scalar interactions

We can square the above three-point amplitudes on each side, $2X \to \phi$ on the left and $2X \to G$, on the right side, where G is the graviton. The comparison concerns then two $XX^* \to XX^*$ processes, at the energy scale m_X, one through scalar and the other through graviton exchange. This leads to the following potentials for X:

$$V_{scalar}(r) = -\frac{\mu^2}{4m_X^2}\frac{1}{r}, \qquad V_{grav}(r) = -\frac{m_X^2}{\tilde{M}_{Pl}^2}\frac{1}{r}$$

Now, both scalar and gravitational interactions have similar dependence in the inter-particles distance and the comparison is straightforward:

$$\frac{\mu^2}{4m_X^2} \geq \frac{m_X^2}{\tilde{M}_{Pl}^2}$$

which can be written:

$$\partial_\phi m_X \partial_\phi m_X \geq \frac{m_X^2}{\tilde{M}_{Pl}^2}$$

In the extremal case saturating the above inequality, the solution is given by:

$$m_X^2(\phi) = m_0^2 \; e^{\pm 2\phi/\tilde{M}_{Pl}}.$$

This is the Swampland Distance Conjecture (SDC).

Example 2: Complex Moduli Fields

Complex modulus Φ and scalar field X with a potential of the form:

$$V(X,\Phi) = m_X^2(\Phi)|X|^2 + \cdots \qquad m_X^2 = m_{X0}^2 + \lambda_\Phi |\Phi|^2 + \cdots$$

where

$$\lambda_\Phi = \partial_\Phi \partial_{\bar{\Phi}} m_X^2(\Phi, \bar{\Phi})$$

The weakness of gravitational interaction:

$$\left|\Phi\Phi^* \leftrightarrow XX^*\right|_{E\sim 2m_X} \geq \left|\Phi\Phi^* \leftrightarrow \text{graviton} \leftrightarrow XX^*\right|_{E\sim 2m_X}$$

$$\partial_\Phi \partial_{\bar{\Phi}} m_X^2 \geq 2\frac{m_X^2}{\tilde{M}_{Pl}^2}$$

If the state X has a self-quartic interaction, then we will also have to check a similar constraint on the self coupling $|\tilde{\lambda}_4|\tilde{M}_{Pl}^2 \geq m_X^2$.

Extremal states

The extremal case that corresponds to the case of equality is solved for:

$$m_X^2(\Phi, \bar{\Phi}) = m_-^2 e^{-\sqrt{2}\frac{\Phi+\bar{\Phi}}{\tilde{M}_{Pl}}} + m_+^2 e^{\sqrt{2}\frac{\Phi+\bar{\Phi}}{\tilde{M}_{Pl}}}$$

We can use the following parametrization:

$$\Phi = \frac{1}{\sqrt{2}}(\phi + i\chi), \qquad e^{\sqrt{2}\frac{\Phi+\bar{\Phi}}{\tilde{M}_{Pl}}} = e^{2\frac{\phi}{\tilde{M}_{Pl}}}, \qquad \text{and} \qquad e^{\frac{\phi}{\tilde{M}_{Pl}}} = R$$

then:

$$m_X^2(R) = \frac{m_-^2}{R^2} + m_+^2 R^2$$

which is the well known formula for string states squared masses with the $\frac{m_-^2}{R^2}$ as the low energy Kaluza-Klein modes and $m_+^2 R^2$ the winding modes.

Looks like the Refined de Sitter Swampland Conjecture

Now, consider the case where the field ϕ is a modulus appearing only as a parameter in the couplings of the massive scalar X ($\langle X \rangle = 0$), through

$$V(X,\phi) = m_X^2(\phi)X^2 + \sum_{n\geq 4} \lambda_n(\phi)X^n$$

Then, the first condition can be written as:

$$\left.\frac{|\partial_\phi V(X,\phi)|}{V}\right|_{X=0} \geq \frac{\sqrt{\tilde{c}}}{M_{Pl}}$$

while the other condition reads now:

$$\left.\frac{|\partial_\phi \partial_{\bar{\phi}} V(X,\phi)|}{V}\right|_{X=0} \geq \frac{2\tilde{c}}{M_{Pl}^2}$$

where we note the similarity with the Refined de Sitter Conjectures when the second derivative is negative.

Conclusions

- The Magnetic Weak Gravity Conjecture: a hyper-weak $U(1)$ coupling predicts new physics at low energy.

  Example: fitting the XENON1T with a very weakly coupled dark photon predicts extra dimensions accessible at a 100 TeV collider.

- We have investigated the implication that "For any scalar field its leading interaction is never gravity".

- We have recovered in some way the Axionic WGC, the Swampland Distance Conjecture and that the Refined de Sitter Conjecture.