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Emergent gravitons and dark photons from strongly coupled sectors.

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Introduction

• Fundamental (particle) physics and cosmology face major problems:

♠ Those associated to observation: dark matter, dark energy, matter-antimatter asymmetry, as well as the completion of the neutrino sector.

♠ Theoretical: the cosmological constant problem and the black-hole information paradox that can be both viewed as a clash between semiclassical gravity and QFT.

• Most researchers agree that, most probably, the weak link in this clash is gravity and its marriage with quantum mechanics.

• Perturbative string theory is the only context so far that can provide at the same time a quantized theory of gravity and a semiclassical space time.

• However, it is not a UV complete theory: it has a cutoff, the Planck scale.

• One of the most important breakthroughs on the last few decades is the concept of the holographic correspondence: it intimately linked two frameworks that were thought to be different: string theory and QFT.

• In the 80's+90's we used to think that string theory was encompassing QFT as a low-energy limit.

• Today we believe that string theory and QFT are two different aspects of a single framework, whose universal description is harder than QFT or string theory alone, and it is not known.

• Holography did however provide a non-perturbative formulation of string theory around asymptotically AdS backgrounds.

• One of the lessons we learned from the holographic correspondence is that gravity and gravitons can be composites of QFT fields (generalized gluons).

• This is an idea much older than holography, that physicists pursued, motivated by the analogy between pions and QCD.

• The low-energy theory of pions (and baryons) is a bit like gravity: non-renormalizable, IR-free, and unclear (until the advent of QCD) on how to treat the quantum theory.

• QCD gave a clear picture: pions are composites of quarks and gluons that are strongly interacting in the IR.

• But the UV properties of the theory are controlled by the quarks and gluons.

• Similarly, it was attempted (in the sixties and seventies) to make gravitons composites of QFT fields.

• Not much progress was achieved, because you need strong coupling at least in the IR, and the Weinberg-Witten theorem severely constrained the outcome.

• 't Hooft's observation in 1974 that there maybe a weakly-coupled string theory describing large N QCD did not appear as a surprise: it was already suspected that QCD flux tubes must have string-like dynamics.

• Nobody dared however fantasize at the time, that the string theory of QCD would describe (among other things) the quantum gravity of the spin 2^{++} glueball.

• When Maldacena proposed AdS/CFT, he provided us with two items:

♠ A string theory candidate dual for a large-N four-dimensional gauge theory.

♠ And a big surprise: the string theory lived in ten (not four) dimensions.

• Moreover it made it clear, that the string theory fields are the gauge-invariant single-trace composites of the gauge theory fields.

• It provided the first non-trivial hole in the WW theorem, that prohibited a massless emergent/composite graviton.

• Since then, theories with discrete spectra like those of real YM have been constructed violating WW in an interesting way:

♠ The emergent graviton is massless in the higher dimensions (this is not forbidden by WW).

♠ When reduced to four-dimensions, it becomes massive via the "gravitational Higgs effect".

• It is interesting to use what we learned from the holographic correspondence, to study the emergence of gravitons and other particles (typically axions and "graviphotons") as avatars of a hidden sector coupled at high energy to the SM.

• This gives a complementary view to the older approach of obtaining gravity (and the SM) from string theory.

A hidden sector generating gravity

• The relevant QFT framework that is UV-complete is

$$(\text{hidden}\quad \text{large}-\text{N}\quad \text{gauge}\quad \text{theory})\times(\text{messengers})\times\widetilde{\text{SM}}$$

where the hidden large-N gauge theory is assumed to be at strong coupling.

• The messengers are bi-fundamentals under the hidden gauge theory gauge group and the SM gauge group. They are massive with mass M that is assumed to be much larger than all the scales of the SM and the hidden theory.

• The complete theory is a standard four-dimensional quantum field theory (QFT) defined on a flat Minkowski background metric.

• There is another picture of the same theory that is valid well below the messenger mass M. In this regime, we can integrate out the messengers and obtain an effective theory with the structure

$$(\text{hidden}\quad \text{large}-\text{N}\quad \text{gauge}\quad \text{theory})\times\widetilde{\text{SM}}$$

• Now the coupling between the hidden theory and the SM is given by products of gauge invariant operators of the two theories. Almost all such couplings are "irrelevant" in the IR.

• Various low-dimensions gauge-invariant operators of the hidden theory couple now directly to the SM.

• One of them, the energy momentum tensor, appears as an external dynamical graviton from the point of view of the SM:

$$S_{int} = \frac{1}{M^4} \int d^4x \; T^{hidden}_{\mu\nu} \; T^{SM,\mu\nu} \quad \to \quad \int d^4x \; h_{\mu\nu} \; T^{SM,\mu\nu}$$

$$h_{\mu\nu} = \frac{1}{M^4} \langle T^{hidden}_{\mu\nu} \rangle$$

• In the generic case, this composite "graviton" looks nowhere near what we usually call a graviton.

• However, if the hidden theory is holographic, then this graviton is a massless graviton in a higher dimension, and in a non-trivial background as we learn from the AdS/CFT correspondence.

• The emerging gravitational interaction in this case has the standard features of gravity: it is weakly coupled, has diffeomorphism invariance and is accompanied by a small number of additional fields, the analogue of graviphotons and light scalars.

• When the gravitational interaction is reduced back to four-dimensions, the graviton acquires massive features, due to the non-trivial gravitational background in the higher dimensions.

• A holographic dual picture of the setup is given by a bulk gravitational theory living in more than four dimensions, dual to the holographic hidden QFT, and coupled to a four-dimensional "SM" brane, embedded in the bulk geometry.

• It resembles the simplified Randall-Sundrum picture, but with important differences.

• The main difference from RS realizations of this idea is that there is no UV cutoff for the bulk description of the hidden holographic QFT and no RS Z_2 boundary conditions for the brane.

• It can be shown in general, that:

♠ Any emergent gravity theory has a unique diffeomorphism invariance.

♠ It is a bigravity theory: there is a dynamical graviton that captures the interactions of the energy-momentum tensor composites, and a classical fiducial (non-fluctuating metric) that is the fixed metric of the dual QFT.

♠ In standard holography, the dynamical graviton lives in the bulk, and the fiducial metric of the dual QFT couples via the boundary conditions at the AdS boundary.

♠ If the dual QFT is defined in a flat metric, then the associated emergent gravitational theory always admits a flat-metric solution for the dynamical graviton, independent of quantum corrections!

Betzios+E.K.+Niarchos

♠ This can be also shown in the dual gravitational (brane+bulk) language, via the self-tuning mechanism for the cosmological constant.

Charmousis+E.K.+Nitti

♠ In this last context we can calculate the effective four-dimensional graviton mass as a function of the theory parameters.

♠ The presence of large N in the holographic theory is important, as it implies weak effective coupling between the composites, in particular the graviton.

♠ It is also responsible for a suppression of the graviton effective four-dimensional mass.

• There are many open questions that need to be addressed until this setup can be confronted with data.

Other avatars of the hidden sector

• Such a hidden sector, beyond emergent gravity coupled to the SM can also provide:

♠ A universal axion coupled to the SM. This is a composite of the instanton density of the hidden theory.

Anastasopoulos+Betzios+Bianchi+Consoli+E.K.

♠ Its low-energy characteristics are given by

$$f_a^2 \sim \frac{M^8}{m_h^6} + \mathcal{O}\left(\Lambda^2_{QCD}\right) \quad , \quad m_a^2 \sim m_h^2 + \mathcal{O}\left(\frac{m_h^6}{M^8}\Lambda^4_{QCD}\right)$$

♠ A (simpler) RS version has been discussed in the past

Bonnefoy+Dudas+Cox+Gherghetta+Nguyen

♠ In the holographic case, the interaction mediated by this axion is four-dimensional at short and long enough distances, but may be five-dimensional at intermediate distances.

Charmousis+E.K.+Nitti

♠ Such a composite axion can provide a new distinct portal to dark matter.

Anastasopoulos+Kaneta+Mambrini+Pierre

Emergent graviphotons

• Another avatar of this framework are emergent (composite) vectors, originating in the hidden theory, that eventually couple to the SM *Betzios+E.K.+Niarchos+Papadoulaki, Anastasopoulos+Betzios+Bianchi+Consoli+E.K.*

♠ Such vectors are associated with exact global symmetries of the hidden theory.

♠ A U(1) composite vector is generated from a hidden conserved U(1) current.

♠ Under the holographic correspondence, such vectors correspond to closed string U(1)'s that we usually call "graviphotons".

♠ Generically, at large N, such a composite vector is weakly-coupled and massive.

♠ However, as with gravitons, formulated as a higher-dimensional vector, it carries an associated gauge symmetry, and is "massless".

♠ If the hidden theory is holographic, then a simple action describing its IR dynamics is of the form*

Gherghetta+von Harling

$$S_{total} = S_{bulk} + S_{brane}$$

$$S_{bulk} = \int d^5x\sqrt{g}\ \frac{1}{4g_5^2}F_{mn}F^{mn} \quad , \quad S_{brane} = \int d^4x\sqrt{\gamma}\frac{1}{4g_4^2}\widehat{F}_{\mu\nu}\widehat{F}^{\mu\nu}$$

$$g_5^2 \sim \frac{1}{M_5} \quad , \quad g_4^2 = \frac{M_5^2}{M_P^2} \ll 1$$

- There are several issues that may be of a phenomenological relevance:

♠ The interaction mediated by this emergent vector is four-dimensional both at short

$$p \quad \gg \quad \frac{M_5^3}{M_P^2}$$

and long distances

$$p \quad \ll \quad m_h$$

♠ The vector is always massive at long distances with a mass of order the scale of the hidden theory, m_h.

♠ The effective four-dimensional coupling constant g_{IR} and effective mass m_{IR} in the IR are given by

$$\frac{1}{g_{IR}^2} \simeq \frac{1}{m_h g_5^2} + \frac{1}{g_4^2} \quad , \quad \frac{m_{IR}^2}{g_{IR}^2} \simeq \frac{m_h}{g_5^2} + \frac{m_0^2}{g_4^2}$$

♠ The effective coupling is small, for both reasons: large N and $M \gg m_h$.

♠ Therefore this type of emergent vector is the class of what we call "dark photons".

The hypercharge portal

• Typically such emergent vectors are not minimally coupled to SM fields.

• Therefore, the most sensitive coupling of such a U(1) vector to the SM is via the hypercharge portal:

$$S_{int} = x \int d^4x \; F^{hidden}_{\mu\nu} F^{\mu\nu}_{Y}$$

• The (dimensionless) coupling x controls via diagonalization the (small) electric charges of the chargeless fields.

Holdom

• Because of this, it is severely constrained $x \ll 10^{-10}$.

• We can estimate x non-perturbatively from general EFT arguments:

$$x \sim \frac{1}{N}$$

• The case of a holographic hidden theory is however special.

• To estimate this, we can use a weakly coupled string theory, and we can calculate the mixing between a graviphoton and a brane photon (hypercharge).

• This has been calculated in the absence of RR fluxes.

Camara+Ibanez+Marchesano,Marchesano+REgalado+Zoccarato

• We calculated this mixing in the presence of RR fluxes

Anastasopoulos+Bianchi+Consoli+E.K.

• We find that for a holographic hidden theory,

$$x \sim \frac{1}{N^{\frac{3}{2}}}$$

• There is a possible exception: if the (extended) SM contains also an adjoint Higgs with respect to SU(2).

• Therefore, in such theories it is easier to have weakly-coupled dark U(1)'s with naturally small mixing to the hypercharge.

Conclusions and Outlook

• A strongly-coupled (holographic) hidden sector can provide emergent gravity coupled to the SM.

• Emergent (holographic) gravity seems to be the promising route to quantum gravity.

• It has the potential of alleviating several of the serious problems quantum gravity faces.

• But many issues remain to be understood until a phenomenologically viable theory exists.

• Along with gravity a composite universal axion and dark (gravi)photons appear naturally from this setup.

• Such dark photons have unexpectedly small couplings and mixings to hypercharge.

• The phenomenological signatures and implications of such composite axions and dark photons need further analysis.

THANK YOU!

The EFT analysis

• The effective action for the emergent vector, obtained after integrating out the hidden theory contains a sequence of low dimension terms.

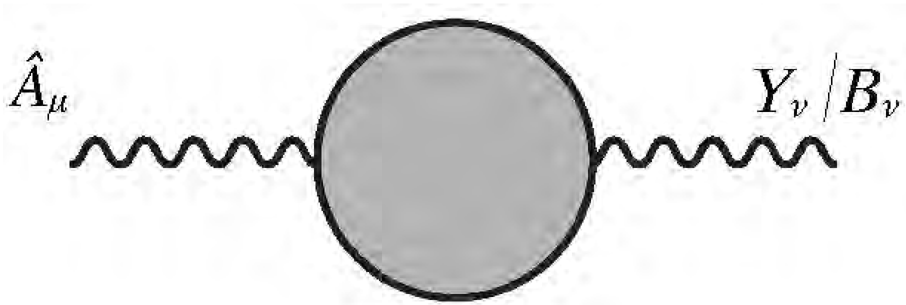
$$W_5(\hat{A}_\mu, \phi_{ij}, \psi_{ij}, A^\mu_{ij}) \sim \frac{1}{NM^2} F^{\mu\nu}_{\hat{A}} D_\mu H^\dagger D_\nu H + \frac{1}{N^{\frac{3}{2}} M^2} F^{\mu\nu}_{\hat{A}} \left[\bar{\psi}\gamma_{\mu\nu} H \psi + c.c.\right]$$

$$+ \frac{1}{N^{\frac{3}{2}} M^2} F^{\hat{A}}_{\mu\nu} F^{Y,\mu\nu} H^\dagger H + \frac{1}{N^2 M^4} F^{\hat{A}}_{\mu\nu} F^{Y,\mu\nu} \left[\bar{\psi} H \psi + c.c.\right] \cdots$$

where H is the Higgs doublet, ψ collectively denotes the standard model fermions and $F^{Y,\mu\nu}$ is the hypercharge field strength.

• Such terms via SM quantum effects produce the effective mixing term

$$S_{mix} \sim \int d^4x \; F^{\hat{A}}_{\mu\nu} F^{Y,\mu\nu}$$



- The general structure of the amplitude that provides the mixing between the hidden gauge field $\widehat{A}$ and non-anomalous Y

Comparison of EFT and string theory

EFT coupling	EFT estimate	graviphoton ~~bulk fluxes~~	graviphoton +bulk fluxes	dark brane photon
$F\widehat{F}$	$\mathcal{O}\left(\frac{1}{N}\right)$	$\mathcal{O}\left(g_s^2\right)$	$\mathcal{O}(g_s^{3/2})$	$\mathcal{O}\left(g_s\right)$
$\phi\ F\widehat{F}$	$\mathcal{O}\left(\frac{1}{N}\right)$	$\mathcal{O}\left(g_s\right)$		
$DHDH^\dagger\widehat{F}$	$\mathcal{O}\left(\frac{1}{N}\right)$	$\mathcal{O}\left(g_s^2\right)$	$\mathcal{O}\left(g_s^2\right)$	$\mathcal{O}(g_s^{3/2})$
$HH^\dagger F\widehat{F}$	$\mathcal{O}\left(\frac{1}{N^{3/2}}\right)$	$\mathcal{O}(g_s^{5/2})$	$\mathcal{O}(g_s^{5/2})$	$\mathcal{O}\left(g_s^2\right)$
$\bar{\psi}H\gamma^{\mu\nu}\psi\widehat{F}_{\mu\nu}$	$\mathcal{O}\left(\frac{1}{N^{3/2}}\right)$	$\mathcal{O}(g_s^{3/2})$	$\mathcal{O}(g_s^{5/2})$	$\mathcal{O}\left(g_s^2\right)$
$\bar{\psi}H\psi F\widehat{F}_{\mu\nu}$	$\mathcal{O}\left(\frac{1}{N^2}\right)$	$\mathcal{O}\left(g_s^2\right)$	$\mathcal{O}\left(g_s^3\right)$	$\mathcal{O}(g_s^{5/2})$

- Comparison of the kinetic mixing contributions and other generating terms in large-N EFT and weakly coupled string theory.

F denotes the hypercharge field strength, $\widehat{F}$ the dark photon field strength, H is the Higgs doublet, ϕ is an adjoint scalar field on the SM stack and ψ collectively denotes SM fermions.

The Weinberg-Witten Theorem

• The WW theorem assumes Lorentz invariance and a conserved Lorentz-covariant Energy-Momentum tensor.

• It proceeds to prove that no massless particle with spin $S > 1$ can couple to the stress tensor and no massless particles with $S > 1/2$ to a global conserved current.

• This does not rule out a theory that contains a "fundamental" massless graviton, as there exists a loop-hole: The stress tensor is not conserved in the presence of a metric, and projecting on helicity-2 is also non-covariant in a general metric.

• There are also other ways of avoiding the theorem:

• In the case of massless vectors the statement says that no massless (non-abelian) vectors can couple to a conserved Lorentz-covariant global current. It seems that Yang-Mills theory is excluded.

• This is avoided in standard non-abelian gauge theories as the conserved current is not Lorentz-covariant (only up to a gauge transformation).

• There are more interesting counter-examples:

$*$ At the lower end of the conformal window in N=1 sQCD: the ρ-mesons become massless but also develop a gauge invariance at the same time.

Komargodski

• These are the "magnetic" gauge bosons of Seiberg.

• Their effective theory is renormalizable (being a standard non-abelian gauge theory).

• A final caveat: Lorentz invariance is crucial: otherwise the notion of masslessness is not well-defined. (even in dS or AdS the notion changes)

• In conclusion: WW can be evaded but it is a serious litmus test for all emergent graviton theories.

• We shall find that although the essence of the WW theorem remains true, the effective theories for composite gravitons are very rich.

The energy momentum tensor

• The state generated out of the vacuum by the (conserved) energy-momentum tensor has the quantum numbers of the graviton

$$T_{\mu\nu}(p)|0\rangle \equiv |\epsilon_{\mu\nu}, p\rangle$$

• It is transverse because of energy conservation.

• In weakly-coupled theories, this is a spin-2 multi-particle state and therefore its interactions are expected to be non-local.

• If however, if the interactions are strong and make this state a true tightly-bound state with a "size" L, then maybe we can reproduce gravity at scales $\gg L$.

• In particular, in the limit of infinitely-strong interactions we would expect to obtain a good point-like interaction theory for this bound-state graviton.

• Of course, WW constrains such bound-states but we will come later to such constraints, as they can be subtle.

• We must remember however that, in the presence of strong attractive interactions in the spin-two channel, there may be, generically speaking, a tower of states generated from the $T_{\mu\nu}$ acting on $|0\rangle$.

• If the theory is not conformal, such a spectrum will be discrete, and would be associated with the (generically complex) poles of the two-point function of the stress tensor.

• Again, generically speaking, many such states will be unstable.

• If the theory is conformal, such states will form a continuum.

• This is the case in AdS/CFT which provided the first concrete and workable example of a composite/emergent graviton.

The AdS/CFT paradigm

• AdS/CFT relates QFT to string theory and therefore to a theory of "quantum gravity"

• That a gauge theory at large-N can be described by a weakly-coupled string theory was anticipated since the work of 't Hooft.

• Emergent dimensions are the avatar of the large N limit. Eigenvalue distributions become continuous extra dimensions as it was already seen in simpler matrix models.

• It is still a puzzle however, why the higher-dimensional theory has diffeomorphism invariance.

• The masslessness of the higher-dimensional graviton, as we understand it now, is related to energy conservation of the dual QFT.

Kiritsis, Adams+Aharony+Karch

• The holographic duality essentially implements what we discussed already: the graviton (and all other bulk fields) are composites of (generalized) gluons.

• Strong coupling in the QFT, and the higher dimensionality, as expected, is important in making the gravitational theory local (by suppressing string corrections)

• The other important ingredient is the large-N limit. It makes bulk fields (composites) interact weakly (despite the fact that the constituents interact strongly)

We have learned that:

♠ Strong coupling in QFT makes gravitons tightly bound states.

♠ Large N makes gravitons weakly interacting.

and both of the above give an effective semiclassical theory of (composite) quantum gravity.

• We believe that the duality can be used to define string theory and gravity non-perturbatively, by using the QFT to define the physics beyond the obvious cutoff of the string theory.

• This however, needs to be understood much better and it is a very difficult question, as in many cases it requires controlling non-perturbative physics

The stress tensor vev as a (classical) dynamical metric

• We would like to implement directly the idea of an emergent graviton as the state generated by the energy-momentum tensor.

• We will construct the theory that describes the dynamics of such a graviton in any QFT.

• As a warm-up, we consider a translationally invariant QFT at a fixed background metric $\mathbf{g}_{\mu\nu}$ and a scalar source J coupled to a scalar operator O (for purposes of illustration).

• The presence of an arbitrary background metric $\mathbf{g}_{\mu\nu}(x)$ breaks translation invariance.

• A redefinition of the derivative→ covariant derivative "restores" energy-momentum conservation (in the absence of other non-constant sources):

$$T_{\mu\nu} \equiv \frac{1}{\sqrt{\mathbf{g}}} \frac{\delta S(\mathbf{g}, J)}{\delta \mathbf{g}^{\mu\nu}} \quad , \quad \nabla^{\mu}_{\mathbf{g}} \langle T_{\mu\nu} \rangle \sim \partial_{\nu} J$$

where $S(\mathbf{g}, J)$ is the action of the theory coupled to the fixed metric $\mathbf{g}$ and to the scalar source J.

• Consider the Schwinger functional $W(g_{\mu\nu}, J)$:

$$e^{-W(g_{\mu\nu},J)} = \int \mathcal{D}\phi \; e^{-S(\phi, g_{\mu\nu}, J)}$$

• $g_{\mu\nu}$ is an arbitrary background metric, ϕ are the "quantum fields".

• $W(g_{\mu\nu}, J)$ has (naive) diffeomorphism invariance.

• We assume the presence of a cutoff that preserves diff invariance so that the quantities above are finite.

• This is tricky business but for the moment we can have dim-reg in mind.

• Sometimes $W(g, J)$ is unique (modulo renormalization) at the linearized level, sometimes it is not (improvement).

• Moreover there are ambiguities at the non-linear level.

• One can add diff-invariant functionals of the curvature for example.

• These correspond to "improvements" (ie alternative definitions of the stress tensor), both at the linear as also the non-linear level.

• We will call all of this "the scheme dependence" of the Schwinger functional.

• $W(g,J)$ is now diff-invariant if the original theory is translation invariant*:

$$W(g'_{\mu\nu}(x'),J(x')) = W(g_{\mu\nu}(x),J(x)) \quad , \quad g'_{\mu\nu} = g_{\rho\sigma}\frac{\partial x^\rho}{\partial x'^\mu}\frac{\partial x^\sigma}{\partial x'^\nu}$$

• The interaction energy between energy-momentum sources $t_{\mu\nu}$ with $g_{\mu\nu} = \mathbf{g}_{\mu\nu} + t_{\mu\nu}$ is encoded in $W(t)$.

• The (quantum) vev of the stress tensor is:

$$h_{\mu\nu} \equiv \frac{1}{\sqrt{\det g}}\frac{\delta W(g,J)}{\delta g^{\mu\nu}}$$

and we will use it to define the associated effective action:

$$\Gamma(h,J,\mathbf{g}) \equiv -W(g,J) + \int d^4x\sqrt{g}\ h_{\mu\nu}\ (g^{\mu\nu} - \mathbf{g}^{\mu\nu})$$

via a modified Legendre transform.

• Γ is the generating functional of 1-PI energy-momentum tensor correlators and is extremal,

$$\left.\frac{\delta\Gamma(h_{\mu\nu},J)}{\delta h_{\mu\nu}}\right|_{g=\mathbf{g}} = 0 \quad , \quad \Gamma(h^*_{\mu\nu},J)\Big|_{g=\mathbf{g}} = W(\mathbf{g},J)$$

• The description above in terms of the energy-momentum tensor "effective action" is a theory of (classical) dynamical gravity.

• The dynamical metric is (almost) the energy-momentum tensor vev, $h_{\mu\nu}$.

• Other sources like J represent energy-momentum carrying sources.

• This description is diff-invariant by construction. The related theory is a bi-gravity as it involves a dynamical metric $h_{\mu\nu}$ and a fixed fiducial metric, $\mathbf{g}_{\mu\nu}$.

• The interactions mediated by this (emergent) graviton are essentially summarizing exchanges of the energy-momentum tensor as we had postulated.

• The emergent graviton propagator (by construction) is generated by the poles of the energy-momentum tensor two-point function in the original theory.

We obtain at quadratic order, around flat space, by definition

$$S_{int} = \int \frac{d^4k}{(2\pi)^4}\ t_{\mu\nu}(k)\ \langle T^{\mu\nu}T^{\rho\sigma}\rangle\ t_{\rho\sigma}(-k) \tag{1}$$

where the general form of the TT two-point function in momentum space with $\mathbf{g}_{\mu\nu} = \eta_{\mu\nu}$ is

$$\langle T_{\mu\nu}T_{\rho\sigma}\rangle(k) = -\frac{V}{2}\left(\eta_{\mu\nu}\eta_{\rho\sigma} + \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho}\right)$$

$$+B_2(k)\ \left[\pi_{\mu\rho}\pi_{\nu\sigma} + \pi_{\mu\sigma}\pi_{\mu\rho} - \frac{2}{3}\pi_{\mu\nu}\pi_{\rho\sigma}\right] + \frac{B_0(k)}{3}\ \pi_{\mu\nu}\pi_{\rho\sigma}$$

$$B_0 = \frac{\pi^2}{40}k^4\ \int_0^\infty d\mu^2 \frac{\rho_0(\mu^2)}{k^2+\mu^2} \quad , \quad B_2 = \frac{3\pi^2}{80}k^4\ \int_0^\infty d\mu^2 \frac{\rho_2(\mu^2)}{k^2+\mu^2}$$

where

$$\langle T_{\mu\nu}\rangle \equiv V\eta_{\mu\nu} \quad , \quad \pi_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \quad , \quad k^\mu\pi_{\mu\nu} = 0\ . \tag{2}$$

• There are three types of contact terms in $\langle TT \rangle$. The $O(k^0)$ are fixed by the translational Ward identity.

• There are $O(k^2)$ terms

$$\delta\langle T_{\mu\nu}T_{\rho\sigma}\rangle(k) = \frac{3\pi^2}{80}\ k^2\ \left[\pi_{\mu\rho}\pi_{\nu\sigma} + \pi_{\mu\sigma}\pi_{\mu\rho} - \frac{2}{3}\pi_{\mu\nu}\pi_{\rho\sigma}\right]\delta_2 + \frac{\pi^2}{120}k^2\ \pi_{\mu\nu}\pi_{\rho\sigma}\ \delta_0$$

For IR regularity:

$$6\delta_2 + \delta_0 = 0$$

• There are $O(k^4)$ terms (scheme dependent)

$$\delta\langle T_{\mu\nu}T_{\rho\sigma}\rangle(k) = \left[\pi_{\mu\rho}\pi_{\nu\sigma} + \pi_{\mu\sigma}\pi_{\mu\rho} - \frac{2}{3}\pi_{\mu\nu}\pi_{\rho\sigma}\right]k^4 A_2 + \frac{B_0(k)}{3}\pi_{\mu\nu}\pi_{\rho\sigma}k^4 A_0$$

• Ignoring the contact terms, the interaction mediated by $T_{\mu\nu}$ is given at the quadratic level by

$$W_2^{nc} = \frac{1}{2}\int\frac{d^4k}{(2\pi)^4}\left[2B_2^{nc}(k)\left(t^{\mu\nu}(k)t_{\mu\nu}(-k) - \frac{1}{3}t(k)t(-k)\right) + \frac{B_0^{nc}(k)}{3}t(k)t(-k)\right]$$

The tensor structure is that of a massive spin-2 exchange. For the non-contact contributions at small k

$$B_2(k) = c_{IR}^{(2)}k^4\log\frac{k^2}{M^2} + \mathcal{O}(k^6) \quad , \quad B_0 \simeq c_{IR}^{(0)}k^4\log\frac{k^2}{M^2} + \mathcal{O}(k^6)$$

• The interaction depends crucially on the structure of $B^{nc}_{2,0}$. If there is a mass gap and discrete states, then near a pole we can approximate

$$B_{2,0} \simeq \frac{R_{2,0}}{k^2 + m^2_{2,0}}$$

where the residue $R_{2,0}$ has mass dimension six as B has mass dimension four.

• The resulting interaction involves a massive spin-2 particle of mass m_2 and a massive spin-0 particle with mass m_0.

• In a unitary theory all residues are positive and the exchanges are never ghostlike.

• By an appropriate rescaling of the interacting densities, we find the associated "Planck scales" to be given by

$$M^2_{0,2} \sim \frac{V^2}{R_{2,0}}$$

• The associated field theory is a bi-gravity theory.

A low-energy effective action

• To try to discern the non-linear theory, we consider a theory with a gap and we write the most general Schwinger functional valid below the gap energy. We keep the metric $g_{\mu\nu}$, and a scalar source, ϕ

$$S_{\rm Schwinger}(g,\phi) = \int d^4x\sqrt{g}\left[-V(\phi) + M^2(\phi)\,R - Z(\phi)\,(\partial\phi)^2 + \mathcal{O}(\partial^4)\right]$$

We now define $h_{\mu\nu}$ as the expectation value of the stress tensor

$$h_{\mu\nu} \equiv \langle T_{\mu\nu}\rangle = \frac{V}{2}g_{\mu\nu} + M^2 G_{\mu\nu} - \mathcal{T}^{\phi}_{\mu\nu} + \mathcal{O}(\partial^4)$$

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{R}{2}g_{\mu\nu} \quad , \quad \mathcal{T}^{\phi}_{\mu\nu} \equiv T^{\phi}_{\mu\nu} + (\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box)M^2$$

and the (emergent) dimensionless metric $\tilde{h}_{\mu\nu}$ as

$$\tilde{h}_{\mu\nu} \equiv \frac{2}{V} h_{\mu\nu} = g_{\mu\nu} + \frac{2M^2}{V} G_{\mu\nu}(g) - \frac{2}{V} \mathcal{T}^{\phi}_{\mu\nu} + \mathcal{O}(\partial^4) \ .$$

• We can now solve g as a function of $\tilde{h}$:

$$g_{\mu\nu} = \tilde{h}_{\mu\nu} - \frac{2M^2}{V} \tilde{G}_{\mu\nu} + \frac{2}{V} \tilde{\mathcal{T}}^{\phi}_{\mu\nu} + \mathcal{O}(\partial^4) \ .$$

• This is the dynamical equation stemming from the effective action Γ for the emergent metric that we rewrite as,

$$M^2 \ \tilde{G}_{\mu\nu} = \frac{V}{2} (\tilde{h}_{\mu\nu} - \mathbf{g}_{\mu\nu}) + \tilde{\mathcal{T}}^{\phi}_{\mu\nu} + \mathcal{O}(\partial^4) \ .$$

• This is the equation of a bi-gravity theory with $\mathbf{g}_{\mu\nu}$ as the fiducial metric.

• Here we see a general property of emergent gravity: $\tilde{h}_{\mu\nu} \sim \mathbf{g}_{\mu\nu}$ is always a solution if $\mathbf{g}_{\mu\nu}$ is a constant curvature metric and corresponds to the vacuum solution of the theory (independently of any quantum corrections) .

• However, and not surprisingly, when we linearize this equation and calculate the interaction it mediates, we obtain

$$S_{int}(T,T') = \frac{T^{\mu\nu}T'_{\mu\nu} - \frac{1}{3}TT'}{M^2(p^2 - \Lambda)} - \frac{1}{6}\frac{TT'}{M^2\left(p^2 + \frac{\Lambda}{2}\right)} \quad , \quad \Lambda = \frac{V}{M^2} \, ,$$

• From this interaction we conclude that the spin-zero mode is always a ghost. Moreover, depending on the sign of the vev Λ, either the spin-2 or the spin-0 exchange behaves as a tachyon.

• This discrepancy exists because expanding in derivatives the Schwinger functional, computing Γ and then computing the induced interaction back, mixes contact terms with pole terms, and therefore misidentifies masses and residues.

Gravitons from hidden sectors

• We have presented so far a proof of principle, on how to describe an emergent graviton in a generic QFT.

• In the real world, the graviton that couples to the SM stress tensor appears to be an additional dynamical field.

• How can we describe this as an emergent degree of freedom?

♠ It can emerge in similar way from a "hidden sector".

♠ The hidden sector will be coupled to the SM at some high scale.

♠ Only a few interactions must survive in the IR between the two theories provided interactions are IR-irrelevant.

♠ This will match with the IR-freedom and non-renormalizability of gravity.

• If we want this graviton to be tightly bound and weakly coupled, then this hidden sector theory must be a large-N, strongly coupled (ie holographic) QFT.

• We are led therefore to couple a large-N theory to the SM in a UV complete fashion.

• One therefore should postulate a massive bifundamental messenger sector to couple the two theories together.

$$QFT_N \quad \times \quad Bifundamentals \quad \times \quad StandardModel$$

• If the bifundamentals have mass M, we can integrate them out to obtain a theory below M with contact interactions between QFT_N and the SM.

The linearized coupling

• We consider a hidden theory and a visible theory defined on the Minkowski metric $\mathbf{g}_{\mu\nu} = \eta_{\mu\nu}$.

• We consider a coupling between the "hidden theory" and the "visible theory" of the form

$$S_{int} = \int d^4x \left(\lambda \mathsf{T}_{\mu\nu}(x)\,\widehat{\mathsf{T}}^{\mu\nu}(x) + \lambda' \mathsf{T}(x)\widehat{\mathsf{T}}(x)\right)$$

at a high scale M where $\mathsf{T} \equiv \eta^{\mu\nu}\mathsf{T}_{\mu\nu}$.

This is an irrelevant coupling with $\lambda, \lambda' \sim M^{-4}$.

• $\mathsf{T}_{\mu\nu}$ is the SM energy-momentum tensor, $\widehat{\mathsf{T}}_{\mu\nu}$ is the hidden one.

• We also define

$$\mathfrak{c} \equiv \frac{\lambda'}{\lambda} \quad , \quad \mathbf{T}_{\mu\nu} \equiv \mathsf{T}_{\mu\nu} + \mathfrak{c}\,\mathsf{T}\,\eta_{\mu\nu}$$

so that

$$S_{int} = \lambda \int d^4x\, \mathbf{T}_{\mu\nu}(x)\,\widehat{\mathsf{T}}^{\mu\nu}(x)$$

• Note that the expectation value of the hidden energy momentum tensor, acts as an external metric for the SM.

$$\int d^4x \ \mathbf{T}_{\mu\nu}(x)\,\widehat{\mathsf{T}}^{\mu\nu}(x) \quad \to \quad \int d^4x \ \mathbf{T}_{\mu\nu}(x) h^{\mu\nu}$$

• We assume that $\langle \widehat{T}_{\mu\nu} \rangle = \widehat{\Lambda}\eta_{\mu\nu}$.

• The coupling has introduced the following effective interactions in the visible theory:

$$\delta S_{vis} = \lambda\widehat{\Lambda} \int d^4x\, \mathbf{T}(x) - \frac{1}{2}\lambda^2 \int d^4x_1 d^4x_2\, \mathbf{T}_{\mu\nu}(x_1)\, \mathbf{T}_{\rho\sigma}(x_2)\, \widehat{\mathsf{G}}^{\mu\nu,\rho\sigma}(x_1 - x_2)$$

• The second term is an induced quadratic energy-momentum interaction in the visible theory.

• This interaction can be reformulated in terms of a classical spin-2 field $h_{\mu\nu}$

$$\delta S_{eff}^{TT} = \int d^4k \left[-h_{\mu\nu}(-k)\mathbf{T}^{\mu\nu}(k) + \frac{(2\pi)^4}{2\lambda^2} h_{\mu\nu}(-k) \quad \mathcal{P}^{\mu\nu,\rho\sigma}(k) \quad h_{\rho\sigma}(k) \right]$$

• The inverse propagator $\mathcal{P}^{\mu\nu,\rho\sigma}(k)$ of the emerging spin-2 field is the inverse of the hidden sector 2-point function $\widehat{\mathsf{G}}^{\mu\nu,\rho\sigma}(k)$.

• It remains to examine under what circumstances $\mathcal{P}^{\mu\nu,\rho\sigma}(k)$ is well-defined and what tensor structures it involves.

• We assume that the hidden theory is a Lorentz-invariant QFT.

$$\widehat{\mathsf{G}}^{\mu\nu,\rho\sigma}(k) = \widehat{\Lambda}\left(\eta^{\mu\nu}\eta^{\rho\sigma} + \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\rho\nu}\right) + \widehat{\mathsf{b}}(k^2)\Pi^{\mu\nu\rho\sigma}(k) + \hat{\mathsf{c}}(k^2)\pi^{\mu\nu}(k)\pi^{\rho\sigma}(k)$$

with

$$\pi^{\mu\nu} = \eta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \quad , \quad \Pi^{\mu\nu,\rho\sigma}(k) = \pi^{\mu\rho}(k)\pi^{\nu\sigma}(k) + \pi^{\mu\sigma}(k)\pi^{\nu\rho}(k)$$

• The only combination of tensor structures which is analytic at quadratic order in momentum, in the long-wavelength limit $k^2 \to 0$, is the one that has

$$\widehat{\mathsf{b}}(k^2) = \widehat{\mathsf{b}}_0 \; k^2 + \mathcal{O}(k^4) \; , \quad \hat{\mathsf{c}}(k^2) = -2\widehat{\mathsf{b}}_0 \; k^2 + \mathcal{O}(k^4)$$

• If $\widehat{\Lambda} = 0$, the two-point function has zero modes which are proportional to k^μ and is therefore not invertible.

• In this case, one must invert in the space orthogonal to the zero modes. This gives rise to a non-local effective theory for the graviton.

• Up to quadratic order in the momentum expansion

$$\mathcal{P}^{\mu\nu\rho\sigma}(k) = -\frac{1}{4\widehat{\Lambda}}(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho})$$

$$+2\widehat{b}_0\widehat{\Lambda}^{-2}\left[\frac{k^2}{8}(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho})\right.$$

$$\left.+\frac{1}{8}(\eta^{\nu\sigma}k^\mu k^\rho + \eta^{\nu\rho}k^\mu k^\sigma + \eta^{\mu\sigma}k^\nu k^\rho + \eta^{\mu\rho}k^\nu k^\sigma)\right] + \mathcal{O}(k^4)$$

Emergent quadratic gravity

- We now re-define:

$$h_{\mu\nu} = -\mathfrak{h}_{\mu\nu} + \frac{1}{2}\mathfrak{h}\,\eta_{\mu\nu} + \lambda\widehat{\Lambda}\,\eta_{\mu\nu}\ , \quad \mathfrak{h} = \mathfrak{h}^{\rho\sigma}\eta_{\rho\sigma}$$

$$\mathfrak{T}^{\mu\nu} \equiv \mathbf{T}^{\mu\nu} - \frac{1}{\lambda}\left(1 + \frac{1}{2\lambda\ \widehat{\Lambda}}\right)\eta^{\mu\nu}\ , \quad \mathfrak{T} = \mathfrak{T}^{\mu\nu}\eta_{\mu\nu}$$

- The full effective action of the visible QFT at this order in the λ-expansion and at the two-derivative level is

$$S_{eff} = S_{vis} + \int d^4x\,\left(\mathfrak{h}_{\mu\nu}\mathfrak{T}^{\mu\nu} - \frac{1}{2}\mathfrak{h}\,\mathfrak{T}\right) + \frac{1}{16\pi G}\int d^4x\left[\sqrt{g}\,(R+\Lambda)\right]^{(2)}_{g_{\mu\nu}=\eta_{\mu\nu}+\mathfrak{h}_{\mu\nu}}$$

with the identification of parameters

$$\Lambda = \frac{\widehat{\Lambda}}{\widehat{b}_0}\ , \quad \frac{1}{16\pi G} \equiv M_P^2 = -\frac{(2\pi)^8\ \widehat{\mathrm{b}}_0}{\lambda^2\widehat{\Lambda}^2}$$

- The sign of Newton's constant is positive when $\widehat{\mathrm{b}}_0$ is negative

• This seems to be the case with simple QFTs but we have no general proof.

• The second term, which describes the coupling of the visible QFT to the emergent graviton, can be expressed in terms of the original energy-momentum tensor of the visible QFT

$$\int d^4x \left(\mathfrak{h}_{\mu\nu}\mathfrak{T}^{\mu\nu} - \frac{1}{2}\mathfrak{h}\,\mathfrak{T}\right) = \int d^4x\sqrt{g}\ g^{\mu\nu}\ \mathfrak{T}_{\mu\nu}\Big|_{g_{\mu\nu}=\eta_{\mu\nu}+\mathfrak{h}_{\mu\nu}}$$

• There is a non-trivial shift of the energy due to the coupling of the two theories.

• Because of the presence of "dark energy" the flat (fiducial) metric is always a solution to the equations of emergent gravity.

• However as before, we will do the computation without expanding in momenta.

Emergent quadratic gravity II

We can compute the (non-contact part of the) induced interaction between SM sources without expanding in momenta

$$L_{int} = -\frac{\lambda^2}{2}\left[2B_2(k)\left(T_{\mu\nu}(-k)T^{\mu\nu}(k) - \frac{1}{3}T(-k)T(k)\right) + \frac{(1+3\mathfrak{c})^2}{3}B_0(k)\right] + \cdots$$

where $\mathfrak{c}$ is defined by

$$S_{int} = \lambda \int d^4x \left[\hat{T}_{\mu\nu}T^{\mu\nu} + \mathfrak{c}\ \hat{T}T\right]$$

- The tensor structure of the interaction is that of massive gravity.

- At the special (integrable) value $\mathfrak{c} = -\frac{1}{3}$ the scalar dilaton decouples.

Taylor

- Both the spin-2 and spin-0 interactions are always attractive and stable.

- Around a massive pole, of mass m_2 (assuming $R \sim m_2^6$) we obtain

$$M_P^2 \sim \frac{M^8}{R_2} \sim M^2 \left(\frac{M}{m_2}\right)^6$$

- A generalization of the formalism of the effective action allows us to (formally) construct the full non-linear theory.

The non-linear analysis

• We start again from the Schwinger functional of the coupled QFTs $W\left(\mathcal{J},\hat{\mathcal{J}},\mathbf{g}\right)$

• The interaction is defined as general as possible:

$$S_{int}=\int d^4x\sqrt{\mathbf{g}}\sum_i\lambda_i\,\mathcal{O}_i(x)\,\widehat{\mathcal{O}}_i(x)$$

• Via similar techniques a functional $S_{eff}(h)$ can be constructed and satisfies

$$\left.\frac{\delta S_{eff}}{\delta h_{\mu\nu}}\right|_{g_{\mu\nu}=\mathbf{g}_{\mu\nu}}=0$$

♠ These are the emerging non-linear gravitational equations.

♠ When evaluated in the solution of the above equation gives the original action.

• Therefore, $S_{eff}(h,\Phi,\mathcal{J},\hat{\mathcal{J}},\mathbf{g})$ is the emergent gravity action that generalizes the linearized computation.

The holographic hidden QFT

• The general action is

$$S = S_{hidden} + S_{T\hat{T}} + S_{visible}$$

• Using the holographic correspondence

$$\langle e^{iS_{T\hat{T}}} \rangle_{hidden} = \int_{\lim_{z\to z_0} G_{\mu\nu}(x,z)=\mathbf{g}_{\mu\nu}} \mathcal{D}G \; e^{iS_{\text{bulk}}[G]+i\lambda\int d^4x\,\sqrt{\mathbf{g}}\,\widehat{T}_{\mu\nu}\mathbf{T}^{\mu\nu}}$$

with $z_0 \sim \frac{1}{M}$.

• It is also true that

$$\langle e^{iS_{T\hat{T}}} \rangle_1 = \int_{\lim_{z\to z_0} G_{\mu\nu}(x,z)=\mathbf{g}_{\mu\nu}+\lambda\mathbf{T}_{\mu\nu}} \mathcal{D}G \; e^{iS_{\text{bulk}}[G]}$$

• By a series of formal manipulations we can show that this is equivalent to a brane (visible theory) coupled to the holographic bulk, but with Neumann bcs.

The brane-world picture

$$S_{total} = S_{bulk} + S_{brane}$$

$$S_{bulk} = M_P^3 \int d^5x\sqrt{G}\left[-V(\phi) + R_5(G) - \frac{1}{2}(\partial\phi)^2 + \cdots\right]$$

$$S_{brane} = M_P^2 \int dz\delta(z-z_0)\left(\int d^4x\sqrt{\gamma}\left[-W_B(\phi) + U_B(\phi)\ R_4(\gamma) - \frac{1}{2}Z_B(\partial\phi)^2 + \cdots\right.\right.$$

$$+S_{SM}(\gamma))$$

with standard bc at the shifted boundary.

- Bulk equations plus Israel conditions give all dynamical equations.

- These have been studied recently and shown to generically have "self-tuning" solutions if the boundary metric is a flat metric.

- This implies that even if $W_B(\phi) \neq 0$, the brane is stabilised at a fixed bulk position $z = z_0$ with an induced flat metric.

Charmousis+Kiritsis+Nitti

The brane graviton

- The induced interaction due to the transverse-traceless fluctuation is

$$S_{int} = -\frac{1}{2M^3}\int d^4x\, d^4x'\, G(r_0, x; r_0, x')\left(T^{\mu\nu}(x)T_{\mu\nu}(x') - \frac{1}{3}T(x)T(x')\right)$$

$$G(r, x; r_0, x') = \frac{1}{\frac{1}{G_{bulk}(r,x;r_0,x')} + G_{brane}(x, x')}$$

Dvali+Gabadadze+Porrati

- This should be contrasted with the field-theoretical formula

$$Interaction\ of\ energy\ sources = \frac{1}{\frac{1}{\lambda^2\langle\hat{T}\hat{T}\rangle_{hidden}} + \langle TT\rangle_{SM}} = \frac{\langle\hat{T}\hat{T}\rangle_{hidden}}{1 + \langle\hat{T}\hat{T}\rangle_{hidden}\langle TT\rangle_{S}}$$

- As $\langle\hat{T}\hat{T}\rangle_{hidden} \sim \mathcal{O}(1)$, $\lambda \sim \mathcal{O}(N^{-1})$ the SM corrections shift slightly the poles of $\langle\hat{T}\hat{T}\rangle_{hidden}$ that are at $m \sim \mathcal{O}(1)$.

- There are the following characteristic distance scales.

- The *transition scale* r_t around which $G_{bulk}(r_0, p)$ changes from small to large momentum asymptotics:

- The DGP scale, r_c:

$$r_c \equiv \frac{U_0}{2};$$

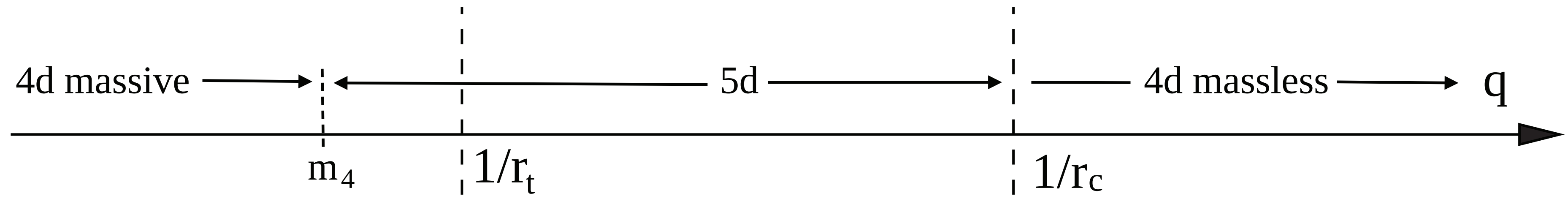
This scale determines the crossover between 5-dimensional and 4-dimensional behavior, and enters the 4D Planck scale and the graviton mass.

- The *gap scale* d_0

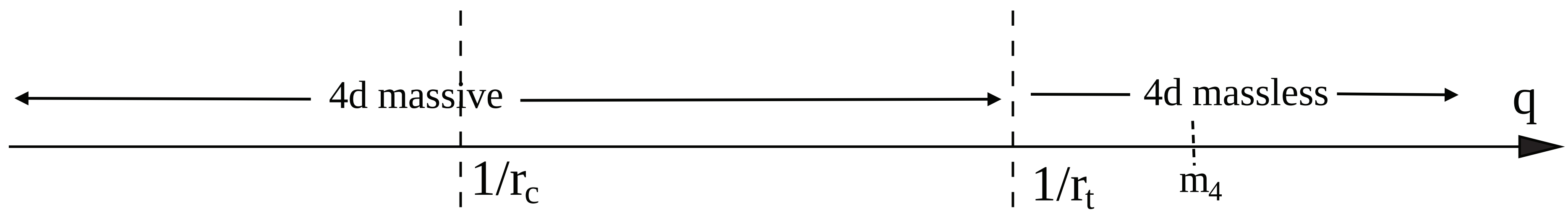
$$d_0 \equiv G_{bulk}(r_0, 0) = \int_0^{r_0} dr' e^{-3A_{UV}(r')},$$

which governs the propagator at the largest distances (in particular it sets the graviton mass).

• When $r_t > r_c$ we have three regimes for the gravitational interaction on the brane:



• Massive 4d gravity ($r_t < r_c$)



• There is a vDVZ discontinuity that (as usual) cannot be cancelled at the linearized order if the theory is positive. It should be cancelled by the Vainshtein mechanism.

Scalar Perturbations

• The equations for the scalar perturbations can be derived and they are complicated.

• There are two scalar modes on the brane:

• In general the two scalar modes couple to two charges:

(a) the “scalar charge” and

(b) the trace of the brane stress tensor.

• The mode that couples to the trace of the stress-tensor has a mass that is of order the graviton mass and is the lightest of the two scalars.

• All the stability conditions for the scalars depend on more details of the brane induced functions $W_B(\Phi)$, $U_B(\Phi)$, $Z_B(\Phi)$.

Comments and Open ends

- We can integrate-in many other fields. Most of them however will have large masses of $\mathcal{O}(M) \sim M_P$. The only generically protected ones, are the graviton, the universal axion and global conserved currents (graviphotons).

Anastasopoulos+Betzios+Bianchi+Consoli+Kiritsis

- It is not yet clear how to bridge the ultimate IR non-linear theory with one of the massive graviton theories.

- In all emergent graviton theories, after including all quantum corrections the background fiducial flat metric is always a solution. Therefore there is no standard CC problem.

- There seems to be a "dark energy" that originates in the hidden theory.

- Additional sources in the hidden theory may provide new sources of "dark" components: energy, matter etc.

- When the hidden theory is a holographic QFT then this description transforms into the brane-in-bulk (or brane-world) description (with Neumann boundary conditions at the AdS boundary).

- In this case, one can self-tune the cosmological constant and always obtain a massive graviton on the brane. Its mass is determined by the holographic hidden QFT.

Charmousis+Kiritsis+Nitti

- Adding also the natural (bulk) axion one can, in principle, correlate the self-tuning of the brane CC to a solution of the hierarchy problem.

Hamada+Kiritsis+Nitti+Witkowski

- There is always a scalar mode (the "dilaton") beyond a massive graviton that is always positive in a unitary QFT.

- This is also a natural dark matter or dark energy candidate. Can one make it comply with the principle of equivalence?

- It is not known if there is a hidden theory that will give an emergent graviton theory that is in agreement with data. The constraints are many.

- Boostrap techniques might answer the question whether spectral properties of the energy-momentum tensor that are necessary, can appear in a QFT.

- Black holes depend on non-linear dynamics. There is a certain puzzle here, as massive gravity seems it cannot give static black-holes but brane-world black holes can be static.

- The signature of the metric in emergent gravity can change. If a stress tensor is close to that of a cosmological constant then the signature is Minkowski. If it is of the photonic type, it has Euclidean signature.

- Therefore we conclude that if the hidden theory is near its ground state, it will be dominated by its cosmological constant and the signature is Minkowski. If the hidden theory is in a highly excited state, then the emerging metric will become of Euclidean signature.

The Weinberg-Witten loop-hole

- In GR the stress tensor is not conserved but covariantly conserved.

- One can add corrections to the tress tensor (involving also the flat metric) to make is strictly conserved and Lorentz covariant. This is however NOT a tensor under general coordinate transformations (but this is OK with WW).

- To make a pure helicity-two state, we must project out the (unphysical) helicity 1 and 0 states. This projection is NOT Lorentz covariant (but only up to a gauge transformation).

- We may appeal to diff-invariance to decouple the helicity 0 and 1 states but then we are stuck: $T_{\mu\nu}$ is now NOT fully covariant.

- Therefore GR and many other theories with an explicit dynamical graviton avoid the WW theorem.

Translation Ward identity

• We consider a theory with Lagrangian $\mathcal{L}$. For concreteness, we focus on four-dimensional QFTs.

• Under an infinitesimal diffeomorphism generated by a vector ξ_μ

$$\delta_\xi \mathcal{L} = \frac{1}{2}\left(\partial_\mu \xi_\nu + \partial_\nu \xi_\mu\right) T^{\mu\nu}$$

$$\delta_\xi T^{\mu\nu} = \xi^\sigma \partial_\sigma T^{\mu\nu} + T^{\sigma\nu}\partial^\mu \xi_\sigma + T^{\mu\sigma}\partial^\nu \xi_\sigma$$

• The invariance of the partition function $Z = e^{i\int d^4x\, \mathcal{L}}$ under the infinitesimal translation implies the conservation equation

$$\partial_\mu \langle T^{\mu\nu}\rangle = 0$$

• Similarly, the invariance of the one-point function of the energy-momentum tensor

$$\langle T^{\rho\sigma}(y)\rangle = \frac{\int D\Phi\, e^{i\int d^4x\,\mathcal{L}}\, T^{\rho\sigma}(y)}{\int D\Phi\, e^{i\int d^4x\,\mathcal{L}}}$$

under the infinitesimal translations implies the Ward identity

$$-i\langle\partial_\mu T^{\mu\nu}(x)T^{\rho\sigma}(y)\rangle + \delta(x-y)\langle\partial^\nu T^{\rho\sigma}(x)\rangle + \partial^\nu\delta(x-y)\langle T^{\rho\sigma}(x)\rangle$$

$$-\partial^\rho\left(\delta(x-y)\langle T^{\nu\sigma}(x)\rangle\right) - \partial^\sigma\left(\delta(x-y)\langle T^{\rho\nu}(x)\rangle\right) = 0$$

• In addition, Lorentz invariance implies that the one-point function of the energy-momentum tensor is

$$\langle T^{\mu\nu}(x)\rangle = a\eta^{\mu\nu}$$

where a is a dimensionfull constant.

Consequently, we set

$$\langle\partial^\nu T^{\rho\sigma}(x)\rangle = 0$$

and use it to simplify the Ward identity

$$i\langle\partial_\mu T^{\mu\nu}(x)T^{\rho\sigma}(y)\rangle - \partial^\nu\delta(x-y)\langle T^{\rho\sigma}(x)\rangle$$

$$+\partial^\rho\left(\delta(x-y)\langle T^{\nu\sigma}(x)\rangle\right) + \partial^\sigma\left(\delta(x-y)\langle T^{\rho\nu}(x)\rangle\right) = 0$$

- In momentum space we obtain instead:

$$k_\mu\langle T^{\mu\nu}(k)T^{\rho\sigma}(-k)\rangle = ia\left(-k^\nu\eta^{\rho\sigma} + k^\rho\eta^{\nu\sigma} + k^\sigma\eta^{\rho\nu}\right)$$

- This allows us to deduce the 2-point function as **??**

$$\langle T^{\mu\nu}(k)T^{\rho\sigma}(-k)\rangle$$

$$= ia\left(-\eta^{\mu\nu}\eta^{\rho\sigma} + \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\rho\nu}\right) + b(k^2)\,\Pi^{\mu\nu\rho\sigma}(k) + c(k^2)\,\pi^{\mu\nu}(k)\pi^{\rho\sigma}(k)$$

with

$$\Pi^{\mu\nu\rho\sigma}(k) = \pi^{\mu\rho}(k)\pi^{\nu\sigma}(k) + \pi^{\mu\sigma}(k)\pi^{\nu\rho}(k) \quad , \quad \pi^{\mu\nu}(k) = \eta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}$$

Aside: String theory vs the swampland

• Conjectures talk about "quantum gravity" but everyone means "string theory"

• The (plausible) assumption that string theory is the space of large-N strongly coupled QFTs, has an automatic avatar:

• The "swampland" corresponds to QFTs that are either weakly-coupled, or are not at large N.

• This explains for example, the generic towers of states that appear at the boundaries of moduli spaces.

• It also suggests why there might be no de Sitter solution in "string theory".

• The notion of string theory used above is certainly more general that the conventional one based on 2d CFTs

• It involves also 3, 4, 5 and 6-dimensional CFTs.

• It might be illuminating to try to see the swampland conjectures via this point of view.

Higher spin

• It is one of the obvious next questions to ask: what about doing this for other operators of your QFT:

• For fields up to $S = 1/2$ this is a standard procedure, and has been done in many contexts.

• The case of $S = 1$ is interesting as it would describe emergent gauge theory. It is qualitatively different than the gravity case.

• When $S > 2$ one can again do the same procedure as here.

• In that case however for interacting theories, higher spin fields are not conserved. The effective theory one obtains will be massive, with characteristic mass the overall cutoff (in string theory this is the string scale).

• They are therefore less interesting for low-energy physics.

• In a free QFT however they are conserved and then one can construct massless actions (of an infinite number of them)

Douglas+Razamat, Leigh

WW versus AdS/CFT

• Is AdS/CFT compatible with the WW theorem?

• The WW theorem involves a subtle limit to define the helicity amplitudes that determine the couplings of massless states to the stress tensor or a local current.

• This limiting procedure is not valid in theories where the states form a continuum.

• This is the case in AdS/CFT.

• From the point of view of the QFT, the effective gravitational coupling is non-local.

• Therefore the WW-theorem does not apply to this case.

• What about non-CFTs?

WW versus nAdS/nCFT

- Consider a familiar example: four-dimensional, large-N YM theory.

- Its string-theory dual is stringy (and nearly tensionless) near the AdS-boundary (weak QFT coupling).

- We expect a gravitational description at low energies (strong QFT coupling).

- The theory has a gap and a discrete spectrum and therefore the emergent gravitational interactions must be local.

- Also gravity must be weakly coupled (and it is, due to large N limit).

• The low energy spectrum contains two stable (lightest) massive scalars (0^{++}, 0^{-+}), and a stable massive graviton (2^{++}). All other glueballs are resonances, and are not asymptotic states.

• The higher cousins of the graviton are unstable.

• A massive graviton is compatible with WW.

• It is also compatible with a fully diff invariant theory of a massless graviton in 5 dimensions.

• The 4d graviton mass is due to the non-trivial 5d background, hence a gravitational "Higgs effect".

• The above gives some credence to the idea that heavy-ion collisions form (unstable) black holes of a massive gravity theory that quickly Hawking evaporate.

Nastase, Kiritsis+Taliotis

An explicit IR parametrization

• We assume that the theory has a uniform mass gap for simplicity.

• We will now parametrize the Schwinger functional W in an IR expansion below the mass gap as

$$W(g,J)=\int\sqrt{g}\left[-V(J)+M^2(J)R(g)-\frac{Z(J)}{2}(\partial J)^2+\mathcal{O}(\partial^4)\right]$$

• We calculate

$$h_{\mu\nu}=\frac{V}{2}g_{\mu\nu}+M^2G_{\mu\nu}-(\nabla_\mu\nabla_\nu-g_{\mu\nu}\Box)M^2-\frac{1}{2}T_{\mu\nu}+\cdots$$

$$G_{\mu\nu}=R_{\mu\nu}-\frac{1}{2}R\;g_{\mu\nu}\quad,\quad T_{\mu\nu}=Z(J)\left(\partial_\mu J\partial_\nu J-\frac{1}{2}g_{\mu\nu}(\partial J)^2\right)$$

• The $h_{\mu\nu}$ appears uniquely determined, but there is a initial+boundary condition dependence in this formula.

• Note that for arbitrary external source J, this energy-momentum tensor vev is not conserved.

$$\nabla_g{}^\mu h_{\mu\nu}=\frac{1}{2}\left[V(J)'-Z(J)\Box_g J-\frac{1}{2}Z'(J)(\partial J)^2-(M(J)^2)'\;R\right]\partial_\nu J$$

• We may now solve $g_{\mu\nu}$ as a function of $h_{\mu\nu}$:

$$g_{\mu\nu} = \tilde{h}_{\mu\nu} - \delta\tilde{h}_{\mu\nu} \quad , \quad \tilde{h}_{\mu\nu} = \frac{2}{V} h_{\mu\nu}$$

$$\delta\tilde{h}_{\mu\nu} = \frac{2}{V}\left[M^2\tilde{G}_{\mu\nu} - (\tilde{\nabla}_\mu\tilde{\nabla}_\nu - \tilde{h}_{\mu\nu}\tilde{\Box})M^2\right] - \frac{1}{V}\tilde{T}_{\mu\nu} + \cdots$$

• All the tensors above are written in terms of $\tilde{h}_{\mu\nu}$.

• $\tilde{h}_{\mu\nu}$ is dimensionless and plays the role of the emergent dynamical metric.

• We may rewrite it as an Einstein equation coupled to "matter"

$$M^2\ \tilde{G}_{\mu\nu} = \frac{V(J)}{2}\left(\tilde{h}_{\mu\nu} - \mathbf{g}_{\mu\nu}\right) + \frac{1}{2}\tilde{T}_{\mu\nu}(J) + (\tilde{\nabla}_\mu\tilde{\nabla}_\nu - \tilde{h}_{\mu\nu}\tilde{\Box})M(J)^2 + \cdots$$

• The effective gravitational equation above is equivalent to $\frac{\delta\Gamma}{\delta h_{\mu\nu}} = 0$.

• The background metric $\mathbf{g}_{\mu\nu}$ appears as an external source and contributes like a cosmological constant.

• This is an "unusual" bigravity theory.

• Other sources act as sources of energy and momentum.

• This description is non-singular only if $V \neq 0$.

• If $V = 0$, then the gravitational theory is non-local but can be constructed.

• Note that when $J(x) \neq 0$ the original QFT is not translationally invariant and its energy-momentum tensor is not conserved.

• The emergent gravity theory is however still diff. invariant, and the diff. invariance is broken "spontaneously" because of the presence of the scalar source $J(x)$ and the fixed (fiducial) metric of the original QFT.

BACK

Emerging quadratic gravity:Comments

• A coupling of stress tensors between two theories induces gravity at the quadratic level.

• This is true in the generic case: $\widehat{\Lambda} \neq 0$.

• Otherwise the graviton theory is non-local.

• There is always an effective cosmological constant for the emerging gravity in the local case.

• There is also a shift of the stress tensor giving a "dark" energy. It is a reflection of the coupling to the hidden theory.

• We parametrize $\lambda = \frac{1}{NM^4}$ where M a large scale controlling the coupling of the two theories and N the number of colors of the hidden theory.

• Also from calculations

$$\hat{b}_0 = -\kappa N^2 m^2 \quad , \quad \kappa \sim O(1) \quad , \quad \hat{\Lambda} = \epsilon \; N^2 \; m^4 \quad , \quad \epsilon = \pm 1 \tag{3}$$

We may now calculate the relevant ratios of scales

$$\frac{\Lambda}{M_P^2} = -\frac{\epsilon}{\kappa^2 x^2} \quad , \quad \frac{\Lambda_{dark}}{M_P^2} = -\frac{\frac{N}{x} + \frac{\epsilon}{2(2\pi)^4}}{(1+4\mathfrak{c})\kappa^2 \; x^2} \quad , \quad \frac{m}{M_P} = \frac{1}{\sqrt{\kappa} \; x} \tag{4}$$

$$\frac{\Lambda_{dark}}{\Lambda} = \frac{\epsilon\frac{N}{x} + \frac{1}{2(2\pi)^4}}{(1+4\mathfrak{c})} \quad , \quad \frac{M^4}{M_P^4} = \frac{1}{\kappa^2 x^3} \quad , \quad x \equiv \frac{M^4}{m^4} \gg 1 \tag{5}$$

• We always have semiclassical gravity, $\Lambda \ll M_P^2$.

• If $N \lesssim x$ then

$$\Lambda \quad \sim \quad \Lambda_{\rm dark} \quad \sim \quad \mathcal{O}(m^2) \quad \ll \quad M^2 \quad \ll \quad M_P^2$$

• If $x \ll N \ll x^{\frac{3}{2}}$ then

$$\Lambda \quad \ll \quad \Lambda_{\rm dark} \quad \ll \quad M^2 \quad \ll \quad M_P^2$$

• If $x^{\frac{3}{2}} \ll N \ll x^3$ then

$$\Lambda \quad \ll \quad M^2 \ll \quad \Lambda_{\rm dark} \quad \ll \quad M_P^2$$

• If $N \gg x^3$ then

$$\Lambda \quad \ll \quad M^2 \quad \ll \quad M_P^2 \quad \ll \quad \Lambda_{\rm dark}$$

• For phenomenological purposes $x \lesssim 10^{20}$ so that the messenger scale is above experimental thresholds.

• Note that so far the SM quantum effects are not included.

Renormalization and contact terms in $\langle TT \rangle$

• There can exist various issues when trying to formulate a spectral representation of correlators in momentum space and the integral over the spectral factor will generically exhibit divergences.

In momentum space we have (without the constant contact term)

$$\langle T_{\mu\nu} T_{\rho\sigma} \rangle(k) = \frac{(d-1)^2 \mathcal{A}_d}{2\Gamma(d)} \; k^4 \; \left[\pi_{\mu\rho}\pi_{\nu\sigma} + \pi_{\mu\sigma}\pi_{\mu\rho} - \frac{2}{d-1}\pi_{\mu\nu}\pi_{\rho\sigma} \right] \bar{G}_2 +$$

$$+ \frac{\mathcal{A}_d}{\Gamma(d)} k^4 \; \pi_{\mu\nu}\pi_{\rho\sigma} \; \bar{G}_0 \equiv$$

$$\equiv B_2(k) \left[\pi_{\mu\rho}\pi_{\nu\sigma} + \pi_{\mu\sigma}\pi_{\mu\rho} - \frac{2}{d-1}\pi_{\mu\nu}\pi_{\rho\sigma} \right] + \frac{B_0(k)}{3} \pi_{\mu\nu}\pi_{\rho\sigma}$$

with

$$\bar{G}_i(k) = \int_0^\infty d\mu^2 \frac{\rho_i(\mu^2)}{k^2 + \mu^2} \quad , \quad i = 0, 2$$

$$\mathcal{A}_d = \frac{2\pi^{d/2}}{(d+1)2^{d-1}\Gamma(d/2)} \quad , \quad \pi_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \quad , \quad k^\mu \pi_{\mu\nu} = 0$$

• In $d = 4$ the spectral functions $B_{2,0}$ are related to the rest as

$$B_0 = \frac{\pi^2}{40} k^4 \ \bar{G}_0(k) \quad , \quad B_2 = \frac{3\pi^2}{80} k^4 \bar{G}_2$$

• Typically, the integral over μ^2 does not converge either at zero or infinity.

• We can rearrange the integral so that we can separate the UV and IR divergences by using the identity

$$\frac{\rho_i(\mu^2)}{k^2+\mu^2} = \frac{\rho_i(\mu^2)}{\mu^2 + m_{IR}{}^2} - (k^2 - m_{IR}{}^2)\frac{\rho_i(\mu^2)}{(\mu^2 + m_{IR}{}^2)(k^2+\mu^2)}$$

and rewrite

$$\bar{G}_i(k) = A_i - (k^2 - m_{IR}^2)\int_0^\infty \frac{d\mu^2}{(\mu^2 + m_{IR}^2)}\frac{\rho_i(\mu^2)}{(k^2+\mu^2)}$$

with

$$A_i \equiv \int_0^\infty d\mu^2 \frac{\rho_i(\mu^2)}{\mu^2 + m_{IR}^2}$$

• m_{IR} acts as an IR cutoff and is needed if the theory in question is massless

• Convergence in the IR assumes that $\lim_{\mu\to 0}\mu^2\rho_i(\mu^2)=0.$. This happens if the IR CFT is non-empty.

• On the other hand, all UV divergences are now hidden in A_i. We may introduce a UV cutoff Λ and define

$$A_i^c(\Lambda, m_{IR}) \equiv \int_0^{\Lambda^2} d\mu^2 \frac{\rho_i(\mu^2)}{\mu^2+m_{IR}^2}$$

so that the cutoff spectral functions are

$$\bar{G}_i^c(k) = A_i^c - (k^2-m_{IR}^2)\int_0^{\infty} \frac{d\mu^2}{(\mu^2+m_{IR}^2)}\frac{\rho_i(\mu^2)}{(k^2+\mu^2)}$$

• As $\Lambda\to\infty$, we have a finite number of divergent terms, starting with a single logarithm in $d=4$,

$$A_i^c \simeq c_i^{UV}\Lambda^{d-4}+d_i^{UV}\Lambda^{d-6}+\cdots+e_i^{UV}\log\Lambda^2+\cdots \quad , \quad d\geq 4 \quad , \quad d=even$$

$$A_i^c \simeq c_i^{UV}\Lambda^{d-4}+d_i^{UV}\Lambda^{d-6}+\cdots+e_i^{UV}\Lambda+\cdots \quad , \quad d>4 \quad , \quad d=odd$$

• We then define the renormalized A_i by subtracting the divergences and eventually a finite piece, and then taking the UV cutoff to infinity.

$$A_i^{ren}(m_{IR}) = \lim_{\Lambda\to\infty} (A_i^c - \text{UV} \quad \text{divergences})$$

• $A_i^{ren}(m_{IR})$ is now a finite contact term that still depends in general on m_{IR}, if the IR theory is a non-trivial CFT.

• It is important to mention that the UV divergences do not depend on m_{IR}, and therefore the subtracted piece does not depend on m_{IR}. This will guarantee that the final renormalized density is m_{IR} -independent.

• Finally the renormalized $\bar{G}_i$ is given by

$$\bar{G}_i^{ren} \equiv A_i^{ren}(m_{IR}) - (k^2 - m_{IR}^2) \int_0^\infty \frac{d\mu^2}{(\mu^2 + m_{IR}^2)} \frac{\rho_i(\mu^2)}{(k^2 + \mu^2)}$$

and is independent of m_{IR}.

• For a CFT$_4$ we have $\rho_i(\mu^2) = c_i$ and we obtain

$$A_i^c = c_i \log \frac{\Lambda^2 + m_{IR}^2}{m_{IR}^2}$$

• This can be renormalized by subtracting the leading UV divergence

$$A_i^{ren} \equiv \lim_{\Lambda\to\infty}\left(A_i^c - c_i \log\frac{\Lambda^2}{M^2}\right) = c_i \log\frac{M^2}{m_{IR}^2}$$

• The scheme dependence is associated with the value of M.

• The renormalized $\bar{G}$ for a CFT$_4$ is then

$$\bar{G}_i^{ren} = A_i^{ren} - (k^2 - m_{IR}^2)\int_0^\infty \frac{d\mu^2}{(\mu^2+m_{IR}^2)}\frac{\rho_i(\mu^2)}{(k^2+\mu^2)} =$$

$$= c_i\left[\log\frac{M^2}{m_{IR}^2} - (k^2-m_{IR}^2)\int_0^\infty \frac{d\mu^2}{(\mu^2+m_{IR}^2)(k^2+\mu^2)}\right] = -c_i \log\frac{k^2}{M^2}$$

where M is a renormalization group scale.

• The appearance of the arbitrary scale M in the momentum space correlator is another avatar of the conformal anomaly.

• For a theory with a mass gap, we can set the scale $m_{IR} = 0$ and we can rewrite

$$\bar{G}_i^{ren} \equiv A_i^{ren} - k^2\int_0^\infty \frac{d\mu^2}{\mu^2}\frac{\rho_i(\mu^2)}{(k^2+\mu^2)}$$

• In $d=4$ the A_i^{ren} are dimensionless contact terms whose value depends on the renormalization scheme.

• The low momentum expansion becomes

$$\bar{G}_i^{ren} \equiv A_i^{ren} - B_i\ k^2 + \mathcal{O}(k^4) \quad , \quad B_i \equiv \int_{m_0^2}^{\infty} \frac{d\mu^2}{\mu^4} \rho_i(\mu^2)$$

where m_0 is the mass gap of the correlator.

• For a general four-dimensional theory without a mass gap we have that

$$\rho_i(\mu^2) \simeq c_i^{UV} \qquad for \qquad \mu \to \infty$$

while

$$\rho_i(\mu^2) \simeq c_i^{IR} \qquad for \qquad \mu \to 0$$

• We pick two scales, $m_1 \to 0$ so that it is much smaller that all the scale of the theory, while $m_2 \to \infty$ is much larger than all scales of the theory (except the UV cutoff) and write

$$\bar{G}_i^{ren} \equiv A_i^{ren}(m_{IR}) - I_{IR}^i - I_{UV}^i - I_{inter}^i$$

with

$$I^i_{IR} \equiv (k^2 - m_{IR}^2) \int_0^{m_1^2} \frac{d\mu^2}{(\mu^2 + m_{IR}^2)} \frac{\rho_i(\mu^2)}{(k^2+\mu^2)} \simeq c_i^{IR} (k^2 - m_{IR}^2) \int_0^{m_1^2} \frac{d\mu^2}{(\mu^2 + m_{IR}^2)(k^2 \ldots}$$

$$= c_i^{IR} \left[\log \frac{(m_1^2 + k^2)}{k^2} + \log \frac{m_{IR}^2}{(m_1^2 + m_{IR}^2)} \right]$$

$$I^i_{UV} \equiv (k^2 - m_{IR}^2) \int_{m_2^2}^{\infty} \frac{d\mu^2}{(\mu^2 + m_{IR}^2)} \frac{\rho_i(\mu^2)}{(k^2+\mu^2)} \simeq c_i^{UV} (k^2 - m_{IR}^2) \int_{m_2^2}^{\infty} \frac{d\mu^2}{(\mu^2 + m_{IR}^2)(k^2 \ldots}$$

$$= c_i^{UV} \log \frac{m_2^2 + k^2}{m_2^2 + m_{IR}^2}$$

and

$$I_{inter} \equiv (k^2 - m_{IR}^2) \int_{m_1^2}^{m_2^2} \frac{d\mu^2}{(\mu^2 + m_{IR}^2)} \frac{\rho_i(\mu^2)}{(k^2+\mu^2)}$$

- From these expressions, we deduce that I_{inter} is a regular power series in k^2 for k^2 small.

- Therefore in $\bar{G}_i$ there is only a log k^2 divergence that is appearing due to the IR CFT.

• For a gapless theory, we can write a small k^2 expansion that is of the form

$$\bar{G}_i = c_i^{IR} \log \frac{M^2}{k^2} + \text{regular} \quad \text{expansion} \quad \text{in} \quad k^2$$

and where M^2 is some scale of the theory.

• On the other hand, as $k^2 \to \infty$ we obtain

$$I_{IR} \simeq \text{regular} \quad \text{series} \quad \text{in} \quad \frac{1}{k^2} \quad , \quad I_{UV} \simeq c_i^{UV} \log k^2 + \text{regular} \quad \text{series} \quad \text{in} \quad \frac{1}{k^2}$$

$$I_{inter} = \text{regular} \quad \text{series} \quad \text{in} \quad \frac{1}{k^2}$$

so that

$$\bar{G}_i = c_i^{UV} \log \frac{k^2}{M'^2} + \text{regular} \quad \text{expansion} \quad \text{in} \quad \frac{1}{k^2}$$

as $k^2 \to \infty$.

• As $k^2 \to 0$, $\bar{G}_i^{ren}$ are regular functions of k^2 with an exception of a $\log k^2$ appearance, if the theory is gapless.

• There is, however, a set of contact terms, compatible with stress tensor conservation and IR regularity that are not included.

$$\bar{G}_i^{ren}(k) \to G_i^{ren} + \frac{\delta_i}{k^2}$$

Then

$$\delta\langle T_{\mu\nu}T_{\rho\sigma}\rangle(k) = \frac{3\mathcal{A}_4}{4}\ k^2\ \left[\pi_{\mu\rho}\pi_{\nu\sigma} + \pi_{\mu\sigma}\pi_{\mu\rho} - \frac{2}{3}\pi_{\mu\nu}\pi_{\rho\sigma}\right]\delta_2 + \frac{\mathcal{A}_4}{6}k^2\ \pi_{\mu\nu}\pi_{\rho\sigma}\ \delta_0$$

and the absence of the $\frac{k_\mu k_\nu k_\rho k_\sigma}{k^2}$ term implies that

$$6\delta_2 + \delta_0 = 0$$

and inserting in $\langle TT\rangle$ we obtain

$$\delta\langle T_{\mu\nu}T_{\rho\sigma}\rangle(k) = \frac{3\pi^2\delta_2}{80}\left[k^2(\delta_{\mu\rho}\delta_{\nu\sigma} + \delta_{\mu\sigma}\delta_{\mu\rho} - 2\delta_{\mu\nu}\delta_{\rho\sigma}) - \right.$$

$$\left. -(\delta_{\mu\rho}k_\nu k_\sigma + \delta_{\nu\sigma}k_\mu k_\rho + \delta_{\mu\sigma}k_\nu k_\rho + \delta_{\nu\rho}k_\mu k_\sigma) + 2\delta_{\mu\nu}k_\rho k_\sigma + 2\delta_{\rho\sigma}k_\mu k_\nu\right]$$

• It is clear that if $\delta_2 > 0$, then $\delta_0 < 0$ and the spin-zero piece of this particular term is ghost-like.

• Summarizing, the explicit contact contributions in the renormalized stress tensor functions $\bar{G}_i^{ren}$ in four-dimensions are

$$\bar{G}_2^{ren,contact}(k) = A_2^{ren} + \frac{\delta_2}{k^2} \quad , \quad \bar{G}_0^{ren,contact}(k) = A_0^{ren} - \frac{6\delta_2}{k^2}$$

Mixing with contact terms

We consider a quadratic source functional

$$W(J) = \int d^4p\ J(-p)G(p)J(p) \quad , \quad G(p) = G_0 + \frac{R}{p^2 - m^2}$$

where we took the two-point correlator to have a pole and a constant contact term. It is clear that the interaction of the source J contains an innocuous contact term contribution and the effect of the exchange of a particle of mass m and residue R.

• Consider now the following sequence of steps. Expand $W(J)$ up to $\mathcal{O}(p^2)$, construct the effective action Γ to order $\mathcal{O}(p^2)$ and then recompute the interaction of sources.

$$W(J) = \int d^4p\ J(-p)J(p)\left[\tilde{G}_0 - \frac{Rp^2}{m^4} + \mathcal{O}(p^4)\right] \quad , \quad \tilde{G}_0 = G_0 - \frac{R}{m^2}$$

$$h(p) = \frac{\delta W}{\delta J(-p)} = 2J(p)\left[\tilde{G}_0 - \frac{Rp^2}{m^4} + \mathcal{O}(p^4)\right]$$

$$\Gamma(h) = \int Jh - W = \frac{1}{4}\int d^4p\ h(-p)\left[\tilde{G}_0 - \frac{Rp^2}{m^4} + \mathcal{O}(p^4)\right]^{-1} h(p) =$$

$$= \frac{1}{4\tilde{G}_0}\int d^4p\ h(-p)\left[1 + \frac{Rp^2}{m^4\tilde{G}_0} + \mathcal{O}(p^4)\right] h(p)$$

Recomputing the original interaction we obtain instead

$$W(J) = \frac{m^4\tilde{G}_0^2}{R}\int d^4p\ \frac{J(-p)J(p)}{p^2 + \frac{m^4}{R}\tilde{G}_0} + \mathcal{O}(p^4)$$

• Comparing we observe that now both the residue and the position of the pole has changed.

• The reason is that the position of the pole is now not reliable in the momentum expansion. Moreover, depending on the sign and size of the initial contact term, G_0, the pole now may become a tachyon.

The non-linear analysis

- We start again from the Schwinger functional of the coupled QFTs

$$e^{-W(\mathcal{J},\hat{\mathcal{J}},\mathbf{g})} = \int [D\Phi]\,[D\widehat{\Phi}]\,e^{-S_{visible}(\Phi,\mathcal{J},\mathbf{g})-S_{hidden}\left(\widehat{\Phi},\mathbf{g},\hat{\mathcal{J}}\right)-S_{int}\left(\mathcal{O}^i,\widehat{\mathcal{O}}^i,\mathbf{g}\right)}$$

- Φ^i and $\widehat{\Phi}^i$ are respectively the (quantum) fields of the visible QFT and the hidden $\widehat{\text{QFT}}$.

- $\mathcal{J}$ and $\hat{\mathcal{J}}$ are (scalar) sources in the visible and hidden theories respectively.

- The interaction part is defined as:

$$S_{int} = \int d^4x\sqrt{\mathbf{g}}\sum_i \lambda_i\,\mathcal{O}_i(x)\,\widehat{\mathcal{O}}_i(x)$$

- For energies $E \ll M$, we can integrate out the hidden theory and obtain

$$e^{-W(\mathcal{J},\hat{\mathcal{J}},\mathbf{g})} = \int [D\Phi][D\widehat{\Phi}]\,e^{-S_{visible}(\Phi,\mathcal{J},\mathbf{g})-S_{hidden}\left(\widehat{\Phi},\hat{\mathcal{J}},\mathbf{g}\right)-S_{int}}$$

$$= \int [D\Phi]\,e^{-S_{visible}(\Phi,\mathcal{J},\mathbf{g})-\mathcal{W}(\mathcal{O}^i+\hat{\mathcal{J}}^i,\mathbf{g},)}$$

• We now put the full theory on a curved manifold with metric $g_{\mu\nu}$ and define again the generating functional in the presence of the background metric as

$$e^{-W(\mathcal{J},g,\hat{\mathcal{J}})} = \int [D\Phi]\, e^{-S_{visible}(\Phi,\mathcal{J},g) - \mathcal{W}(\mathcal{O}^i+\hat{\mathcal{J}}^i,g)}$$

• We define

$$h_{\mu\nu} \equiv \frac{1}{\sqrt{g}} \frac{\delta \mathcal{W}\left(\mathcal{O}^i, g, \hat{\mathcal{J}}\right)}{\delta g^{\mu\nu}}\Big|_{g_{\mu\nu}=\mathbf{g}_{\mu\nu}} = \langle \widehat{\mathsf{T}}_{\mu\nu} \rangle$$

• This will eventually play the role of an emergent metric for the visible theory.

• The diffeomorphism invariance of the functional $W(\mathcal{J}, g, \hat{\mathcal{J}})$ is reflecting (as usual) the translational invariance of the underlying QFT.

• We define the Legendre-transformed functional

$$S_{eff}(h,\Phi,\mathcal{J},\hat{\mathcal{J}},\mathbf{g}) = S_{vis}(\mathbf{g},\Phi,\mathcal{J}) - \int d^4x \sqrt{g(\mathcal{O}^i+\hat{\mathcal{J}}^i,h)}\, h_{\mu\nu}\times$$

$$\times\left[g^{\mu\nu}(\mathcal{O}^i+\hat{\mathcal{J}}^i,h) - \mathbf{g}^{\mu\nu}\right] + \mathcal{W}\left(\mathcal{O}^i+\hat{\mathcal{J}}^i, g(\mathcal{O}^i+\hat{\mathcal{J}}^i,h)\right)$$

We can show that:

♠ This functional satisfies

$$\left.\frac{\delta S_{eff}}{\delta h_{\mu\nu}}\right|_{g_{\mu\nu}=\mathbf{g}_{\mu\nu}} = 0$$

♠ These are the emerging non-linear gravitational equations.

♠ When evaluated in the solution of the above equation gives the original action.

$$, \quad \left. S_{eff}\right|_{g_{\mu\nu}=\mathbf{g}_{\mu\nu}} = S_{visible} + \mathcal{W}\left(\mathcal{O}^i+\hat{\mathcal{J}}^i,\mathbf{g}\right)$$

• Therefore, $S_{eff}(h,\Phi,\mathcal{J},\hat{\mathcal{J}},\mathbf{g})$ is the emergent gravity action that generalizes the linearized computation.

The brane-bulk setup

- The general action is

$$S = S_{hidden} + S_{T\hat{T}} + S_{visible}$$

- Using the holographic correspondence

$$\langle e^{iS_{T\hat{T}}} \rangle_{hidden} = \int_{\lim_{z \to z_0} G_{\mu\nu}(x,z) = \mathbf{g}_{\mu\nu}} \mathcal{D}G \; e^{iS_{\text{bulk}}[G] + i\lambda \int d^4x \sqrt{\mathbf{g}}\, \widehat{T}_{\mu\nu} \mathbf{T}^{\mu\nu}}$$

with $z_0 \sim \frac{1}{M}$.

- It is also true that

$$\langle e^{iS_{T\hat{T}}} \rangle_1 = \int_{\lim_{z \to z_0} G_{\mu\nu}(x,z) = \mathbf{g}_{\mu\nu} + \lambda \mathbf{T}_{\mu\nu}} \mathcal{D}G \; e^{iS_{\text{bulk}}[G]}$$

- By inserting a functional δ-function we may rewrite it as

$$\langle e^{iS_{T\hat{T}}} \rangle_1 = \int \mathcal{D}\chi \mathcal{D}h \int_{\lim_{z \to z_0} G_{\mu\nu}(x,z) = \chi_{\mu\nu}} \mathcal{D}G \; e^{iS_{\text{bulk}}[G] - i \int d^4x h^{\mu\nu}(x)(\chi_{\mu\nu}(x) - \mathbf{g}_{\mu\nu} - \lambda \mathbf{T}_{\mu\nu}(x))}$$

• The total Schwinger functional is represented semi-holographically by substituting the previous equation into

$$e^{iW(\mathbf{g})} = \int \mathcal{D}\Phi_{vis}\, e^{iS_{visible}(\Phi_{vis},\mathbf{g})}\, \langle e^{iS_{T\hat{T}}} \rangle$$

• We now change perspective and integrate $\chi_{\mu\nu}(x)$ first in the path integral

$$\langle e^{iS_{T\hat{T}}} \rangle_1 = \int \mathcal{D}\chi \int \mathcal{D}h e^{i\int d^4x\ h^{\mu\nu}(x)(\mathbf{g}_{\mu\nu}+\lambda \mathbf{T}_{\mu\nu}(x))} \times$$

$$\times \int_{\lim_{z\to z_0} G_{\mu\nu}(x,z)=\chi_{\mu\nu}} \mathcal{D}G\ e^{iS_{\text{bulk}}[G]-i\int d^4x\ h^{\mu\nu}(x)\chi_{\mu\nu}(x)}\,.$$

This is equivalent to

$$\langle e^{iS_{T\hat{T}}} \rangle_1 = \int \mathcal{D}h e^{i\int d^4x\ h^{\mu\nu}(x)(\mathbf{g}_{\mu\nu}+\lambda \mathbf{T}_{\mu\nu}(x))} \int \mathcal{D}\chi\, e^{iW_{hid}(\chi)-i\int d^4x h^{\mu\nu}(x)\chi_{\mu\nu}(x)}$$

$$= \int \mathcal{D}h e^{i\int d^4x\ h^{\mu\nu}(x)(\mathbf{g}_{\mu\nu}+\lambda \mathbf{T}_{\mu\nu}(x))}\, e^{i\Gamma^{eff}_{hid}(h)}\,,$$

that involves the effective action $\Gamma^{eff}_{hid}(h)$ of the (hidden) bulk theory.

• At the saddle point, this reduces to the Legendre transform of the Schwinger functional of the bulk graviton.

• This corresponds in holography to switching boundary conditions at the AdS boundary from Dirichlet to Neumann for the graviton.

Compere+Marolf

• We can then rewrite the effective action part, holographically, using Neumann boundary conditions

$$\langle e^{iS_{12}}\rangle_1 = \int_{G_{\mu\nu}(x,z_0)\,:\,N.B.C.} \mathcal{D}G_{MN}(x,z)\mathcal{D}h_{\mu\nu}(x)\ e^{iS_N[G]+i\int h^{\mu\nu}(x)(\mathbf{g}_{\mu\nu}+\lambda\mathbf{T}_{\mu\nu}(x))}$$

and hence

$$e^{iW(\mathbf{g})} = \int \mathcal{D}h_{\mu\nu}\int_{G_{\mu\nu}(x,z_0)\,:\,N.B.C.} \mathcal{D}G_{MN}\mathcal{D}\Phi_{SM}\ e^{iS_N[G]+i\int h^{\mu\nu}(x)(\mathbf{g}_{\mu\nu}+\lambda\mathbf{T}_{\mu\nu}(x))+iS}$$

• This setup corresponds to our linearized computation and describes a four-dimensional visible QFT, whose stress tensor ($\mathbf{T}_{\mu\nu}$) is linearly coupled to a dynamical boundary graviton denoted by $h_{\mu\nu}(x)$.

• In addition, the original background metric $\mathbf{g}$, plays the role of a "Dark" stress energy tensor that shifts the SM stress energy tensor $\mathbf{T}_{\mu\nu}$.

• The non-linear completion is quite simple and just involves setting $\mathbf{g}_{\mu\nu} = \mathbf{g}_{\mu\nu}(h)$ in the SM action so that the total system is self-consistently coupled to the dynamical boundary metric $h_{\mu\nu}(x)$.

• The end-result is that we obtain a holographic bulk with a SM brane embedded, coupled to the bulk fields, but with Neumann bcs

The characteristic scales

• There are the following characteristic distance scales that play a role, besides r_0 set by the brane position.

• The *transition scale* r_t around which $D(r_0, p)$ changes from small to large momentum asymptotics:

$$D(r_0, p) \simeq \begin{cases} \dfrac{1}{2p} & p \gg \frac{1}{r_t}, \\ \\ d_0 + O(p^2) & p \ll \frac{1}{r_t} \end{cases}$$

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• The *crossover scale,* or DGP scale, r_c:

$$r_c \equiv \frac{U_0}{2};$$

This scale determines the crossover between 5-dimensional and 4-dimensional behavior, and enters the 4D Planck scale and the graviton mass.

• The *gap scale* d_0

$$d_0 \equiv D(r_0,0) = e^{3A_0}\int_0^{r_0} dr' e^{-3A_{UV}(r')},$$

which governs the propagator at the largest distances (in particular it sets the graviton mass as we will see).

• In generic cases, $d_0 \lesssim r_0$

• In confining bulk backgrounds we have instead

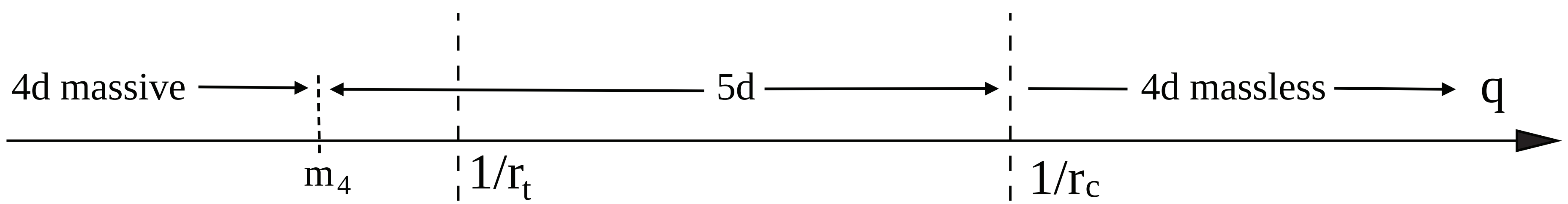
$$d_0 \simeq \frac{1}{6\Lambda_{QCD}^2\ r_0}$$

• In the far IR, $\Lambda r_0 \gg 1$ and d_0 can be made arbitrarily small.

DGP and massive gravity

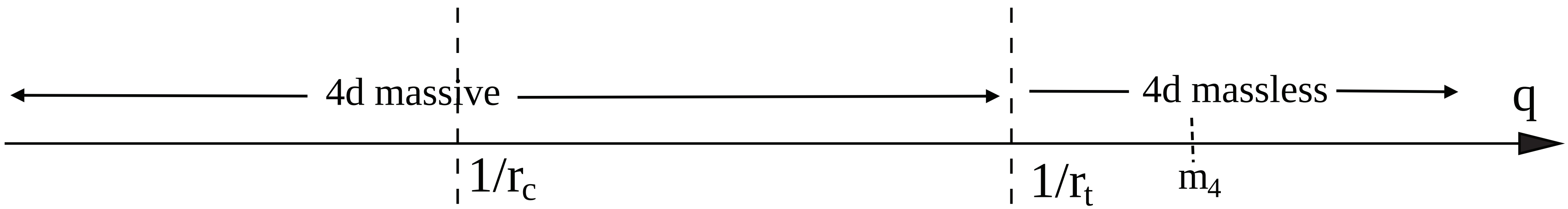
• When $r_t > r_c$ we have three regimes for the gravitational interaction on the brane:

$$\tilde{G}_4(p) \simeq \begin{cases} -\frac{1}{2M_P^2} \quad \frac{1}{p^2} & p \gg \frac{1}{r_c}, \qquad , M_P^2 = r_c M^3 \\ -\frac{1}{2M^3} \quad \frac{1}{p} & \frac{1}{r_c} \gg p \gg m_0 \\ -\frac{1}{2M_P^2} \quad \frac{1}{p^2 + m_0^2} & p \ll m_0, \qquad m_0^2 \equiv \frac{1}{2r_c d_0} \end{cases}$$



• Massive 4d gravity ($r_t < r_c$)

• In this case, at all momenta above the transition scale, $p \gg 1/r_t > 1/r_c$, we are in the 4-dimensional regime of the DGP-like propagator.



• Below the transition, $p \ll 1/r_t$, we have again a massive-graviton propagator.

• The behavior is four-dimensional at all scales, and it interpolates between massless and massive four-dimensional gravity.

Kiritsis+Tetradis+Tomaras

More on scales

• Scales depend on the bulk dynamics=the nature of the RG flow.

• They depend on "SM" data (the brane potential and the cutoff scale Λ).

• They can depend on boundary conditions = the UV coupling constant of the bulk QFT.

• Φ_0 at the position of the brane is fixed by the Israel conditions and is independent of boundary conditions.

• The two important parameters for 4d gravity do not depend on b.c.

$$\frac{m_0}{M_P} \sim \left(\frac{M}{\Lambda}\right)^2 \frac{1}{N^{\frac{2}{3}}} \quad , \quad m_0 \, M_P = \left(\frac{M^3}{\bar{d}}\right)^{\frac{1}{2}}$$

• $\bar{d}$ is the "rescaled" value of the bulk propagator at $p = 0$ at the position of the brane (so that it is independent of boundary conditions). It depends only on the bulk action.

• The choice of a small ratio $\frac{m_0}{M_P} \sim 10^{-60}$ is (technically) natural from the QFT point of view.

• There is important numerology to be analyzed for typical classes of holographic theories.

The brane graviton

The graviton fluctuation satisfies

$$\partial_r\left(e^{3A(r)}\partial_r\hat{h}\right)+\left[e^{3A(r)}+U_0\delta(r-r_0)\right]\partial_\mu\partial^\mu\hat{h}=\delta(r-r_0)\frac{\hat{T}}{M^3}$$

- Then, the solution is given by:

$$\hat{h}_{\mu\nu}(x,r)=\frac{1}{M^3}\int d^dx'\,G(r,x;r_0,x')\hat{T}_{\mu\nu}(x'),$$

- The induced interaction is

$$S_{int}=-\frac{1}{2M^3}\int d^4x\,d^4x'\,G(r_0,x;r_0,x')\left(T^{\mu\nu}(x)T_{\mu\nu}(x')-\frac{1}{3}T(x)T(x')\right)$$

$$G(r,x;r_0,x')=\frac{1}{\frac{1}{G_{bulk}(r,x;r_0,x')}+G_{brane}(x,x')}$$

Dvali+Gabadadze+Porrati

- This should be contrasted with the field-theoretical formula

$$Interaction\ \ of\ \ energy\ \ sources=\frac{1}{\frac{1}{\langle\hat{T}\hat{T}\rangle_{hidden}}+\langle TT\rangle_{SM}}=\frac{\langle\hat{T}\hat{T}\rangle_{hidden}}{1+\langle\hat{T}\hat{T}\rangle_{hidden}\langle TT\rangle_{SM}}$$

• There are the following characteristic distance scales.

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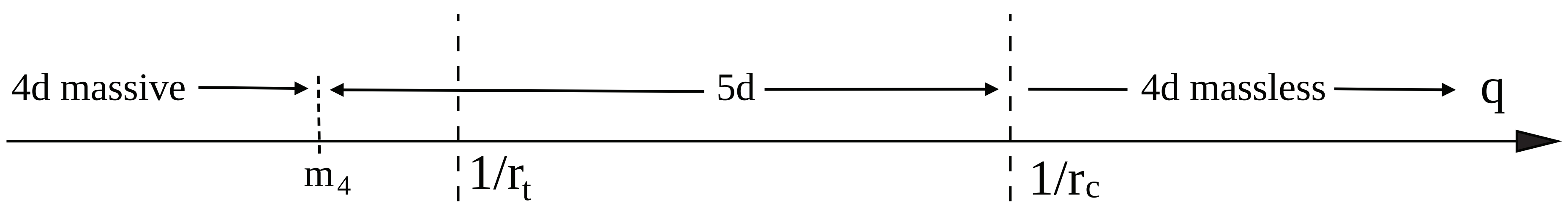

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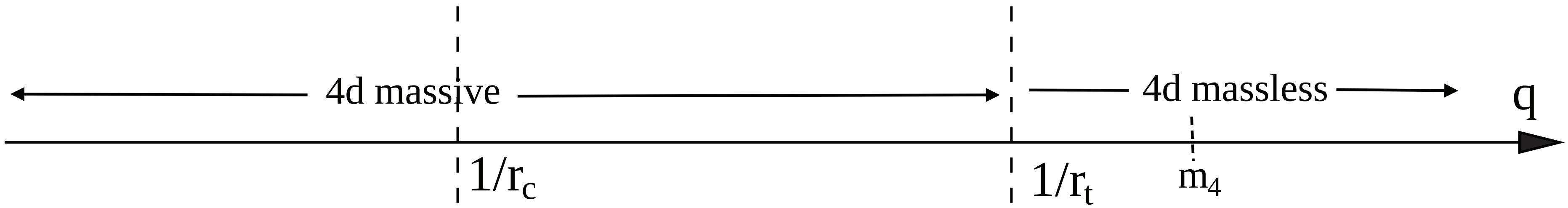

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Kiritsis+Tetradis+Tomaras

• There is a vDVZ discontinuity that (as usual) cannot be cancelled at the linearized order if the theory is positive. It should be cancelled by the Vainshtein mechanism.

Scalar Perturbations

• The scalar perturbations are of interest, as they might destroy the equivalence principle.

• The equations for the scalar perturbations can be derived and they are complicated.

• Unlike previous analysis of similar systems they cannot be factorized to a relatively simple system as the graviton.

• There are two scalar modes on the brane:

• In one gauge, the brane bedding mode can be "eliminated" but the scalar perturbation is discontinuous on the brane.

• In another gauge the perturbation is continuous but the brane bending mode is present.

The effective quadratic interactions for the scalar modes are of the form

$$S_4 = -\frac{\mathcal{N}}{2}\int d^4x\sqrt{\gamma}((\partial\phi)^2 + m^2\phi^2)$$

- We need both $\mathcal{N} > 0$ and $m^2 > 0$.

• In general the two scalar modes couple to two charges:

(a) the "scalar charge" and

(b) the trace of the brane stress tensor.

• The mode that couples to the scalar charge has a "heavy" mass of the order of the cutoff/Planck Scale.

• The mode that couples to the trace of the stress-tensor has a mass that is of order the graviton mass.

• All the stability conditions for the scalars depend on more details of the brane induced functions $W_B(\Phi)$, $U_B(\Phi)$, $Z_B(\Phi)$.

Scalar Perturbations

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• The mode that couples to the scalar charge has a "heavy" mass of the order of the cutoff/Planck Scale.

• The mode that couples to the trace of the stress-tensor has a mass that is O(1) in cutoff units (like the graviton mass).

• All the stability conditions for the scalars depend on more details of the brane induced functions $W_B(\Phi)$, $U_B(\Phi)$, $Z_B(\Phi)$.

• They can be investigated further from the known parameter dependence of the vacuum energy in the SM.

Kounnas+Pavel+Zwirner, Dimopoulos+Giudince+Tetradis

• There is a vDVZ discontinuity that (as usual) cannot be cancelled at the linearized order if the theory is positive.

• It should be cancelled by the Vainshtein mechanism. To derive the relevant constraints on parameters, we must study the non-linear interactions of the scalar-graviton modes.

Scalar Perturbations (details)

We introduce perturbations of the metric and scalar field, on each side of the brane, in the form:

$$ds^2 = a^2(r)\left[(1+2\phi)dr^2 + 2A_\mu dx^\mu dr + (\eta_{\mu\nu}+h_{\mu\nu})dx^\mu dx^\nu\right], \quad \varphi = \bar{\varphi}(r) + \chi$$

where the fields $\phi, A_\mu, h_{\mu\nu}, \chi$ depend on r, x_μ.

• We further decompose the 5 dimensional bulk modes into tensor, vector and scalar perturbations with respect to the 4 dimensional diffeomorphism group,

$$A_\mu = \partial_\mu W + A_\mu^T, \quad h_{\mu\nu} = 2\eta_{\mu\nu}\psi + 2\partial_\mu\partial_\nu E + 2\partial_{(\mu}V_{\nu)}^T + \hat{h}_{\mu\nu}$$

with $\partial^\mu A_\mu^T = \partial^\mu V_\mu^T = \partial^\mu \hat{h}_{\mu\nu} = \hat{h}_\mu^\mu = 0$. All indices μ, ν are raised and lowered with the flat Minkowski metric $\eta_{\mu\nu}$.

• Therefore, we have one bulk tensor $\hat{h}_{\mu\nu}$, two bulk transverse vectors (A_μ^T, V_μ^T), five bulk scalars (ϕ, ψ, χ, W, E) (plus one brane scalar, describing brane bending as we will see later).

• At the linearized level, general coordinate transformations $(\delta r, \delta x^{\mu}) = (\xi^5, g^{\mu\nu}\xi_{\nu})$ act as gauge transformations, under which:

$$\delta\psi = -\frac{a'}{a}\xi^5 \qquad \delta\phi = -(\xi^5)' - \frac{a'}{a}\xi^5$$

$$\delta B = -\xi' - \xi^5, \qquad \delta E = -\xi, \qquad \delta\chi = -\bar{\varphi}'\xi^5, \tag{6}$$

$$\delta A^T_{\mu} = -(\xi^T_{\mu})', \qquad \delta V^T_{\mu} = -\xi^T_{\mu}$$

$$\delta\widehat{h}_{\mu\nu} = 0 \tag{7}$$

where we have introduced a decomposition of the diffeomorphism parameter ξ_{μ} in its transverse and longitudinal components, i.e. $\xi_{\mu} = \xi^T_{\mu} + \partial_{\mu}\xi$ with $\partial^{\mu}\xi^T_{\mu} = 0$.

• The tensor mode $\widehat{h}_{\mu\nu}$ is gauge-invariant, and gauge symmetry plus constraints allow to eliminate the two vectors and four of the bulk scalars.

• The remaining physical bulk scalar can be identified with the gauge-invariant combination:

$$\zeta = \psi - \frac{1}{z}\chi,$$

where $z(r)$ is the background quantity:

$$z \equiv \frac{a\bar{\Phi}'}{a'}.$$

• However ζ is not continuous along the brane so we choose to work with ψ by setting $\chi = 0$.

• We also use a residual transformation to set the brane bending mode to zero at the expense of making ψ discontinuous.

• The bulk gauge-invariant fluctuations satisfy the second order equations:

$$\widehat{h}''_{\mu\nu} + (d-1)\frac{a'}{a}\widehat{h}'_{\mu\nu} + \partial^\rho\partial_\rho\widehat{h}_{\mu\nu} = 0 \qquad (8)$$

$$\zeta'' + \left[(d-1)\frac{a'}{a} + 2\frac{z'}{z}\right]\zeta' + \partial^\rho\partial_\rho\zeta = 0. \qquad (9)$$

• After solving the constraints for E and ϕ, and after eliminating the brane-bending field ρ, one is left with only the scalar mode ψ, which satisfies the bulk field equation (on each side of the brane) as well as the Israel conditions

$$\begin{pmatrix} \psi'_{UV}(r_0) \\ \psi'_{IR}(r_0) \end{pmatrix} = (\Gamma_1 + \Gamma_2 \partial^\mu \partial_\mu) \begin{pmatrix} \psi_{UV}(r_0) \\ \psi_{IR}(r_0) \end{pmatrix}$$

where the matrices Γ_1 and Γ_2 are given by:

$$\Gamma_1 = \frac{a_0 \tilde{\mathcal{M}}^2}{[z]^2} \begin{pmatrix} -z_{IR}^2 & z_{IR}^2 \\ -z_{UV}^2 & z_{UV}^2 \end{pmatrix},$$

$$\Gamma_2 = \frac{1}{[z]^2 a_0} \begin{pmatrix} -12 z_{IR} \frac{dU_B}{d\Phi}\Big|_{\Phi_0} + \tau_0 + Z_0 z_{IR}^2 & 6 z_{IR} \left(\frac{z_{IR}}{z_{UV}} + 1\right) \frac{dU_B}{d\Phi}\Big|_{\Phi_0} - \tau_0 \frac{z_{IR}}{z_{UV}} - Z_0 z_{IR}^2 \\ -6 z_{UV} \left(\frac{z_{UV}}{z_{IR}} + 1\right) \frac{dU_B}{d\Phi}\Big|_{\Phi_0} + \tau_0 \frac{z_{UV}}{z_{IR}} + Z_0 z_{UV}^2 & 12 z_{UV} \frac{dU_B}{d\Phi}\Big|_{\Phi_0} - \tau_0 - Z_0 z_{UV}^2 \end{pmatrix} \quad (10$$

where

$$\tilde{\mathcal{M}}^2 = \frac{d^2 W_B}{d\Phi^2}\Big|_{\Phi_0} - \left[\frac{d^2 W}{d\Phi^2}\right], \qquad \tau_0 = 6\left(6 \frac{W_B}{W_{IR} W_{UV}}\Big|_{\Phi_0} - U_0\right).$$

Detailed plan of the presentation

- Title page 0 minutes
- Bibliography 1 minutes
- Introduction 6 minutes
- A hidden sector generating gravity 12 minutes
- Other avatars of the hidden sector 15 minutes
- Emergent graviphotons 19 minutes
- The hypercharge portal 21 minutes
- Conclusions 22 minutes

- The EFT analysis 24 minutes
- The Witten-Weinberg theorem 26 minutes
- The energy-momentum tensor 28 minutes
- The AdS/CFT paradigm 30 minutes
- The stress-tensor state as a (classical) dynamical graviton 41 minutes
- A low-energy effective action 46 minutes
- Gravitons from hidden sectors 48 minutes
- The linearized coupling 55 minutes
- Emergent quadratic gravity 59 minutes
- Emergent quadratic gravity, II 61 minutes
- The non-linear analysis 67 minutes
- The holographic hidden QFT 68 minutes
- The brane-world picture 69 minutes
- The brane graviton 71 minutes
- The scalar perturbations 72 minutes
- Comments and open ends 74 minutes

- Subtleties of WW 75 minutes
- Translation Ward Identities 76 minutes
- Aside: String theory and the Swampland 77 minutes
- Higher Spin 78 minutes
- WW versus AdS/CFT 79 minutes
- WW versus nAdS/nCFT 80 minutes
- An explicit IR parametrization 85 minutes
- Emerging quadratic gravity:Comments 89 minutes
- Renormalization and contact terms in $\langle TT\rangle$ 93 minutes
- Mixing with contact terms 97 minutes
- The non-linear analysis 100 minutes
- The brane bulk setup 104 minutes
- The characteristic scales 108 minutes
- DGP and massive gravity 112 minutes
- More on scales 116 minutes
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- The scalar perturbations 128 minutes
- Scalar Perturbations 132 minutes
- Scalar perturbations (details) 136 minutes