Resolving singularity by quantum gravity

Bouhmadi-Lopez, Chen, Chew, Ong and DY, 2005.13260

Bouhmadi-Lopez, Brahma, Chen, Chen and DY, 1911.02129

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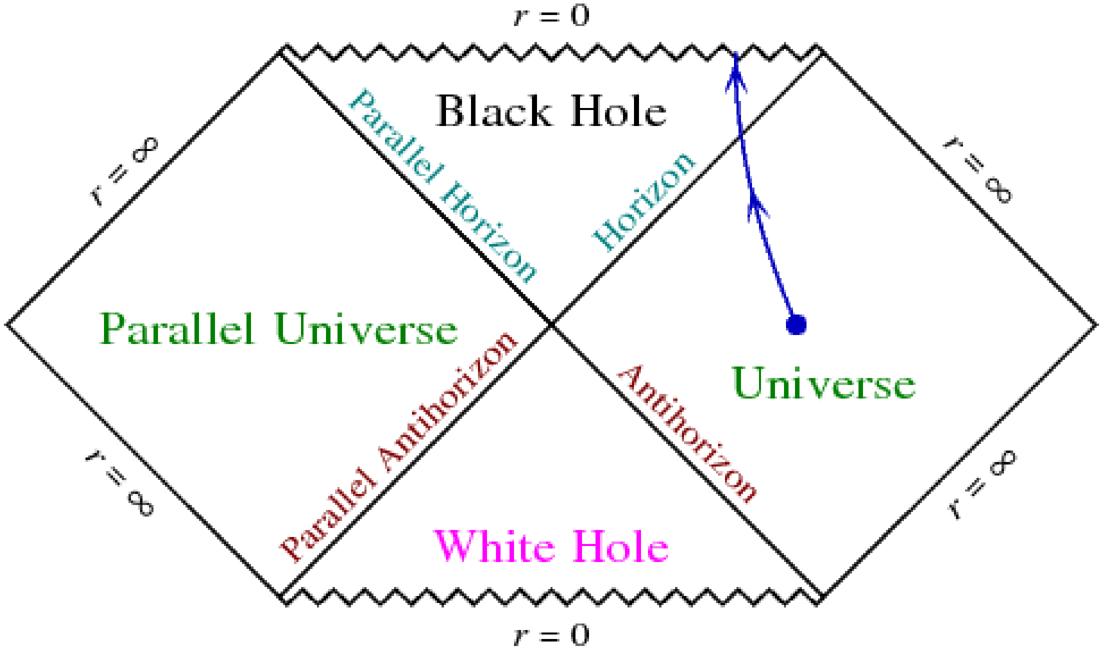
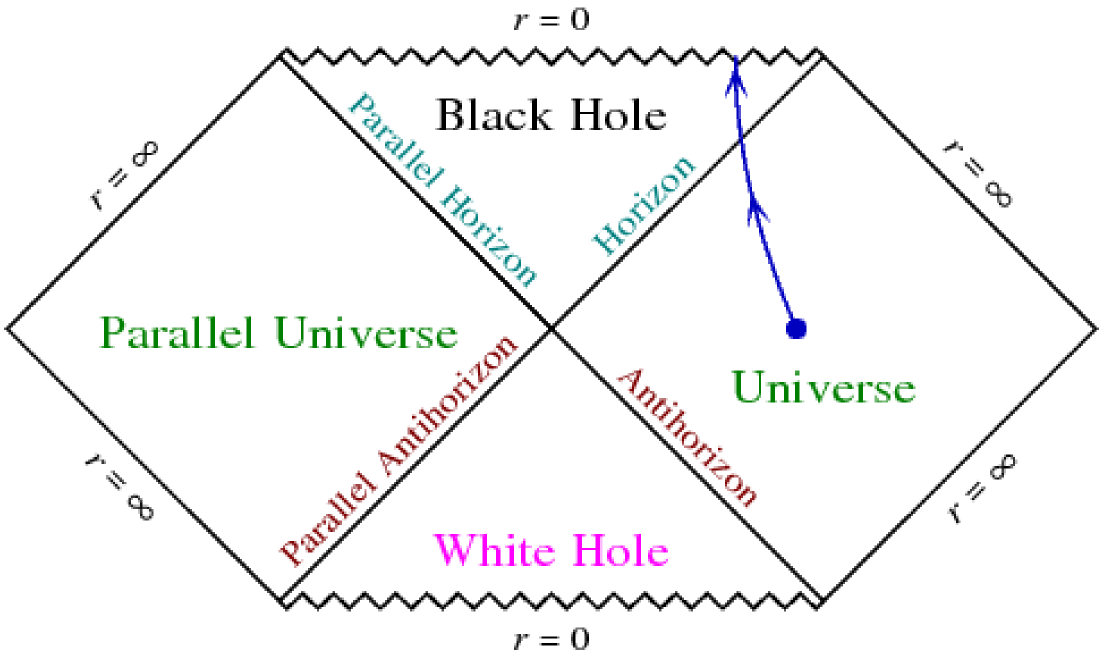
Cosmology INSIDE a black hole?

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$



Let us see inside a Schwarzschild black hole.

$$ds^2 = -\left(\frac{2M}{r} - 1\right)^{-1} dr^2 + \left(\frac{2M}{r} - 1\right) dt^2 + r^2 d\Omega^2$$



The spacetime looks like a **time−dependent** universe.

$$ds^2 = -N^2(t)dt^2 + e^{2X(t)}dR^2 + r_s^2 e^{2(Y(t)-X(t))}d\Omega_2^2$$

One way to present a metric inside a black hole

$$X(t) = \log\tan\frac{t}{2}$$

$$Y(t) = \log\frac{1}{2}\sin t$$

After redefining parameters, one can present the metric again.

Can we resolve the singularity inside a black hole using the same technique for cosmology?

We provide **three** possible applications
for cosmology inside the horizon.

Modified gravity

Canonical quantum gravity

Euclidean quantum gravity

Modified gravity

Canonical quantum gravity

Euclidean quantum gravity

Consider the action

$$S = \int d^4x \left[\frac{R}{2\kappa} - \frac{1}{48} F_{abcd} F^{abcd} - V(A^2)\right]$$

with a 3−form field

$$F_{abcd} = 4\nabla_{[a} A_{bcd]}$$

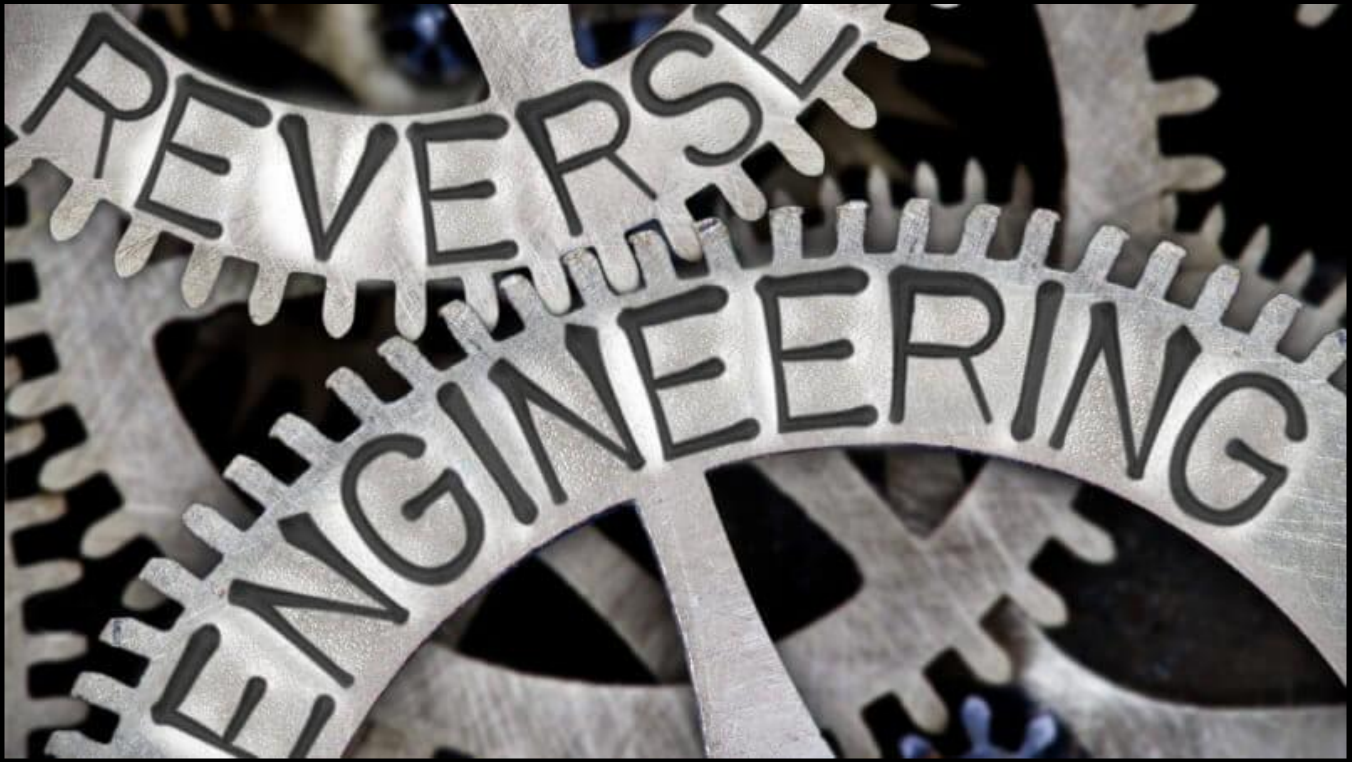
One can solve the equation with the metric ansatz

$$ds^2 = -dt^2 + a^2(t)dR^2 + r_s^2 b^2(t)d\Omega_2^2$$

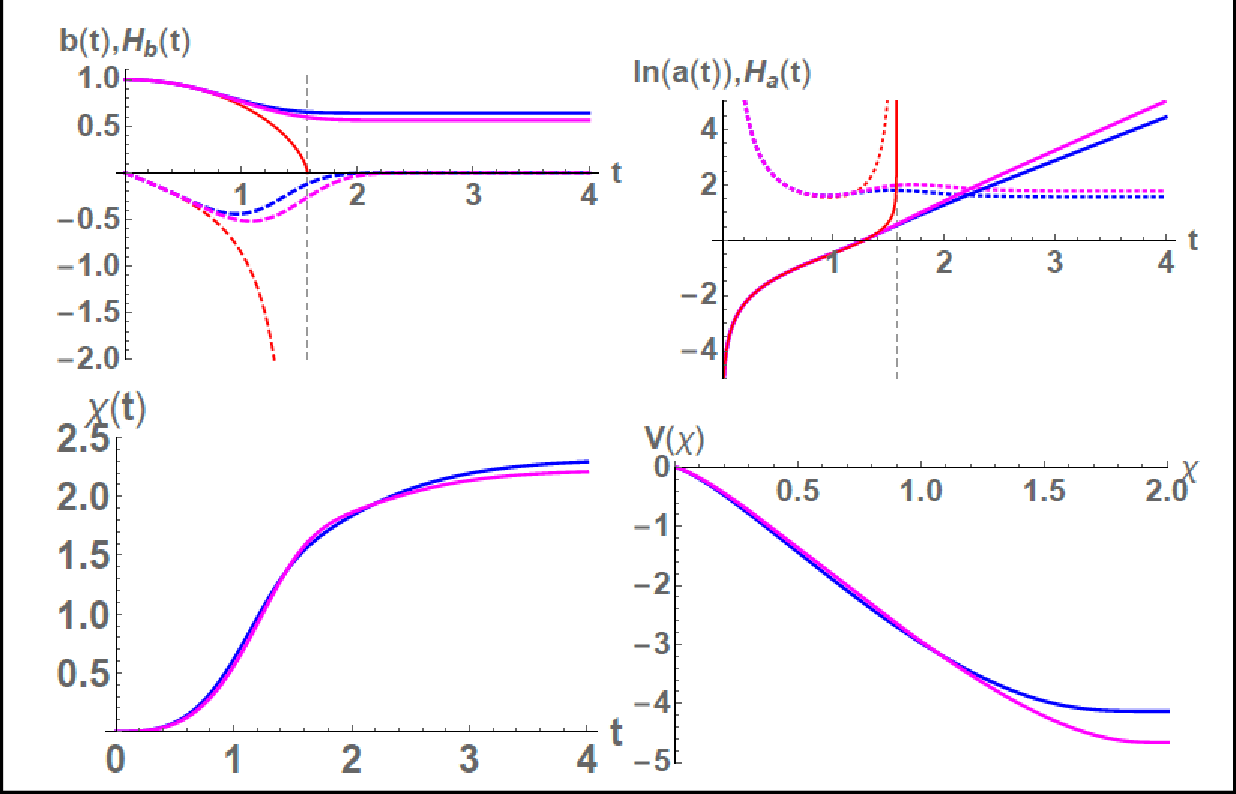
and the field ansatz

$$A_{R\theta\phi} = r_s^2 a(t) b^2(t) \chi(t) \epsilon_{R\theta\phi}$$

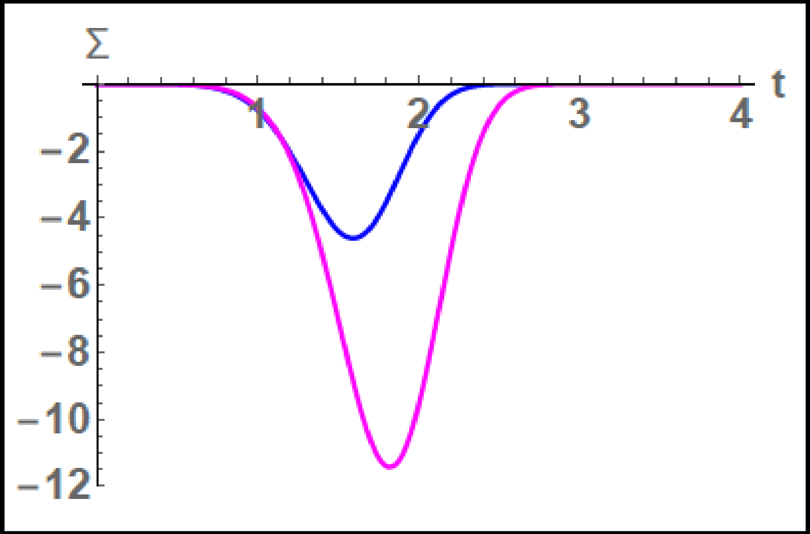
while the initial conditions correspond to the event horizon of the Schwarzschild solution.



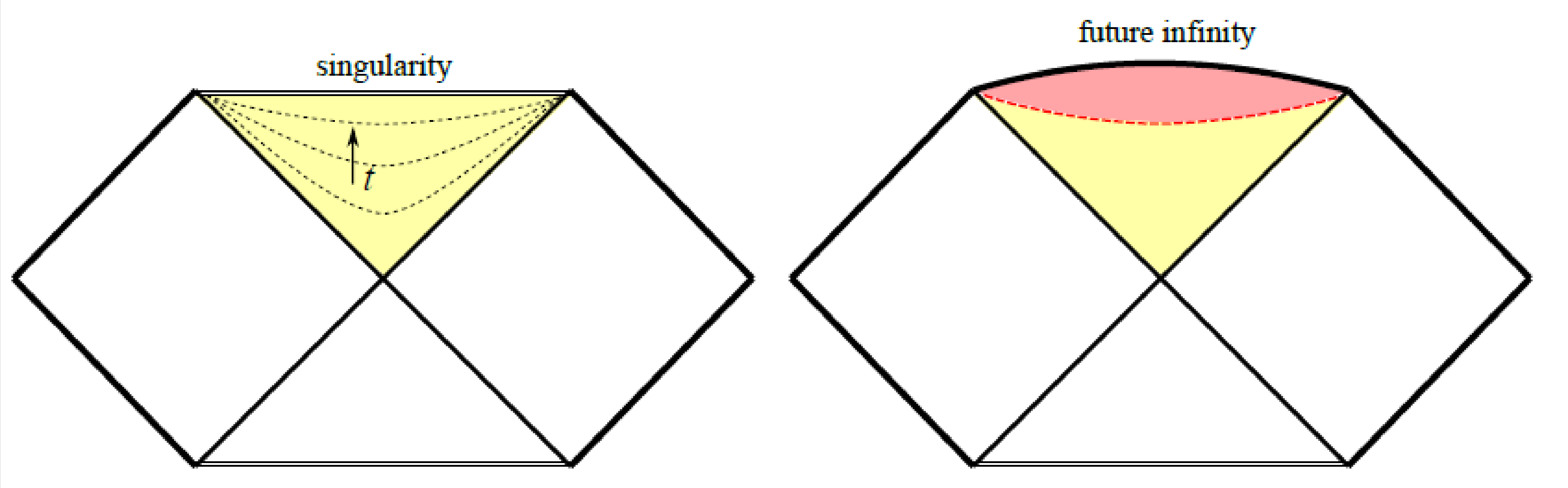
Like the reverse engineering, we first fix the form of $b(t)$ and find the corresponding solution of $a(t)$, $\chi(t)$, and $V(t)$.



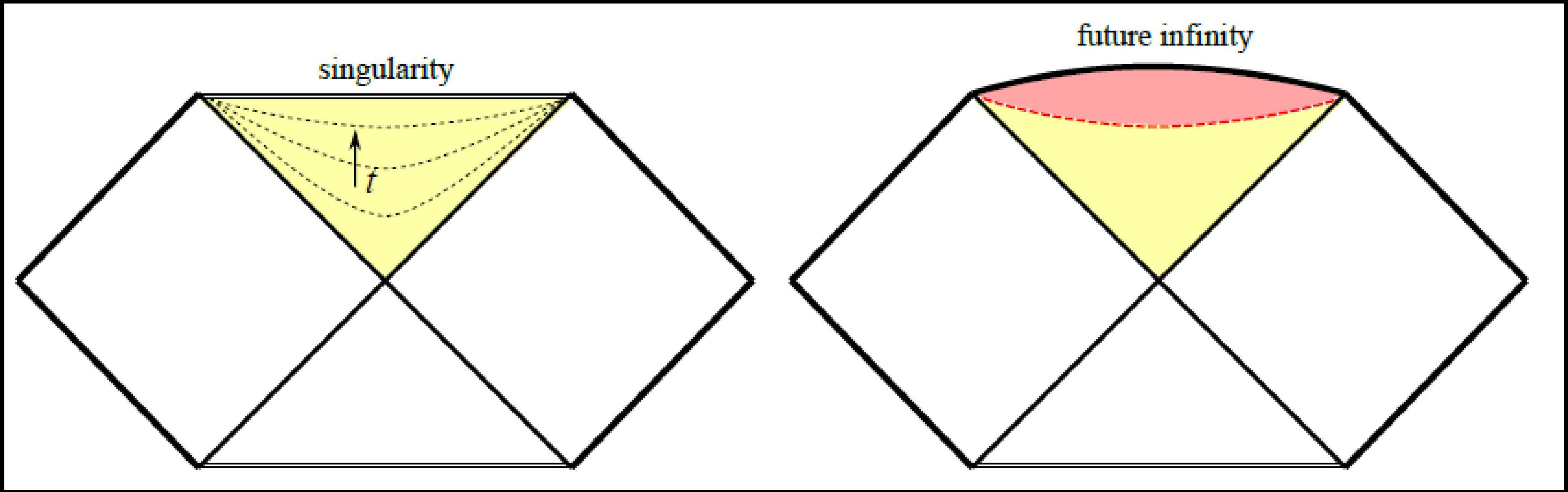
A regular black hole solution is possible.



The singularity is avoided, because the null energy condition is effectively violated.



The causal structure is unique (different from two-horizon model, different from any model of LQC, etc.), where it does not suffer from the mass inflation problem inside the inner horizon.



The interior geometry asymptotically approaches to

$$dS_2 \times S_2$$

i.e., dynamical compactification realized.

Modified gravity

Canonical quantum gravity

Euclidean quantum gravity

The Wheeler–DeWitt equation

$$\widehat{\mathcal{H}}\Psi = 0$$

quantum Hamiltonian constraint

wave function of field space

Let us study the quantum gravitational wave function inside a black hole.

$$ds^2 = -N^2(t)dt^2 + e^{2X(t)}dR^2 + r_s^2 e^{2(Y(t)-X(t))}d\Omega_2^2$$

One way to present a metric inside a black hole

$$Y = -\log(e^X + e^{-X})$$

One can remove time and show the on−shell solution as a relation between $X(t)$ and $Y(t)$.

$$ds^2 = -N^2(t)dt^2 + e^{2X(t)}dR^2 + r_s^2 e^{2(Y(t)-X(t))}d\Omega_2^2$$

One way to present a metric inside a black hole

$$\left(\frac{\partial^2}{\partial X^2} - \frac{\partial^2}{\partial Y^2} + 4r_s^2 e^{2Y}\right)\Psi[X, Y] = 0$$

The Wheeler−DeWitt equation presented by X and Y.
This was also known previously,
e.g., gr−qc/9411070, hep−th/0107250, etc.

$$ds^2 = -N^2(t)dt^2 + e^{2X(t)}dR^2 + r_s^2 e^{2(Y(t)-X(t))}d\Omega_2^2$$

One way to present a metric inside a black hole

$$\left(\frac{d^2}{dX^2} + k^2\right)\phi[X] = 0$$

$$\left(\frac{d^2}{dY^2} - 4r_s^2 e^{2Y} + k^2\right)\psi[Y] = 0$$

Separation of variable

$$ds^2 = -N^2(t)dt^2 + e^{2X(t)}dR^2 + r_s^2 e^{2(Y(t)-X(t))}d\Omega_2^2$$

One way to present a metric inside a black hole

$$\phi_k[X] = e^{\pm ikX}$$

$$\psi_k[Y] = C_1 I_{ik}(2r_s e^Y) + C_2 K_{ik}(2r_s e^Y)$$

hyperbolic Bessel function

General analytic solution

$$ds^2 = -N^2(t)dt^2 + e^{2X(t)}dR^2 + r_s^2 e^{2(Y(t)-X(t))}d\Omega_2^2$$

One way to present a metric inside a black hole

$$\phi_k[X] = e^{\pm ikX}$$

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hyperbolic Bessel function

Since the second term diverges as Y increases,
we will ignore the term.

$$\Psi[X, Y] = \int_{-\infty}^{\infty} f(k)\, e^{-ikX} I_{ik}(2r_s e^{Y}) dk$$

This is the most generic solution of the WDW equation
for inside a Schwarzschild black hole.

$$ds^2 = -N^2(t)dt^2 + e^{2X(t)}dR^2 + r_s^2 e^{2(Y(t)-X(t))}d\Omega_2^2$$

One way to present a metric inside a black hole

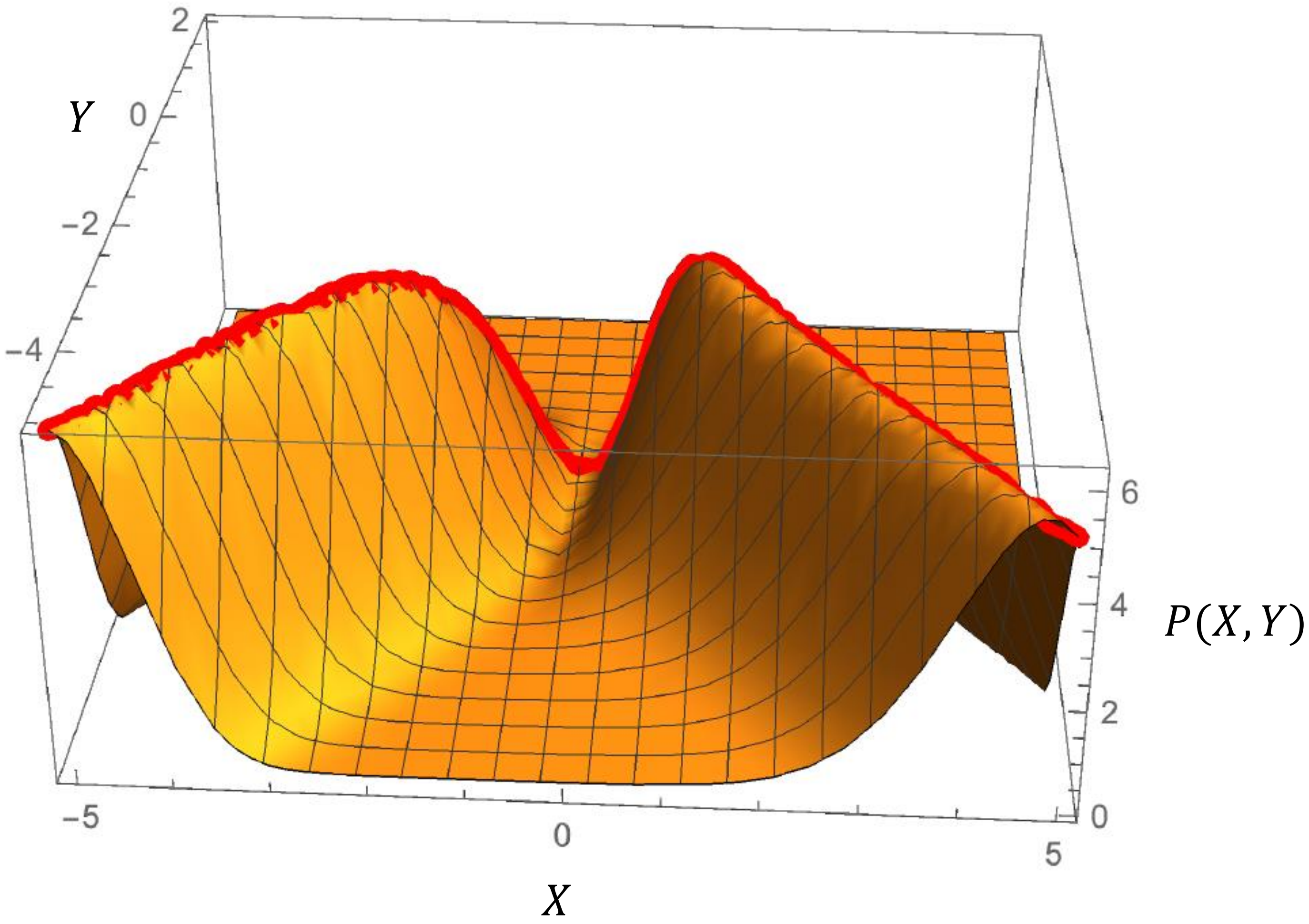
$$Y = -\log(e^X + e^{-X})$$

In this solution, the event horizon is located at $X, Y \to -\infty$, while the singularity is located at $X \to \infty$ and $Y \to -\infty$.

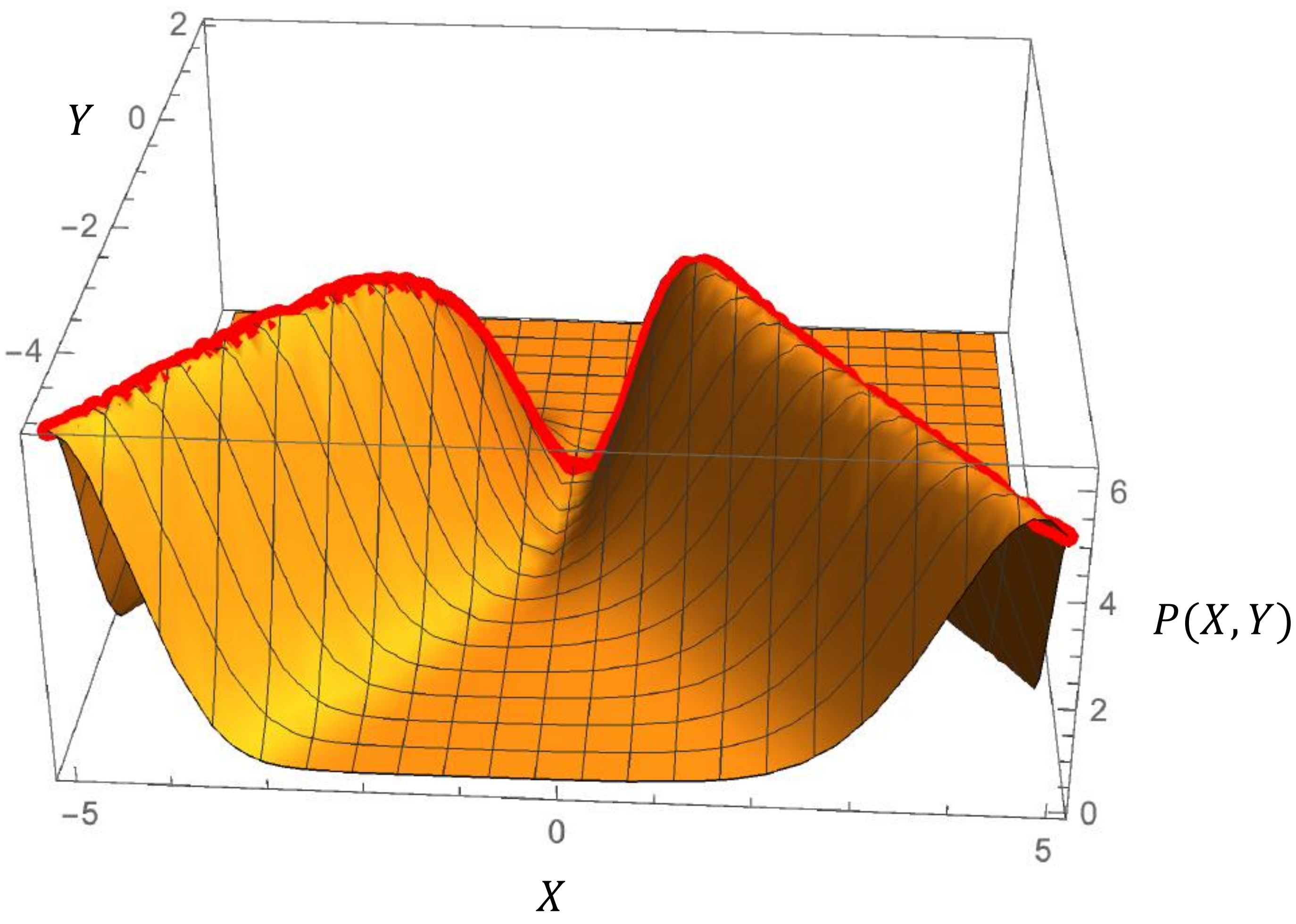
$$\Psi[X,Y] = \int_{-\infty}^{\infty} f(k)\, e^{-ikX} I_{ik}(2r_s e^{Y})dk$$

$$f(k) = \frac{2Ae^{-\sigma^2 k^2/2}}{\Gamma(-ik) r_s^{ik}}$$

We will impose the boundary condition such that
the wave function as a (Gaussian) peak at the event horizon,
because it is reasonable to assume that
the solution is classical at the horizon.

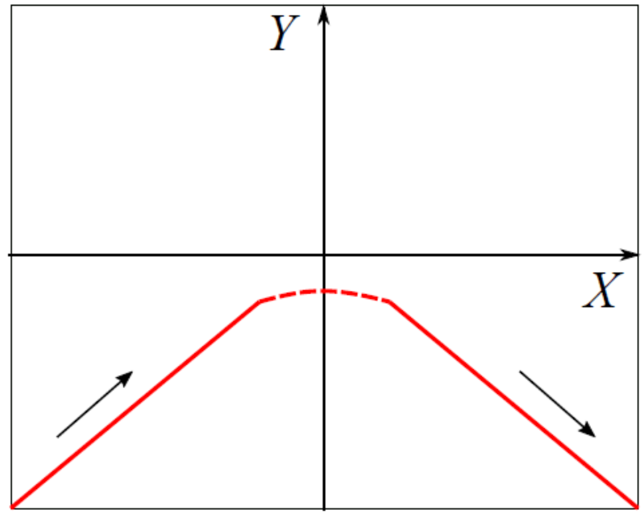


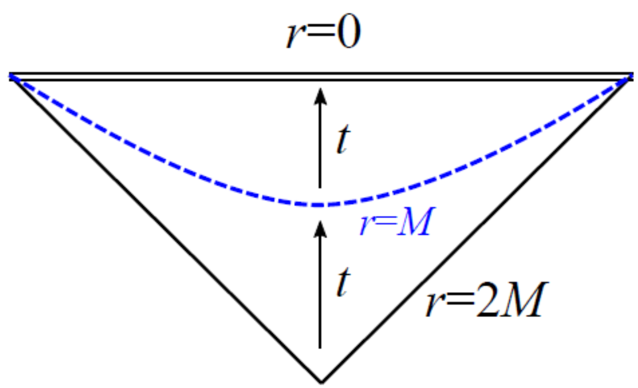
This is the numerical plot of the wave function.
The red curve is the peak of the wave function, i.e.,
the steepest−descent.



This steepest−descent coincides well with the classical trajectory.

$$Y = -\log(e^{X} + e^{-X})$$





This steepest−descent coincides well with the classical trajectory.

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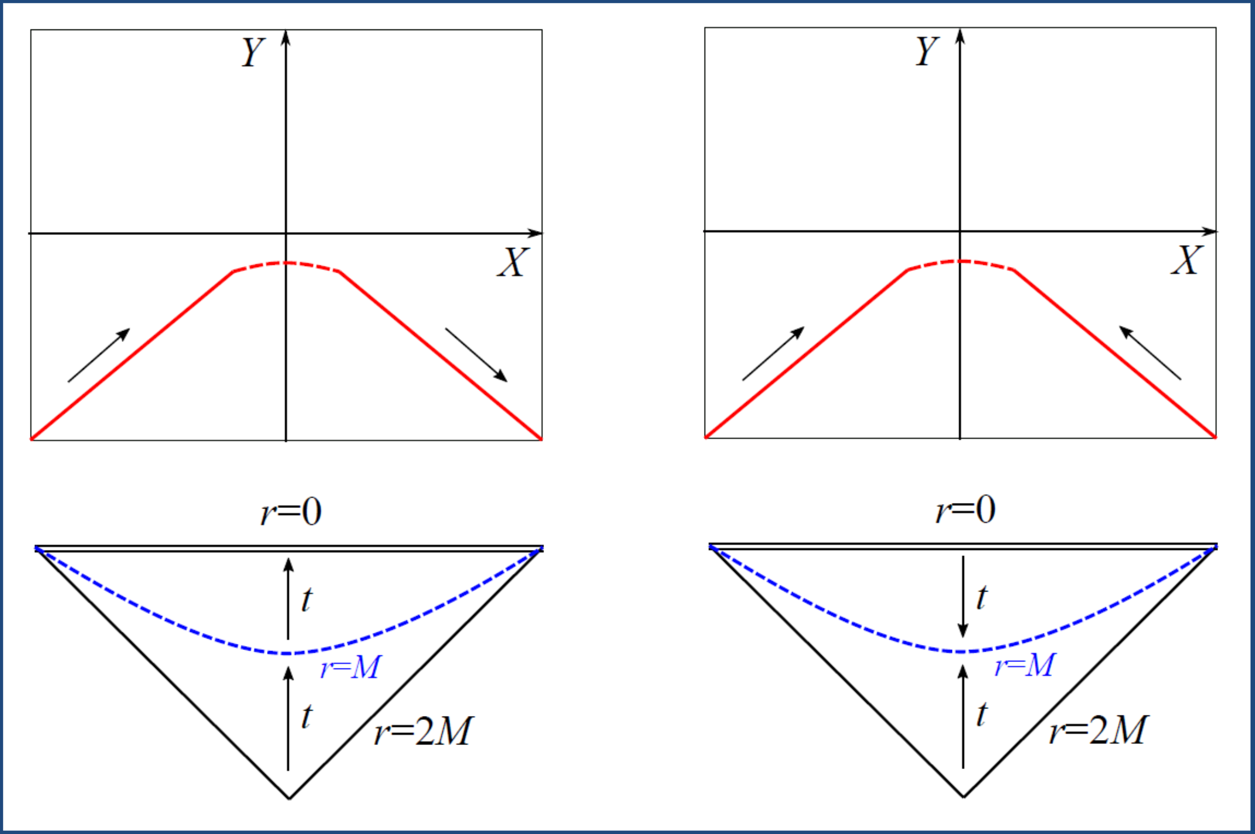
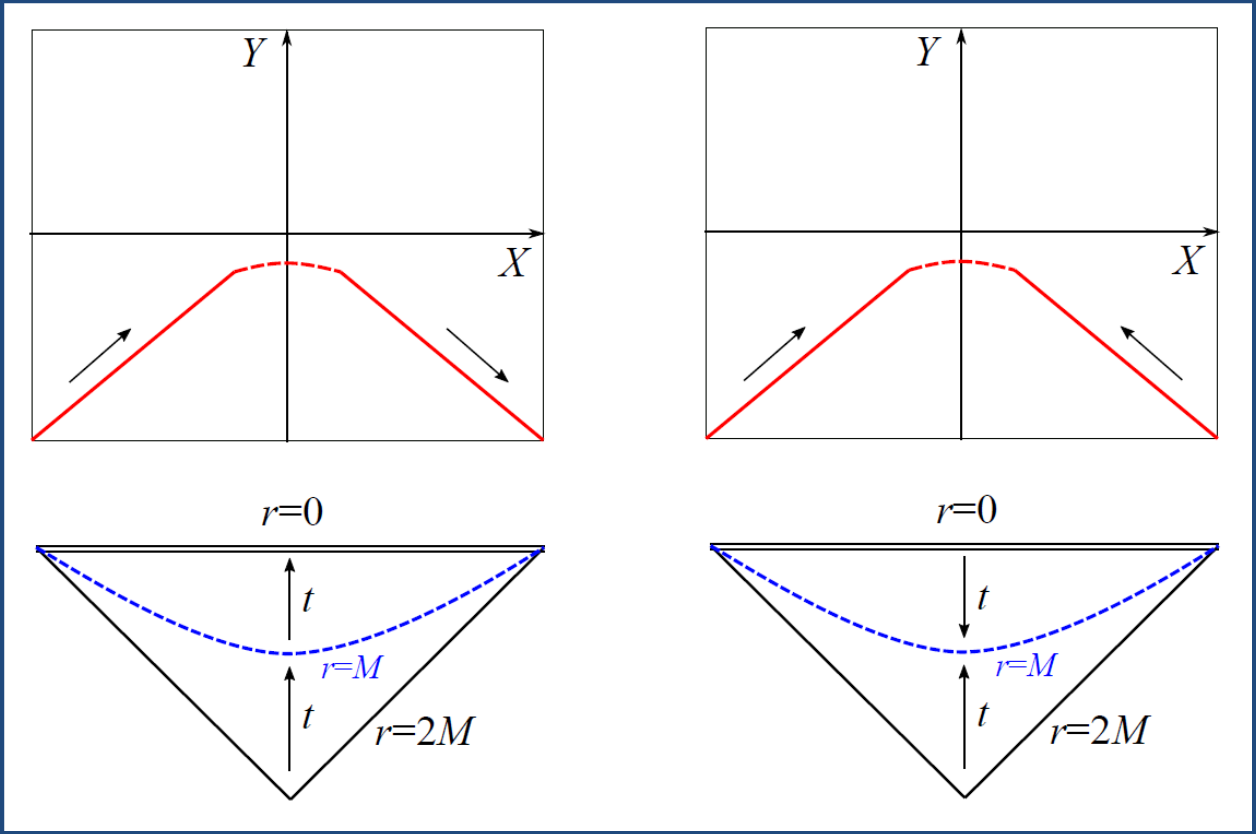
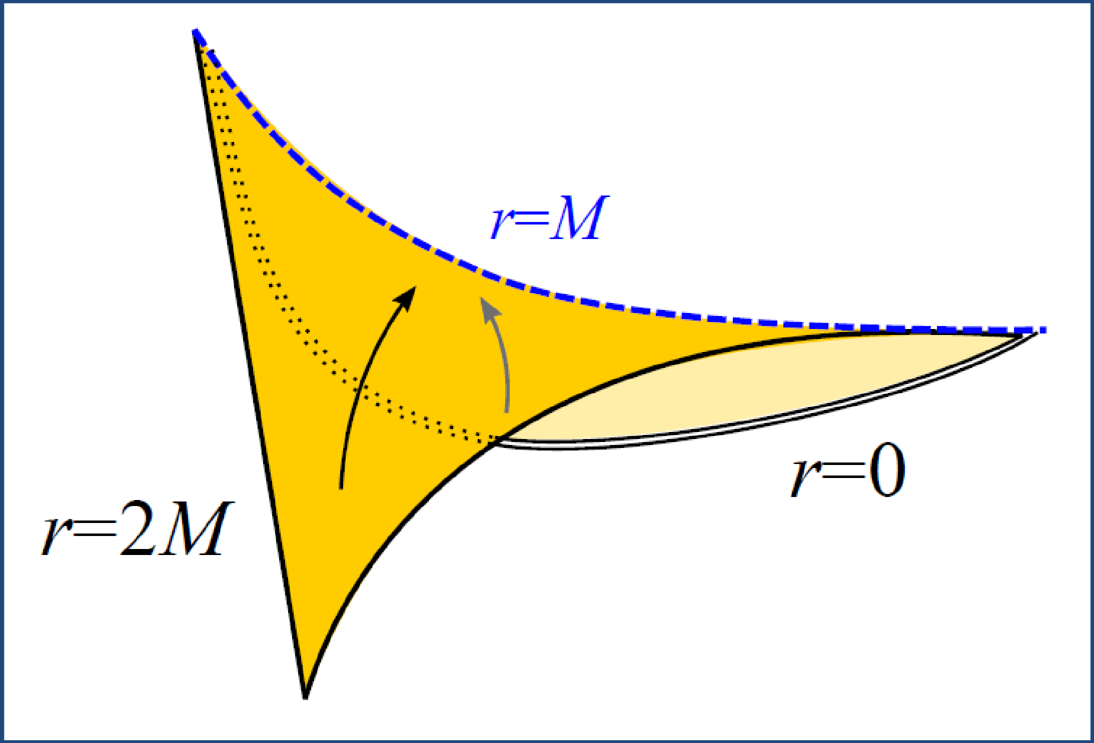


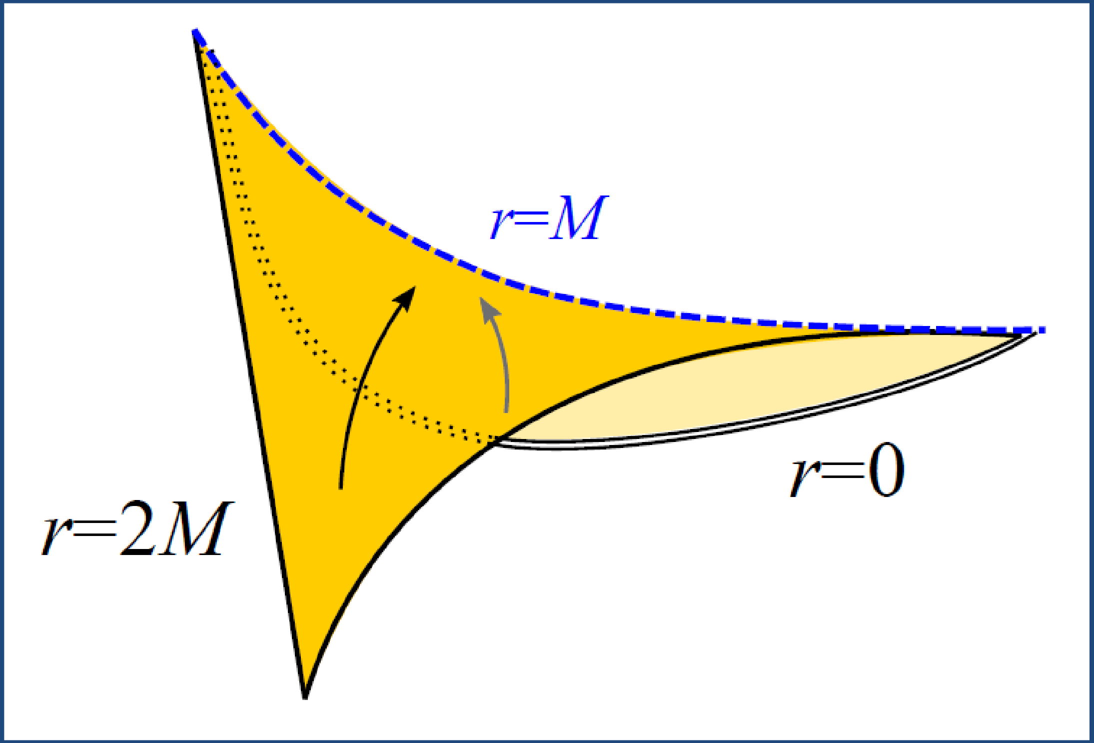
One can interpret that there is only one arrow of time.
However, it is fair to say that
there are two arrows of time.



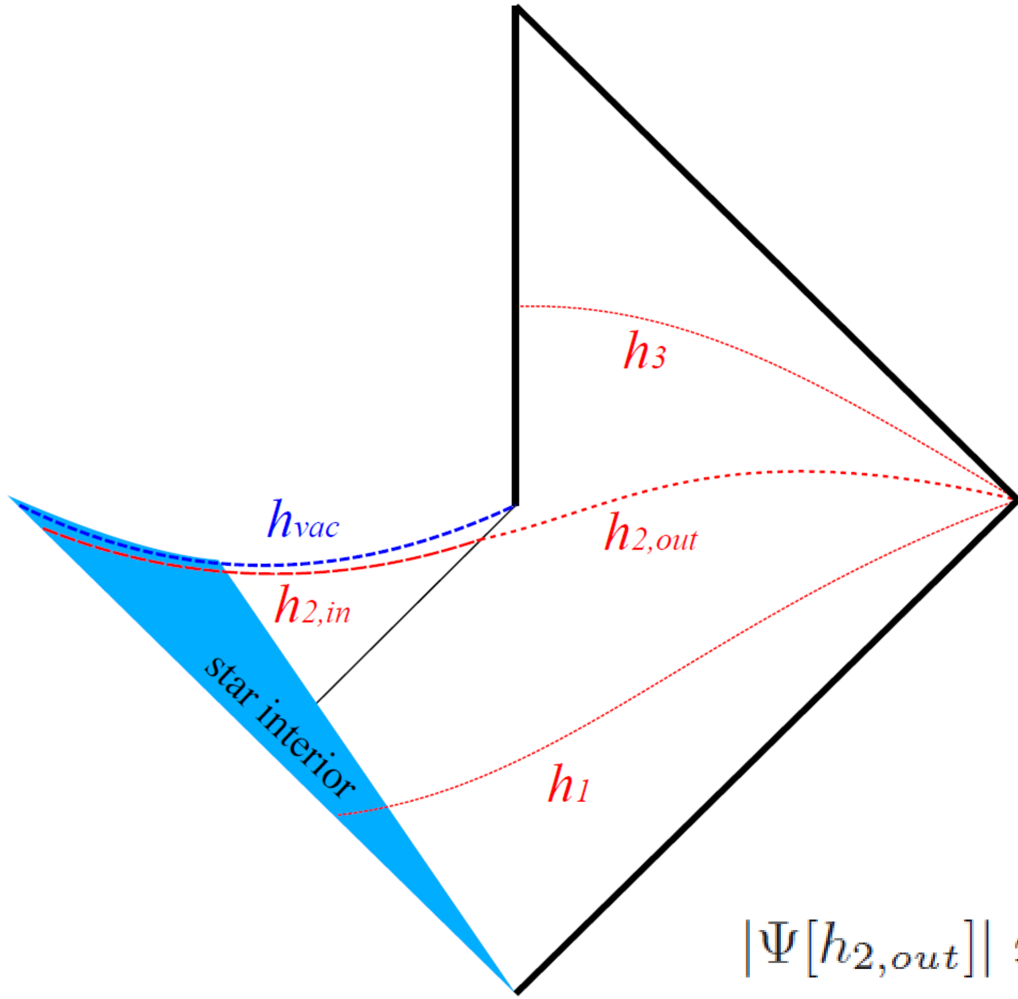
If we interpret following the latter way, it is very interesting.
The two parts of black hole spacetime is annihilated at a hypersurface.



Annihilation to nothing.

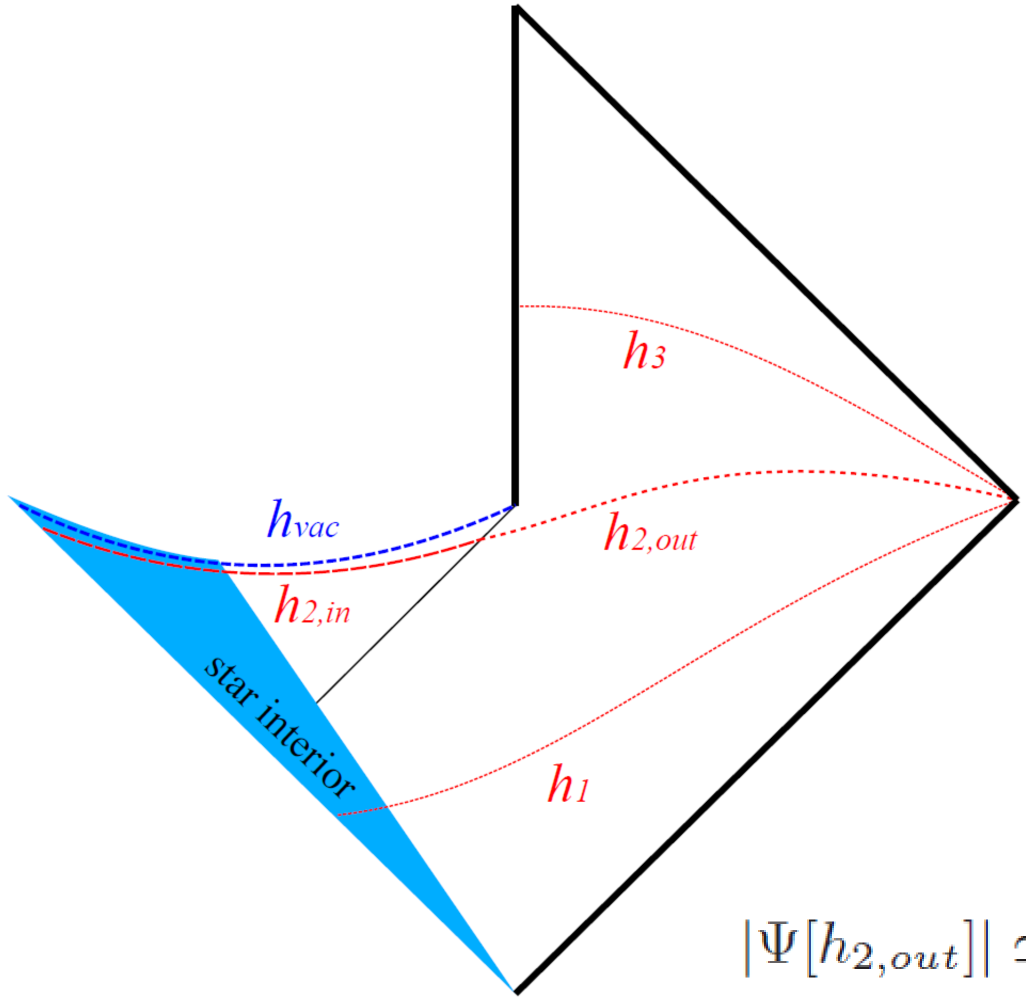


This may be an opposite process of the creation from nothing.



$$|\Psi[h_{2,out}]| \simeq |\Psi[h_3]|$$

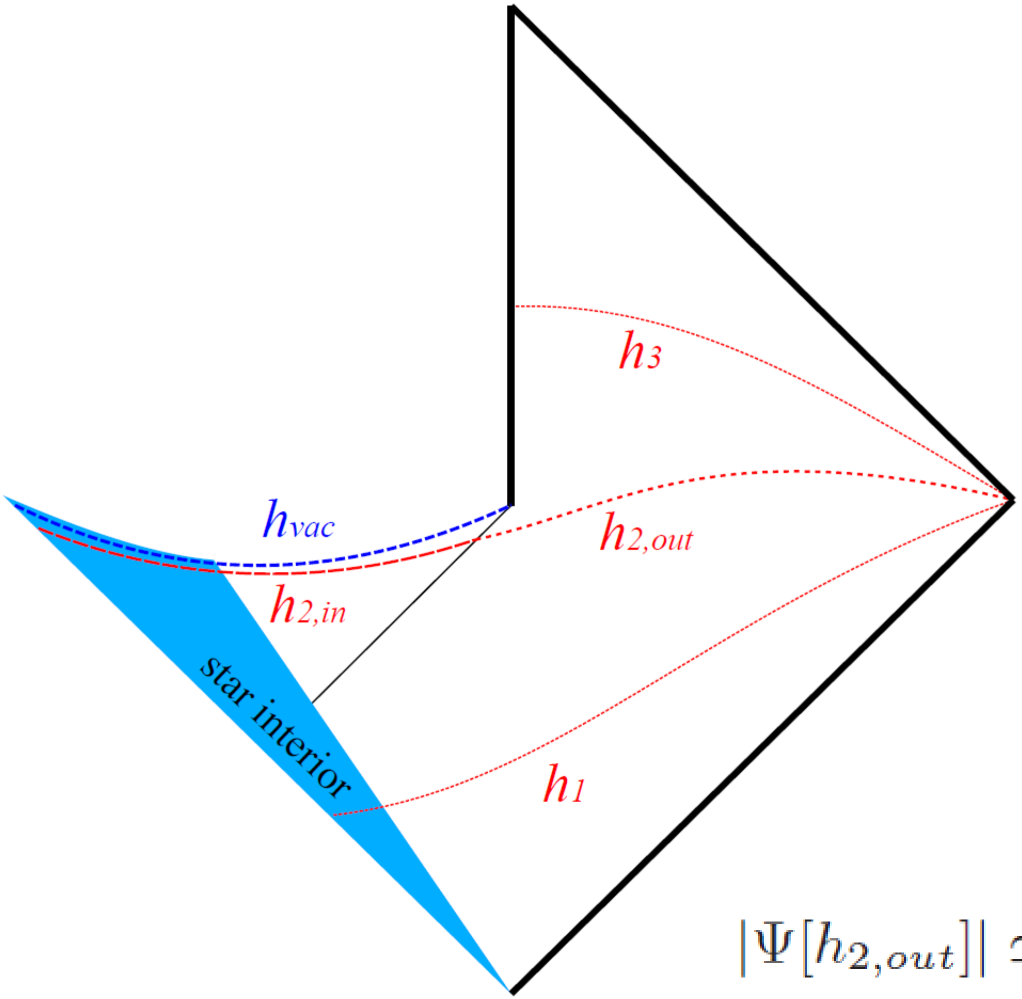
Now let us see both of inside and outside.
If we only focus outside, then it is semi−classical and the probability of each slice will not vary.



$$|\Psi[h_{2,out}]| \simeq |\Psi[h_3]|$$

$$\Psi[h_{vac} \cup h_3] \to 0$$

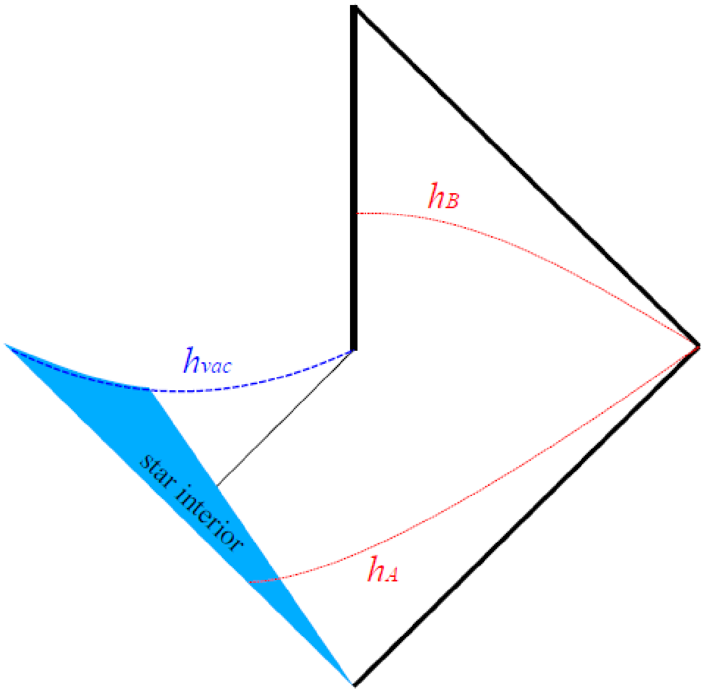
However, if we evaluate the probability of outside and inside together, it will approach to zero.



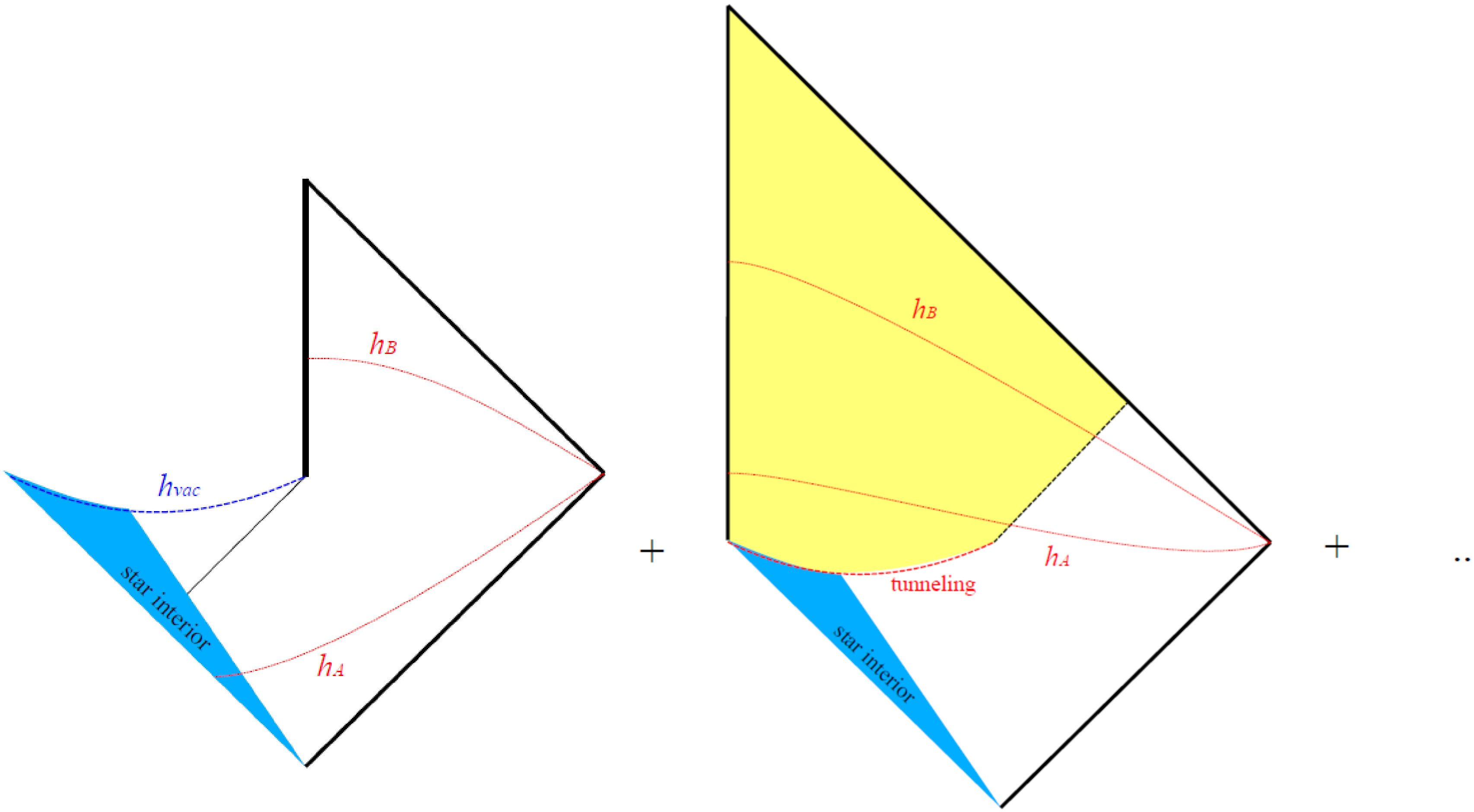
$$|\Psi[h_{2,out}]| \simeq |\Psi[h_3]|$$

$$\Psi[h_{vac} \cup h_3] \to 0$$

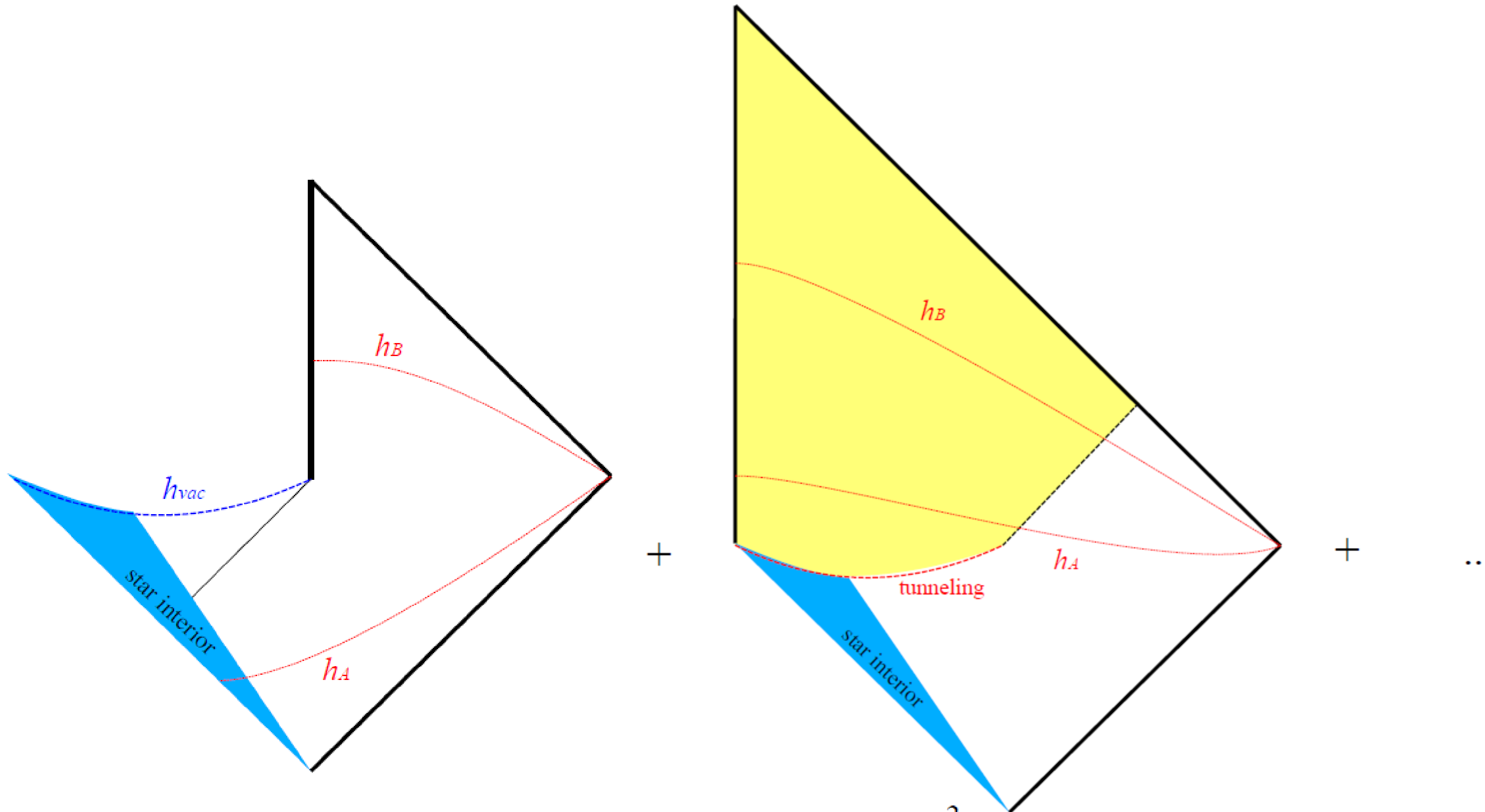
This process is definitely non−unitary and
we will lost information.



Let us see the entire wave function.

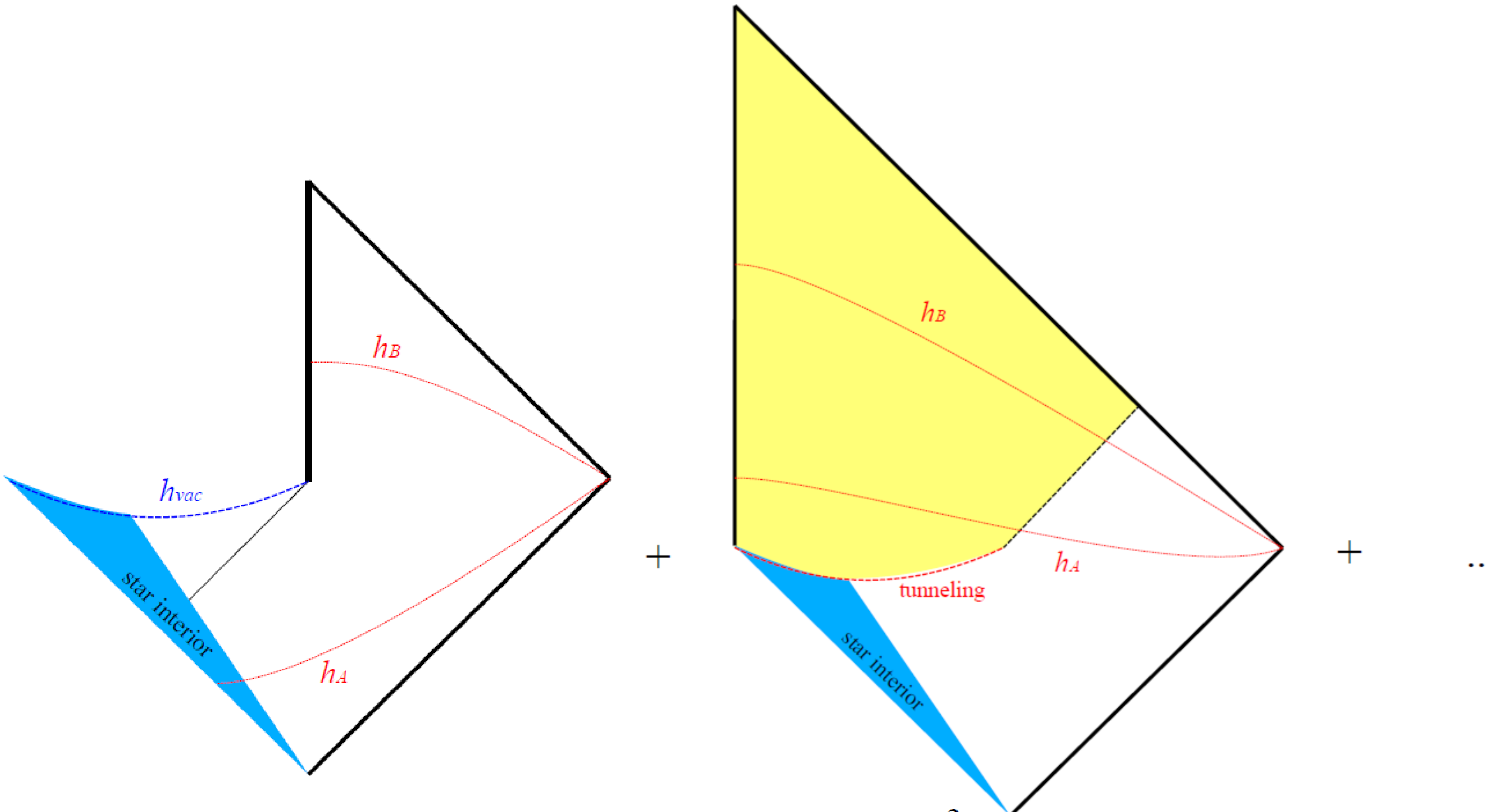


In the path integral, there exists a tunneling channel such that there is no formation of a black hole, even though the probability is very low (Chen, Sasaki and DY, 1806.03766).



$p_1[h_A] \sim 1$ $\qquad$ $p_2[h_A] \sim e^{-M^2}$

From the beginning, the first history is dominant in terms of probability.



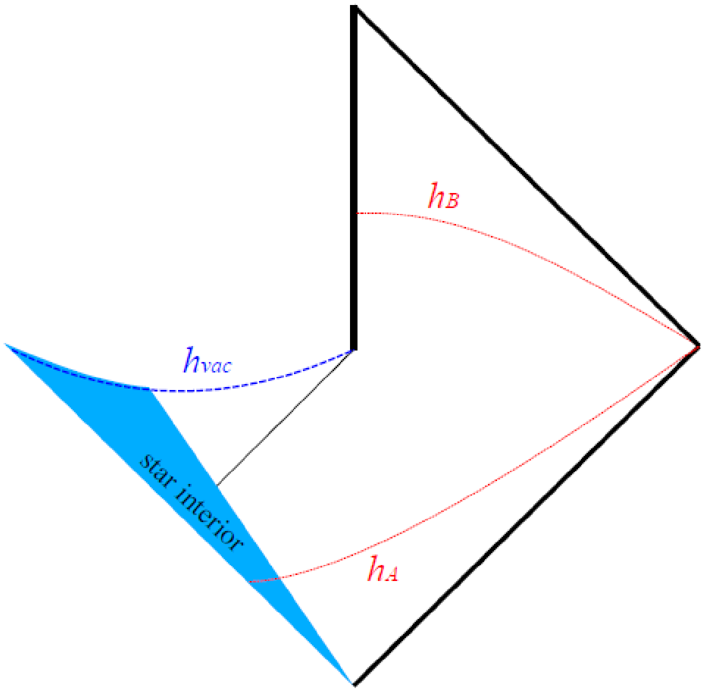
$p_1[h_A] \sim 1$

$\rightarrow \quad p_1[h_{vac} \text{ U } h_B] = 0$

$p_2[h_A] \sim e^{-M^2}$

However, as the time slice evolves,
the probability of the first history will decay to zero.

$$p_1 + p_2 + \ \ldots = 1$$



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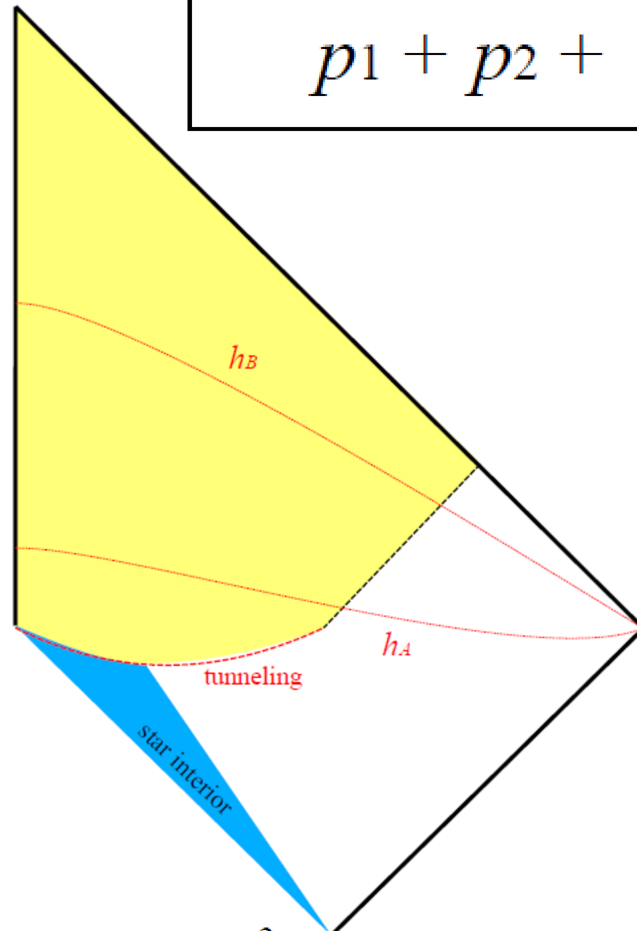


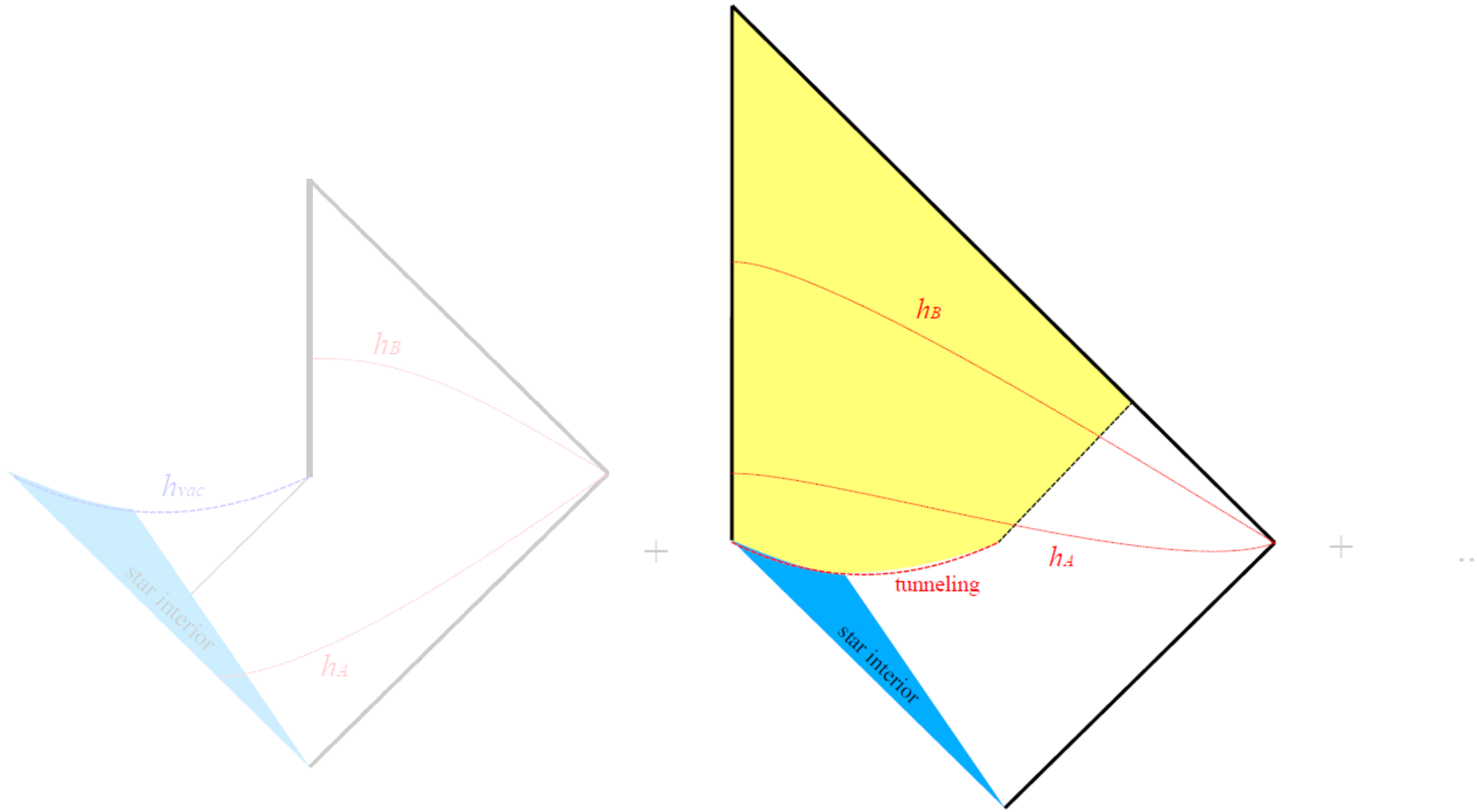
+ ...

$p_1[h_A] \sim 1$

$\rightarrow \quad p_1[h_{vac} \cup h_B] = 0$

$p_2[h_A] \sim e^{-M^2} \quad \rightarrow \quad p_2[h_B] \sim 1$

If the sum of the probabilities should be preserved, then
the probability of the second history should be dominated later.



So, in the late time, the wave function is dominated by the trivial geometry which has no loss of information.

Revisit the conceptual picture of Hartle and Hertog.

This will shed some light to find the resolution of the information loss paradox.

Modified gravity

Canonical quantum gravity

Euclidean quantum gravity

Can we provide a Euclidean boundary condition
for inside the horizon?

Can we provide a Euclidean boundary condition
for inside the horizon?

Maybe!

What if we Wick-rotate inside the horizon

$$ds^2 = -\left(\frac{2M}{r} - 1\right)^{-1} dr^2 + \left(\frac{2M}{r} - 1\right) dt^2 + r^2 d\Omega^2$$

using $r = M - i\rho$

What if we Wick−rotate inside the horizon

$$ds^2 = -\left(\frac{2M}{r} - 1\right)^{-1} dr^2 + \left(\frac{2M}{r} - 1\right) dt^2 + r^2 d\Omega^2$$

using $$r = M - i\rho$$

Then, asymptotically approaches to

$$ds^2 = -(dt^2 + d\rho^2 + \rho^2 d\Omega^2)$$

However, the problem is,

the Euclidean action is not well−defined,

since the volume is infinite.

$$S_E = \lim_{\Lambda \longrightarrow 0^+} \frac{\Lambda}{8\pi} \times (volume)$$

Perhaps, the analysis using canonical quantum gravity can provide a clue!

$$\Psi[X,Y] = \int_0^\infty \frac{2Ae^{-\sigma^2 k^2/2}}{\Gamma(-ik)r_s^{ik}} e^{-ikX} I_{ik}(2r_s e^Y)\,dk$$

First, let us (arbitrarily) integrate from 0.

$$\Psi[x, Y] = \int_0^\infty \frac{2Ae^{-\sigma^2 k^2/2}}{\Gamma(-ik) r_s^{ik}} e^{-kx} I_{ik}(2 r_s e^Y) dk$$

Second, Wick-rotate $X = -ix$.

Then, we choose only exponentially decreasing modes.

The steepest–descent can be described as follows.

$$Y = -\log(e^{X} + e^{-X})$$

$$\downarrow$$

$$Y = -\log\left(e^{-ix} + e^{ix}\right)$$

The asymptotic limit:

$$Y = -\log\left(e^{-ix} + e^{ix}\right)$$

$$\Big\downarrow \; x = \frac{\pi}{2}$$

$$ds^2 = -(dT^2 + dR^2 + r_s^2 b^2(T) d\Omega^2)$$

Can this provide a good relationship with
the previous analytic continuation?

$$ds^2 \to -(dT^2 + dR^2 + r_s^2 b^2(T) d\Omega^2)$$

$$ds^2 \to -(dt^2 + d\rho^2 + \rho^2 d\Omega^2)$$

One more speculation: for asymptotic de Sitter space,

$$ds^2 = -\left(\frac{2M}{r} - 1 + \frac{r^2}{\ell^2}\right)^{-1} dr^2 + \left(\frac{2M}{r} - 1 + \frac{r^2}{\ell^2}\right) dt^2 + r^2 d\Omega^2$$

using $r = r_0 - i\rho$

Then, asymptotically approaches to Euclidean AdS

$$ds^2 = -\left(\frac{\rho^2}{\ell^2} dt^2 + \frac{\ell^2}{\rho^2} d\rho^2 + \rho^2 d\Omega^2\right)$$

Alternative approaches shed some lights toward the ultimate understanding.

Thank you very much