Beyond Standard Model: From Theory to Experiment

Dynamics and Emergent Spacetime.

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Contents

Background

The Atoms of Spacetime?

- Length scales less than the Planck Length ($l_p = 1.6 \times 10^{-35}$) are *meaningless* [Hossenfelder 2013].
- Discrete spacetime at l_p scale is a *potential* route to Quantum Gravity.
- Lorentz invariance can be preserved ([Snyder 1947] and Doubly Special Relativity [Amelino-Camelia 2002])
- Starting assumption: *spacetime is discrete*, physics is *emergent*.

Glossary

Definitions

- A_{ij} adjacency matrix of the graph $G(V, E)$, e_{ij} is an edge
- Clustering Coefficient C_g: τ_G number of triangles, λ_G the number of corners $C_G = \frac{\tau_G}{\lambda_G}$
- L_{ij} is Laplacian matrix and $\Delta_{ij} = \delta_{ij} k_j$ the degree matrix, $L_{ij} = \Delta_{ij} - A_{ij}$.
- Hausdorff Dimension - extrinsic embedding dimension. Volume, area scaling parameter
- Spectral Dimension - intrinsic 'experienced' dimension. Random walk return probability
- Degree Dimension - lattice dimension. Half the number of links per node.

The Emergent Spacetime Model

The Set-up [Carlo A. Trugenberger 2015; Carlo A. Trugenberger 2016; Tee 2020] - Frustrated Ising

- A Hilbert Space $(\{|0\rangle, |1\rangle\})$ on each edge and vertex, total space $\mathcal{H}_{total} = \bigotimes_{e_{ij} \in E} \mathcal{H}_{edge} \bigotimes_{N} \mathcal{H}_i$
- $\hat{a}^{\dagger}_{ij}$, $\hat{a}_{ij}$ edge creation/annhilation operators, $\hat{s}_i$ vertex spin operators (assume $\equiv \hat{s}_z$), $\hat{s}^{\pm}$ normal ladder operators.
- Hamiltonian $(A_{ij} = \hat{a}^{\dagger}_{ij}\hat{a}_{ij})$

$$H = \frac{g^2}{2}\left(\mathrm{Tr}\left(A^3_{ij}\right) + \sum_{i \neq j}^{N} \sum_{k \neq i,j}^{N} A_{ik}A_{kj}\right) - \frac{g}{2}\sum_{i,j} \hat{s}_i A_{ij} \hat{s}_j \quad (1)$$

- Last term favors links between spins at lattice points i,j
- First terms frustrate links $(A_{ik}A_{kj})$, and triangles $\mathrm{Tr}\left\{A^3_{ij}\right\}$
- Simulation used - Minimization of Hamiltonian using Glauber dynamics

Simulated Ground State

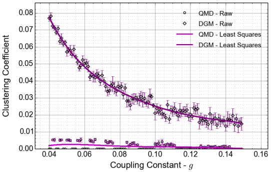


Figure: Ground state is effectively cluster free - a 'large world'

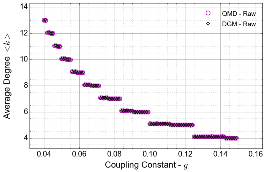


Figure: Average node degree reduces in steps with increasing g

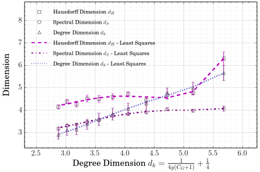


Figure: Measures of graph dimensional agree in the range $3.0 - 5.0$

Some Observations

Desirable Properties

- The ground state is Forman-Ricci and Ollivier-Ricci flat.
- Convergence of dimensions at 3.0 to 5.0
- Physicality of the model.
- Graphs have informational Entropy [Simonyi 1995; Passerini and Severini 2008]. Regular graphs, maximise graph entropy.

Questions & Problems

- **Time:** No 'special' temporal dimension.
  Speculate higher Hausdorff dimension could accommodate time?

Modeling Matter and Dynamics

Stable Defects

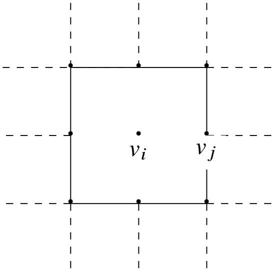


Figure: A section of the emerged quantum mesh with $\langle k \rangle = 2d = 4$. Isolated defect surrounds v_i. Here $s_i = |i, 0\rangle$, all other $j \neq i$, $s_j = |j, 1\rangle$.

- Defect energy $\Delta E = g^2/4(\langle k \rangle - 1) - g/2$.
- $\langle k \rangle \propto 1/g$ energy increases with more edges.
- Speculate $T \propto 1/g$ (micro-canonical ensemble arguments, defects stable to low T.
- localized, number conserved, finite 'intrinsic' energy, i.e. *a particle*.

Time and Dynamics

Assumption

Assumption: Time is a label (could be discrete, we assume continuous) on graph states. Ideally time should emerge with space.

Simplification

Consider only evolution of vertex spin states (edges dealt with in [Tee 2020]).

Unitary Evolution

- Assume unitary time evolution from time t by small $t+\tau$, $|v_i, t+\tau\rangle = e^{-i\hat{H}_d \tau/\hbar} |v_i, t\rangle$, according to Hamiltonian $\hat{H}$.

Choosing a Dynamical Hamiltonian

Desiderata

- **Locality:** Interaction between sites should decay with distance
- **Ground State Preserving:** No interactions between connected vertices
- **Dimensions:** Have correct dimensions of an Hamiltonian
- **Elements:** Valid for any graph state, defined independent of state

Our choice, for vertices v_i,v_j, separated by r_{ij}, summed over all pairs.

$$\hat{H}_{ij} = -\frac{gc^2}{2\epsilon_m r_{ij}^2}\hat{s}_i^+(1 + L_{ij})\hat{s}_j^-. \tag{2}$$

Laplacian matrix L_{ij}, r_{ij} can be expressed as

$$L_{ij} = \sum_{k,j=0}^{k,j=N} \delta_i^j \hat{a}_{ik}^\dagger \hat{a}_{kj} - \hat{a}_{ij}^\dagger \hat{a}_{ij} \quad r_{ij} = \sum_{n=0}^{n=\infty} \delta(A_{ij}^n) \tag{3}$$

Taking the Continuum Limit

Expanding Time Evolution

- Expand Unitary Time Evolution $|v_i, t + \tau\rangle = e^{-i\hat{H}_d \tau/\hbar} |v_i, t\rangle$ and Taylor expand $|v_i, t\rangle$.
- Gather terms to first order in τ, sub in $\hat{H}$:

$$-\frac{\hbar}{i} \frac{\partial |v_i, t\rangle}{\partial t} = \frac{gc^2\hbar^2}{8\epsilon_m r_{ij}^2} |v_i, t\rangle + \frac{gc^2\hbar^2}{8\epsilon_m r_{ij}^2} L_{ij} |v_i, t\rangle , \tag{4}$$

- Note that $L_{ij} \iff -\nabla^2$ [Chung 1997], in continuum limit. (e.g. heat conduction on a chicken wire fence)

Recovering Schrödinger

Renormalizing g and dealing with Laplace

- In the continuum limit $r_{ij} \to 0$ (shrink edges to length 0).
- We 'renormalize' $g \to g_p$.
- Replace $\epsilon_m = mc^2$ by its mass.
- Each lattice point v_i becomes a point $\vec{x}$, and $|v_i, t\rangle \to \psi(\vec{x}, t)$.
- Recover Schrödinger for a free particle in a constant potential $\frac{g_p \hbar^2}{2m}$

$$-\frac{\hbar}{i}\frac{\partial \psi(\vec{x}, t)}{\partial t} = \frac{g_p \hbar^2}{2m}\psi(\vec{x}, t) + \frac{g_p}{2m}\left(\frac{\hbar}{i}\nabla\right)^2 \psi(\vec{x}, t) \quad (5)$$

- Our dynamic Hamiltonian is consistent with QM!

Some observations...

Is dynamic Hamiltonian discrete QM?

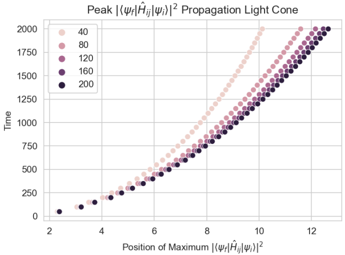
- Our choice of dynamic Hamiltonian implies QM, why?
- L_{ij} is in $\hat{H}_{ij}$ to preserve ground state and move a defect.
- It is the presence of L_{ij}, a kind of ‘kinetic’ term that yields $-\nabla^2$
- $\hat{H}_{ij} \rightarrow$ Schödinger equation (i.e. Unitary evolution)

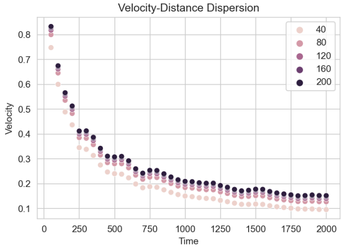
Approximate Causality

Lieb-Robinson Bounds

- Hamiltonian Eq. (2) could admit a Lieb-Robinson (LR) bound [Nachtergaele, Sims, and Young 2019; Tran et al. 2020].
- LR bound limits propagation speed.
- Simulation introduces defect at time $t = 0$.
- Propagate forward in time i.e. compute transition probability of moving defect from location v_i to v_j separated by distance r_{ij} by acting on spins using $\hat{H}_{ij}$.

Simulations in a $2d$ Mesh





Convergence to a bound?

To early to say, but there appears to be approximate bounding to an emergent light-cone.

Have we Achieved Dynamics?

Quantum Mechanics from Spacetime?

- Starting point is a Hilbert Space and ‘double-Ising’ Hamiltonian that produces an acceptable ground state.
- Model admits stable matter like defects.
- Unitary Evolution and minimal dynamic Hamiltonian produces QM in the continuum limit.
- A dynamical discrete model of emergent QM?

Concluding Remarks

Curvature and Canonical GR

- Hamiltonian Eq. (1) potentially has relationship to discrete curvature (Forman-Ricci/Ollivier-Ricci).
- How to link discrete model to canonical Quantum Gravity using Hamiltonian GR?
- Recent results on Forman-Ricci and Ollivier-Ricci equivalence (Tee and C. Trugenberger 2021) establishes discrete curvature framework
- Current work is proving a link between Eq. (1) to Canonical GR, using Eq. (2) as the kinetic term

Thank You & Questions