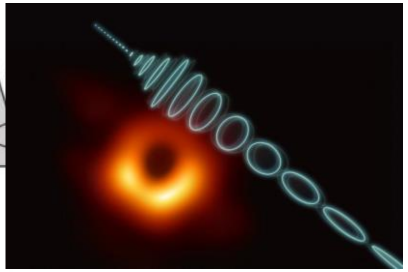

WEAK FIELD DEFLECTION ANGLE BY REGULAR BLACK HOLES WITH COSMIC STRINGS

ASSOC. PROF. DR. ALİ ÖVGÜN

EASTERN MEDITERRANEAN UNIVERSITY, FAMAGUSTA, NORTH CYPRUS

@BSM-2021

www.aovgun.com

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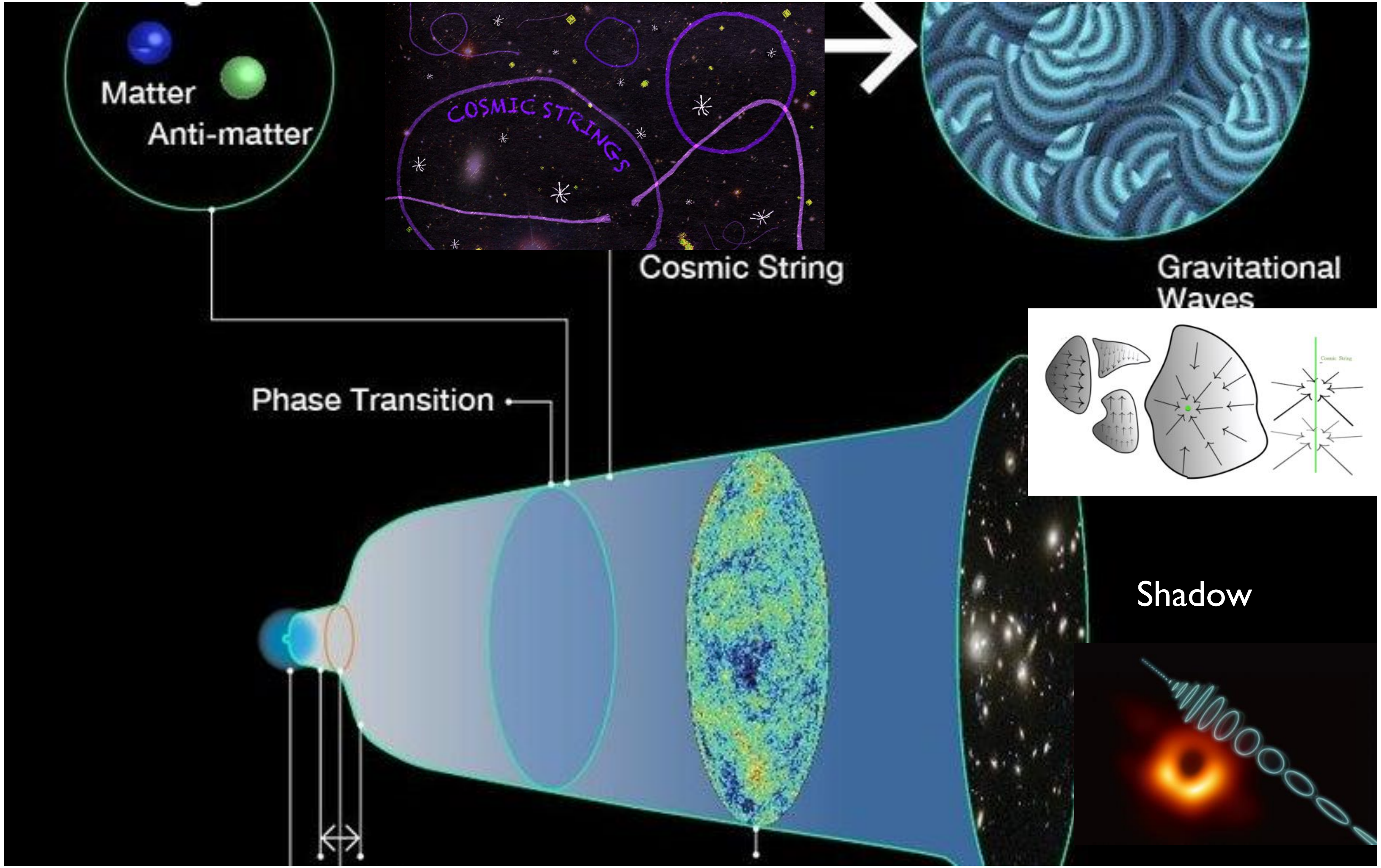
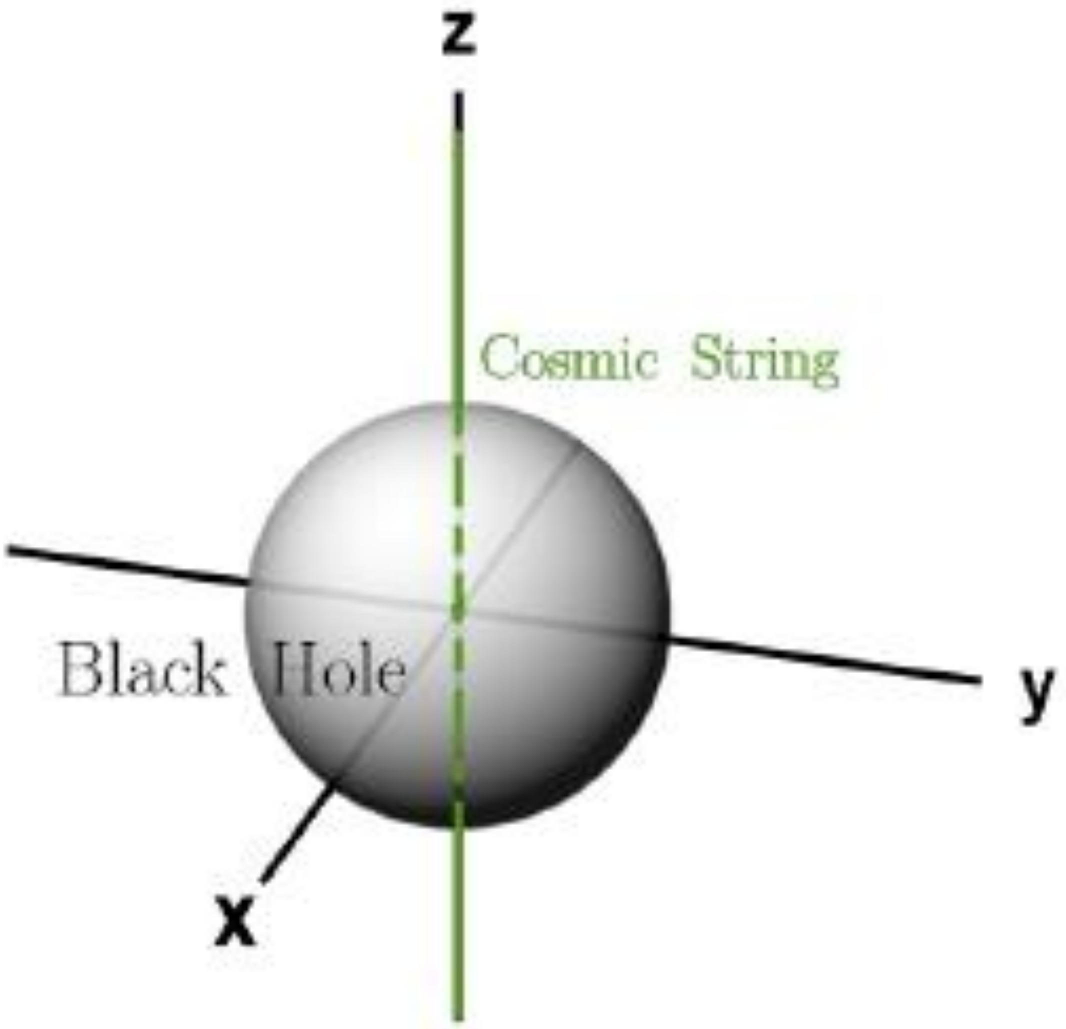
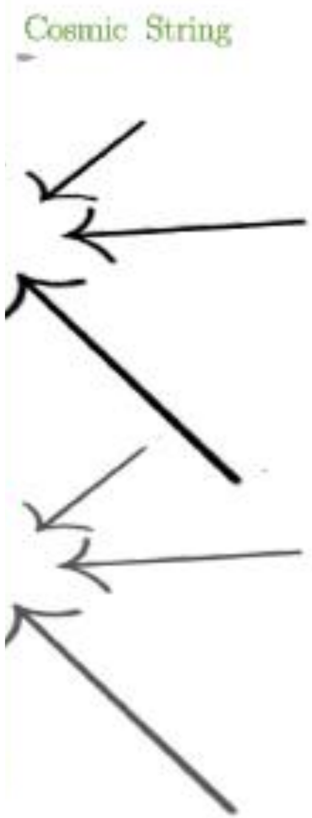


(Original credit: R. Hurt/Caltech-JPL, NASA, and ESA Credit: Kavli IPMU - Kavli IPMU modified this figure based on the image credited by R.Hurt/Caltech-JPL, NASA, and ESA)

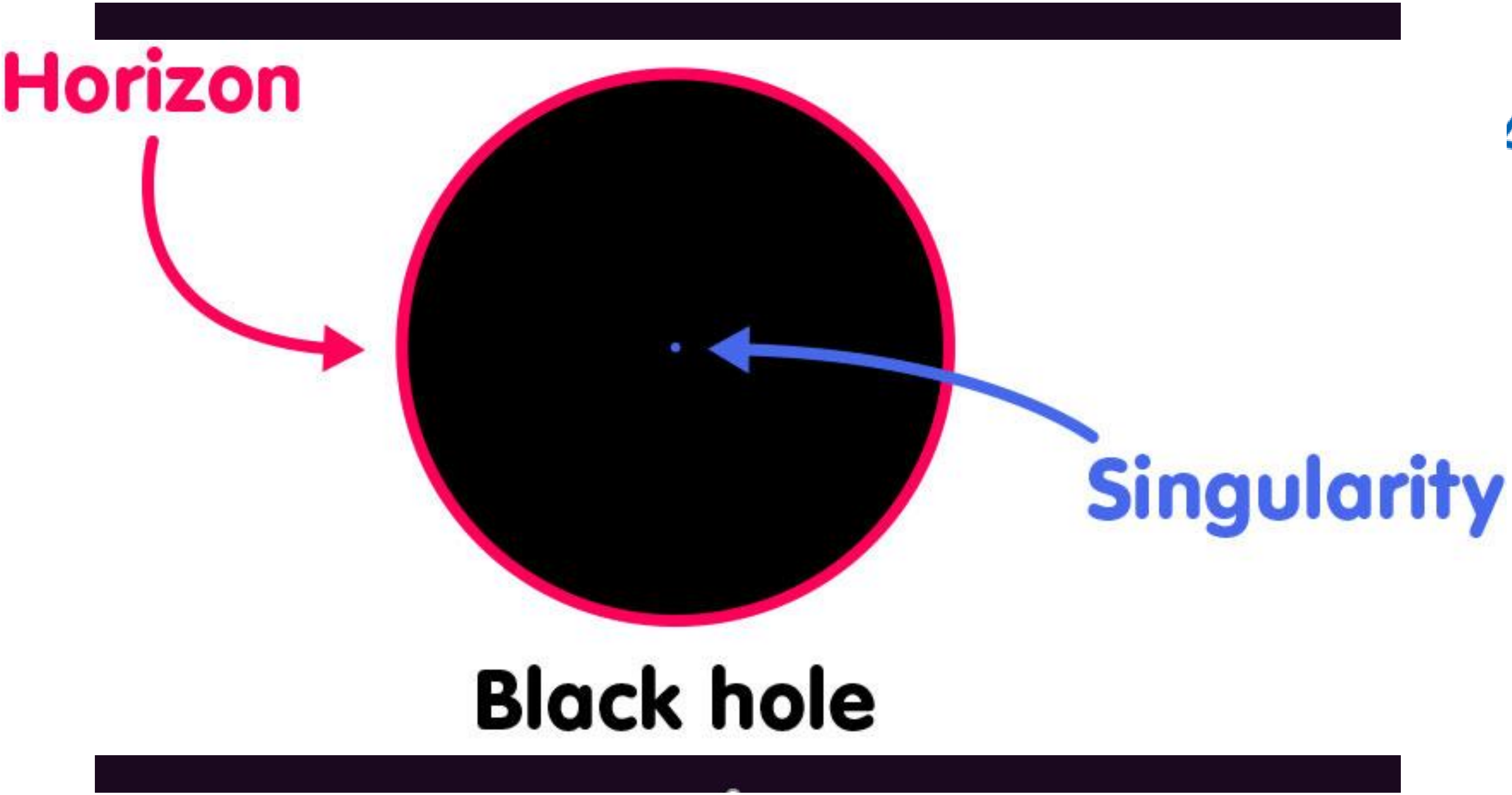




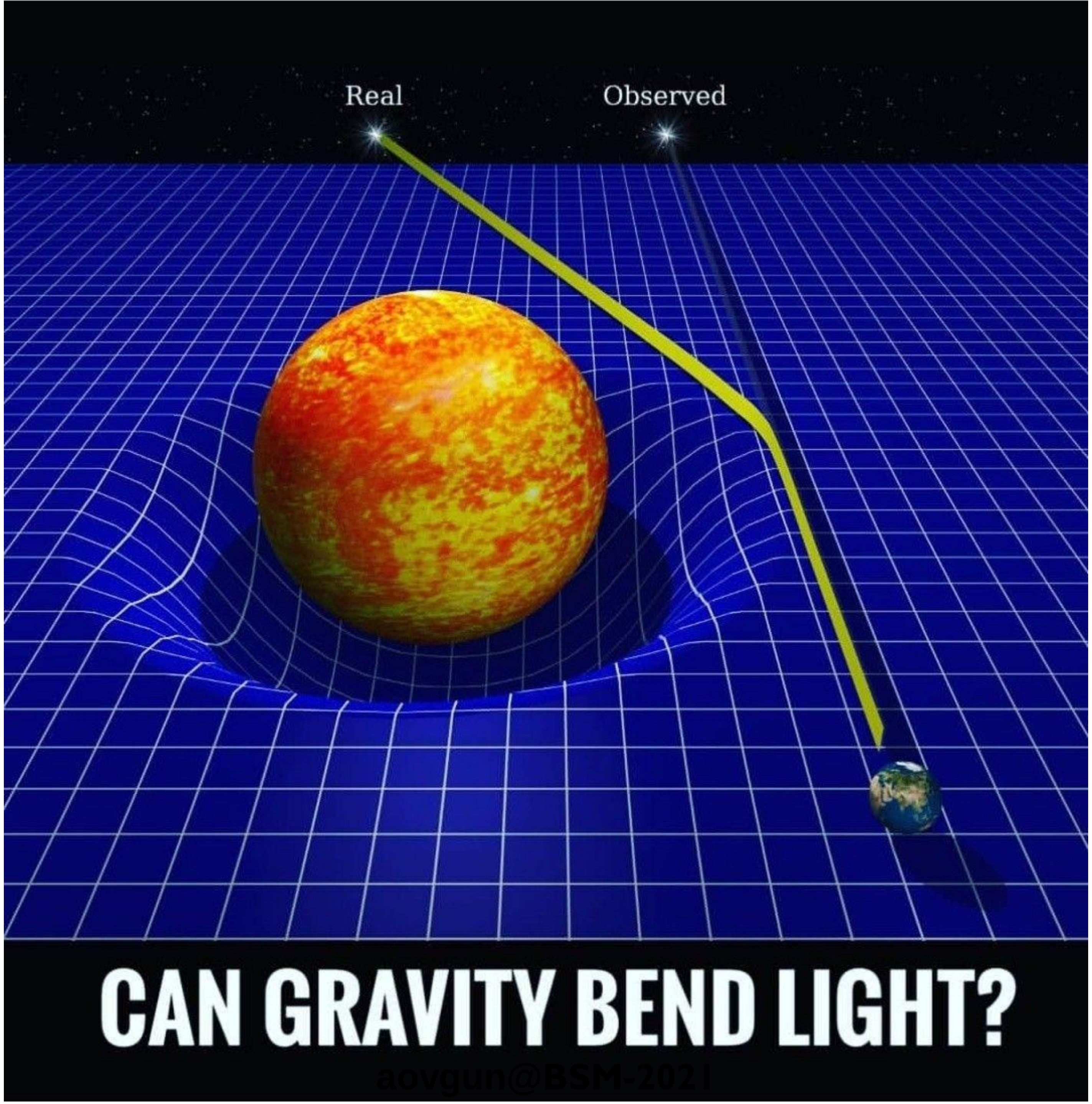
- Different regions of space creating a defect through symmetry breaking.

Original credit:

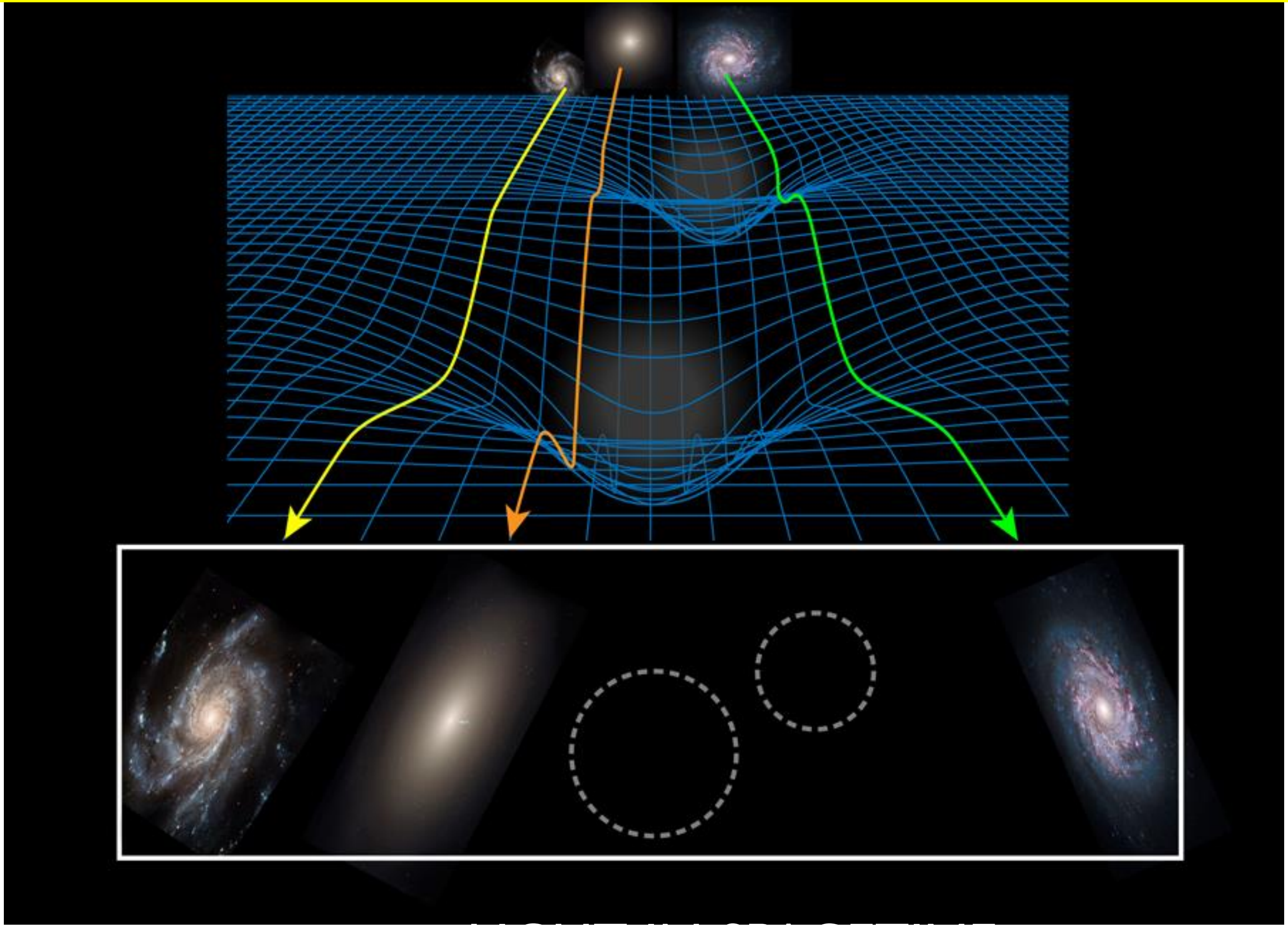
Matheus do Carmo Teodoro


Horizon
Singularity
Black hole

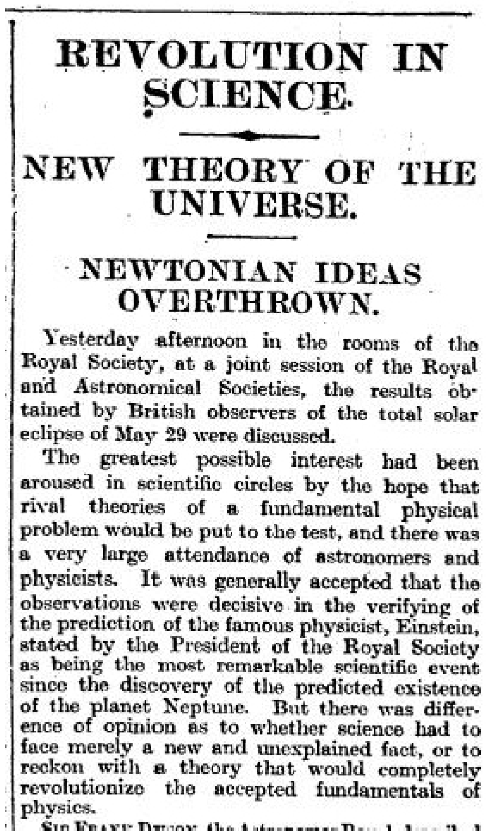

Sun
Black hole


Real
Observed
CAN GRAVITY BEND LIGHT?

GRAVITATIONAL LENSING USING GAUSS-BONNET THEOREM



1919:



REVOLUTION IN SCIENCE.

NEW THEORY OF THE UNIVERSE.

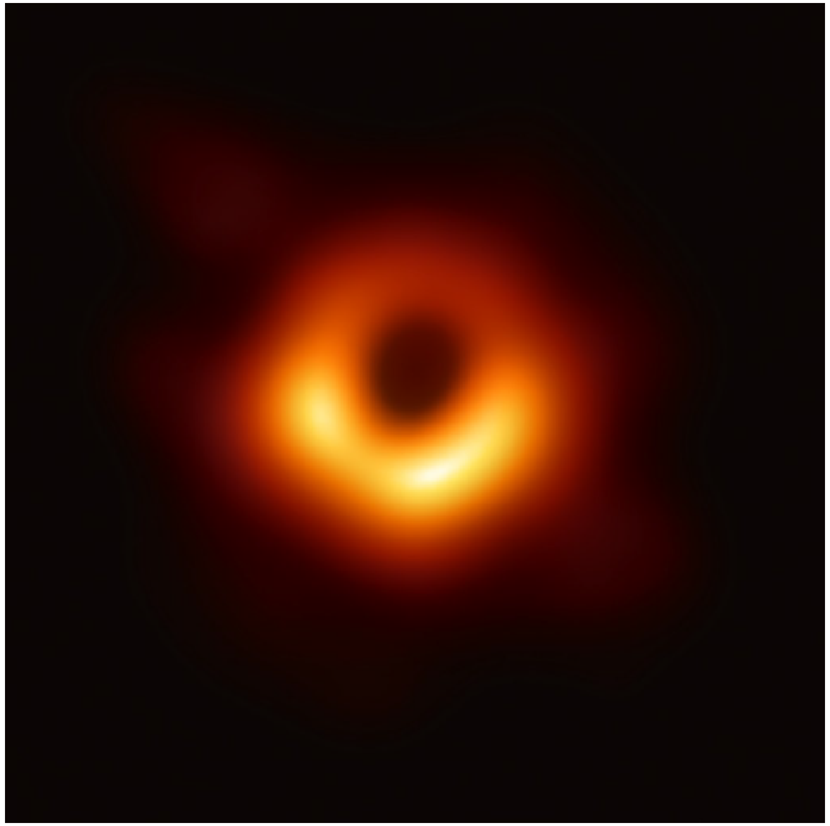
NEWTONIAN IDEAS OVERTHROWN.

Yesterday afternoon in the rooms of the Royal Society, at a joint session of the Royal and Astronomical Societies, the results obtained by British observers of the total solar eclipse of May 29 were discussed.

The greatest possible interest had been aroused in scientific circles by the hope that rival theories of a fundamental physical problem would be put to the test, and there was a very large attendance of astronomers and physicists. It was generally accepted that the observations were decisive in the verifying of the prediction of the famous physicist, Einstein, stated by the President of the Royal Society as being the most remarkable scientific event since the discovery of the predicted existence of the planet Neptune. But there was difference of opinion as to whether science had to face merely a new and unexplained fact, or to reckon with a theory that would completely revolutionize the accepted fundamentals of physics.

Eddington's eclipse expedition corroborating GR lensing

2019:

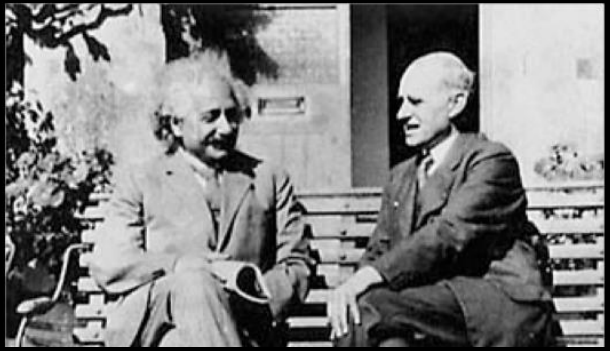


M87* photon sphere shadow, Event Horizon Telescope

GR test: 1919 Eclipse

- 1919: Arthur Eddington went to Island of Principe, off west to observe a total eclipse. Another team went to Sobral, Brazil.

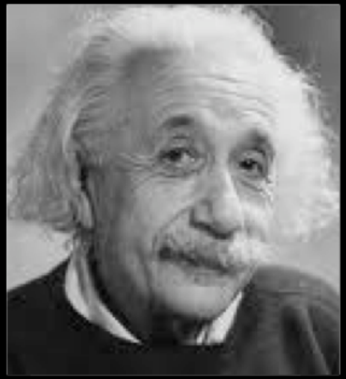
  -2-plates: (Principe) measured $\delta = 1.61 \pm 0.30$ arcsec



Einstein

-1912: predicted that a geometric theory of gravity would exhibit light deflection.

-1915: publication of the General Theory of Relativity and determined a new value for the deflection angle $\delta = \dfrac{4GM}{Rc^2}$



Solar deflection angle is now 1.75 arcsecs, double the Newotnian (corpuscle) value.

Gravitational lensing theory can be approached in three ways:

1. null geodesics in 4-dimensional spacetime;
2. standard approximation used in astronomy: quasi-Newtonian impulse approximation in Euclidean 3-space;
3. optical geometry: 2-/3-dimensional space whose geodesics are spatial projections of null geodesics, by Fermat's Principle.

Static spacetime: Riemannian optical geometry.

Null curves obey $\mathrm{d}t^2 = h_{ij}\mathrm{d}x^i\mathrm{d}x^j$,

with optical metric $h_{ij} = -\frac{g_{ij}}{g_{tt}}$.

Stationary spacetime: Finslerian optical geometry.

I.e., $\mathrm{d}t = \sqrt{h_{ij}\mathrm{d}x^i\mathrm{d}x^j} + b_i\mathrm{d}x^i$.

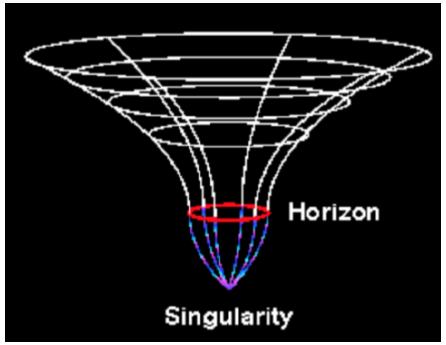
OPTICAL GEOMETRY OF SCHWARZSCHILD

Given the line element of the Schwarzschild solution:

$$ds^2 = -\left(1 - \frac{2\mu}{r}\right) dt^2 + \left(1 - \frac{2\mu}{r}\right)^{-1} dr^2 + r^2 \left(d\theta^2 + sin^2\theta d\phi^2\right) \quad (6)$$

the metric of the optical geometry can be read off from $(\theta = \pi/2)$:

$$dt^2 = \left(1 - \frac{2\mu}{r}\right)^{-2} dr^2 + \left(1 - \frac{2\mu}{r}\right)^{-1} r^2 d\phi^2 \quad (7)$$



Gibbons G W and Werner M C 2008
Class. Quantum Grav. **25** 235009

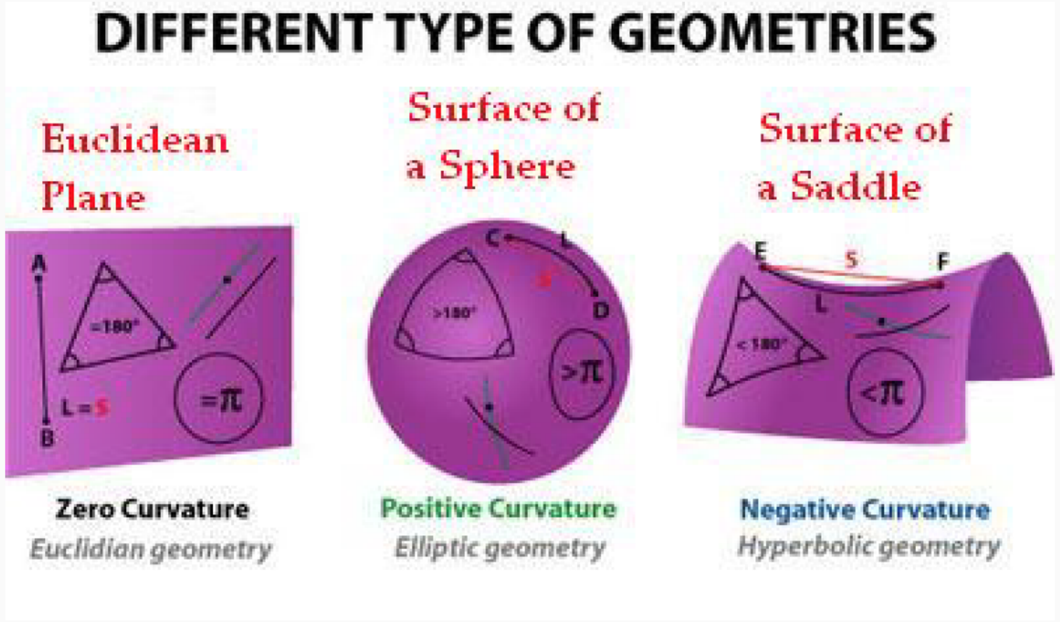
LENSING IN THIS OPTICAL GEOMETRY

Geodesics on this surface correspond to spatial light rays. However, the Gaussian curvature at every point

$$K < 0 \quad (8)$$

so geodesics must locally diverge.

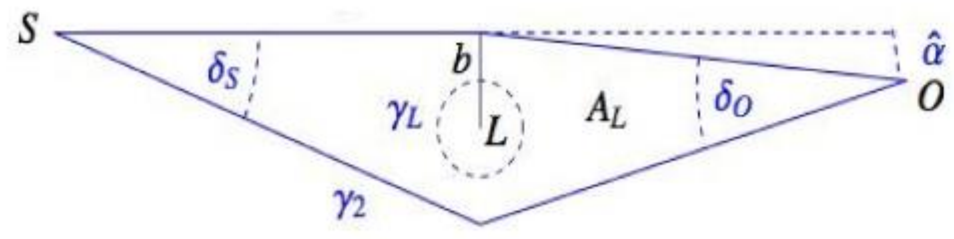
Then how can two light rays from a light source refocus at the observer, so that the two images of the Schwarzschild lens are obtained?



- This method relies on the fact that the deflection angle can be calculated using a domain outside of the light ray.
- It is known that the effect of lensing strongly depends on the mass of the enclosed region body on spacetime.
- to calculate the Gaussian curvature of K, so that the GBT is found as follows:

$$\int\int_A K\mathrm{dS} + \int_{\partial\mathrm{A}} \kappa\mathrm{dt} + \sum_{\mathrm{i}} \alpha_{\mathrm{i}} = 2\pi\chi(\mathrm{A}). \tag{3}$$

- κ stands for the geodesics curvature of $\partial A : \{t\} \rightarrow A$
- α_i is the exterior angle with the i^{th} vertex.
- Euler characteristic of χ
- a Riemannian metric of g
- This technique is for asymptotically flat observers and sources.
- The resulting deflection angle is expected to be too small, which is also a joint point in astronomy.



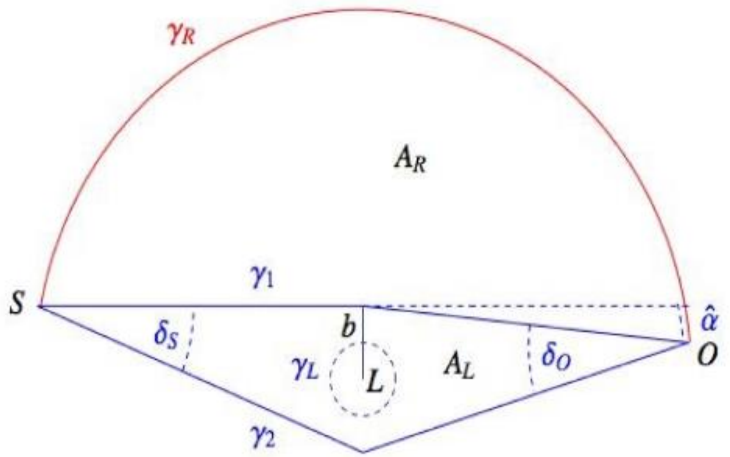
Gibbons G W and Werner M C 2008 *Class. Quantum Grav.* **25** 235009

We will calculate the Gaussian curvature of K in an optical geometry to obtain the asymptotic deflecting angle of alpha:

$$\hat{\alpha} = -\int\int_{A_\infty} K \mathrm{d}S. \qquad (4)$$

Note that we use the infinite region of the surface A_∞ bounded by the light ray to calculate our integral.

Consider the circular domain A_R, with $R \to \infty$:



The deflection angle in asymptotically flat limits can be obtained by

([E.g., Werner (2012); Ono, Ishihara & Asada (2017 & 2018); Jusufi, Werner, Banerjee & Övgün (2017); Crisnejo & Gallo (2018); Jusufi & Övgün (2018); Övgün, Jusufi & Sakallı (2019); Övgün (2019); de Leon & Vega (2019)]

$$\hat{\alpha} = -\int_{0}^{\pi}\int_{\frac{b}{\sin\varphi}}^{\infty} \mathcal{K}\mathrm{d}A.$$

integrating outside (!) the light ray; extensions are possible.

Deflection of light from black hole

The Gaussian optical curvature of Schwarzschild black hole is

$$\mathcal{K} \approx -\frac{2\mu}{r^3}$$

$$\alpha \approx \int_0^{\pi}\int_{\frac{b}{\sin\varphi}}^{\infty}\left(\frac{2\mu}{r^3}\right) r dr d\phi \approx \frac{4\mu}{b}$$

where b is a impact parameter.

Light deflection by Damour-Solodukhin wormholes and Gauss-Bonnet theorem

Ali Övgün*

The spacetime metric of the Schwarzschild-like wormhole's solution is:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{g(r)} + r^2 d\Omega^2_{(2)} \tag{22}$$

where $f(r) = 1 - \frac{2M}{r}$ and $g(r) = 1 - \frac{2M(1+\lambda^2)}{r}$.

T. Damour and S. N. Solodukhin. Phys. Phys. Rev. D 76, 024016 (2007).

The deflection angle as

$$\hat{\alpha} \simeq \frac{4M}{b} + \frac{2M\lambda^2}{b}$$

Deflection angle of rotating Damour-Solodukhin wormhole

$$\hat{\alpha} = \frac{2M(\lambda^2+2)}{b} \pm \frac{4Ma}{b^2}.$$

- The deflection angle by DSW is increased with the ratio of the parameter λ with compared to Schwarzschild BH.

Physics Letters B 308 (1993) 237–239
North-Holland

PHYSICS LETTERS B

No glory in cosmic string theory

G.W. Gibbons

DAMTP, Silver Street, Cambridge CB3 9EW, UK

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Editor: P.V. Landshoff

Light deflection by *non-axisymmetric* cosmic strings is studied. Various properties of the geodesics are obtained including the non-existence of closed or almost closed geodesics. It is shown that the behaviour of the geodesics resembles the case of the axisymmetric cosmic strings usually studied. In particular no exotic behaviour such as glory scattering is possible. The usual formula for the deflection of distant geodesics is obtained. An application to the slow motion of SU(3) monopoles, considered as geodesic motion on a moduli space, is also given.

PHYSICAL REVIEW D **99**, 104075 (2019)

Can we see the effects of the cosmic strings on deflection angle?

Weak field deflection angle by regular black holes with cosmic strings using the Gauss-Bonnet theorem

A. Övgün*

Instituto de Física, Pontificia Universidad Católica de Valparaíso, Casilla 4950, Valparaíso, Chile and Physics Department, Faculty of Arts and Sciences, Eastern Mediterranean University, Famagusta, North Cyprus, via Mersin 10, Turkey

(Received 20 March 2019; published 28 May 2019)

In this paper, we investigate light bending in the spacetime of regular black holes with cosmic strings in weak field limits. To do so, we apply the Gauss-Bonnet theorem to the optical geometry of the black hole; and, using the Gibbons-Werner method, we obtain the deflection angle of light in the weak field limits which shows that the bending of light is a global and topological effect. Afterwards, we demonstrate the effect of a plasma medium on the deflection of light by a regular black hole with cosmic strings. We discuss that increasing cosmic string parameter μ and mass M_0 will increase the bending angle.

DOI: 10.1103/PhysRevD.99.104075

II. REGULAR BLACK HOLES WITH COSMIC STRINGS

[17] C. H. Bayraktar, Thermodynamics of regular black holes with cosmic strings, Eur. Phys. J. Plus **133**, 377 (2018).

The RBCS metric in spherical coordinates is given by the equations [17]

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \zeta^2 \sin^2\theta d\phi^2), \quad (1)$$

$$f(r) = 1 - \frac{2m(r)}{r} \quad (2)$$

with the cosmic string parameter $\zeta = 1$–4μ. It is noted that the mass function [16] is given by

$$m(r) = \frac{M_0}{[1 + (\frac{r_0}{r})^q]^{\frac{p}{q}}}, \quad (3)$$

where M_0 and r_0 are mass and length

above metric reduces as follows for Bardeen black holes ($p = 3$, $q = 2$) and Hayward black holes ($p = q = 3$) [6]. There are two solutions for $r_0 < M_0$, where $r = r_\pm$. Note that the inner horizon is r_- and the outer horizon is $r_+ \approx 2m(r)$.

III. CALCULATION OF DEFLECTION ANGLE BY RBCS OPTICAL SPACETIME

The RBCS optical spacetime can be simply written in equatorial plane $\theta = \pi/2$, to obtain null geodesics ($ds^2 = 0$):

$$dt^2 = \frac{dr^2}{f(r)^2} + \frac{\zeta^2 r^2 d\varphi^2}{f(r)}. \tag{4}$$

Radial component of the geodesic curvature

$$K = \frac{R_{\text{icci Scalar}}}{2} \approx -\frac{2M_0}{r^3} + \frac{3{M_0}^2}{r^4}. \tag{5}$$

$$\kappa(C_R)dt = \frac{1}{R}(\zeta R)d\varphi.$$

Note that it is NOT flat because of the cosmic strings at asymptotic limits. The Gauss-Bonnet equation reduces to:

$$\iint_{D_R} K dS + \oint_{\partial D_R} \kappa dt = \pi.$$

$$\pi = \iint_{S_\infty} K dS + \zeta \int_0^{\pi+\alpha} d\varphi.$$

where α is a deflection angle and the optical surface area of RBCS is $dS = \zeta r dr d\varphi$.

In the weak field regions, the light ray follows a straight line approximation, so that we can use the condition of $r = b/\sin\varphi$ at zeroth-order.

$$\alpha = \frac{\pi - \pi\zeta}{\zeta} - \int_{0}^{\pi} \int_{\frac{b}{\sin\varphi}}^{\infty} K r dr d\varphi. \qquad (11)$$

The deflection angle α of RBCS in weak field limits is found as follows:

$$\alpha \simeq \frac{4M_0}{b} + 4\pi\mu. \qquad (12)$$

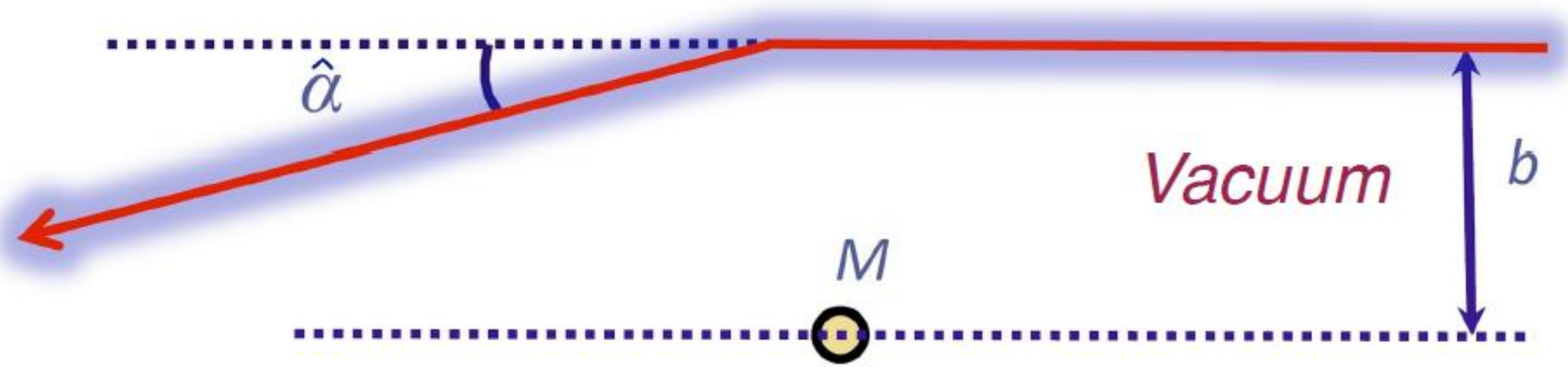
$\zeta = 1–4\mu.$

Note that the cosmic string parameter μ, and the mass term increase the deflection angle.

Vacuum:

Einstein angle:

$$\hat{\alpha} = \frac{4GM}{c^2 b} = \frac{2R_S}{b} \qquad b \gg R_S = \frac{2GM}{c^2}$$



Deflection angle of the photon in vacuum does not depend on the photon frequency (or energy). Deflection in vacuum is **achromatic**.

Gravitational lensing in vacuum is achromatic.

Plasma:

How is this situation changed in presence of plasma?

In outer space, **the rays of light travel through the plasma**.

In plasma, photons undergo various effects.

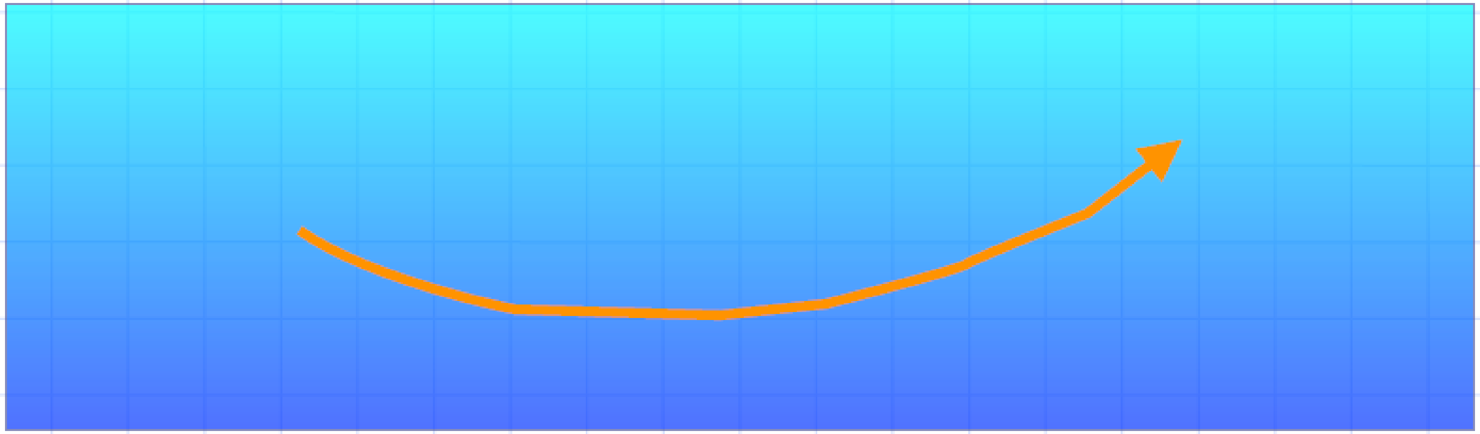
For gravitational lensing the main interest is the **change in the angle of deflection** of the light ray.

Due to dispersion properties of plasma, we may expect that **chromatic effects will arise**.

Refraction

Light rays in a transparent, inhomogeneous medium propagate along curved trajectories. This phenomenon is called refraction and is well known from everyday life.

The bending of the light rays due to refraction is not related to relativity or gravity and occurs only if the medium is optically non-homogeneous.



$$n=n(x)$$

'Non-homogeneous' means here that the refractive index depends explicitly on space coordinates

Gravitational deflection of light rays in presence of homogenous plasma

Refraction index of plasma:

$$n^2 = 1 - \frac{\omega_e^2}{[\omega(r)]^2}, \qquad \omega_e^2 = \frac{4\pi e^2 N_0}{m} = \text{const}$$

Here, the frequency of the photon $\omega(r)$ depends on the spatial coordinate r due to the presence of a gravitational field (the gravitational redshift). We will use the following notation: $\omega(\infty) \equiv \omega$, e is the charge of the electron, m is the electron mass, ω_e is the electron plasma frequency, and $N_0 = \text{const}$ is the electron concentration in a homogeneous plasma. This for-

We have shown for the first time, that the gravitational deflection in homogeneous plasma differs from the vacuum deflection angle, and depends on frequency of the photon:

$$\hat{\alpha} = \frac{2R_S}{b} = \frac{4GM}{c^2 b}$$

in vacuum

$$\hat{\alpha} = \frac{R_S}{b}\left(1 + \frac{1}{1 - (\omega_e^2/\omega^2)}\right)$$

in homogeneous plasma

Chromatic gravitational deflection!

Bisnovatyi-Kogan and Tsupko 2009, 2010, 2013

IV. WEAK GRAVITATIONAL LENSING BY RBCS IN A PLASMA MEDIUM

In this section, we investigate the effect of a plasma medium on the weak gravitational lensing by RBCS.

The refractive index $n(r)$ for RBCS is obtained as [39],

$$n(r) = \sqrt{1 - \frac{\omega_e^2}{\omega_\infty^2}\left(1 - \frac{2m(r)}{r}\right)}, \tag{13}$$

ω_e is the electron plasma frequency and ω_∞ is the photon frequency measured by an observer at infinity.

$$d\sigma^2 = g_{ij}^{\text{opt}} dx^i dx^j = \frac{n^2(r)}{f(r)}\left(\frac{dr^2}{f(r)} + \zeta^2 r^2 d\varphi^2\right). \tag{14}$$

$$\mathcal{K} = \frac{M_0(\omega_e{}^2 - 2\omega_\infty{}^2)\omega_\infty{}^2}{(\omega_e{}^2 - \omega_\infty{}^2)^2 r^3} - 3\frac{M_0{}^2(\omega_e{}^2 + \omega_\infty{}^2)\omega_\infty{}^4}{(\omega_e{}^2 - \omega_\infty{}^2)^3 r^4}. \tag{15}$$

$$\lim_{R\to\infty} \kappa_g \frac{d\sigma}{d\varphi}\bigg|_{C_R} = \zeta.$$

For the limit of $R \to \infty$, and using the straight light approximation $r = b/\sin\varphi$, the Gauss-Bonnet theorem becomes [39]:

$$\lim_{R\to\infty} \int_0^{\pi+\alpha} \left[\kappa_g \frac{d\sigma}{d\varphi}\right]\bigg|_{C_R} d\varphi = \pi - \lim_{R\to\infty} \int_0^{\pi} \int_{\frac{b}{\sin\varphi}}^{R} \mathcal{K} dS. \tag{19}$$

Hence, the deflection angle with the approximate expression for the weak field limits reads as:

$$\alpha = 4\pi\mu + \frac{4M_0}{b} + 4\frac{M_0{\omega_e}^2}{{\omega_\infty}^2 b}. \qquad (20)$$

These results show that the photon rays move in a medium of homogeneous plasma. It is noted that $\omega_e/\omega_\infty \to 0$, (20)

Conclusions

- In this paper, we performed a comprehensive study of RBCS's deflection angle of photons in weak field approximation.
- To this end, we have used the Gauss-Bonnet theorem and a straight-line approximation to obtain the deflection angle of light at leading order terms.
- Then, we calculated the deflection angle of light by RBCS in a plasma medium up to the first order with the approximate expression for the weak deflection.
- For both cases, the cosmic string parameter μ and the mass term increase the deflection angle.
- After neglecting the plasma effects, $\omega_e/\omega_\infty \to 0$, the deflection angle reduces to the value in vacuum case.
- The deflection angle using the Gauss-Bonnet theorem is calculated by integrating over a domain outside the impact parameter, which shows that gravitational lensing is a global effect and is a powerful tool to research the nature of black hole singularities.

Thank you!

VERY MUCH