WEAK DEFLECTION ANGLE OF EXTENDED UNCERTAINTY PRINCPLE BLACK HOLES

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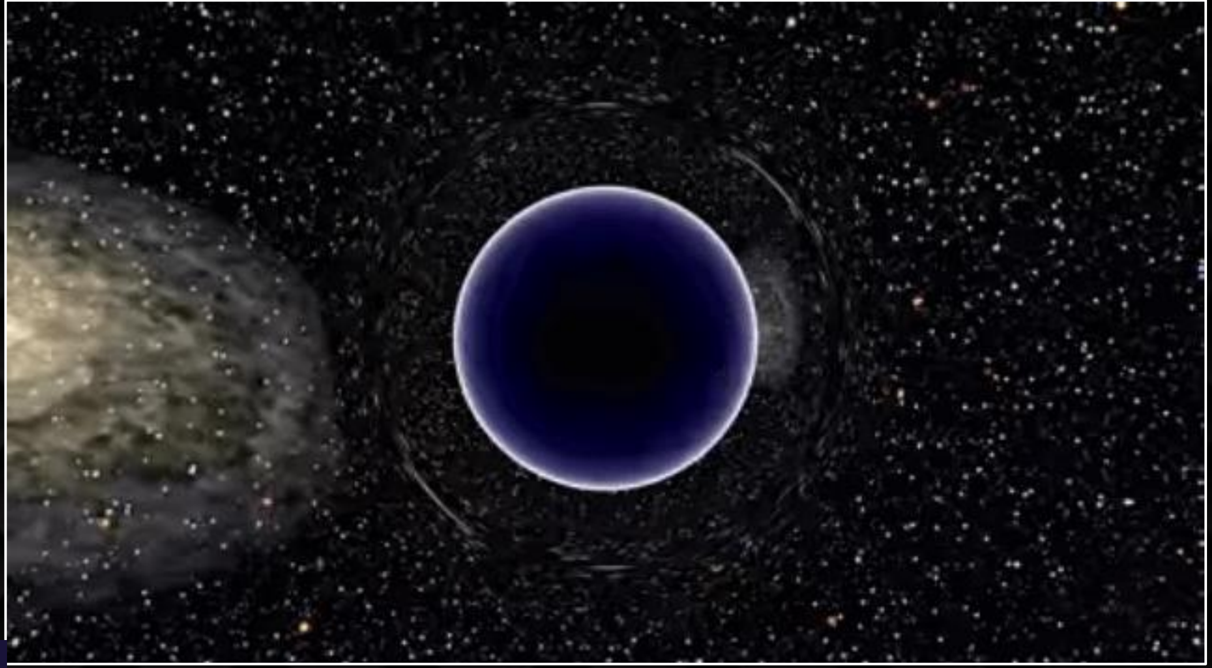
Recurring Abbreviations:
EUP: Extended Uncertainty Principle
GBT: Gauss-Bonnet Theorem

CONTENTS

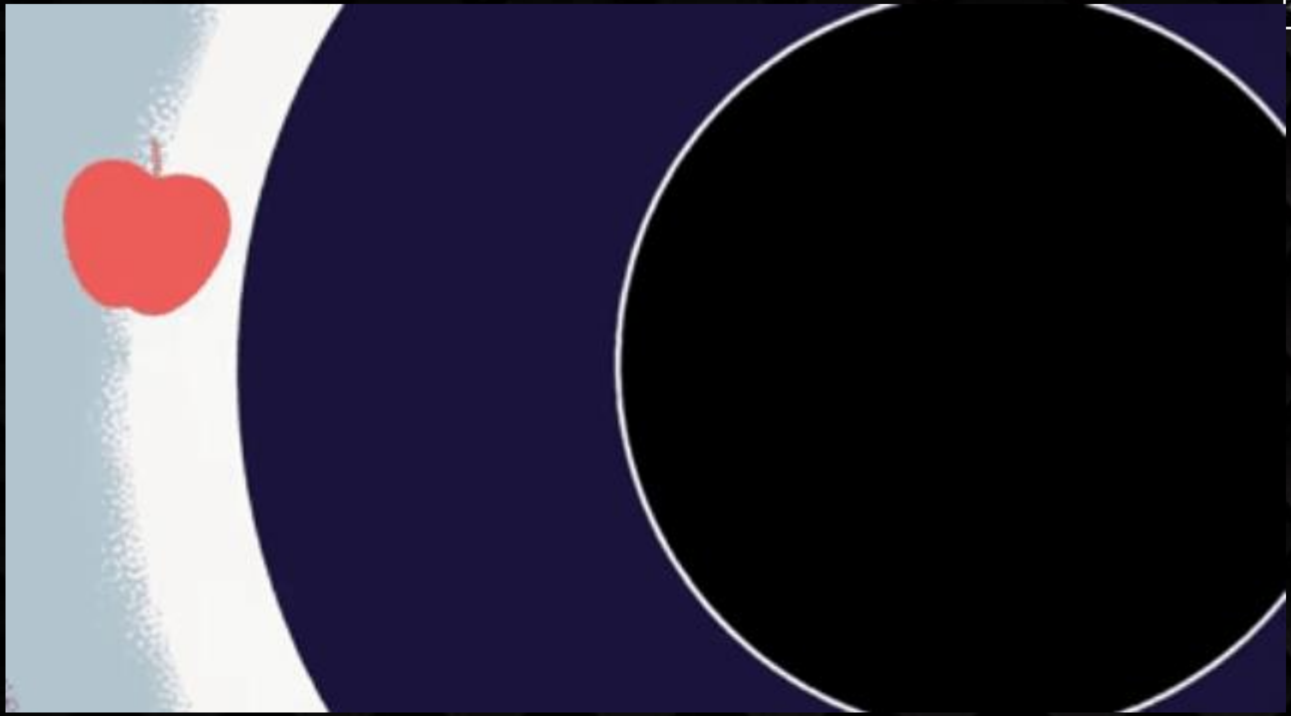
INTRODUCTION

- Gravitational lensing

→ Strong and weak lensing



Source: https://www.youtube.com/watch?v=hLTGGBxj-SM



Source: https://www.youtube.com/watch?v=r5Pcqkhmp_0

- Black hole information paradox

Solution – quantum fluctuations

EUP BLACK HOLE

x: Position
p: Momentum

α: Coupling constant
L: Fundamental distance

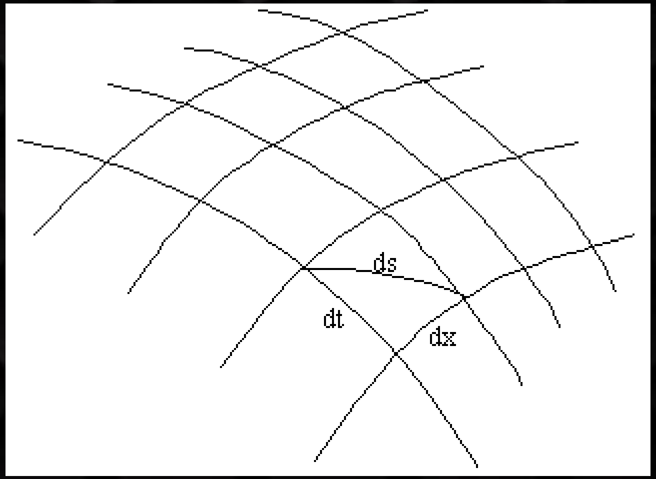
M: Mass of the black hole
r_0: Closest approach

- Heisenberg's Uncertainty Principle (extended)

$$\Delta x\, \Delta p \geq 1 + \alpha \frac{\Delta x^2}{L^2}$$

- Line element of a spherically symmetric black hole:

$$ds^2 = -f(r)\, dt^2 + f(r)^{-1}\, dr^2 + r^2\, (d\theta^2 + \sin^2\theta\ d\phi^2)$$



Source: jimhaldenwang.com/black_hole.htm

where, the function $f(r)$ for an EUP black hole is given by:

$$f(r) \equiv 1 - \frac{2M}{r}\left(1 + \cancel{\frac{4\alpha M^2}{L^2}}\right)$$

Schwarzschild case

→ $M/r_0 \ll 1 \Rightarrow$ weak lensing due to small deflection angle.

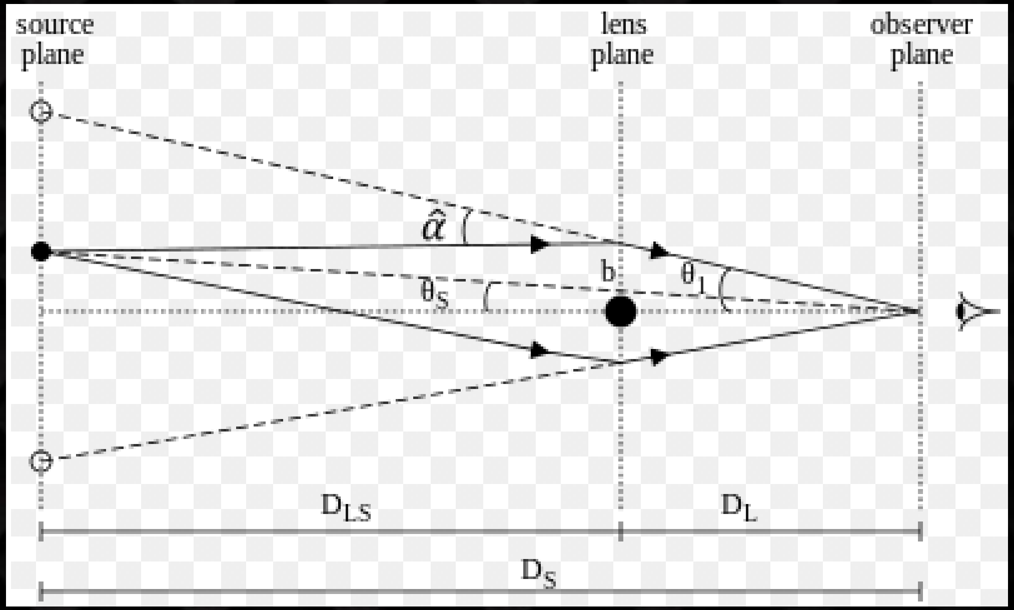
Gauss-Bonnet Theorem

K: Gaussian curvature
κ: Geodesic curvature
α: Exterior angle with ith vertex
χ: Euler characteristic
g: Riemannian metric
(D, χ, g): Surface domain

$$\iint_D K\,dS + \int_{\partial D} \kappa\,dt + \sum_i \alpha_i = 2\pi\chi(D)$$

Deflection angle given by Gibbons and Werner :

$$\Longrightarrow \hat{\alpha} = -\iint_D K\,dS$$



Source: https://en.wikipedia.org/wiki/Einstein_radius

R: Ricci scalar

u: Impact parameter

Weak Deflection Angle

- Non-zero Christoffel symbols of the optical metric for null geodesics give:

$$K = \frac{R}{2} = -\frac{8\alpha M^3}{L^2 r^3} - \frac{2M}{r^3}$$

- Straight line approximation sets $r = u/\sin\theta$ and the weak deflection angle becomes:

$$\hat{\alpha}_w = \frac{4M}{u} + \cancel{\frac{16\alpha M^3}{uL^2}}$$

Schwarzschild case

PLASMA MEDIUM

ω_e: Electron plasma frequency
ω_∞: Photon frequency

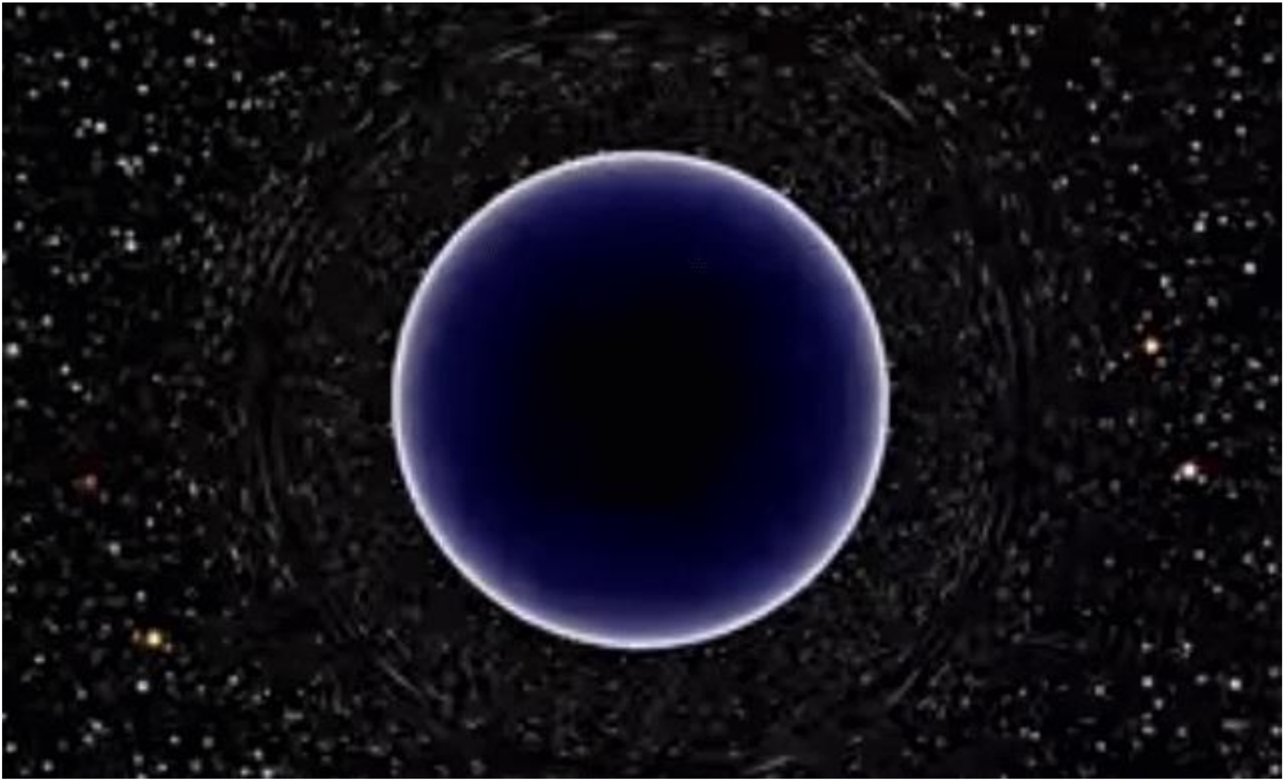
- Refractive index defined as $n \equiv c/v$

  for an EUP corrected black hole:

$$n(r) = \sqrt{1 - \frac{\omega_e^2}{\omega_\infty^2}\left[1 - \frac{2M}{r}\left(1 + \frac{4\alpha M^2}{L^2}\right)\right]}$$

- Consequently,

$$\hat{\alpha}_{w_n} = \boxed{\frac{4M}{u}} + \cancel{\frac{2M}{u}\frac{\omega_e^2}{\omega_\infty^2}} + \boxed{\frac{16\alpha M^3}{uL^2}} + \cancel{\frac{8\alpha M^3}{uL^2}\frac{\omega_e^2}{\omega_\infty^2}}$$



Source: https://www.youtube.com/watch?v=hLTGGBxj-SM

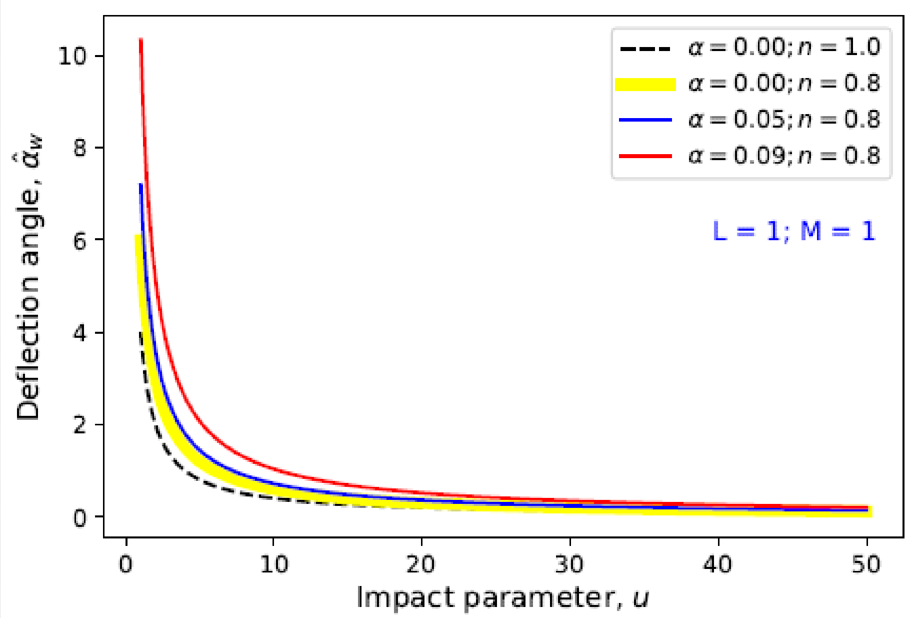
These terms can be combined into:

$$\frac{4M}{u}\left(1 + \alpha l^2\right)$$

called **Deflection angle of the post-Newtonian approach**

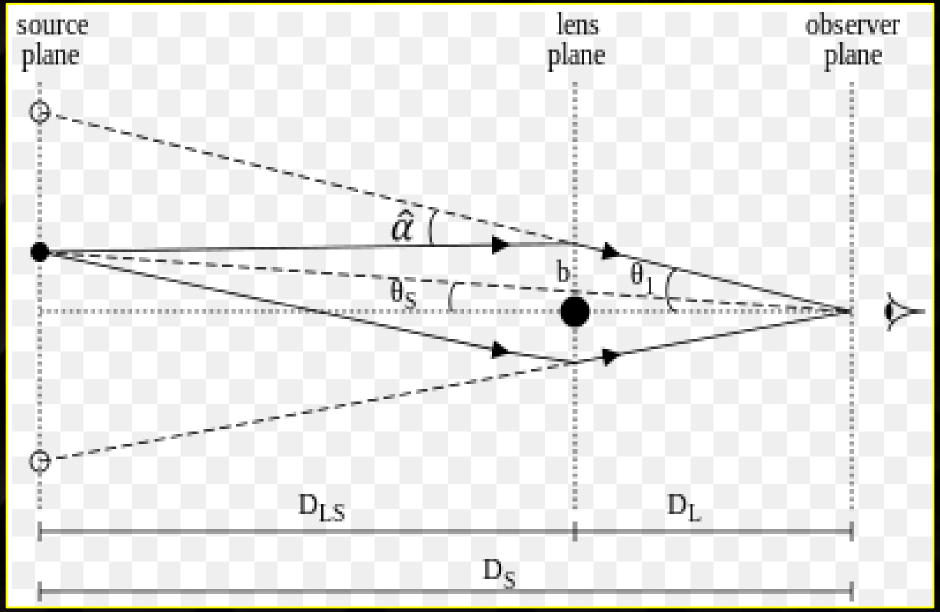
where, $l \equiv 2M/L$.

Observations



Deflection angle vs impact parameter for various values of coupling constant, α , with refractive index, n = 0.8

Observables



- Magnification: $\mu_\pm = \frac{\beta^2+2\left(1+\alpha l^2\right)\theta_E^2}{2\beta\sqrt{\beta^2+4(1+\alpha l^2)\theta_E^2}} \pm \frac{1}{2}$

- Time delay: $\Delta T = 4M\,\left[\frac{\beta}{2\theta_E^2}\sqrt{\beta^2+4(1+\alpha l^2)\theta_E^2}\right] + (1+\alpha l^2)\ln\left(\frac{\sqrt{\beta^2+4(1+\alpha l^2)\theta_E^2}+\beta}{\sqrt{\beta^2+4(1+\alpha l^2)\theta_E^2}-\beta}\right)$

- Angular separation: $\Theta_\pm = \theta_E\sqrt{\frac{1}{2}\left\{1+\left[1-\frac{\omega_e^2}{\omega_\infty^2}\,(1-\xi(1+\alpha l^2)\right]^{-1}\right\}}$

CONCLUSION

Quantum gravity effects in the vicinity of a black hole were investigated:

- These effects are found to engender substantial changes in the deflection angle and observables in the presence of a plasma.
- This variation can be utilized to develop the precision of differential deflection by the typical assumptions of weak lensing.
- The nature of equation of $\hat{\alpha}_w$ to adapt between vacuum and plasma advocates flexibility to modify the result for a wide range of analyses.
- **It is concluded that the EUP corrections are indeed high enough to determine various parameters on a large scale.**

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APPENDIX

- The corresponding deflection angle of a photon hurling across an EUP black hole:

$$\hat{\alpha}(r_0) \equiv -\pi + 2\int_{r_0}^{\infty} dr \, \frac{\sqrt{f(r)^{-1}}}{r\sqrt{\frac{r^2}{r_0^2}\frac{f(r_0)}{f(r)} - 1}}$$

- Optical Gaussian curvature for an EUP black hole corrected for plasma:

$$K = -\frac{2M}{r^3} + \frac{3M^2}{r^4} - \frac{8\alpha M^3}{L^2 r^3} - \frac{M}{r^3}\frac{\omega_e^2}{\omega_\infty^2} + \frac{4M^2}{r^4}\frac{\omega_e^2}{\omega_\infty^2} - \frac{4\alpha M^3}{L^2 r^3}\frac{\omega_e^2}{\omega_\infty^2} - \frac{4M^3}{r^5}\frac{\omega_e^2}{\omega_\infty^2}$$