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Time evolution of Lepton family Number carried by Majorana neutrinos

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1 Introduction and Motivation

The properties of neutrinos are not fully understood yet.

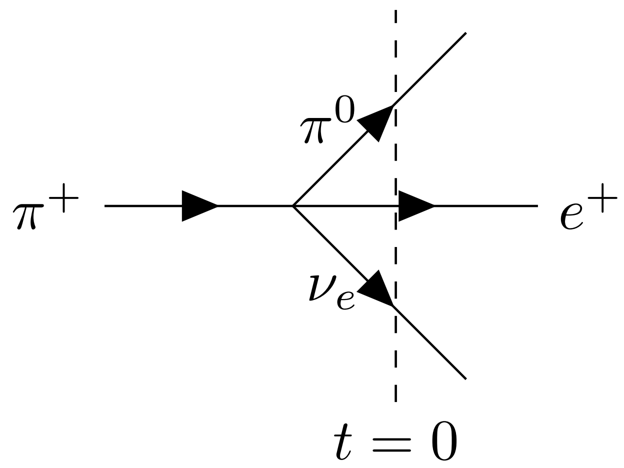
- One central question is that whether neutrino is Majorana or Dirac particle ?
- How one can discriminate Majorana and Dirac ?
- To discriminate them,
  one needs to go to very low momentum $|q| < m_\nu$.
- Temperture of CNB (Cosmic Neutrino Background) is
  $T_{CNB} \simeq 2K \sim 2 \times 10^{-4}$ (eV) $< m_\nu = 10^{-3}$ (eV).
- The formula for the time evolution of lepton family number is derived for Majorana neutrinos.
- It is valid for relativistic and non-relativistic case.

2 Our set up

To investigate the Lepton Number under the presence of Majorana mass term, we consider the situation that the Majorana mass term is turned on at $t = 0$.

For $t < 0$:massless, $t > 0$: massive.

The situation corresponds to the following Lagrangian: $(\alpha = e, \mu, \tau, i = 1, 2, 3)$.



$$\mathcal{L} = \overline{\nu_{\alpha L}} i\gamma^\mu \partial_\mu \nu_{\alpha L} - \theta(t) \frac{m_{\alpha\beta}}{2} \left(\overline{(\nu_{\alpha L})^c} \nu_{\beta L} + h.c. \right)$$

$$= \frac{1}{2} \overline{\psi_i} i\gamma^\mu \partial_\mu \psi_i - \theta(t) \frac{m_i}{2} \overline{\psi_i} \psi_i .$$

where the second line is written in terms of the mass eigenstate for the Majorana field, $\psi_i^c = \psi_i$,

$$\psi_i = \nu_{iL} + (\nu_{iL})^c, \quad \nu_{\alpha L} = V_{\alpha i} \nu_{iL},$$

$$m_i \delta_{ij} = \left(V^T \right)_{i\alpha} m_{\alpha\beta} V_{\beta j} .$$

3 Continuity Condition

The field operator is continuous at $t = 0$.

Massless Massive

$$\nu_{L\alpha}(t = 0_- < 0, \mathbf{x}) = V_{\alpha i} P_L \psi_i(t = 0_+ > 0, \mathbf{x}), \quad P_L = \frac{1-\gamma_5}{2}.$$

$$LHS = \int' \frac{d^3\mathbf{p}}{(2\pi)^3 2|\mathbf{p}|} \left(a_\alpha(\mathbf{p}) e^{i\mathbf{p}\cdot\mathbf{x}} u_L(\mathbf{p}) + b^\dagger_\alpha(\mathbf{p}) e^{-i\mathbf{p}\cdot\mathbf{x}} v_L(\mathbf{p}) \right),$$

$$RHS = V_{\alpha i} P_L \int' \frac{\sum_{\lambda=\pm 1} d^3\mathbf{p}}{(2\pi)^3 2E(\mathbf{p})} \left(a_{Mi}(\mathbf{p},\lambda) u_i(\mathbf{p},\lambda) e^{i\mathbf{p}\cdot\mathbf{x}} + a^\dagger_{Mi}(\mathbf{p},\lambda) v_i(\mathbf{p},\lambda) e^{-i\mathbf{p}\cdot\mathbf{x}} \right).$$

Massless neutrino $a_\alpha(\mathbf{p})$ (LH neutrino) $b_\alpha(\mathbf{p})$ (RH anti-neutrino).

Massive Majorana $a_{Mi}(\mathbf{p}, \lambda = \pm 1)$ (λ: helicity).

4 Relation between operators and time evolution

Relation bet. flavor operator and operator for mass eigenstate:

$$\frac{1}{\sqrt{2|\mathbf{p}|}}\begin{pmatrix} V_{\alpha j}^{*} a_{\alpha}(\mathbf{p}) \\ V_{\alpha j} a_{\alpha}^{\dagger}(-\mathbf{p}) \end{pmatrix} = \frac{\sqrt{N_j(\mathbf{p})}}{2E_j(\mathbf{p})}\begin{pmatrix} 1 & \frac{im_j}{E_j(\mathbf{p})+|\mathbf{p}|} \\ \frac{im_j}{E_j(\mathbf{p})+|\mathbf{p}|} & 1 \end{pmatrix}\begin{pmatrix} a_{Mj}(\mathbf{p},-) \\ a_{Mj}^{\dagger}(-\mathbf{p},-) \end{pmatrix},$$

$N_{jp} = E_j(\mathbf{p}) + |\mathbf{p}|, E_j(\mathbf{p}) = \sqrt{|\mathbf{p}|^2 + m_j^2}.$ $(\alpha, \gamma = e, \mu, \tau,\ j = 1 \sim 3)$

Time evolution of flavor operator:

$$\begin{aligned} a_{\alpha}(\mathbf{p}, t) = & \sum_{j,\gamma} V_{\alpha j} V_{\gamma j}^{*}\left(\cos(E_j(\mathbf{p})t) - \frac{i|\mathbf{p}|\sin(E_j(\mathbf{p})t)}{E_j(\mathbf{p})}\right) a_{\gamma}(\mathbf{p}) \\ & - \sum_{j,\gamma} V_{\alpha j} V_{\gamma j} \frac{m_j \sin(E_j(\mathbf{p})t)}{E_j(\mathbf{p})} a_{\gamma}^{\dagger}(-\mathbf{p}), \end{aligned}$$

For an anti-neutrino operator $b_{\alpha}(\mathbf{p}, t)$, just replace $V \to V^{*}$.

5 Lepton family number operator

The countable lepton family number of neutrinos through charged current interaction: $(\alpha = e, \mu, \tau)$

$$L_\alpha(0) = \int' \frac{d^3p}{(2\pi)^3|2\mathbf{p}|} \Big(a_\alpha^\dagger(\mathbf{p}) a_\alpha(\mathbf{p}) - b_\alpha^\dagger(\mathbf{p}) b_\alpha(\mathbf{p}) \Big).$$

$$L_\alpha(t) = \int' \frac{d^3p}{(2\pi)^3|2\mathbf{p}|} \Big(a_\alpha^\dagger(\mathbf{p},t) a_\alpha(\mathbf{p},t) - b_\alpha^\dagger(\mathbf{p},t) b_\alpha(\mathbf{p},t) \Big).$$

6 Expectation value of the total lepton number

The expectation value with a state with a definite lepton number $\sigma \in (e, \mu, \tau)$

$$|\mathbf{q}, \sigma\rangle = n_q a^\dagger_\sigma(\mathbf{q})|0\rangle$$

$(n_q$ normalization const. $\langle \mathbf{q}, \sigma|\mathbf{q}, \sigma\rangle = 1.)$.

$$\langle \mathbf{q}, \sigma|L(t)|\mathbf{q}, \sigma\rangle = \sum_{i=1}^{3} |V_{\sigma i}|^2 \left(\frac{q^2 + m_i^2 \cos 2E_i t}{E_i^2} \right).$$

where $q = |\mathbf{q}|$. The expectation value of the total lepton number is in the region $[-1, 1]$.

$$-1 \leq \sum_i |V_{\sigma i}|^2 \frac{|\mathbf{q}|^2 - m_i^2}{|\mathbf{q}|^2 + m_i^2} \leq \langle \mathbf{q}, \sigma|L(t)|\mathbf{q}, \sigma\rangle \leq 1.$$

The lower bound can be negative for $q^2 < m^2_{\text{lightest}}$ and reaches to -1 for $\mathbf{q} = 0$.

7 limit (I) Ultrarelativistic $q^2 \gg m_i m_j$

(I) The ultrarelativistic limit of $\langle\sigma|L_\alpha(t)|\sigma\rangle$ leads to the probability of neutrino oscillation. $0 \le P_{\sigma\to\alpha}(t) \le 1$.

$$\langle \mathbf{q},\sigma|L_\alpha(t)|\mathbf{q},\sigma\rangle \to P_{\sigma\to\alpha}(t)$$

$$P_{\sigma\to\alpha}(t) = \delta_{\sigma\alpha} - 4\sum_{(i,j)cyclic} \mathrm{Re.}(V^*_{\alpha i}V_{\sigma i}V_{\alpha j}V^*_{\sigma j})\sin^2\frac{\Delta m^2_{ij}t}{4q}$$

$$-2\mathrm{Im}(V^*_{\alpha 1}V_{\sigma 1}V_{\alpha 2}V^*_{\sigma 2})\sum_{(i,j)cyclic}\sin\frac{\Delta m^2_{ij}t}{2q} \quad (E_i - E_j \to \frac{\Delta m^2_{ij}}{2q}.)$$

V is PMNS matrix for Majorana neutrinos. (CPV: Two Majorana phases α_{31}, α_{21}, one KM type (Dirac) phase δ). In the limit (I), it only depends on δ.

$$V_{\sigma i} = \begin{pmatrix} c_{12}c_{13} & (s_{12}c_{13})e^{i\frac{\alpha_{21}}{2}} & (s_{13}e^{-i\delta})e^{i\frac{\alpha_{31}}{2}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & (c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta})e^{i\frac{\alpha_{21}}{2}} & (s_{23}c_{13})e^{i\frac{\alpha_{31}}{2}} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & (-c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta})e^{i\frac{\alpha_{21}}{2}} & (c_{23}c_{13})e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

M.Doi, T. Kotani, H. Nishiura, K. Okuda, E. Takasugi, Phys.Lett.B (1981)

8 limit (II) Non-relativistic $q^2 \ll m_i m_j$

(II) zero momentum limit: $q \to 0$ (dependent on the Majorana phases)

$$\langle \mathbf{q}, \sigma | L_\alpha(t) | \mathbf{q}, \sigma \rangle = \sum_{i=1}^{3} |V_{\alpha i}|^2 |V_{\sigma i}|^2 m_i^2 \cos 2m_i t$$

$$+ \sum_{(i,j)cyclic} \mathrm{Re}(V_{\alpha i}^* V_{\sigma i} V_{\alpha j} V_{\sigma j}^*) \{ (\cos(m_i - m_j)t)(1 - \mathrm{Re}(\frac{V_{\sigma i}^* V_{\sigma j}}{V_{\sigma i} V_{\sigma j}^*}))$$

$$+ (\cos(m_i + m_j)t)(1 + \mathrm{Re}(\frac{V_{\sigma i}^* V_{\sigma j}}{V_{\sigma i} V_{\sigma j}^*})) \} + \mathrm{Im}(V_{\alpha 1}^* V_{\sigma 1} V_{\alpha 2} V_{\sigma 2}^*)$$

$$\times \sum_{(i,j)cyclic} (\cos(m_i - m_j)t - \cos(m_i + m_j)t) \mathrm{Im}(\frac{V_{\sigma i}^* V_{\sigma j}}{V_{\sigma i} V_{\sigma j}^*}).$$

For $\sigma = e$, the following combinations of PMNS elements directly related to Majorana phases.

$$\frac{V_{e1}^* V_{e2}}{V_{e1} V_{e2}^*} = e^{i\alpha_{21}}, \quad \frac{V_{e1}^* V_{e3}}{V_{e1} V_{e3}^*} = e^{i(\alpha_{31} - 2\delta)}.$$

9 Numerical calculation

Input of neutrino parameters: (NuFIT 5.0(2020), I. Esteban et.al.).

	mass^2 differences	s_{12} s_{23}	s_{13} δ	$(\alpha_{21}, \alpha_{31} - 2\delta)$ $m_{ee}^{max}, m_{ee}^{min}$
normal	$\Delta m_{21}^2 = 7.4 \times 10^{-5}$ $\Delta m_{31}^2 = 2.52 \times 10^{-3}$	0.55 0.76	0.15 1.09π	$(0, 0), (\pi, \pi)$ 0.012, 0.0018
inverted	$\Delta m_{21}^2 = 7.4 \times 10^{-5}$ $\Delta m_{23}^2 = 2.50 \times 10^{-3}$	0.55 0.76	0.15 1.57π	$(0, 0)$, (π, π) 0.050 ,0.018

We assume the lightest neutrino mass to be 0.01[eV]. From that choice of the lightest neutrino mass each mass eigenvalue m_i[eV] is given by,

$$m_1 = 0.0100, \quad m_2 = 0.0132, \quad m_3 = 0.0512, \quad \text{(Normal)},$$
$$m_1 = 0.0502, \quad m_2 = 0.0510, \quad m_3 = 0.0100, \quad \text{(Inverted)}.$$

Numerical calculation I: $q = 0.2$ (eV)$> m_i$

Figure1 normal hierarchy

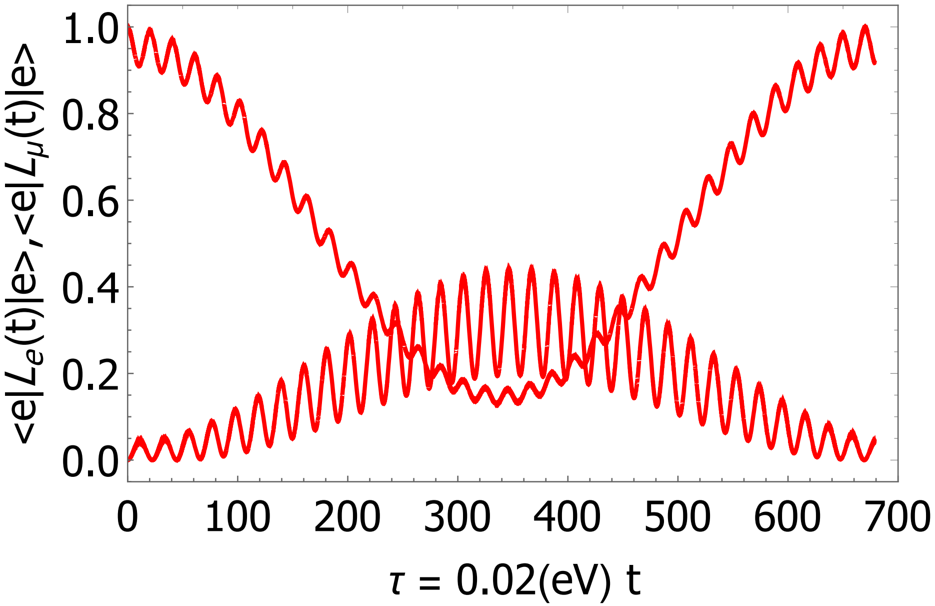
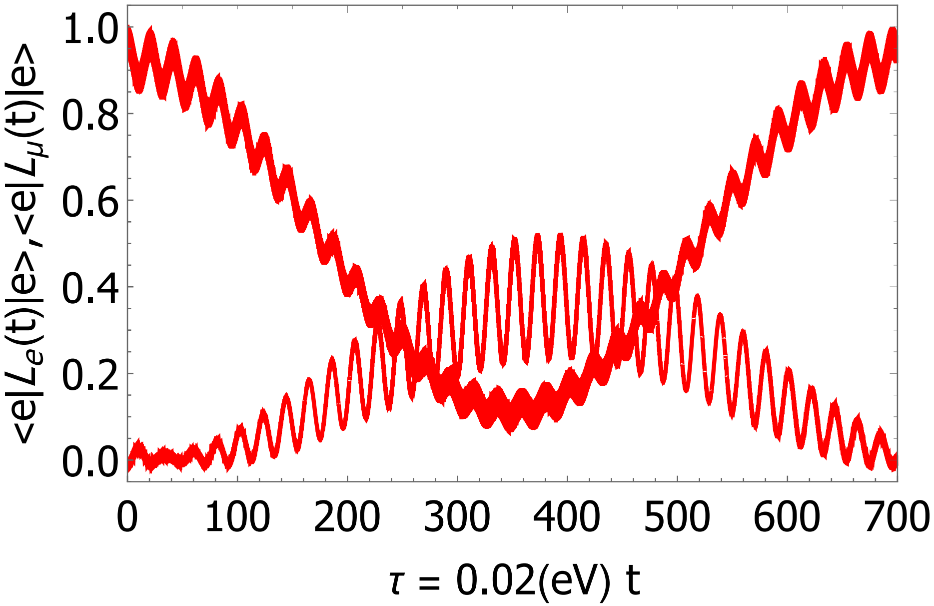


Figure2 inverted hierarchy



(1)The lepton family numbers are always within the range $[0, 1]$.

(2)The long period $\tau_L = \frac{2\pi}{E_2 - E_1} \simeq 680 \sim 700$, $\tau_{S_{1,2}} = 20.3, 20.9 = \frac{2\pi}{E_3 - E_{1,2}}$.

(3)The beat like behavior for muon number with $\tau_{beat} = \tau_L$.

11 Numerical calculation II

($q = 0.0002 < m_1 \sim m_2 \sim 0.01 < q = 0.02 < m_3 \sim 0.03$ eV): NH

Figure3 $\langle e|L_e(t)|e\rangle$ $q = 0.02$(eV)

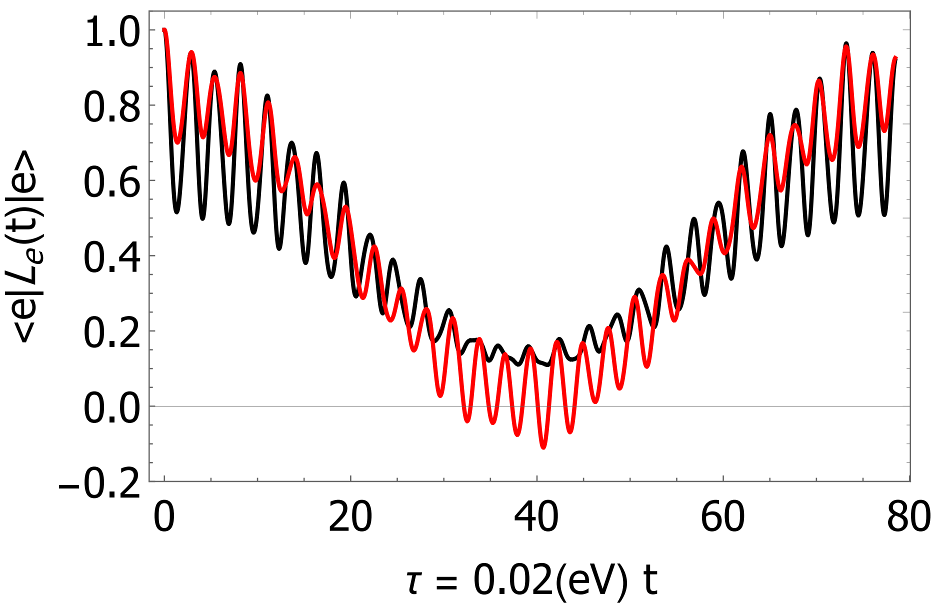
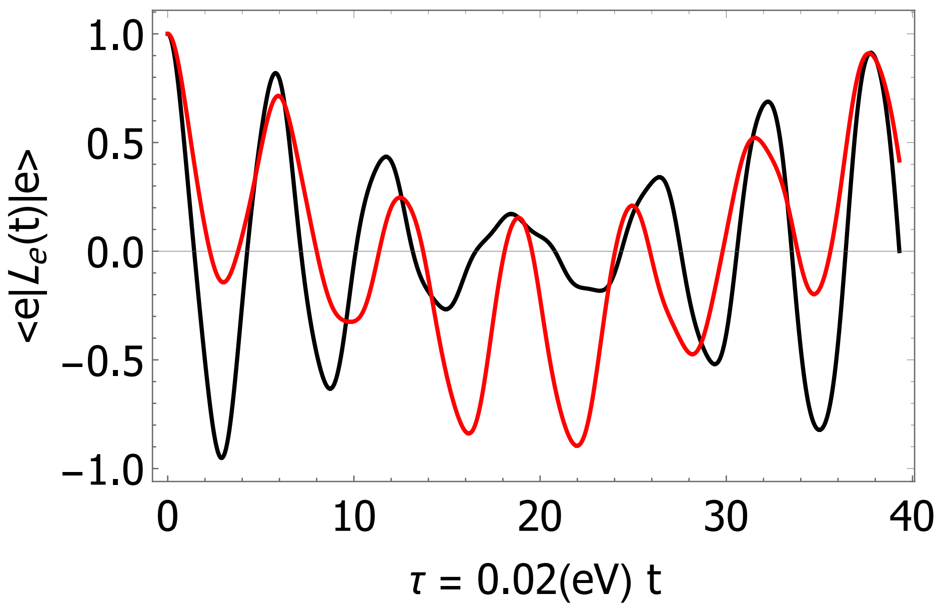


Figure4 $\langle e|L_e(t)|e\rangle$:$q = 0.0002$(eV)



The strong dependence on Majorana phases for the electron number. The black(red)curve shows the case for the Majorana phase such that $|m_{ee}|$ is maximum (minimum). $\langle e|L_e(t \sim 0)|e\rangle \simeq 1 - (|m_{ee}|^2 + (m^\dagger m)_{ee})t^2$.

12 Numerical calculation IV; Normal vs Inverted for L_τ

Figure5 L_τ for NH: $q = 0.0002$

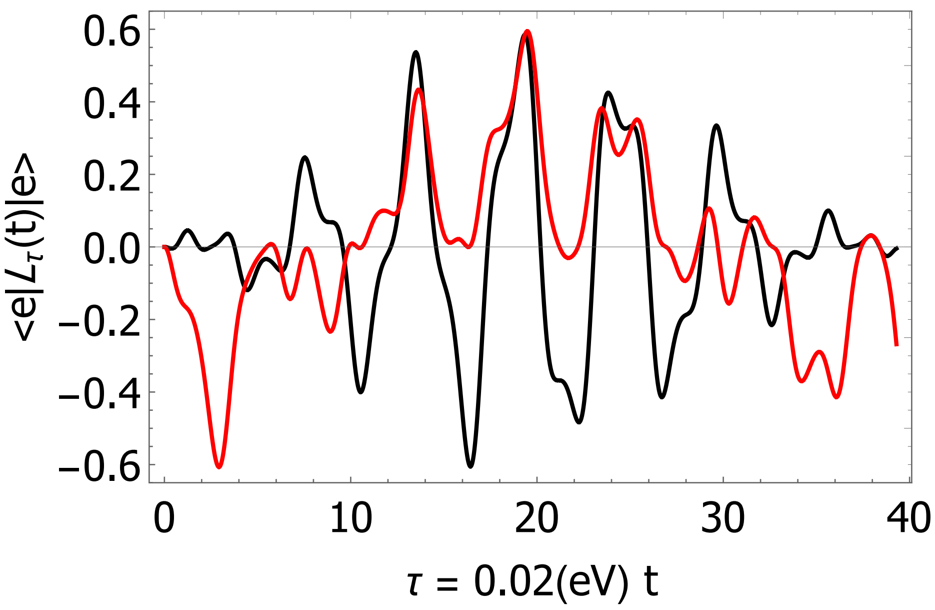
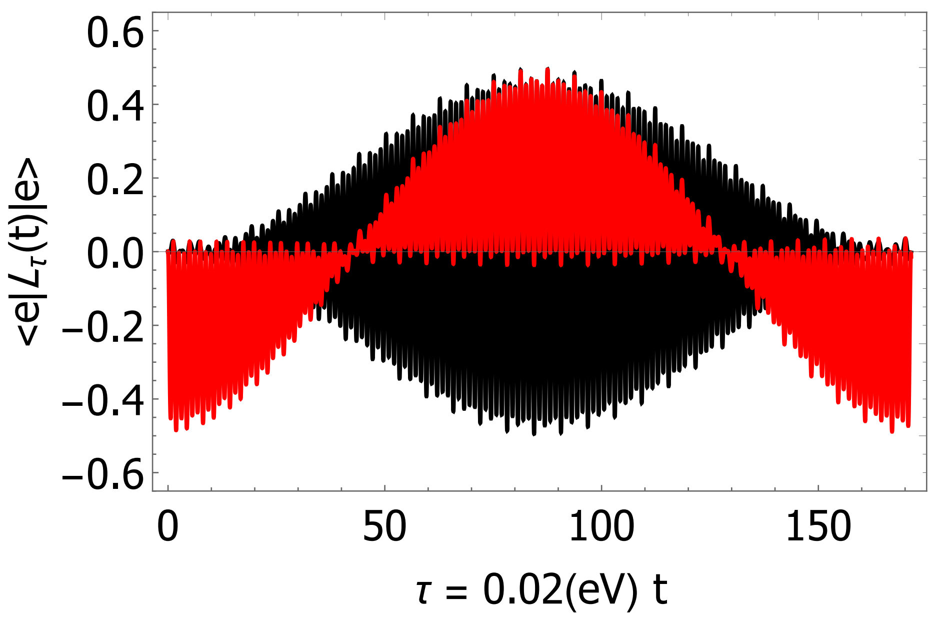


Figure6 L_τ for IH: $q = 0.0002$



$$m_+|_{\rm inverted} \simeq 5 \times m_+|_{\rm normal}, \quad m_-|_{\rm inverted} \simeq \frac{1}{5} \times m_-|_{\rm normal},$$

$$m_\pm = m_2 \pm m_1.$$

13 Numerical calculation V: The momentum dependence

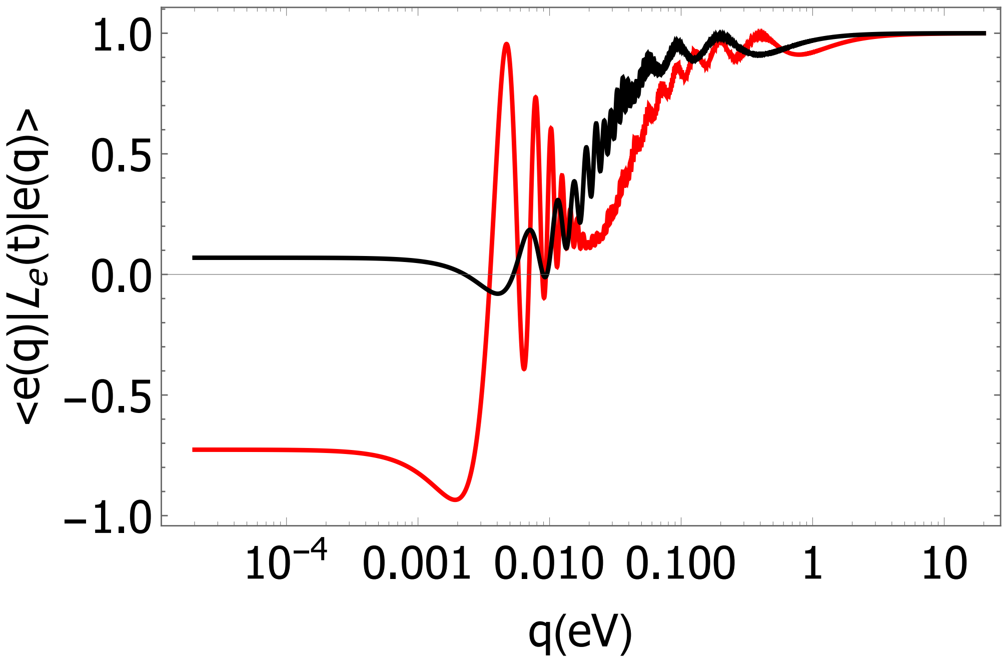


Figure7 q dependence of $\langle e(q)|L_e(t)|e(q)\rangle$ for fixed time $\tau = 40$(red) and $\tau = 20$(black)

As the momentum q decreases from the larger ($q = 20$(eV)) to the smaller (q$= 2 \times 10^{-5}$(eV)), the electron number at $\tau = 40$ changes from $+1$ to -1. This indicates the transition from ν_e to $\bar{\nu}_e$ due to Majorana mass is more pronounced for the low momentum $q < m_\nu$.

14 Implication on CNB and summary

- In the early universe, neutrinos' momentum is large and the lepton family numbers are approximately conserved. While the universe is cooled down, the momentum carried by the neutrinos is red-shifted and they become non-relativistic. Then the effects of Majorana mass term is turned on, and the lepton number may oscillate and alternate its sign . These effects strongly depend on the Majorana phases, neutrino masses, and hierarchies as we have seen.
- We investigated the time evolution of lepton (family) number under the presence of Majorana mass terms.
- With the lepton family number, the neutrino flavor transition and the neutrino and anti-neutrino transition effects is included in a single framework and the formula for lepton numbers is valid for the broad range of the momentum.
- The effect of Majorana mass is more significant at the low momentum region $q < m_\nu$.

15 Background(https://www.nobelprize.org/prizes/physics/2002/davis/facts/) See also Jose Bernabeu , Symmetry 2020,12,1316, S.M. Bilenky, arXiv:Physics/0603039v3)

- Bruno Pontecorvo (1946) suggested the following process:

$$n \to p + e^- + \bar{\nu}_e \quad \text{reactor (anti-)neutrino}$$

$$\searrow$$

$$\bar{\nu} + Cl^{34}_{17} \to Ar^{34}_{18} + e^-$$

  Raymond Davis Jr tried to observe the process and failed to detect it.

- If it would occur, this indicates the neutrino is a Majorana particle through $\bar{\nu}_e \to \nu_e$ oscillation.

$$n \to p + e^- + \bar{\nu}_e \quad \text{reactor (anti-)neutrino}$$

$$\searrow$$

$$\nu_e + Cl^{34}_{17} \to Ar^{34}_{18} + e^-$$

16 Set up for the production and detection of electron (anti-) neutrino at $t = 0$ and $t > 0$. $L_e(t)$: the electron number carried by (anti-) neutrino.

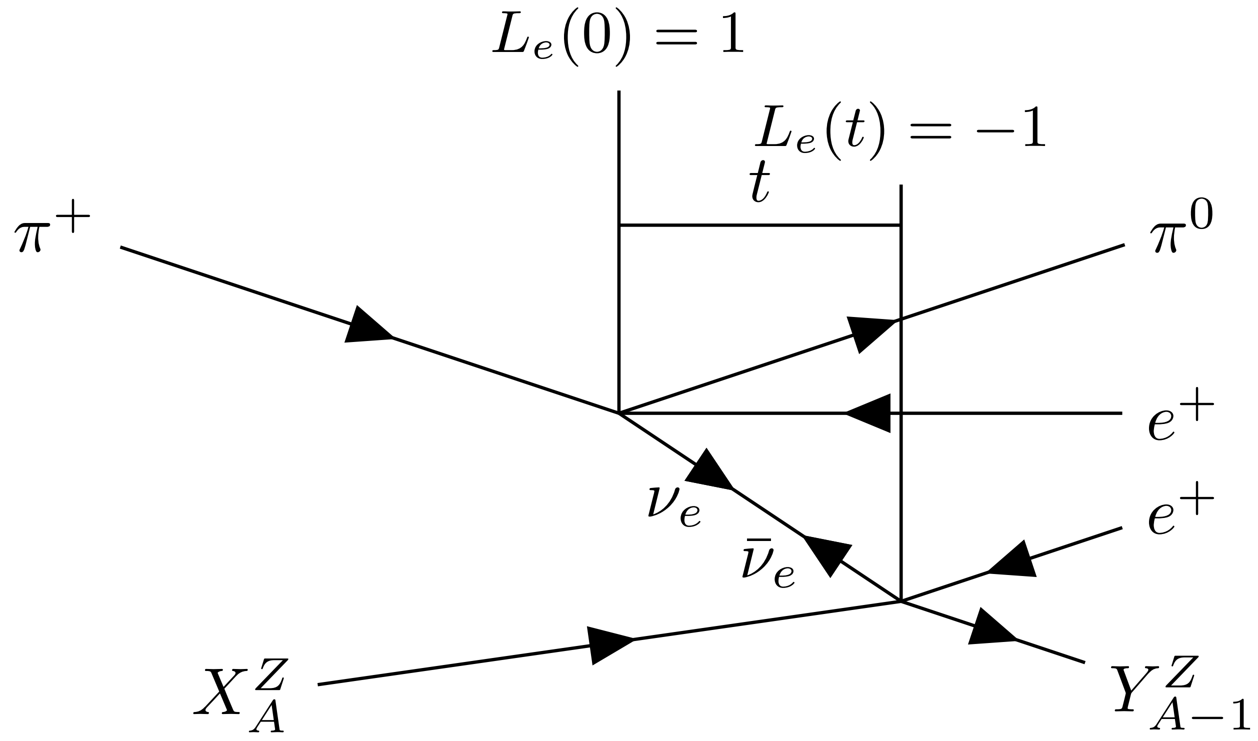


Figure8 $\pi^+ + X \to \pi^0 + e^+ + (\nu_e(0) \to \bar{\nu}_e(t)) + X \to \pi^0 + e^+ + Y + e^+$

17 Time evolution of lepton number operator $L(t) = \sum_{\alpha=e,\mu,\tau} L_\alpha(t)$

$$L(t) = \sum_\alpha L_\alpha(0) \leftarrow \text{Total lepton Number at } t = 0$$

$$- \int' \frac{d^3\mathbf{p}}{(2\pi)^3|2\mathbf{p}|} \frac{2m_i^2 \sin^2(E_i(\mathbf{p})t)}{E_i^2(\mathbf{p})} \Big(V_{\beta i} V_{\gamma i}^* a_\beta^\dagger(\mathbf{p}) a_\gamma(\mathbf{p}) - V_{\beta i}^* V_{\gamma i} b_\beta^\dagger(\mathbf{p}) b_\gamma(\mathbf{p}) \Big)$$

$$+ \int'_{\mathbf{p}\in A} \frac{m_i \sin(2E_i(\mathbf{p})t)}{E_i(\mathbf{p})} \Big(V_{\beta i} V_{\gamma i} a_\beta^\dagger(\mathbf{p}) a_\gamma^\dagger(-\mathbf{p}) - V_{\beta i}^* V_{\gamma i}^* b_\beta^\dagger(\mathbf{p}) b_\gamma^\dagger(-\mathbf{p}) + h.c. \Big)$$

$$+ 2i \int'_{\mathbf{p}\in A} \frac{m_i |\mathbf{p}| \sin^2 (E_i(\mathbf{p})t)}{E_i^2(\mathbf{p})} \Big(V_{\beta i} V_{\gamma i} a_\beta^\dagger(\mathbf{p}) a_\gamma^\dagger(-\mathbf{p}) - V_{\beta i}^* V_{\gamma i}^* b_\beta^\dagger(\mathbf{p}) b_\gamma^\dagger(-\mathbf{p}) - h.c. \Big).$$

$\int'_{\mathbf{p}\in A} = \int'_{\mathbf{p}\in A} \frac{d^3p}{(2\pi)^3 2|\mathbf{p}|}$ and A is a hemisphere region of $\mathbf{p} \neq 0$.

$$\langle \mathbf{q},\sigma|L_\alpha(t)|\mathbf{q},\sigma\rangle = \sum_{i=1}^{3}|V_{\alpha i}|^2|V_{\sigma i}|^2\frac{q^2+m_i^2\cos 2E_i t}{E_i^2}$$

$$+\sum_{(i,j)cyclic}\mathrm{Re}(V_{\alpha i}^*V_{\sigma i}V_{\alpha j}V_{\sigma j}^*)\{\cos(E_i-E_j)t\times$$

$$(1+\frac{q^2-m_im_j\mathrm{Re}(\frac{V_{\sigma i}^*V_{\sigma j}}{V_{\sigma i}V_{\sigma j}^*})}{E_iE_j})+\cos(E_i+E_j)t\times$$

$$(1-\frac{q^2-m_im_j\mathrm{Re}(\frac{V_{\sigma i}^*V_{\sigma j}}{V_{\sigma i}V_{\sigma j}^*})}{E_iE_j})\}+\mathrm{Im}(V_{\alpha 1}^*V_{\sigma 1}V_{\alpha 2}V_{\sigma 2}^*)\times$$

$$\sum_{(i,j)cyclic}\{(\cos(E_i-E_j)t-\cos(E_i+E_j)t)\frac{m_im_j}{E_iE_j}\mathrm{Im}(\frac{V_{\sigma i}^*V_{\sigma j}}{V_{\sigma i}V_{\sigma j}^*})$$

$$-((\frac{q}{E_i}-\frac{q}{E_j})\sin(E_i+E_j)t+(\frac{q}{E_i}+\frac{q}{E_j})\sin(E_i-E_j)t)\}.$$