Theoretical Bounds on Dark Matter

Xavier Calmet

Physics & Astronomy
University of Sussex

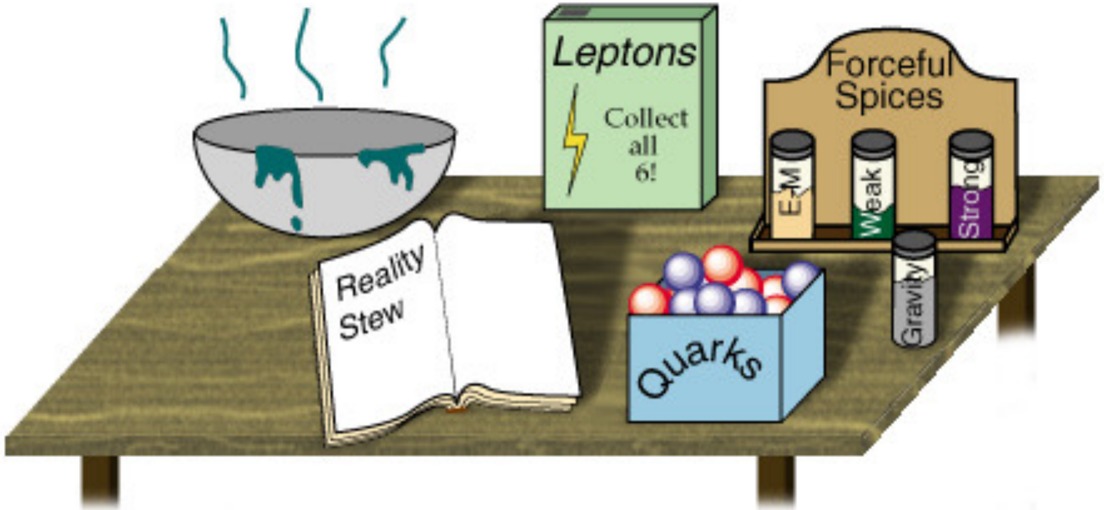
University of Sussex

based on 1907.05680 & 1912.04147 & 2008.06243 (with F. Kuipers)
& 2009.11575 (with F. Kuipers) & 1805.08552 (with B. Latosh)

Outline

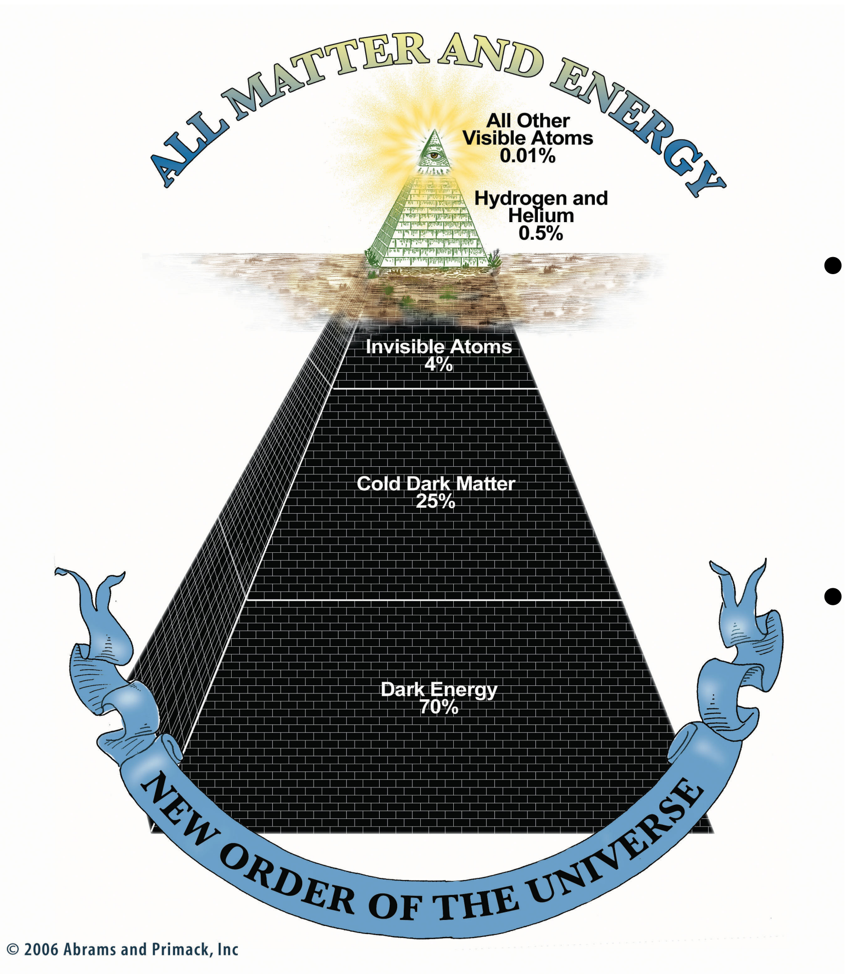
- Introduction
- Can quantum gravity account for dark matter?
- Hidden sector and Dark Matter: quantum gravity matters.
- Conclusions

Introduction



- Gravity is at odd with other forces of nature.
- It has a dimensionful coupling constant: Newton's constant.
- It is a very weak force in comparison to the standard model interactions.
- For this reason, it is usually assumed by most particle physicists that the issue of quantum gravity can safely be ignored at low energy.
- However, I will show you that quantum gravity matters!

Missing ingredient in the Standard Model: dark matter



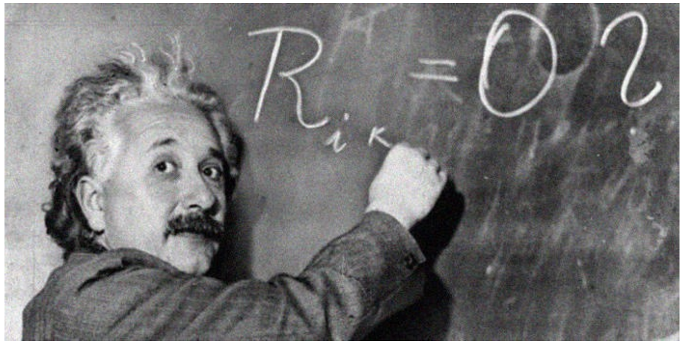
- Can gravity/quantum gravity account for the shortcomings of the standard model?

- For example, can it say anything about dark matter, inflation or the unification of forces?

- Gravity is weak but universal, what can we learn about the dark universe?

What is Dark Matter?

- No candidate in the Standard Model
- Primordial black holes are not ruled out, but difficult to produce them.
- New particle beyond the Standard Model, typically cold dark matter works better than warm for structure formation.
- Spin, mass, interactions (self and with the Standard Model) are unknown.
- Modified gravity: can be mapped to dark matter particles that are gravitationally coupled to the Standard Model.
- It must be a new particle.



Effective action for quantum gravity

The Hilbert-Einstein action

$$S = \int d^4x \sqrt{-g} \frac{2}{\kappa^2} R$$

receives corrections when integrating out fluctuations of the graviton (and any other matter fields depending on the energy under consideration), one obtains:

$$S = \int d^4x \sqrt{-g} \left[\left(\frac{1}{2} M^2 + \xi H^\dagger H \right) R - \Lambda_C^4 + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + c_4 \Box R \right.$$
$$\left. - b_1 R \log \frac{\Box}{\mu_1^2} R - b_2 R_{\mu\nu} \log \frac{\Box}{\mu_2^2} R^{\mu\nu} - b_3 R_{\mu\nu\rho\sigma} \log \frac{\Box}{\mu_3^2} R^{\mu\nu\rho\sigma} + \mathcal{O}(M_\star^{-2}) + \mathcal{L}_{SM} \right]$$

The non-local part of the EFT

- The Wilson coefficients of the non-local operators are universal predictions of quantum gravity:

$$-b_1 R \log \frac{\Box}{\mu_1^2} R - b_2 R_{\mu\nu} \log \frac{\Box}{\mu_2^2} R^{\mu\nu} - b_3 R_{\mu\nu\rho\sigma} \log \frac{\Box}{\mu_3^2} R^{\mu\nu\rho\sigma}$$

	b_1	b_2	b_3
Scalar	$5(6\xi-1)^2$	-2	2
Fermion	-5	8	7
Vector	-50	176	-26
Graviton	250	-244	424

NB: they are calculated using dim-reg.

All numbers should be divided by 11520 π^2

- The Wilson coefficients of the local operators on the other hand are not calculable: this is the price to pay.

Field content of the EFT

- By linearizing the EFT (or mapping it to the Einstein frame), one can easily identify the field content.
- Calculating

$$T^{(1)\mu\nu}(k) D_{\mu\nu\alpha\beta}(k) T^{(2)\alpha\beta}(k)$$

- we find

$$\frac{\kappa^2}{4}\left[\frac{T^{(1)}_{\mu\nu}T^{(2)\mu\nu} - \frac{1}{2}T^{(1)\mu}_{\ \mu}T^{(2)\nu}_{\ \nu}}{k^2}\right.$$

$$\left. - \frac{T^{(1)}_{\mu\nu}T^{(2)\mu\nu} - \frac{1}{3}T^{(1)\mu}_{\ \mu}T^{(2)\nu}_{\ \nu}}{k^2 - \frac{2}{\kappa^2\left(-c_2+(b_2+4b_3)\log\left(\frac{-k^2}{\mu^2}\right)\right)}} + \frac{T^{(1)\mu}_{\ \mu}T^{(2)\nu}_{\ \nu}}{k^2 - \frac{1}{\kappa^2\left((3c_1+c_2)-(3b_1+b_2+b_3)\log\left(\frac{-k^2}{\mu^2}\right)\right)}}\right]$$

Masses of the new states

- The masses are given by the poles of the Green's function.
- We find:
  - Massless spin-2 field (classical graviton)
  - Massive spin-2 field with a complex mass

$$m_2^2 = \frac{2}{(b_2 + 4b_3)\kappa^2 W\left(-\frac{2\exp\frac{-c_2}{(b_2+4b_3)}}{(b_2+4b_3)\kappa^2\mu^2}\right)}$$

$$z = f^{-1}\left(ze^z\right) = W\left(ze^z\right)$$

  - Massive spin-0 field with a complex mass

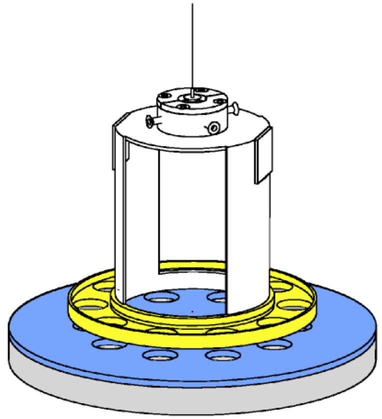
$$m_0^2 = \frac{-1}{(3b_1 + b_2 + b_3)\kappa^2 W\left(\frac{\exp\frac{-3c_1-c_2}{(3b_1+b_2+b_3)}}{(3b_1+b_2+b_3)\kappa^2\mu^2}\right)}$$

  - Note that the poles are complex ones, we can identify a mass and width

$$m_i^2 = (M_i - i\Gamma_i/2)^2$$

  - the badly behaving ones can be eliminated by a proper choice of the contour integrals.

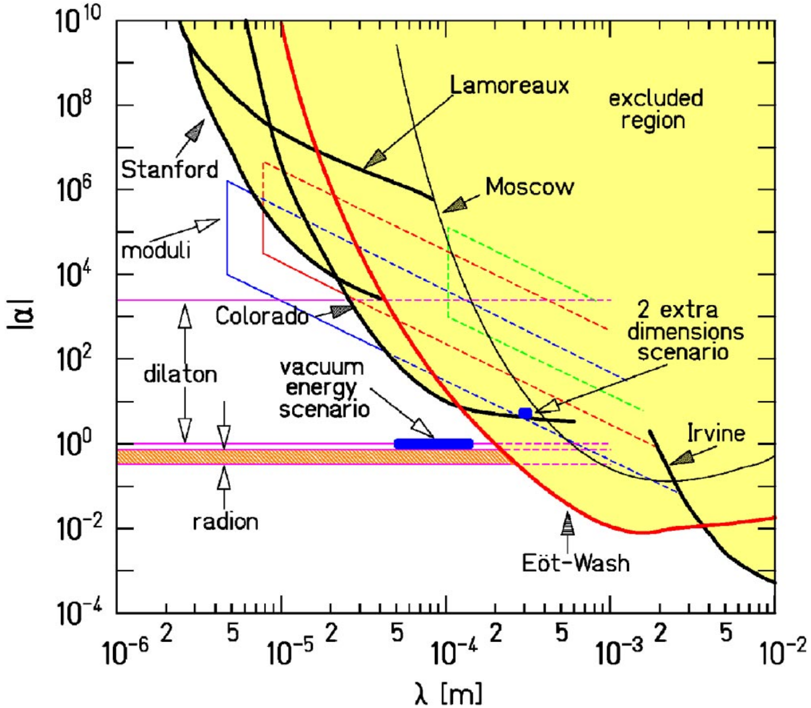
Quantum gravitational correction to Newton's Law



- Quantum gravitational corrections to the Newtonian potential of a point mass

$$\Phi(r) = -\frac{Gm}{r}\left(1 + \frac{1}{3}e^{-Re(m_0)r} - \frac{4}{3}e^{-Re(m_2)r}\right)$$

- In the absence of accidental fine cancellations between both Yukawa terms, the current bounds imply: m_0 , $m_2 > (0.03\ \text{cm})^{-1} = 6.6 \times 10^{-13}$ GeV.



$$V(r) = -G\frac{m_1 m_2}{r}[1 + \alpha e^{-r/\lambda}]$$

Summary of bounds on the EFT parameters

- We can describe any theory of quantum gravity below the Planck scale using effective field theory techniques:

$$S = \int d^4x \sqrt{-g} \left[\left(\frac{1}{2} M^2 + \xi H^\dagger H \right) \mathcal{R} - \Lambda_C^4 + c_1 \mathcal{R}^2 + c_2 \mathcal{R}_{\mu\nu} \mathcal{R}^{\mu\nu} + \mathcal{L}_{SM} + \mathcal{O}(M_\star^{-2}) \right]$$

- Planck scale $(M^2 + \xi v^2) = M_P^2 \qquad M_P = 2.4335 \times 10^{18}$ GeV
- $\Lambda_C \sim 10^{-12}$ GeV; cosmological constant.
- $M_\star >$ few TeVs from QBH searches at LHC and cosmic rays.
- Dimensionless coupling constants ξ, c_1, c_2
  - c_1 and $c_2 < 10^{61}$ [xc, Hsu and Reeb (2008)]

    R^2 inflation requires $c_1 = 5.5 \times 10^8$ (Faulkner et al. astro-ph/0612569]).
  - $\xi < 2.6 \times 10^{15}$ [xc & Atkins, 2013]

    Higgs inflation requires $\xi \sim 10^4$.

Can quantum gravity account for dark matter?

- If the massive spin-0 and spin-2 fields are components of the dark matter content of the universe nowadays, their masses have to be such that none of their partial decay widths enables these fields to decay faster than the current age of the universe.

- From the requirement that the lifetime of the spin-0 field is longer than current age of the universe, we can thus get a bound on c_2 using the gravitational decay width.

- We find $\tau = 1/\Gamma = 7.2 \times 10^{-17} \sqrt{c_2^3}\ \mathrm{GeV}^{-1} > 13.77 \times 10^9 \mathrm{y}$

and thus $c_2 > 4.4 \times 10^{38}$ and a similar bound on $3c_1 + c_2$

Quantum Gravity as Dark Matter

- Note that the Eöt-Wash experiment implies $c_2 < 10^{61}$.

- We thus find a bound:

$$4.4\times10^{38} < c_2 < 10^{61} \quad \text{or} \quad 1\times10^{-12}\,\text{GeV} < m_0 < 0.16\,\text{GeV}.$$

- A similar bound applies to the combination $3c_1 + c_2$ and thus to m_2.

XC & B. Latosh, Eur.Phys.J.C 78 (2018) 6, 520

Production of QG Dark Matter

- The fact that our dark matter candidates are light points towards the vacuum misalignment mechanism.

- Indeed, in an expanding universe both the spin-0 and spin-2 fields have an effective potential in which they oscillate.

- The amount of dark matter produced by this mechanism becomes simply a randomly chosen initial condition for the value of the field in our patch of the universe.

- Quantum gravity could thus easily account for dark matter, maybe in conjunction with primordial black holes.

Hidden sector and Dark Matter: quantum gravity matters

- Let's assume that there is a hidden sector with spin 0, ½,1 or 2 fields with potential feeble interactions with the Standard Model fields.

- The model is described by

$$S = S_{\rm EH} + \int \sqrt{|g|}\,(\mathcal{L}_{\rm SM} + \mathcal{L}_{\rm DM} + \mathcal{L}_{\rm int})\,d^4x,$$

- with

$$\mathcal{L}_{\rm SM} = \sum_i c_i\,\mathcal{O}_{{\rm SM},i},$$

$$\mathcal{L}_{\rm DM} = \sum_j c_j\,\mathcal{O}_{{\rm DM},j},$$

$$\mathcal{L}_{\rm int} = \sum_k c_k\,\mathcal{O}_{{\rm int},k},$$

Quantum Gravity generates portals

- Gravity is universal.

- For every $O_{SM,i}$ and $O_{DM,j}$, perturbative quantum gravity will generate the additional interactions:

$$\mathcal{L}_{\rm int} = \sum_k c_k \, \mathcal{O}_{\rm int,k} + \sum_{i,j} \frac{c_{i,j}}{M_{\rm P}^4} \, \mathcal{O}_{{\rm SM},i} \, \mathcal{O}_{{\rm DM},j} \, ,$$

- These are strongly suppressed effects.

- However, nonperturbative effects can be much more relevant.

- Non-perturbative quantum gravity effects could generate effective operators of any dimension.

- However, any such operator must be suppressed by the scale of quantum gravity as such interactions must vanish in the limit where $M_P \rightarrow \infty$, i.e. when gravity decouples.

- Non-perturbative quantum gravity

$$\sum_{n\geq 0}\sum_{k}\tilde{c}_{n,k}\,\mathcal{O}_{\mathrm{QG},n,k}=\sum_{n\geq 0}\sum_{k}\frac{\tilde{c}_{n,k}}{M_{\mathrm{P}}^{n}}\,O_{\mathrm{QG},n,k}\,,$$

- Note dimension four operator must be exponentially suppressed

$$e^{-M_{\mathrm{P}}/\mu}$$

- We thus get additional quantum contribution to all interactions whether there are already interactions between the hidden sector and the Standard Model or not:

$$\mathcal{L}_{\mathrm{int}}=\sum_{n\geq 0}\sum_{k}\left(\frac{c_{n,k}}{\Lambda_{n,k}^{n}}+\frac{\tilde{c}_{n,k}}{M_{\mathrm{P}}^{n}}\right)O_{\mathrm{int},n,k}$$

Hidden Sector Scalar Field

- Dimension 5 operator $$O_{1,\mathrm{QG}} = \left(\frac{c_1}{\Lambda_1} + \frac{\tilde{c}_1}{M_\mathrm{P}} \right) \phi \, F_{\mu\nu} F^{\mu\nu},$$
- Independently of c_1 from Eöt-Wash: $m_\phi \lesssim 3 \cdot 10^{-3}\,\mathrm{eV}$

Green: limits from light shining through a wall experiments.

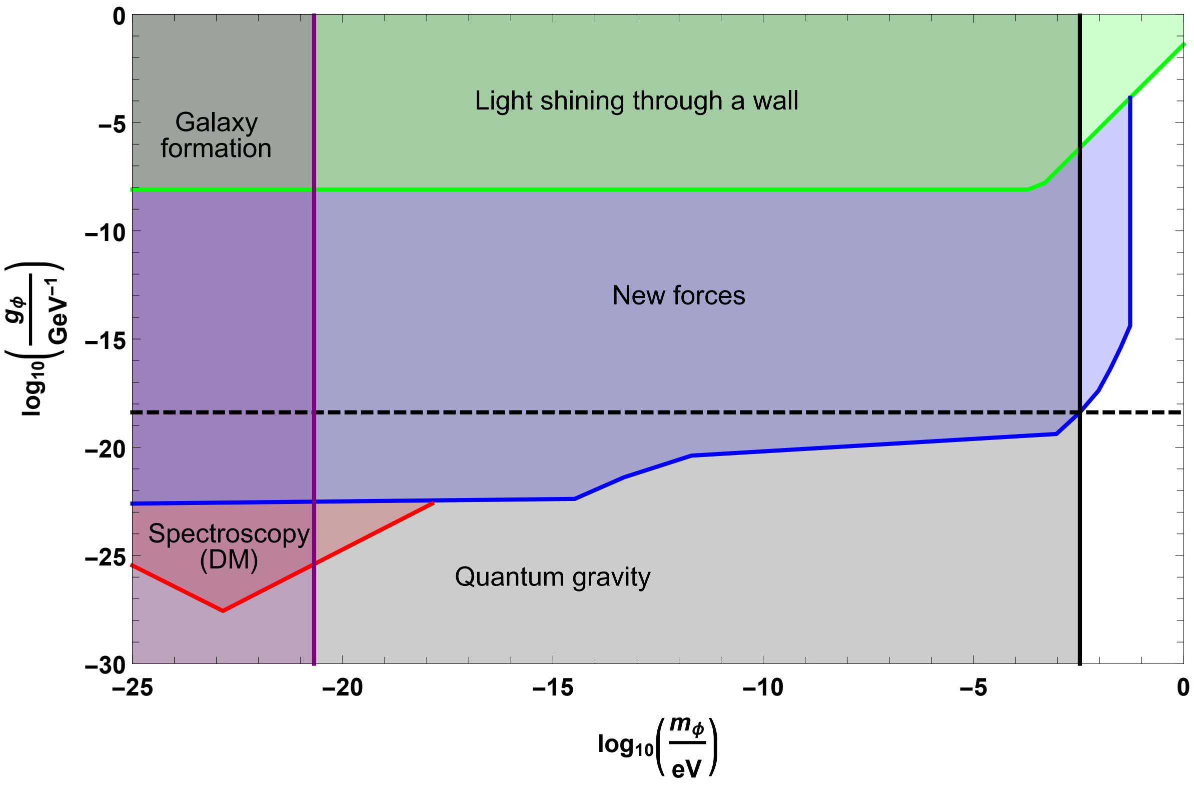
Blue: limits from torsion experiments.

Red: limits from atomic spectroscopy experiments.

Purple: limits from galaxy formation, quasar lensing and stellar streams.

Black: limits from quantum gravity as discussed in this paper.

Dashed black line: reduced Planck scale.



Hidden Sector Pseudoscalar

- Dimension 5 operator

$$O_{2,\mathrm{QG}} = \left(\frac{c_2}{\Lambda_2} + \frac{\tilde{c}_2}{M_\mathrm{P}}\right) a\, G_{\mu\nu}\tilde{G}^{\mu\nu},$$

- Magnetometry measurements:

$$m_a \lesssim 5 \cdot 10^{-21}\,\mathrm{eV}$$

Green: limits from supernovae measurements.

Blue: limits from the big bang nucleosynthesis.

Red: limits from magnetometry experiments.

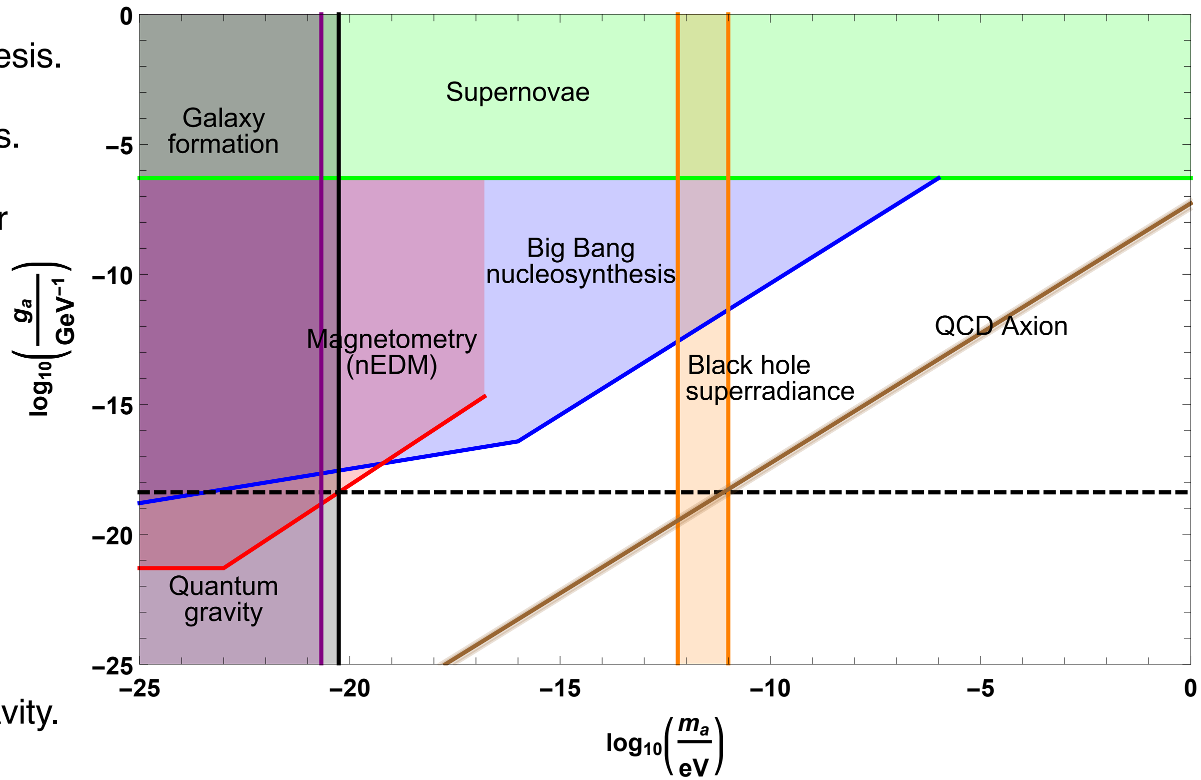
Purple: limits from galaxy formation, quasar lensing and stellar streams.

Orange: limits from the superradiance instability of black holes, however note that these bounds can be avoided, if the self-interaction of the axion-like particle is sufficiently strong.

Brown: predicted value of the QCD axion.

Black: axion masses below 10E–21 eV are excluded by parity conserving quantum gravity.

Dashed black line: reduced Planck scale.



Hidden Sector Pseudoscalar

- Dimension 5 operator, parity violating quantum gravity

$$O_4 = \frac{\tilde{c}_4}{M_\mathrm{P}}\, a\, G_{\mu\nu} G^{\mu\nu}\,,$$

$$O_5 = \frac{\tilde{c}_5}{M_\mathrm{P}}\, a\, F_{\mu\nu} F^{\mu\nu}\,,$$

- From Eöt-Wash : $m_a \lesssim 3 \times 10^{-3}\,\mathrm{eV}$

Green: limits from supernovae measurements.

Blue: limits from the big bang nucleosynthesis.

Red: limits from magnetometry experiments.

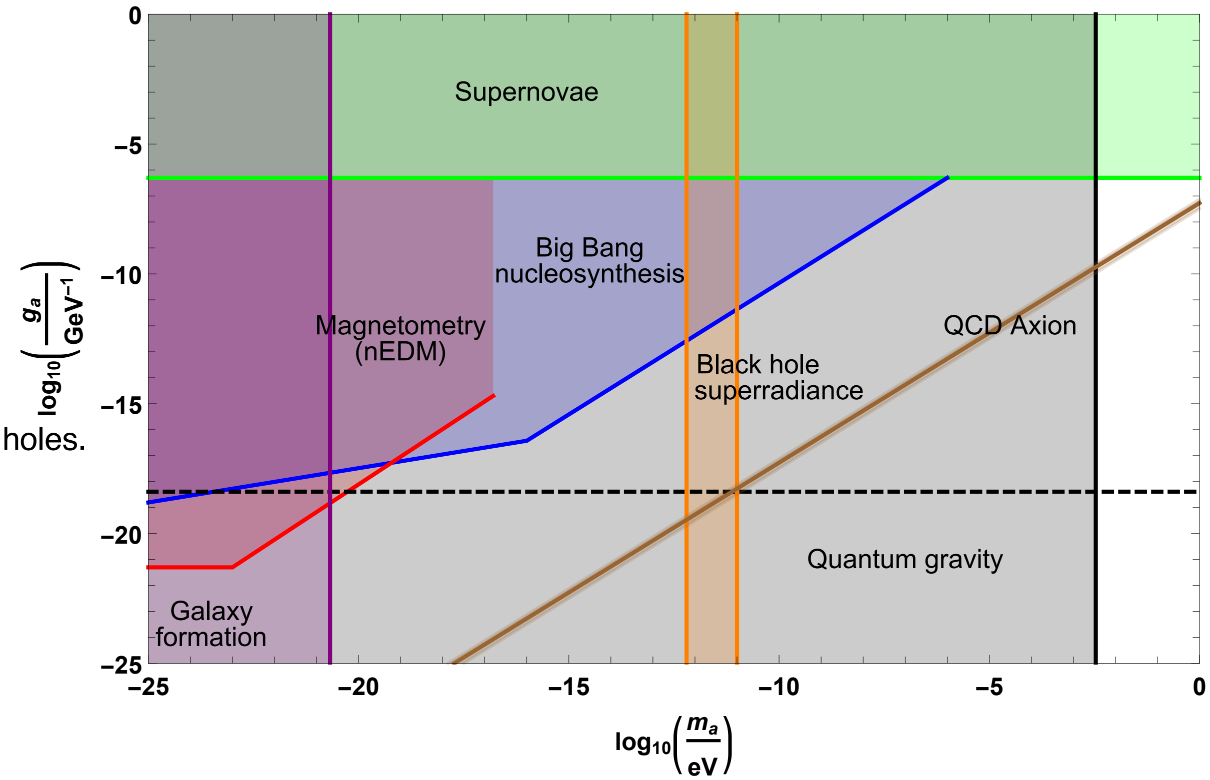
Purple: limits from galaxy formation, quasar lensing and stellar streams.

Orange: limits from the superradiance instability of black holes.

Brown: predicted value of the QCD axion.

Black: axion masses below 3E–3 eV are excluded by parity violating quantum gravity.

Dashed black line: reduced Planck scale.



Gauged scalar field

- To avoid bound from quantum gravity, one needs to impose a gauge symmetry in the hidden sector:

$$O_{7,\mathrm{QG}} = \left(\frac{c_7}{\Lambda_7^2} + \frac{\tilde{c}_7}{M_{\mathrm{P}}^2} \right) \Phi \cdot \Phi \, F_{\mu\nu} F^{\mu\nu} ,$$

- From requirement that the field is DM, one excludes: $m_\Phi \lesssim 10^{-22}\,\mathrm{eV}$

Green: limits from supernovae measurements.

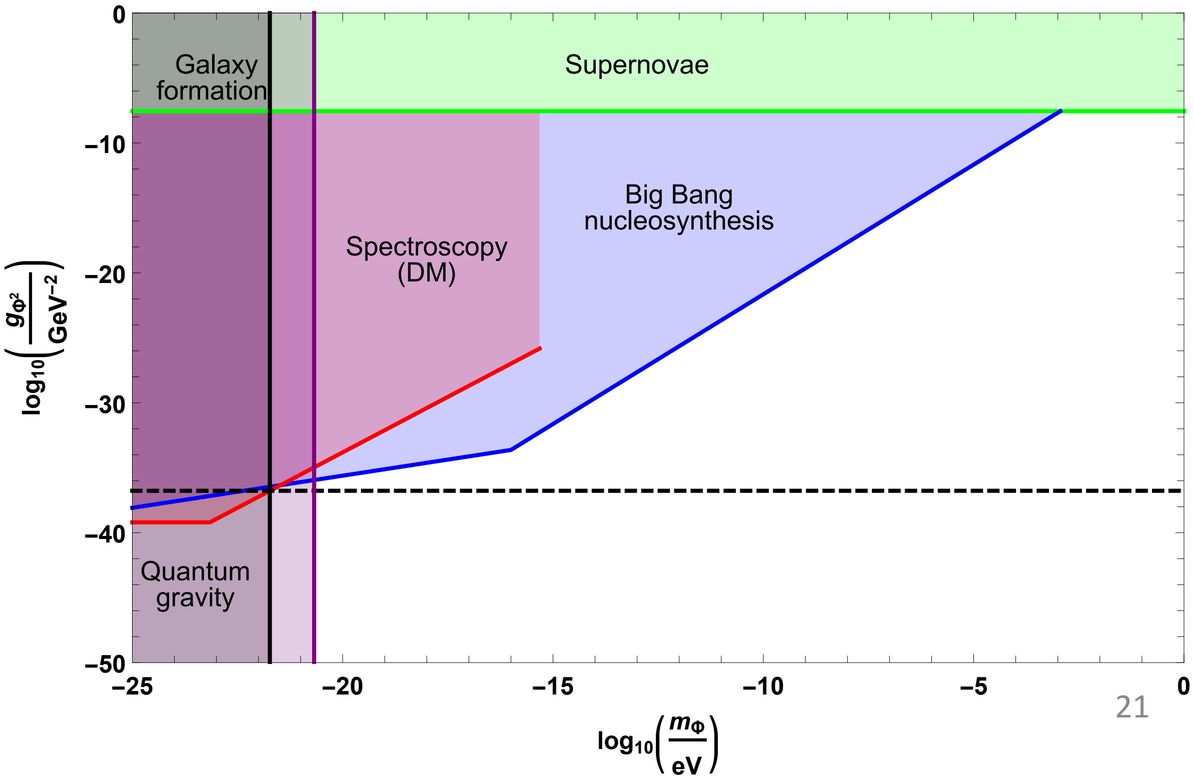
Blue: limits from the big bang nucleosynthesis.

Red: limits from atomic spectroscopy.

Purple: limits from galaxy formation, quasar lensing and stellar streams.

Black: limits from quantum gravity.

Dashed black line: reduced Planck scale.



Quantum Gravity also leads to DM decay!

- Quantum gravity operators will lead to a decay of DM particles.
- We can thus get an upper bound on their masses to insure stability with respect to the age of the universe.
- Together with previous bounds we get a mass range for singlet dark matter particles.
- Singlet scalar fields: $10^{-3}\text{eV} \lesssim m_\phi \lesssim 10^7\text{eV},$
- Singlet pseudoscalars
  - Parity conserving quantum gravity: $10^{-21}\text{eV} \lesssim m_a \lesssim 10^7\text{eV},$
  - Parity violating quantum gravity: $10^{-3}\text{eV} \lesssim m_a \lesssim 10^7\text{eV}$

- For spin ½ fields, we find an upper bound on the mass from the operator

$$O_\psi = \frac{c_\psi}{M_{\rm P}}\,\bar{\psi}\tilde{H}^\dagger \not{D} L,$$

- while the Fermi-Dirac statistics leads to the famous Tremaine and Gunn lower bound on the mass of DM spinors.

$$10^2 \text{eV} \lesssim m_\psi \lesssim 10^{10} \text{eV}$$

- For spin 1 fields, we get an upper bound on the mass of vectors from

$$O_V = \frac{c_V}{M_{\rm P}} h_{\mu\alpha} F^\mu_{\ \nu} B^{\nu\alpha}$$

- and the lower bound comes from the requirement that the de Broglie wavelength fits into the smallest known type of galaxies:

$$10^{-22} \text{eV} \lesssim m_V \lesssim 10^{7} \text{eV}$$

- The spin-2 case is very similar to the spin 0 case and we find:

$$10^{-3} \text{eV} \lesssim m_2 \lesssim 10^{7} \text{eV}.$$

Conclusions

- We have discussed a conservative effective action for quantum gravity within usual QFTs such as the standard model or GUT.
- EFT techniques lead to predictions which can be confronted to data.
- This progress in quantum gravity enables phenomenological applications, e.g. dark matter.
- One cannot ignore quantum gravity, even at low energy! It could explain dark matter.
- We derived bounds on the masses of singlet DM particles.