Gravitino Thermal Production

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[Helmut Eberl, Ioannis Gialamas, VCS, arXiv 2010.14621 (to appear in PRD)]

Outline

- Gravitino as DM
- Background of the calculation
- The setup of the calculation
- Result and cosmological consequences
- Summary

Gravitino as DM

- **Gravitino is the s=3/2 superpartner of graviton. Naturally is in the spectrum of any SUGRA model** [Ellis, Hagelin, Nanopoulos, Olive, Srednicki (1983), Khlopov, Linde (1984)]

- **The "classic" freeze-in DM candidate particle**

- **Naturally escapes all the direct and indirect DM searches**

- **Can be produced non-thermally: (i) inflaton decays** [Giudice, Riotto, Tkachev (1999); Kallosh, Kofman, Linde, Van Proeyen (2000); Nilles, Peloso, Sorbo (2001), Endo, Kawasaki, Takahashi, Yanagida (2006)] **(ii) decays from unstable particles, eg NLSP decays in GDM models** [Cyburt, Ellis, Field, Olive, VSC (2006); Kawasaki, Kohri, Moroi, Yotsuyanagi (2008)]

- **In the later case the BBN constraints should be applied** [Cyburt, Ellis, Field, Luo, Olive, VSC (2012)]

- **In any case the thermal gravitino production rate is vital to apply cosmological constraints**

Background of the calculation

- **Effective theory of light gravitinos, only 1/2 goldstino components,** [Ellis, Kim, Nanopoulos (1984); Moroi, Murayama, Yamagushi, Kawasaki (1993,1994)]

- **Use of Braaten, Pisarksi, Yuan method, including 3/2 components** [Ellis, Nanopoulos, Olive, Rey (1996); Bolz, Buchmuller, Plumacher, Brandenburg (1998,2001); Pradler, Steffen (2007)]

- **Full 1-loop beyond HTL approximation** [Rychkov, Strumia (2007)]

- **Our calculation: corrections of errors and proper parametrisation of the result** [Eberl, Gialamas, VCS, arXiv 2010.14621 (to appear in PRD)]

The setup of the calculation

The Braaten-Yuan prescription

[Braaten, Pisarski, Yuan (1990,1991)]

$$\gamma = \gamma|_{\mathrm{hard}}^{k^*<k} + \gamma|_{\mathrm{soft}}^{k^*>k}$$

where $gT \ll k^* \ll T$ assuming $g \ll 1$

Hard part is calculated from squared matrix elements

$$|\mathcal{M}(a\, b \to c\, \widetilde{G})|^2$$

Soft part is calculated from Imaginary part of the gravitino self-energy

$\gamma|_{\mathrm{hard}}^{k^*<k} = A_{\mathrm{hard}} + B \ln\left(\frac{T}{k^*}\right)$ and $\gamma|_{\mathrm{soft}}^{k^*>k} = A_{\mathrm{soft}} + B' \ln\left(\frac{k^*}{m_{\mathrm{thermal}}}\right)$

Thus

$$\gamma_{\rm BY} = \frac{3\zeta(3)}{16\pi^3}\,\frac{T^6}{M_{\rm P}^2}\sum_{N=1}^{3} c'_N\, g_N^2 \left(1+\frac{m_{\lambda_N}^2}{3m_{3/2}^2}\right)\ln\left(\frac{k'_N}{g_N}\right)$$

$c'_N = (11, 27, 72)$, $k'_N = (1.266, 1.312, 1.271)$ [Pradler, Steffen (2007)]

Analytical result, but valid only for $g \ll 1$

where $\gamma\big|_{\rm soft}$ is calculated in the Hard Thermal Loop (HTL) approx

The condition $g(T) \ll 1$ is not satisfied in the whole temperature range

especially if $g = g_3$

Beyond the HTL approx

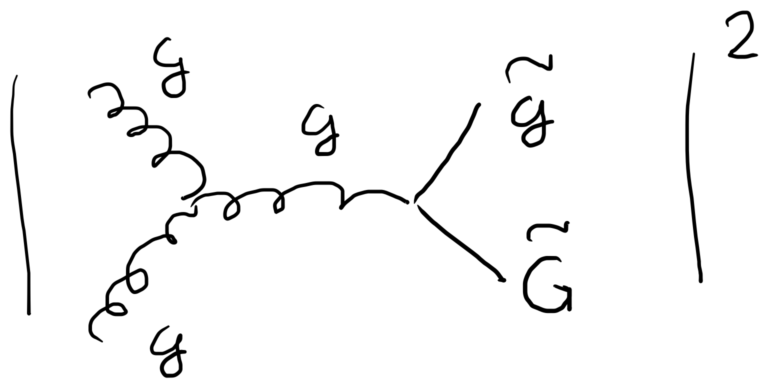
- **Calculate the full 1-loop gravitino self-energy beyond HTL approximation**

- **Calculate the so-called subtracted part of the $|\mathcal{M}|^2$** [Rychkov, Strumia (2007)]

The subtracted part of the squared amplitude is this that cannot be part of the gravitino self-energy

For example if *X:* $gg \to \tilde{g}\widetilde{G}$

$|\mathcal{M}_{X,s}|^2 =$



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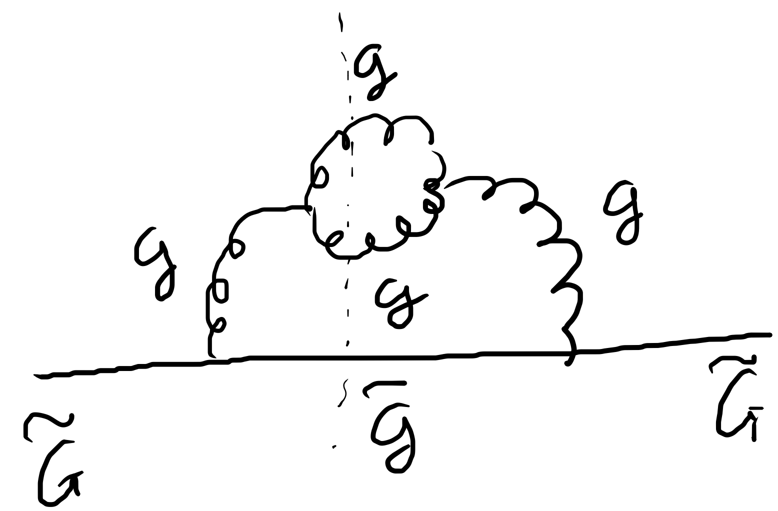
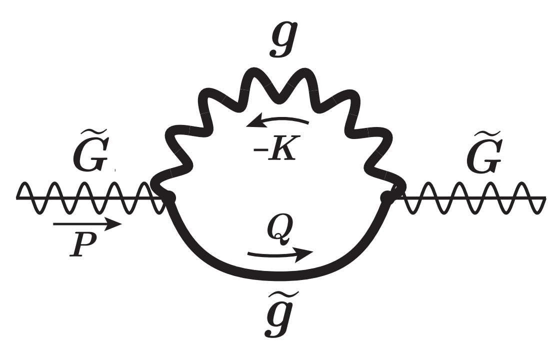


which is part of



(D-graph)

where thick lines denote resumed thermal propagators

Thus	$\gamma_{3/2} = \gamma_{\rm sub} + \gamma_{\rm D} + \gamma_{\rm top}$

X	process	$\lvert\mathcal{M}_{X,\mathrm{full}}\rvert^2$	$\lvert\mathcal{M}_{X,\mathrm{sub}}\rvert^2$
A	$gg \to \tilde{g}\widetilde{G}$	$4C_3(s+2t+2t^2/s)$	$-2sC_3$
B	$g\tilde{g} \to g\widetilde{G}$	$-4C_3(t+2s+2s^2/t)$	$2tC_3$
C	$\tilde{q}g \to q\widetilde{G}$	$2sC_3'$	0
D	$gq \to \tilde{q}\widetilde{G}$	$-2tC_3'$	0
E	$\tilde{q}q \to g\widetilde{G}$	$-2tC_3'$	0
F	$\tilde{g}\tilde{g} \to \tilde{g}\widetilde{G}$	$8C_3(s^2+t^2+u^2)^2/(stu)$	0
G	$q\tilde{g} \to q\widetilde{G}$	$-4C_3'(s+s^2/t)$	0
H	$\tilde{q}\tilde{g} \to \tilde{q}\widetilde{G}$	$-2C_3'(t+2s+2s^2/t)$	0
I	$q\tilde{q} \to \tilde{g}\widetilde{G}$	$-4C_3'(t+t^2/s)$	0
J	$\tilde{q}\tilde{q} \to \tilde{g}\widetilde{G}$	$2C_3'(s+2t+2t^2/s)$	0

Squared matrix elements for gravitino production in $SU(3)_c$ in terms of $g_3^2\, Y_3/M_{\mathrm{P}}^2$

$Y_3 = 1 + m_{\tilde{g}}^2/(3m_{3/2}^2)$, $C_3 = 24$ and $C_3' = 48$

$$|\mathcal{M}_{X,\mathrm{full}}|^2 = |\mathcal{M}_{X,s} + \mathcal{M}_{X,t} + \mathcal{M}_{X,u} + \mathcal{M}_{X,x}|^2$$

$$|\mathcal{M}_{X,D}|^2 = |\mathcal{M}_{X,s}|^2 + |\mathcal{M}_{X,t}|^2 + |\mathcal{M}_{X,u}|^2$$

$$|\mathcal{M}_{X,\mathrm{sub}}|^2 = |\mathcal{M}_{X,\mathrm{full}}|^2 - |\mathcal{M}_{X,D}|^2$$

$$\gamma_{\rm sub}$$

$$\gamma = \frac{1}{(2\pi)^8}\int \frac{d^3\mathbf{p}_a}{2E_a}\,\frac{d^3\mathbf{p}_b}{2E_b}\,\frac{d^3\mathbf{p}_c}{2E_c}\,\frac{d^3\mathbf{p}_{\widetilde{G}}}{2E_{\widetilde{G}}}\;|\mathcal{M}|^2\, f_a\, f_b\,(1\pm f_c)\times\delta^4(P_a+P_b-P_c-P_{\widetilde{G}}) \qquad f_{B|F} = \frac{1}{e^{\frac{E}{T}}\mp 1}$$

$$|\mathcal{M}_{A,\rm sub}|^2+|\mathcal{M}_{B,\rm sub}|^2 = \frac{g_N^2}{M_{\rm P}^2}\left(1+\frac{m_{\lambda_N}^2}{3m_{3/2}^2}\right)C_N(-s+2t)$$ as taken from the Table with the amplitudes

Performing numerical integration

$$\gamma_{\rm sub} = \frac{T^6}{M_{\rm P}^2}\sum_{N=1}^{3} g_N^2\left(1+\frac{m_{\lambda_N}^2}{3m_{3/2}^2}\right)C_N\left(-\mathcal{C}^s_{\rm BBF}+2\,\mathcal{C}^t_{\rm BFB}\right)$$

$$\mathcal{C}^s_{\rm BBF} = 0.25957\times 10^{-3}$$

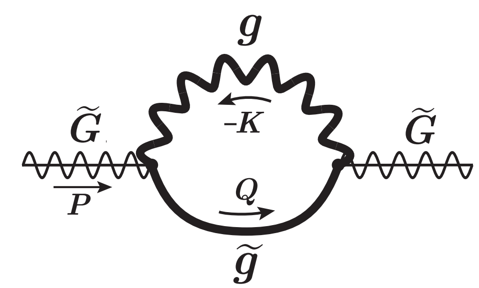
$$\mathcal{C}^t_{\rm BFB} = -0.13286\times 10^{-3}\,.$$

$\gamma_{\rm D}$

$$\Pi^<(P) = \frac{1}{16M_P^2}\sum_{N=1}^{3} n_N \left(1+\frac{m_{\lambda_N}^2}{3m_{3/2}^2}\right)\int \frac{\mathrm{d}^4K}{(2\pi)^4}\,\mathrm{Tr}\Big[\not{P}[\not{K},\gamma^\mu]\,{}^*S^<(Q)[\not{K},\gamma^\nu]\,{}^*D^<_{\mu\nu}(K)\Big]$$

$${}^*S^<(Q) = \frac{f_F(q_0)}{2}\left[(\gamma_0-\boldsymbol{\gamma}\cdot\mathbf{q}/q)\,\rho_+(Q)+(\gamma_0+\boldsymbol{\gamma}\cdot\mathbf{q}/q)\,\rho_-(Q)\right]$$

$${}^*D^<_{\mu\nu}(K) = f_B(k_0)\left[\Pi^T_{\mu\nu}\,\rho_T(K)+\Pi^L_{\mu\nu}\frac{k^2}{K^2}\,\rho_L(K)+\xi\frac{K_\mu K_\nu}{K^4}\right]$$



$$\gamma_D = \int \frac{\mathrm{d}^3\mathbf{p}}{2p_0(2\pi)^3}\;\Pi^<(p)$$

$$\begin{aligned}\gamma_{\rm D} = \frac{1}{4(2\pi)^5M_{\rm P}^2}\sum_{N=1}^{3} n_N\left(1+\frac{m_{\lambda_N}^2}{3m_{3/2}^2}\right)\int_0^\infty \mathrm{d}p\int_{-\infty}^{\infty}\mathrm{d}k_0\int_0^\infty \mathrm{d}k\int_{|k-p|}^{k+p}\mathrm{d}q\;k\,f_B(k_0)\,f_F(q_0)\\
\times\Big[\rho_L(K)\,\rho_-(Q)\,(p-q)^2\big((p+q)^2-k^2\big)+\rho_L(K)\,\rho_+(Q)\,(p+q)^2\big(k^2-(p-q)^2\big)\\
+\,\rho_T(K)\,\rho_-(Q)\,\big(k^2-(p-q)^2\big)\Big(\big(1+k_0^2/k^2\big)\big(k^2+(p+q)^2\big)-4k_0(p+q)\Big)\\
+\,\rho_T(K)\,\rho_+(Q)\,\big((p+q)^2-k^2\big)\Big(\big(1+k_0^2/k^2\big)\big(k^2+(p-q)^2\big)-4k_0(p-q)\Big)\Big]\,,\end{aligned}$$

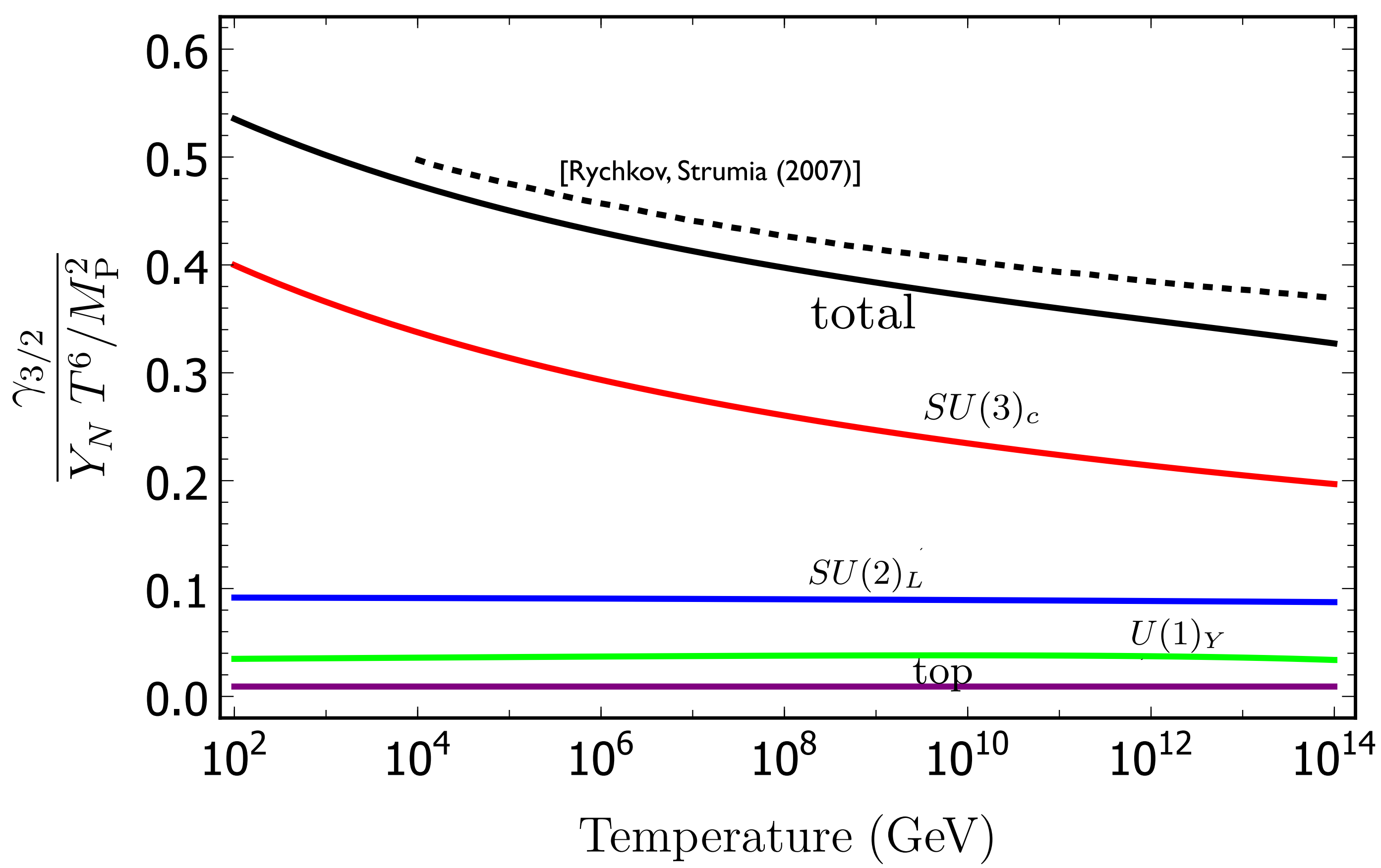
$\gamma_{\rm top}$

$$\gamma_{\rm top} = \frac{T^6}{M_{\rm P}^2}\,72\,\mathcal{C}^s_{\rm BBF}\,\lambda_t^2\left(1+\frac{A_t^2}{3m_{3/2}^2}\right) \qquad \mathcal{C}^s_{\rm BBF} = 0.25957\times10^{-3}.$$

Result and cosmological consequences

$$\gamma_{\mathrm{sub}}+\gamma_{\mathrm{D}} = \frac{3\zeta(3)}{16\pi^3}\,\frac{T^6}{M_{\mathrm{P}}^2}\sum_{N=1}^{3} c_N\, g_N^2 \left(1+\frac{m_{\lambda_N}^2}{3m_{3/2}^2}\right)\ln\left(\frac{k_N}{g_N}\right)$$

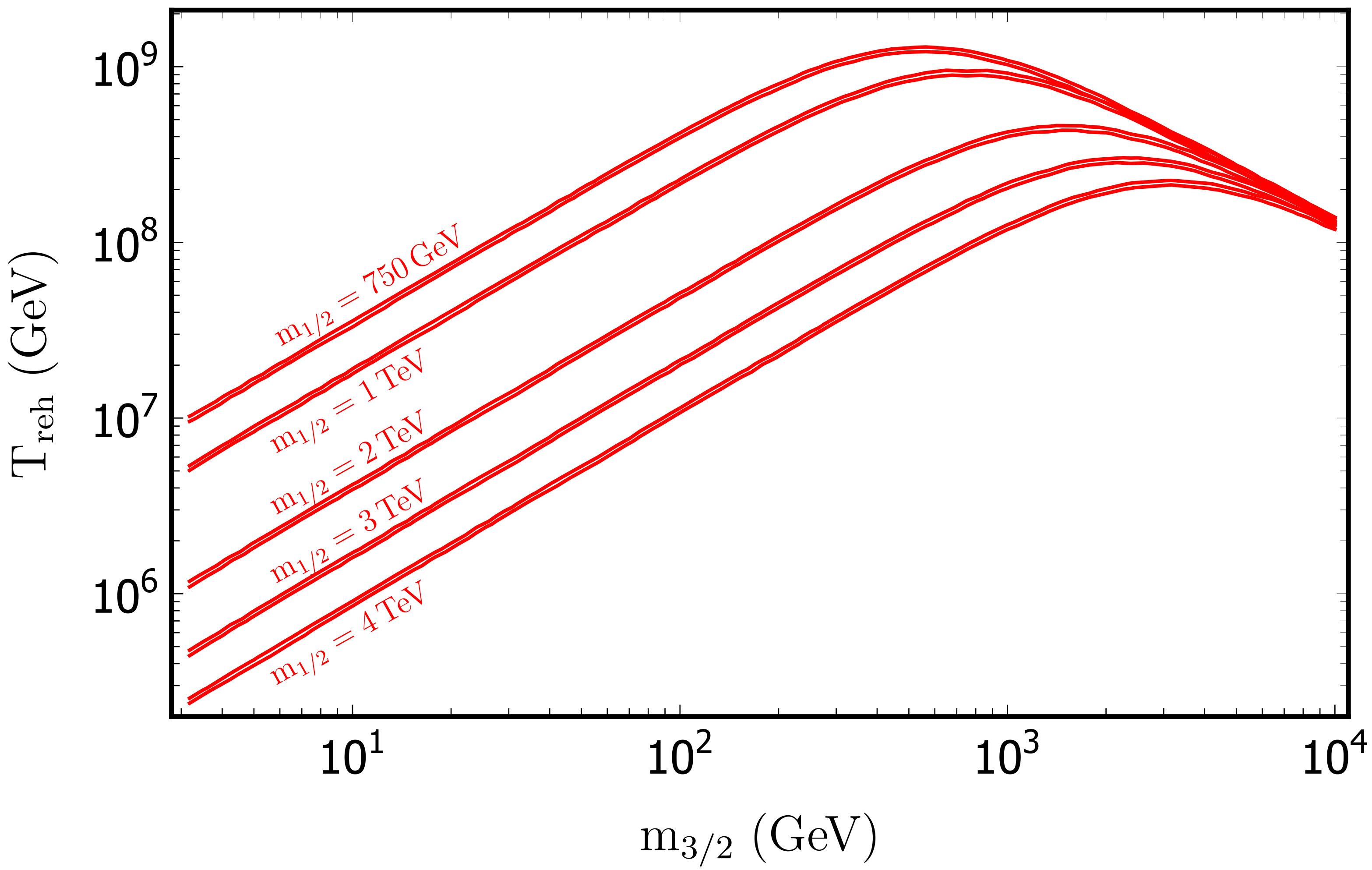
Gauge group	c_N	k_N
$U(1)_Y$	41.937	0.824
$SU(2)_L$	68.228	1.008
$SU(3)_c$	21.067	6.878



Gravitino abundance

$$Y_{3/2}(T) \simeq \frac{\gamma_{3/2}(T_{\rm reh})}{H(T_{\rm reh})\; n_{\rm rad}(T_{\rm reh})} \; \frac{g_{*s}(T)}{g_{*s}(T_{\rm reh})}$$

$$\Omega_{\rm DM} h^2 = \frac{\rho_{3/2}(t_0)\, h^2}{\rho_{\rm cr}} = \frac{m_{3/2}\, Y_{3/2}(T_0)\, n_{\rm rad}(T_0)\; h^2}{\rho_{\rm cr}} \; \simeq 1.33 \times 10^{24}\, \frac{m_{3/2}\;\; \gamma_{3/2}(T_{\rm reh})}{T_{\rm reh}^5}$$



Summary

- Gravitino is natural candidate in SUGRA models for DM.

- We calculated the gravitino thermal production rate beyond the HTL approximation.

- Assuming $m_{1/2}$> 750 GeV (~ LHC current bound), for the maximum value of T_{reh} ~ 10^9 GeV, we get $m_{3/2}$ ~ 550 GeV. If T_{reh} ~ 10^8 GeV for the same $m_{3/2}$, $m_{1/2}$~ 3-4 TeV.

- No-thermal gravitinos should be taken into account, but for this a particular inflation model has to assumed.