Freeze-in produced dark matter in the ultra-relativistic regime

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Beyond Standard Model - from Theory to Experiments

March 30th, 2021

in collaboration with Jacopo Ghiglieri (arXiv 2012.09083, JCAP03(2021)075)

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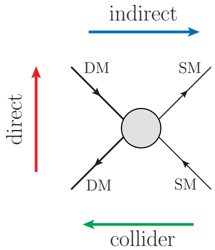
Freeze-in production mechanism



- DM as a particle: many candidates see review G. Bertone 2016
- non-interacting with photons, absolutely stable or long-lived $\sim \tau_{\rm Universe}$
- Any model has to comply with

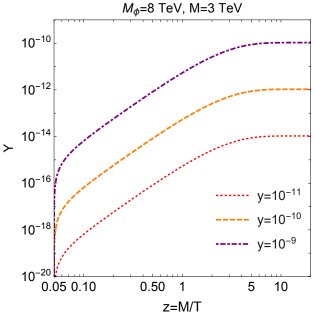
$$\Omega_{\rm DM} h^2(M_{\rm DM}, M_{\rm DM'}, \alpha_{\rm DM}, \alpha_{\rm SM}) = 0.1200 \pm 0.0012$$

Freeze-in mechanism J. McDonald (2002)

- DM never reach thermal equilibrium
- DM from decay and/or annihilations of **equilibrated** species
- for a simple model $\mathcal{L}_{\rm int} = -y\phi\bar{\chi}\chi$, $\phi \to \chi\chi$

$$\frac{dn_\chi}{dt} + 3Hn_\chi = \langle \Gamma_{\phi\to\chi\chi}\rangle n_\phi^{\rm eq} \ , \ Y = n_\chi/s$$

- $\Omega_{\rm DM} h^2 = \frac{M}{\rm GeV}\frac{Y_{\rm fin}}{3.645\times 10^{-9}}$



FRAMING THE PARTICLE PHYSICS MODEL

- Simplified DM models:
  $\Rightarrow$ capture the d.o.f. and parameters needed to study DM phenomenology
- χ Majorana fermion singlet, $\chi \equiv$ DM particle
- η is charged under QCD and $\text{U}(1)_Y$, $\eta \equiv$ mediator with $M_\eta = M + \Delta M$

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{\text{SM}} + \frac{1}{2}\bar{\chi}\left(i\partial\!\!\!/ - M\right)\chi + (D_\mu \eta)^\dagger D^\mu \eta - M_\eta^2\, \eta^\dagger \eta - \lambda_2 (\eta^\dagger \eta)^2 \\ &- \lambda_3\, \eta^\dagger \eta\, \phi^\dagger \phi - y\, \eta^\dagger \bar{\chi} a_R q - y^* \bar{q} a_L \chi\, \eta\end{aligned}$$

same model and freeze-in see M. Garny and J. Heisig (2018) and G. Bélanger et al (2018)

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within the freeze-in $y \lesssim \mathcal{O}(10^{-8})$ and for $T > 150$ GeV

- we shall address the high-temperature dynamics
- multiple soft scatterings and $2 \to 2$ process

$\boxed{T \gg M_\eta \text{ can be very important even for renormalizable interactions}}$

Production rate and rate equation

General approach

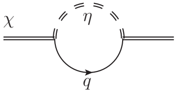
- Given a field χ weakly coupled to a an equilibrated bath, with internal couplings g

  [T. Asaka, M. Laine and M. Shaposhnikov (2006), M. Laine and A. Vuorinen (2017), D. Bödeker, M. Sangel and M. Wörmann (2016)]

- at leading order in y and **all orders in** g one can prove D. Bödeker, M. Sangel and M. Wörmann (2016)

$$\dot{f}_\chi(t,\boldsymbol{k}) = \Gamma(k)[n_{\rm F}(k^0) - f_\chi(t,\boldsymbol{k})], \quad \Gamma(k) = \frac{|y|^2}{2k^0}\int d^4X\, e^{iK\cdot X}\langle[J(X),J(0)]\rangle$$

- $f_\chi(t,\boldsymbol{k})$ is the single-particle phase-space distribution; J made of bath fields



$$\boxed{\Gamma(k) = \frac{|y|^2}{k^0}\mathrm{Im}\Pi_R}$$

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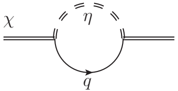


$$\boxed{\Gamma(k) = \frac{|y|^2}{k^0}\mathrm{Im}\Pi_R}$$

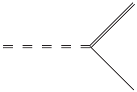
- when doing perturbative expansions $\Rightarrow$ **Boltzmann equation is recovered**
- general framework to include:
  resummation and NLO computations, non-perturbative and thermal effects

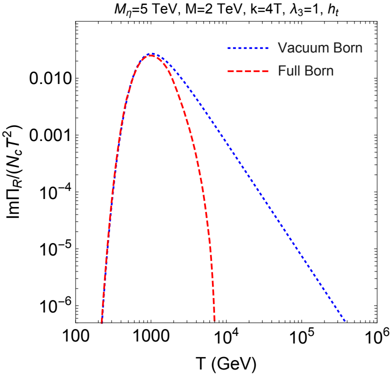
In-vacuum versus finite thermal masses

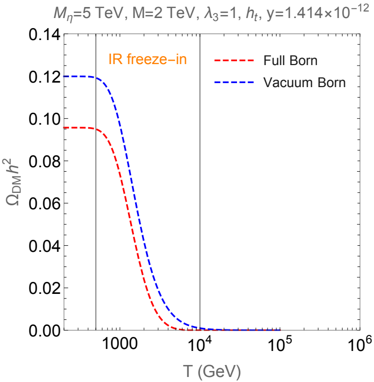
- Born rate $\eta \to \chi + q$ with and without thermal masses (recall m_q is purely thermal)

for thermal masses see also L. Darmé, A. Hryczuk, D. Karamitros and L. Roszkowski (2019)

$$\dot{n}_{\rm DM} + 3H n_{\rm DM} = 2|y|^2 \int_{\boldsymbol{k}} \frac{n_{\rm F}(k^0)}{k^0} \mathrm{Im}\Pi_R^{\rm Born}$$



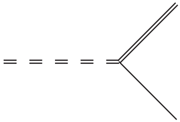




LPM resummation for light-cone kinematics

HIGH-TEMPERATURES $\pi T \gg M_\eta$

- all the particles are seen as *massless*,
- momenta of external particles $p \sim \pi T$,
- particles are ultra-relativistic, and gT is a soft scale
  $\Rightarrow$ collinear kinematics $\approx$ high T



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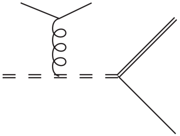

LPM RESUMMATION FOR LIGHT-CONE KINEMATICS

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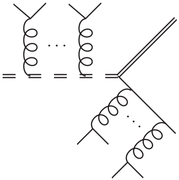

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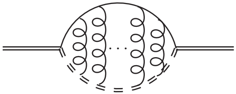

n soft scatterings: LPM resummation

L. Landau and I. Pomeranchuk (1953) and A. B. Migdal (1956)

see e.g. J. Ghiglieri and G. D. Moore (2014) for a review

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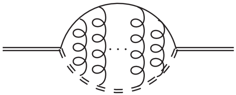

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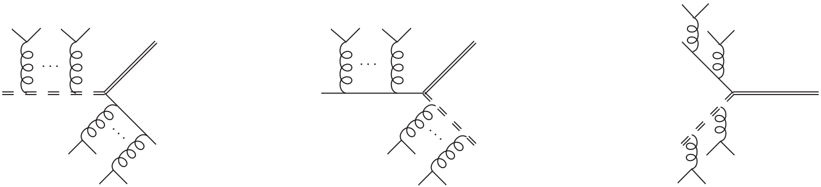
- at $T \gg M_\eta$ three effective processes contribute to the production of χ

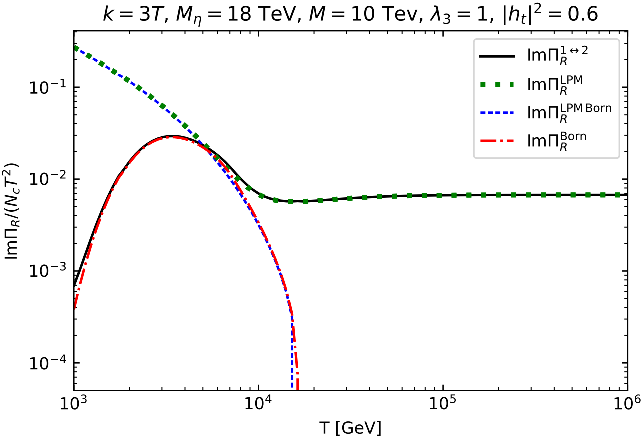
$$\boxed{\eta \to \chi + q, \quad q \to \chi + \eta, \quad q + \eta \to \chi}$$



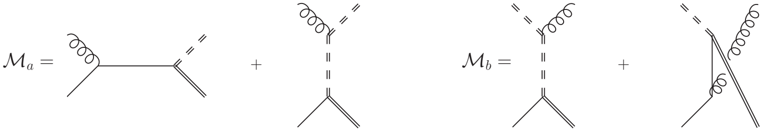
LPM results

- prescription for any temperature (inspired from I. Ghisoiu and M. Laine (2014))

$$\mathrm{Im}\Pi_R^{1\leftrightarrow 2} = \mathrm{Im}\Pi_R^{\mathrm{LPM}} - \mathrm{Im}\Pi_R^{\mathrm{LPM\ Born}} + \mathrm{Im}\Pi_R^{\mathrm{Born}}$$



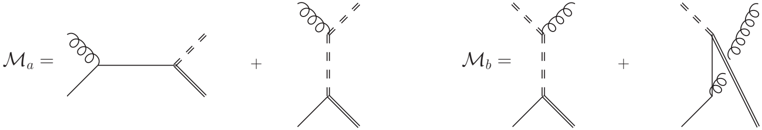
2 → 2 scatterings



- Considered by M. Garny and J. Heisig (1809.10135) for $T \leq M$ (possibly some issues with IR of some processes)
- for s/t and u/t contributions from **both hard and soft momentum regions**

D. Besak and D. Bodeker (2012), J. Ghiglieri and M. Laine (2016)

$2 \to 2$ scatterings



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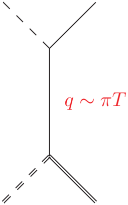
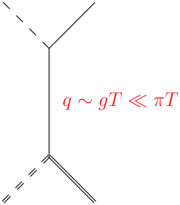
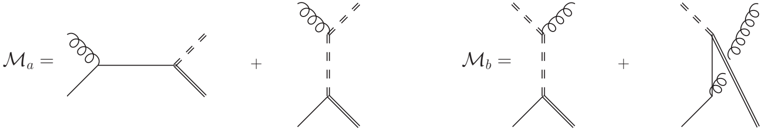




$2 \to 2$ SCATTERINGS



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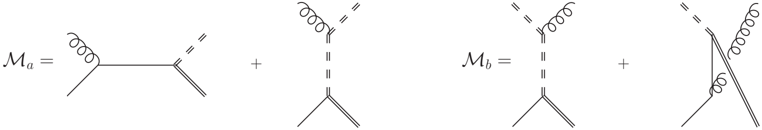

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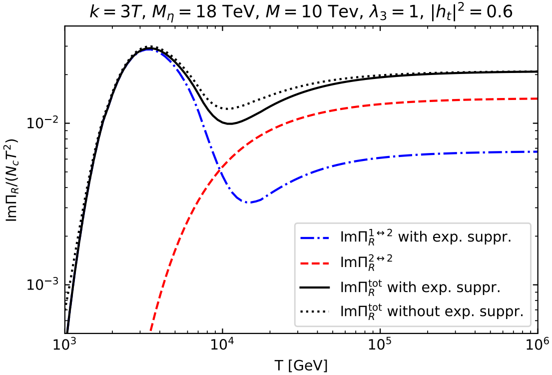
$$
\begin{aligned}
\mathrm{Im}\Pi_R^{2\leftrightarrow 2} &= \frac{2}{(4\pi)^3 k}\int_k^\infty \mathrm{d}q_+ \int_0^k \mathrm{d}q_- \Big\{ \big[n_{\mathrm{F}}(q_0) + n_{\mathrm{B}}(q_0 - k) \big] N_c \left(Y_q^2 g_1^2 + C_F g_3^2 + |h_q|^2 \right) \Phi_{s2} \Big\} \\
&+ \frac{2}{(4\pi)^3 k}\int_0^k \mathrm{d}q_+ \int_{-\infty}^0 \mathrm{d}q_- \Big\{ \big[1 - n_{\mathrm{F}}(q_0) + n_{\mathrm{B}}(k - q_0) \big] N_c \left(Y_q^2 g_1^2 + C_F g_3^2 + |h_q|^2 \right) \Phi_{t2} \\
&- \left[n_{\mathrm{B}}(k) + \frac{1}{2} \right] N_c \left(Y_q^2 g_1^2 + C_F g_3^2 + |h_q|^2 \right) \frac{k \pi^2 T^2}{q^2} \Big\} + N_c \frac{m_q^2}{16\pi} \left[n_{\mathrm{B}}(k) + \frac{1}{2} \right] \ln\left(1 + \frac{4k^2}{m_q^2} \right)
\end{aligned}
$$

Summary of the rates

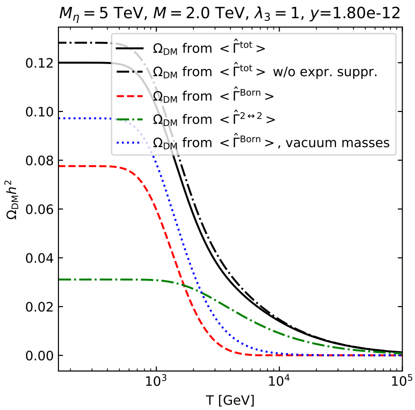
- Phenomenological switch off the high-temperature processes (lack of NLO rates)

$$\kappa(M_\eta) = \frac{3}{\pi^2 T^3} \int_0^\infty dp\, p^2\, n_{\rm B}(E_\eta)[1 + n_{\rm B}(E_\eta)]\,, \qquad \boxed{\Pi_R^{\rm tot} = {\rm Im}\Pi_R^{1\leftrightarrow 2} + {\rm Im}\Pi_R^{2\leftrightarrow 2}}$$

- ${\rm Im}\Pi_R^{1\leftrightarrow 2} = ({\rm Im}\Pi_R^{\rm LPM} - {\rm Im}\Pi_R^{\rm LPM\ Born})\kappa(M_\eta) + {\rm Im}\Pi_R^{\rm Born}$



Benchmark point P0



- at $T \gg M_\eta, M$

  $\to$ **non-negligible production of χ!**
- important high-temperature window

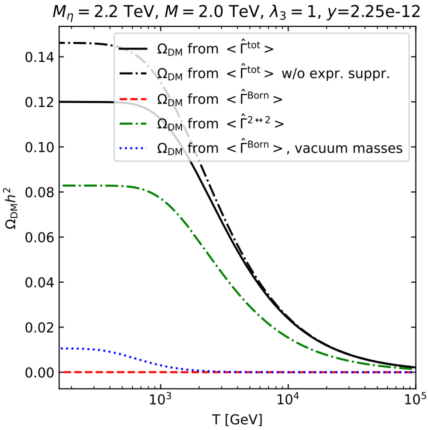
  $2 \to 2$ and effective $1 \leftrightarrow 2$

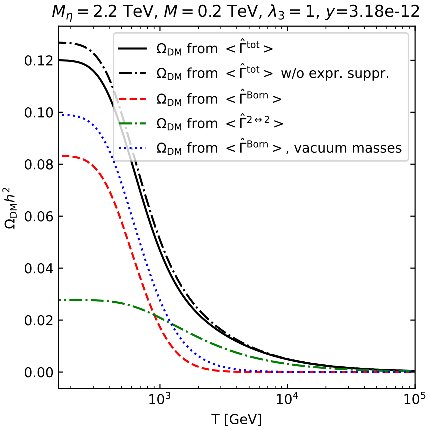
- Born rate with vacuum masses $\Rightarrow$ 20% reduction of $\Omega_{\rm DM}h^2$ with respect to $\Pi_R^{\rm tot}$
- 30% when including thermal masses but excluding $2 \to 2$ and effective $1 \leftrightarrow 2$
- estimation of theoretical error: LPM with and without $\kappa(M_\eta)$, here $\sim 10\%$ effect

LARGE AND SMALL MASS SPLITTINGS

- the smaller $\Delta M/M$ the larger the effect of thermal masses, LPM and $2 \to 2$

  Left plot: $\Delta M/M = 0.1$; Right plot: $\Delta M/M = 10$
- also other parameters of the model are relevant (h_q, λ_3)





Summary

- we studied the impact of the ultra-relativistic regime on the production of a feebly interacting DM particle
- Before: in renormalizable models bulk DM population produced at $T \sim M$
- Our work: this is not always the case
  high-temperature $1 \leftrightarrow 2$, $2 \to 2$ can give $\mathcal{O}(1)$ contribution

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- simplified dark matter model:
  χ Majorana fermion DM and η mediator charged under $\mathrm{SU}(3) \otimes \mathrm{U}(1)_Y$
- freeze-in with large impact from $1 \leftrightarrow 2$, $2 \to 2$

$$M = 2\ \mathrm{TeV}, \Delta M = 0.2\ \mathrm{TeV} \Rightarrow \frac{(\Omega h^2)_{\mathrm{full}}}{(\Omega h^2)_{\mathrm{Born}}} \simeq 10$$

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- similar effects can affect other models if DM comes from particles in equilibrium
- Our main uncertainty comes from the lack of NLO rates (state-of-art M.Laine (2013))

The born rate with vanishing thermal masses

- Let us look at the model at hand

$$\left(\frac{\partial}{\partial t} - Hk_i \frac{\partial}{\partial k_i}\right) f_\chi(t,\boldsymbol{k}) = \Gamma(k)[n_{\mathrm{F}}(k^0) - f_\chi(t,\boldsymbol{k})],$$

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$$\begin{aligned} \Pi^E(K) &\equiv \mathrm{Tr}\left\{ i\not{K} \left[\int_X e^{iK\cdot X} a_R \langle (\eta^\dagger q)(X)(\bar{q}\eta)(0)\rangle a_L \right] \right\} \\ &= N_c \int_{\boldsymbol{p}} T \sum_n \frac{-i\not{P} a_L}{p_n^2 + E_q^2} \frac{1}{(p_n + k_n)^2 + E_\eta^2} \end{aligned}$$

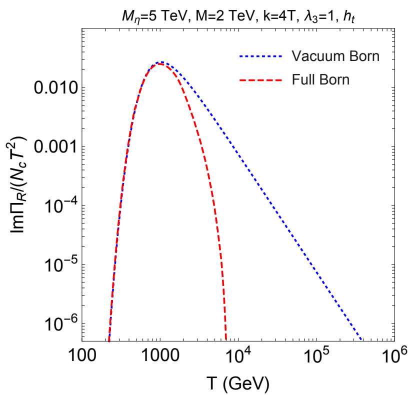
- with $E_q = |\boldsymbol{p}| = p$ and $E_\eta = \sqrt{(\boldsymbol{p}+\boldsymbol{k})^2 + M_\eta^2}$

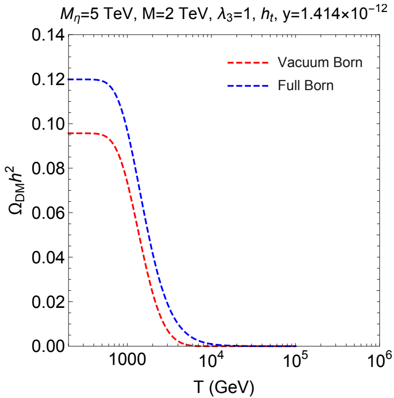
see M. Laine and A. Vuorinen (2017)

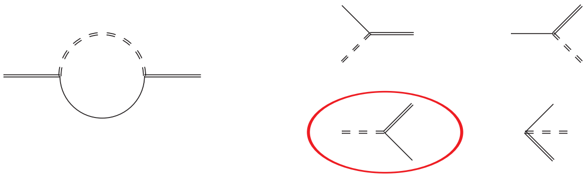
In-vacuum versus finite thermal masses

- Scalar mass: $\mathcal{M}_\eta^2 = M_\eta^2 + m_\eta^2$, for $\boldsymbol{\eta} \to \boldsymbol{\chi} + \boldsymbol{q}$ and $E_p = \sqrt{p^2 + m_q^2}$

$$\mathrm{Im}\Pi^{\mathrm{Born}}_{\mathrm{R},\eta\to\chi q} = \frac{N_c}{16\pi k}\int_{p_{\min}}^{p_{\max}} dp[\mathcal{M}_\eta^2 - M^2 - m_q^2 - 2k^0(E_p - p)][n_{\mathrm{B}}(k^0 + E_p) + n_{\mathrm{F}}(E_p)]$$





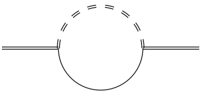


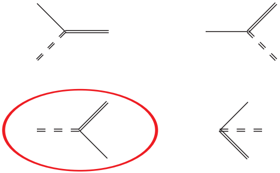
$\eta \to \chi + q$

For us $M_\eta > M_\chi + M_q \ldots$ (first row: $M_\chi > M_\eta + M_q,\ M_q > M_\eta + M_\chi$)

$$\mathrm{Im}\Pi_R^{\mathrm{Born}} = \frac{N_c(M_\eta^2 - M^2)}{8 n_{\mathrm{F}}(k^0)} \int \frac{d^3\boldsymbol{p}_\eta}{(2\pi)^3} \int \frac{d^3\boldsymbol{p}_q}{(2\pi)^3} \frac{(2\pi)^4 \delta^4(\mathcal{P}_\eta - \mathcal{P}_q - \mathcal{K})}{E_\eta E_q} n_{\mathrm{B}}(E_\eta)(1 - n_{\mathrm{F}}(E_q))$$

Born term and Boltzmann equation





$\eta \to \chi + q$

For us $M_\eta > M_\chi + M_q \ldots$ (first row: $M_\chi > M_\eta + M_q$, $M_q > M_\eta + M_\chi$)

$$\mathrm{Im}\Pi_R^{\mathrm{Born}} = \frac{N_c(M_\eta^2 - M^2)}{8 n_{\mathrm{F}}(k^0)} \int \frac{d^3\boldsymbol{p}_\eta}{(2\pi)^3} \int \frac{d^3\boldsymbol{p}_q}{(2\pi)^3} \frac{(2\pi)^4 \delta^4(\mathcal{P}_\eta - \mathcal{P}_q - \mathcal{K})}{E_\eta E_q} n_{\mathrm{B}}(E_\eta)(1 - n_{\mathrm{F}}(E_q))$$

- $n_{\mathrm{DM}} = 2 \int_{\boldsymbol{k}} f_\chi(t, k)$, for $\eta \to \chi + q$

$$\begin{aligned} \dot{n}_{\mathrm{DM}} + 3 H n_{\mathrm{DM}} &= 2|y|^2 \int_{\boldsymbol{k}} \frac{n_{\mathrm{F}}(k^0)}{k^0} \mathrm{Im}\Pi_R \\ &= 2|y|^2 N_c (M_\eta^2 - M^2) \int_{\boldsymbol{p}_\eta, \boldsymbol{p}_q, \boldsymbol{k}} \frac{(2\pi)^4 \delta^4(\mathcal{P}_\eta - \mathcal{P}_q - \mathcal{K})}{8 E_\eta E_q k^0} n_{\mathrm{B}}(E_\eta)\,[1 - n_{\mathrm{F}}(E_q)] \end{aligned}$$

First improvement: thermal masses

- at high temperatures, $\pi T \gg M_\eta$, repeated interactions within the bath change the dispersion relations $\Rightarrow$ *asymptotic masses*

see also L. Darmé, A. Hryczuk, D. Karamitros and L. Roszkowski (2019)

- for $T > T_c \simeq 150$ GeV the quarks only have

$$m_q^2 = \frac{T^2}{4}(g_3^2 C_F + Y_q^2 g_1^2 + |h_q|^2)$$

- for the colored scalar

$$m_\eta^2 = \left(\frac{g_3^2 C_F + Y_q^2 g_1^2}{4} + \frac{\lambda_3}{6} \right) T^2$$

- no thermal mass correction for χ since $y \ll g$

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$$m_{\mathrm{B}}^2 = \left(\frac{n_{\mathrm{S}}}{6} + \frac{5n_{\mathrm{G}}}{9} + \frac{Y_q^2 N_c}{3} \right) g_1^2 T^2 \, , \quad m_g^2 = \left(\frac{N_c}{3} + \frac{n_{\mathrm{G}}}{3} + \frac{1}{6} \right) g_3^2 T^2$$

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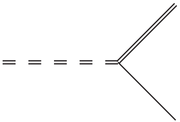
$$\begin{aligned} m_\eta^2(M_\eta, p) \quad &= \quad \frac{Y_q^2 g_1^2 + C_F g_3^2}{2\pi^2} \int_0^\infty dq \left\{ \frac{n_{\mathrm{B}}(E_\eta(q))}{E_\eta(q)} \left[q^2 + \frac{M_\eta^2 q}{2p} \ln\left(\frac{(p+q)^2}{(p-q)^2} \right) \right] + 2\, q\, n_{\mathrm{B}}(q) \right\} \\ &\quad + \frac{\lambda_3}{\pi^2} \int_0^\infty dq\, q\, n_{\mathrm{B}}(q) \end{aligned}$$

$\pi T \gg M_\eta$

- all the particles are seen as *massless*,
- all the momenta of extarnal particles are $p \sim \pi T$,
- particles are ultra-relativistic, and gT is a soft scale
  $\Rightarrow$ collinear kinematics $\approx$ high T



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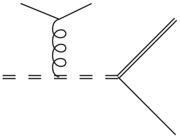

LPM resummation for light-cone kinematics

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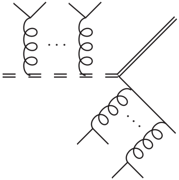

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$$m_g^2 = \left(\frac{N_c}{3} + \frac{n_G}{3} + \frac{1}{6} \right) g_3^2 T^2$$

Long formation time $\sim \frac{1}{g^2 T}$

see e.g. J. Ghiglieri and G. D. Moore (2014)

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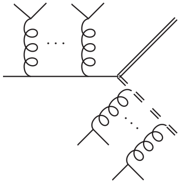

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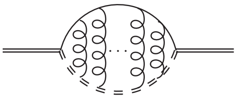


- connection between LPM resummation and $\mathrm{Im}\Pi_R^{\mathrm{LPM}} \Rightarrow \chi$ self-energy
- notation and computational setting from

$$\hat{H} \equiv -\frac{M^2}{2k_0} + \frac{m_q^2 - \nabla_\perp^2}{2E_q} + \frac{\mathcal{M}_\eta^2 - \nabla_\perp^2}{2E_\eta} + i\Gamma(y)\,, \quad y \equiv |\boldsymbol{y}_\perp|$$

$$\Gamma(y) = \frac{T}{2\pi} g_1^2 Y_q^2 \left[\ln\left(\frac{m_{\rm B} y}{2}\right) + \gamma_E + K_0(m_{\rm B} y)\right] + \frac{T}{2\pi} g_3^2 C_F \left[\ln\left(\frac{m_g y}{2}\right) + \gamma_E + K_0(m_g y)\right]$$

Numerical strategy and LPM Born

- $\hat{H}$ enters the inhomogeneous equations for the functions $g(\boldsymbol{y})$ and $\boldsymbol{f}(\boldsymbol{y})$

$$(\hat{H}+i0^+)g(\boldsymbol{y})=\delta^{(2)}(\boldsymbol{y})\,,\quad (\hat{H}+i0^+)\boldsymbol{f}(\boldsymbol{y})=-\nabla_\perp\delta^{(2)}(\boldsymbol{y})$$

$$\begin{aligned}\mathrm{Im}\Pi_{\mathrm{R}}^{\mathrm{LPM}} \quad &= \quad -\frac{N_c}{8\pi}\int_{-\infty}^{+\infty}dE_q\int_{-\infty}^{+\infty}dE_\eta\,\delta(k_0-E_q-E_\eta)[1-n_{\mathrm{F}}(E_q)+n_{\mathrm{B}}(E_\eta)]\\ &\qquad \frac{k_0}{E_\eta}\lim_{\boldsymbol{y}\to 0}\left\{\frac{M^2}{k_0^2}\mathrm{Im}[g(\boldsymbol{y})]+\frac{1}{E_q^2}\mathrm{Im}[\nabla_\perp\cdot\boldsymbol{f}(\boldsymbol{y})]\right\}\end{aligned}$$

Numerical strategy and LPM Born

- $\hat{H}$ enters the inhomogeneous equations for the functions $g(\boldsymbol{y})$ and $\boldsymbol{f}(\boldsymbol{y})$

$$(\hat{H} + i0^+)g(\boldsymbol{y}) = \delta^{(2)}(\boldsymbol{y})\,, \quad (\hat{H} + i0^+)\boldsymbol{f}(\boldsymbol{y}) = -\nabla_\perp \delta^{(2)}(\boldsymbol{y})$$

$$\begin{aligned}\mathrm{Im}\Pi_{\mathrm{R}}^{\mathrm{LPM}} &= -\frac{N_c}{8\pi}\int_{-\infty}^{+\infty} dE_q \int_{-\infty}^{+\infty} dE_\eta\, \delta(k_0 - E_q - E_\eta)[1 - n_{\mathrm{F}}(E_q) + n_{\mathrm{B}}(E_\eta)] \\ &\quad \frac{k_0}{E_\eta} \lim_{\boldsymbol{y}\to 0}\left\{\frac{M^2}{k_0^2}\mathrm{Im}[g(\boldsymbol{y})] + \frac{1}{E_q^2}\mathrm{Im}[\nabla_\perp \cdot \boldsymbol{f}(\boldsymbol{y})]\right\}\end{aligned}$$

Once $E_\eta = k^0 - E_q$ by the δ

1. $k^0 > E_q > 0$: this corresponds to the effective $2 \to 1$ process $\eta, q \to \chi$
2. $E_q < 0$: this corresponds to the effective $1 \to 2$ process $\eta \to q\chi$

   $\Rightarrow$ LPM-Born with thermal masses is the $n = 0$ limit (no scatterings)
3. $E_q > k^0$: this corresponds to the effective $1 \to 2$ process $q \to \eta\chi$.

$$(\Omega_{\rm DM} h^2)_{\rm obs.} = (\Omega_{\rm DM} h^2)_{\rm freeze\text{-}in} + (\Omega_{\rm DM} h^2)_{\rm super\text{-}WIMP}$$

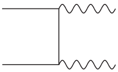
- η particles stays in chemical equilibrium till late times $T \sim M_\eta/25$
- there is a populations of η, as governed by freeze-out, which decays into χ

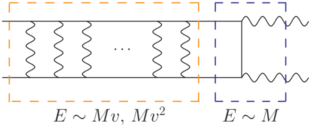
M. Garny, J. Heisig (2018)

$$(\Omega_{\rm DM} h^2)_{\rm super\text{-}WIMP} = \frac{M}{M_\eta} (\Omega h^2)_\eta \, .$$

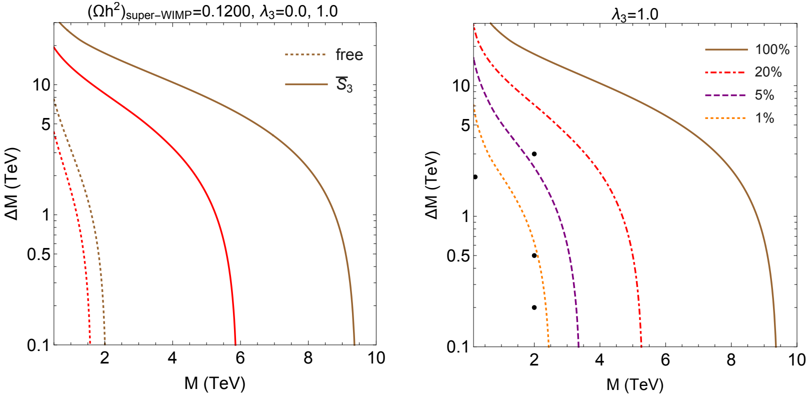
$$\frac{dn_\eta}{dt} + 3Hn_\eta = -\langle \sigma_{\rm eff} v \rangle (n_\eta^2 - n_{\eta,{\rm eq}}^2) \, , \quad \langle \sigma_{\rm eff} v \rangle = \frac{c_3 \bar{S}_3 + c_4 C_{\rm F} \bar{S}_4}{N_c}$$

A. Mitridate et al (2017), S. B. and M. Laine (2018), S. B. and S. Vogl (2019), J. Hartz and K. Petraki (2018)



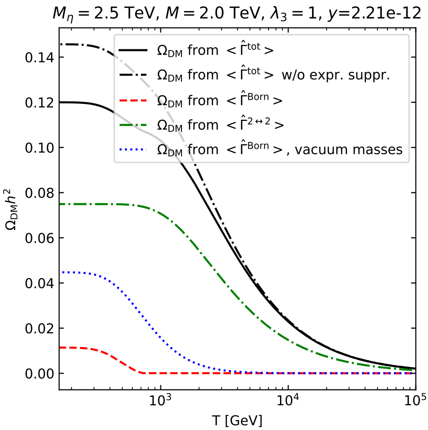


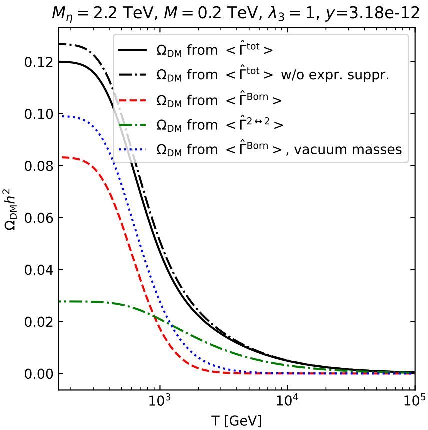
Benchmark values with bound-state effects



- P0 ($M = 2.0$ Tev, $M_\eta = 5.0$ TeV); P1 ($M = 0.2$ Tev, $M_\eta = 2.2$ TeV); P2 ($M = 2.0$ Tev, $M_\eta = 2.5$ TeV); P3 ($M = 2.0$ Tev, $M_\eta = 2.2$ TeV);

Large and small mass splittings II





- for $M_\eta = 2.2$ TeV the freeze-in production stops fairly close to $T_c \simeq 150$ GeV
  $\Rightarrow$ follow DM production in the SM broken phase
- CMS analysis provides us with $M_\eta > 1250$ GeV CMS-PAS-EXO-16-036