Anomaly-free Abelian gauge symmetries

with radiative Dirac neutrino masses

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In collaboration with Julian Calle and Nicolás Bernal

arXiv:2102.06211

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Active Symmetry

$U(1)_X$: Multi-TeV : Z'_μ

SM X-Charges: $f_R \to f$

Field	u_R	d_R	$(Q)^\dagger$	e_R	$(L)^\dagger$
Symbol	u	d	Q	e	L

Dark Symmetry

$U(1)_D$: Dark photon A'_μ

N chiral Fields (CF)

$N = N' + 3$

$\oplus$ N' CF:

$$\underbrace{\psi_1, \psi_2, \ldots, \psi_{N'}}, \psi_{N'+1}\ \psi_{N'+2}\ \psi_{N'+3}$$

Chiral SM $\oplus$ $\psi_1, \psi_2, \cdots, \psi_{N'}, \psi_{N'+1}\ \psi_{N'+2}\ \psi_{N'+3}$

Chiral singlets

$m = e + 2L \to f = f(m, L)$

$N = N' + 3 \geq 5$

U(1)

X-charges $(n_1, n_2 \cdots, n_{N'}, m, m, m)$

D-charges $(n_1, n_2 \cdots, n_{N'}, n_{N'+1}, n_{N'+2}, n_{N'+3})$

$$\sum_{\rho=1}^{N} n_\rho = 0 \qquad \sum_{\rho=1}^{N} n_\rho^3 = 0$$

Example: $N=6 \rightarrow$ $\sum_{\rho=1}^{N} n_\rho = 0, \quad \sum_{\rho=1}^{N} n_\rho^3 = 0$

$$\vec{n} = (-4, -4, 5, 1, 1, 1)$$

$$\vec{X} = (n_1, n_2, n_3, m, m, m)$$

$U(1)_X$

Field	Symbol	f(m,L)	X	B-L
u_R	u	$\frac{4L}{3} - m$	$\frac{4L}{3} - 1$	$\frac{1}{3}$
d_R	d	$-\frac{2L}{3} + m$	$-\frac{2L}{3} + 1$	$\frac{1}{3}$
$(Q)^\dagger$	Q	$-L/3$	$-L/3$	$-\frac{1}{3}$
e_R	e	$m - 2L$	$1 - 2L$	-1
$(L)^\dagger$	L	L	L	1
H	h	$L - m$	$L - 1$	0
ν_{R1}	n_1	-4	-4	-4
ν_{R2}	n_2	-4	-4	-4
ν_{R3}	n_3	-5	-5	-5

$$\vec{D} = (n_1, n_2, n_3, n_4, n_5, n_6)$$

$U(1)_D$

Field	Symbol	X
ψ_1	n_1	-4
ψ_2	n_2	-4
ψ_3	n_3	-5
ψ_4	n_4	1
ψ_5	n_5	1
ψ_6	n_6	1

NEW

$N' > 3$ $\quad U(1)_X$

From: arXiv:1905.13279 [PRL] Costa, *et al*

Let a vector $\boldsymbol{z}$ with N non-zero integer entries such that

$$\sum_{i=1}^{N} z_i = 0\,, \qquad \sum_{i=1}^{N} z_i^3 = 0\,.$$

We like to build this set of N integers from two subsets $\boldsymbol{\ell}$ and $\boldsymbol{k}$ with sizes

$$\dim(\boldsymbol{\ell}) = \begin{cases} \alpha = \frac{N}{2} - 1\,, & \text{if } N \text{ even} \\ \beta = \frac{N-3}{2}\,, & \text{if } N \text{ odd} \end{cases} \;; \qquad \dim(\boldsymbol{k}) = \begin{cases} \alpha = \frac{N}{2} - 1\,, & \text{if } N \text{ even} \\ \beta + 1 = \frac{N-1}{2}\,, & \text{if } N \text{ odd} \end{cases}$$

- N even: Consider the following two vector-like examples of $\boldsymbol{z}$ such that

$$\boldsymbol{x} = (\ell_1, k_1, \ldots, k_\alpha, -\ell_1, -k_1, \ldots, -k_\alpha)$$
$$\boldsymbol{y} = (0, 0, \ell_1, \ldots, \ell_\alpha, -\ell_1, \ldots, -\ell_\alpha)\,.$$

- N odd:

$$\boldsymbol{x} = (0, k_1, \ldots, k_{\beta+1}, -k_1, \ldots, -k_{\beta+1})$$

$$\boldsymbol{y} = (\ell_1, \ldots, \ell_\beta, k_1, 0, -\ell_1, \ldots, -\ell_\beta, -k_1)$$

From any of this, we can build a final $\boldsymbol{z}$ which can includes *chiral* solutions

$$\boldsymbol{x} \oplus \boldsymbol{y} \equiv \left(\sum_{i=1}^{N} x_i y_i^2\right) \boldsymbol{x} - \left(\sum_{i=1}^{N} x_i^2 y_i\right) \boldsymbol{y}\,.$$

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- N even: Consider the following two vector-like examples of $\boldsymbol{z}$ such that

$$\begin{aligned} \boldsymbol{x} &= (\ell_1, k_1, \ldots, k_\alpha, -\ell_1, -k_1, \ldots, -k_\alpha) \\ \boldsymbol{y} &= (0, 0, \ell_1, \ldots, \ell_\alpha, -\ell_1, \ldots, -\ell_\alpha)\ . \end{aligned}$$

- N odd:

$$\boldsymbol{x} = (0, k_1, \ldots, k_{\beta+1}, -k_1, \ldots, -k_{\beta+1})$$

$$\boldsymbol{y} = (\ell_1, \ldots, \ell_\beta, k_1, 0, -\ell_1, \ldots, -\ell_\beta, -k_1)$$

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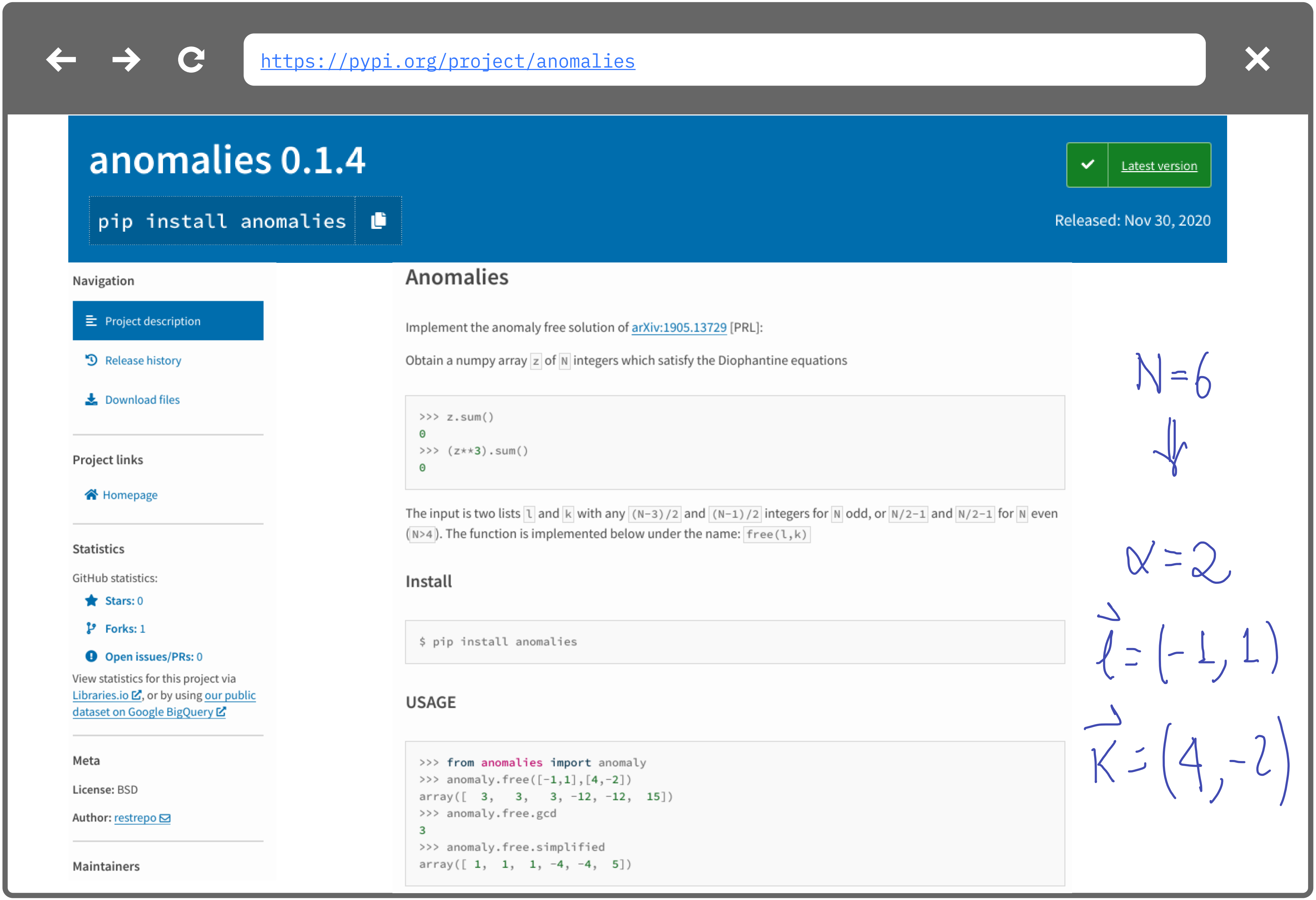


https://pypi.org/project/anomalies

anomalies 0.1.4

```
pip install anomalies
```

Latest version

Released: Nov 30, 2020

Navigation

- Project description
- Release history
- Download files

Project links

- Homepage

Statistics

GitHub statistics:

- Stars: 0
- Forks: 1
- Open issues/PRs: 0

View statistics for this project via Libraries.io, or by using our public dataset on Google BigQuery

Meta

License: BSD

Author: restrepo

Maintainers

Anomalies

Implement the anomaly free solution of arXiv:1905.13729 [PRL]:

Obtain a numpy array `z` of `N` integers which satisfy the Diophantine equations

```
>>> z.sum()
0
>>> (z**3).sum()
0
```

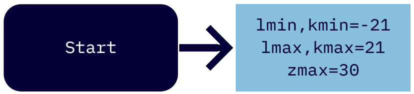
The input is two lists `l` and `k` with any `(N-3)/2` and `(N-1)/2` integers for `N` odd, or `N/2-1` and `N/2-1` for `N` even (`N>4`). The function is implemented below under the name: `free(l,k)`

Install

```
$ pip install anomalies
```

USAGE

```
>>> from anomalies import anomaly
>>> anomaly.free([-1,1],[4,-2])
array([  3,   3,   3, -12, -12,  15])
>>> anomaly.free.gcd
3
>>> anomaly.free.simplified
array([ 1,  1,  1, -4, -4,  5])
```



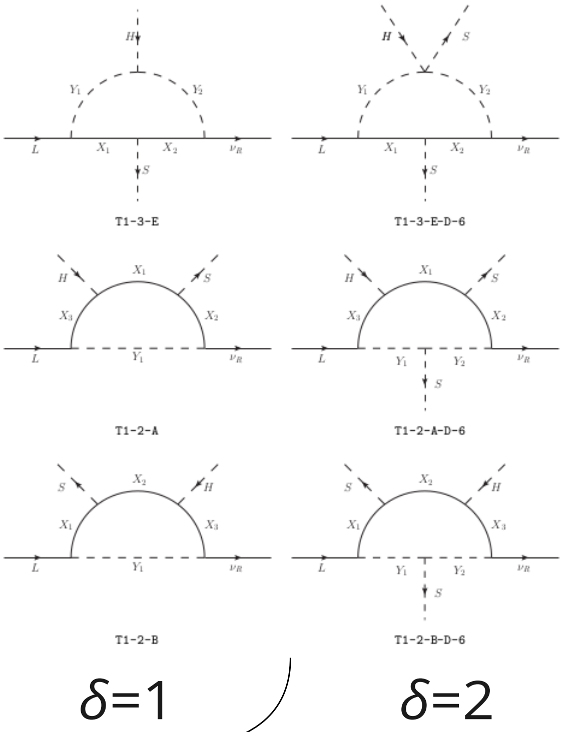
solutions {~30,000 solutions

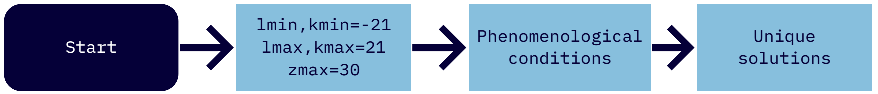
↓

(ν, ν, ...)

with repeated charges

$$\mathcal{L}_{\text{eff}} = h_{\nu}^{\alpha i} \, (\nu_{R\alpha})^{\dagger} \, \epsilon_{ab} \, L_i^a \, H^b \left(\frac{S^*}{\Lambda} \right)^{\delta} + \text{H.c.}, \qquad \text{with } i = 1, 2, 3$$





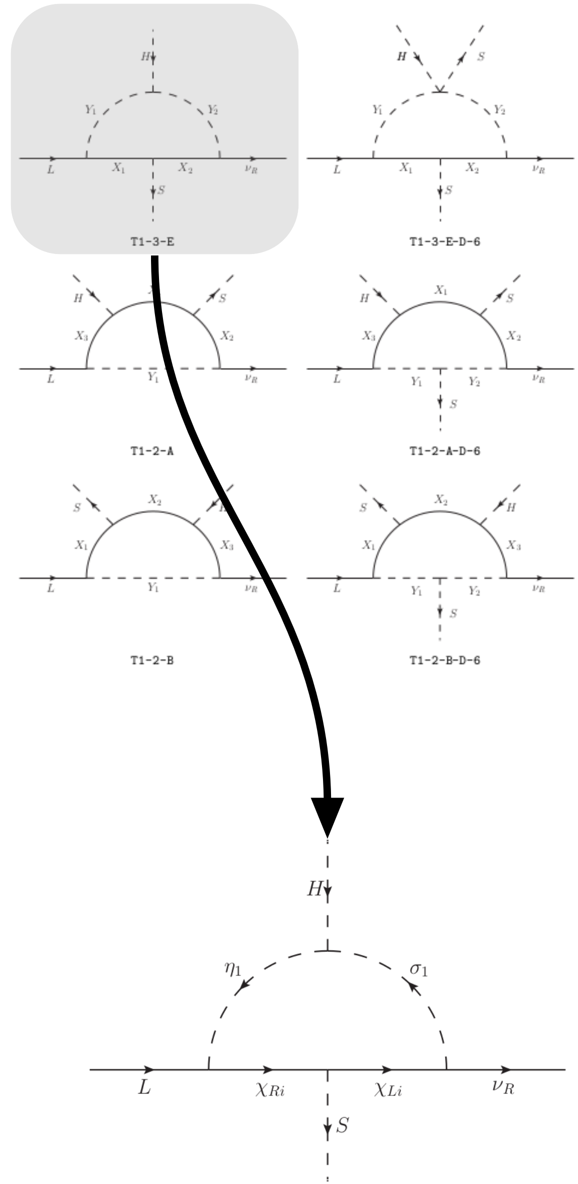
$$\nu + \delta \, (l + r) + m = 0$$

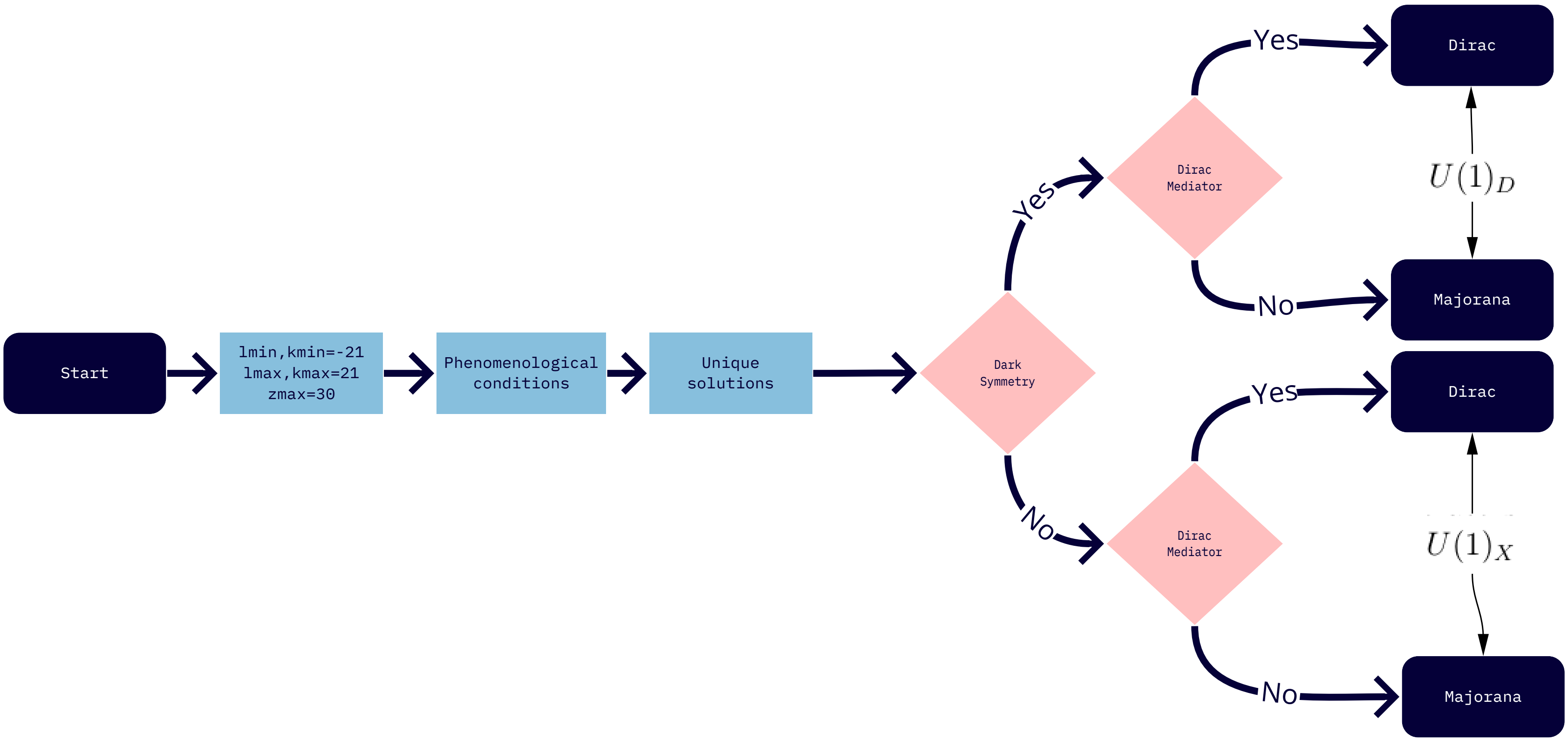
$$\mathcal{L}_{\text{eff}} = h_\nu^{\alpha i}\ (\nu_{R\alpha})^\dagger\ \epsilon_{ab}\, L_i^a\, H^b \left(\frac{S^*}{\Lambda}\right)^\delta + \text{H.c.}, \qquad \text{with } i = 1, 2, 3$$

Start → lmin,kmin=-21 lmax,kmax=21 zmax=30 → Phenomenological conditions → Unique solutions

$$\nu + \delta\,(l + r) + m = 0$$

$$\delta = 1 \rightarrow \begin{cases} \nu + l + r = 0 & \rightarrow U(1)_D : \quad (\nu, \nu, \cdots, l, r, \cdots) \\ \nu + l + r + m = 0 & \rightarrow U(1)_X : \quad (\nu, \nu, \cdots, l, r, \cdots, m, m, m) \end{cases}$$




Start
lmin,kmin=-21
lmax,kmax=21
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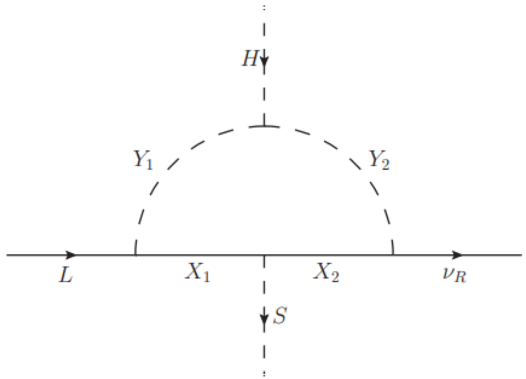
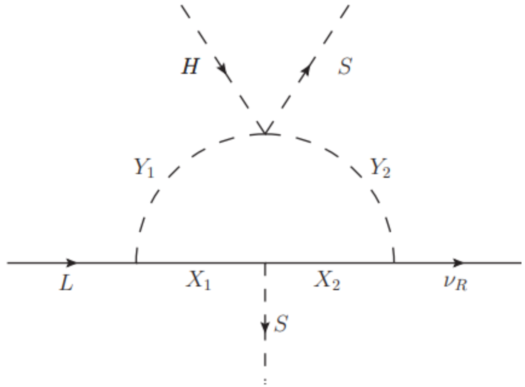
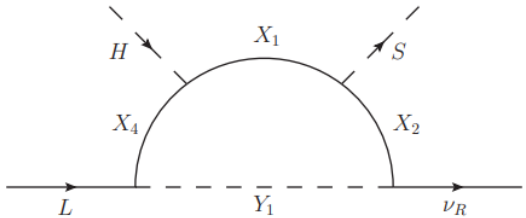
Phenomenological conditions
Unique solutions
Dark Symmetry
Yes
No
Dirac Mediator
Dirac Mediator
Yes
No
Yes
No
Dirac
Majorana
Dirac
Majorana
$U(1)_D$
$U(1)_X$



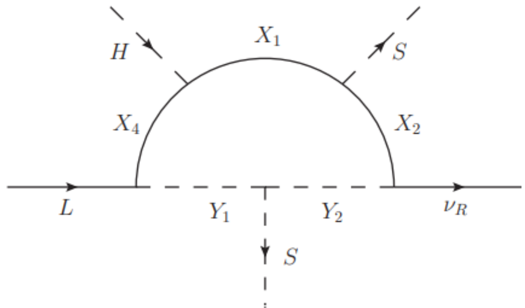
T1-3-E



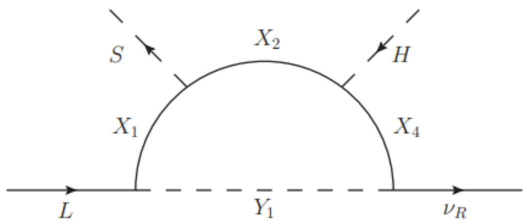
T1-3-E-D-6



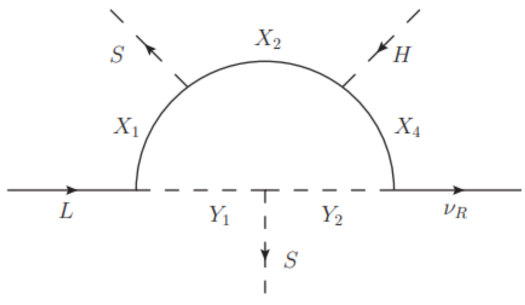
T1-2-A



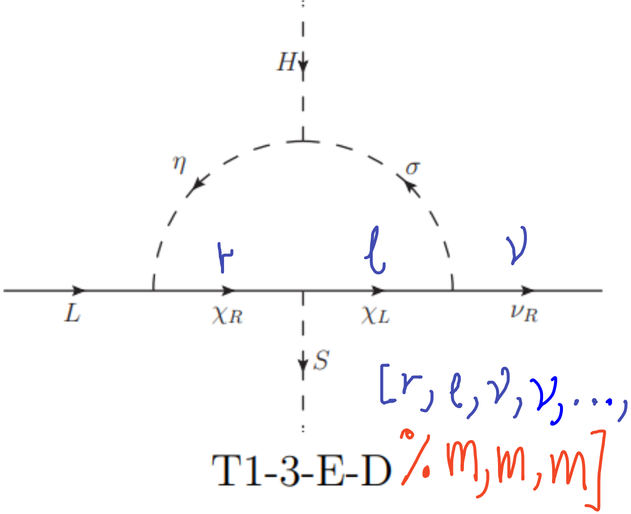
T1-2-A-D-6



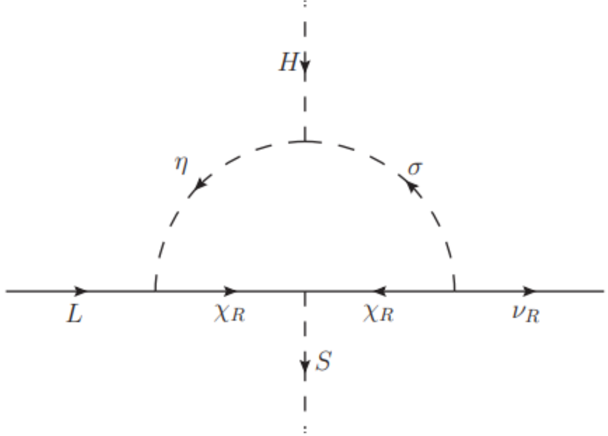
T1-2-B



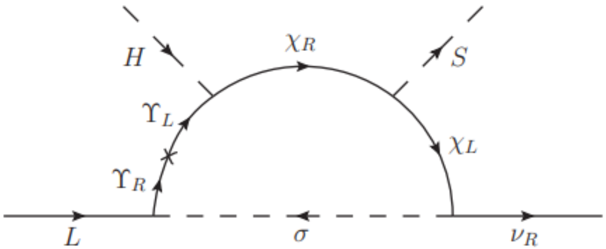
T1-2-B-D-6



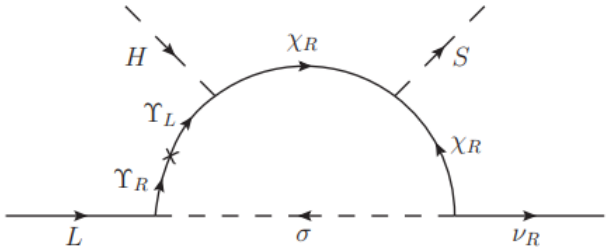
T1-3-E-D



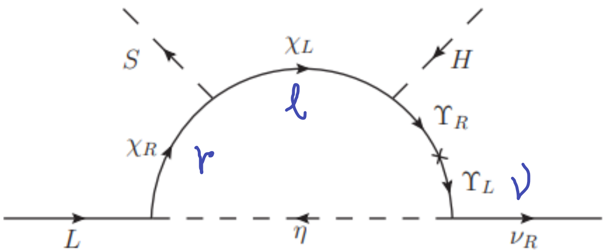
T1-3-E-M



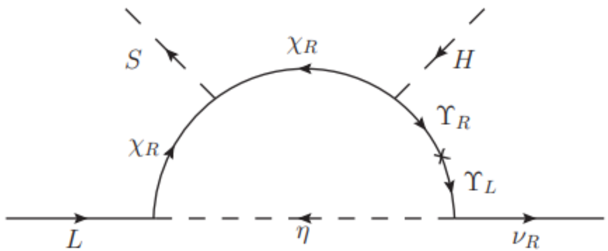
T1-2-A-D



T1-2-A-M



T1-2-B-D



T1-2-B-M

D-5

$$\nu + l + r = \begin{cases} 0\,, & D\text{-Dirac (DD)} \\ -m\,, & X\text{-Dirac (XD)} \end{cases}$$

$$\nu + 2r = \begin{cases} 0\,, & D\text{-Majorana (DM)} \\ -m\,, & X\text{-Majorana (XM)} \end{cases}$$

.

D-6

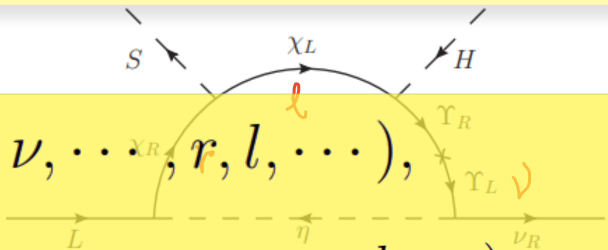
Dirac

$$\nu + 2l + 2r = \begin{cases} 0\,, & \text{DD} \\ -m\,, & \text{XD} \end{cases}$$

Majorana

$$\nu + 4r = \begin{cases} 0\,, & \text{DM} \\ -m\,, & \text{XM} \end{cases}$$

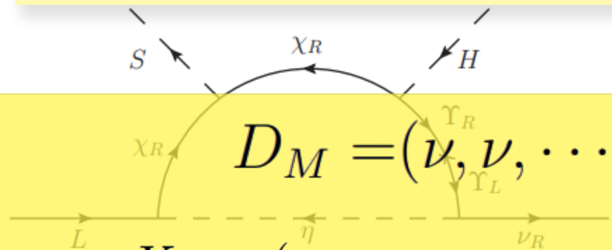
.



$$D_D = (\nu, \nu, \cdots, r, l, \cdots),$$

$$X_D = (\nu, \nu, \cdots, m, m, m, r, l, \cdots),$$

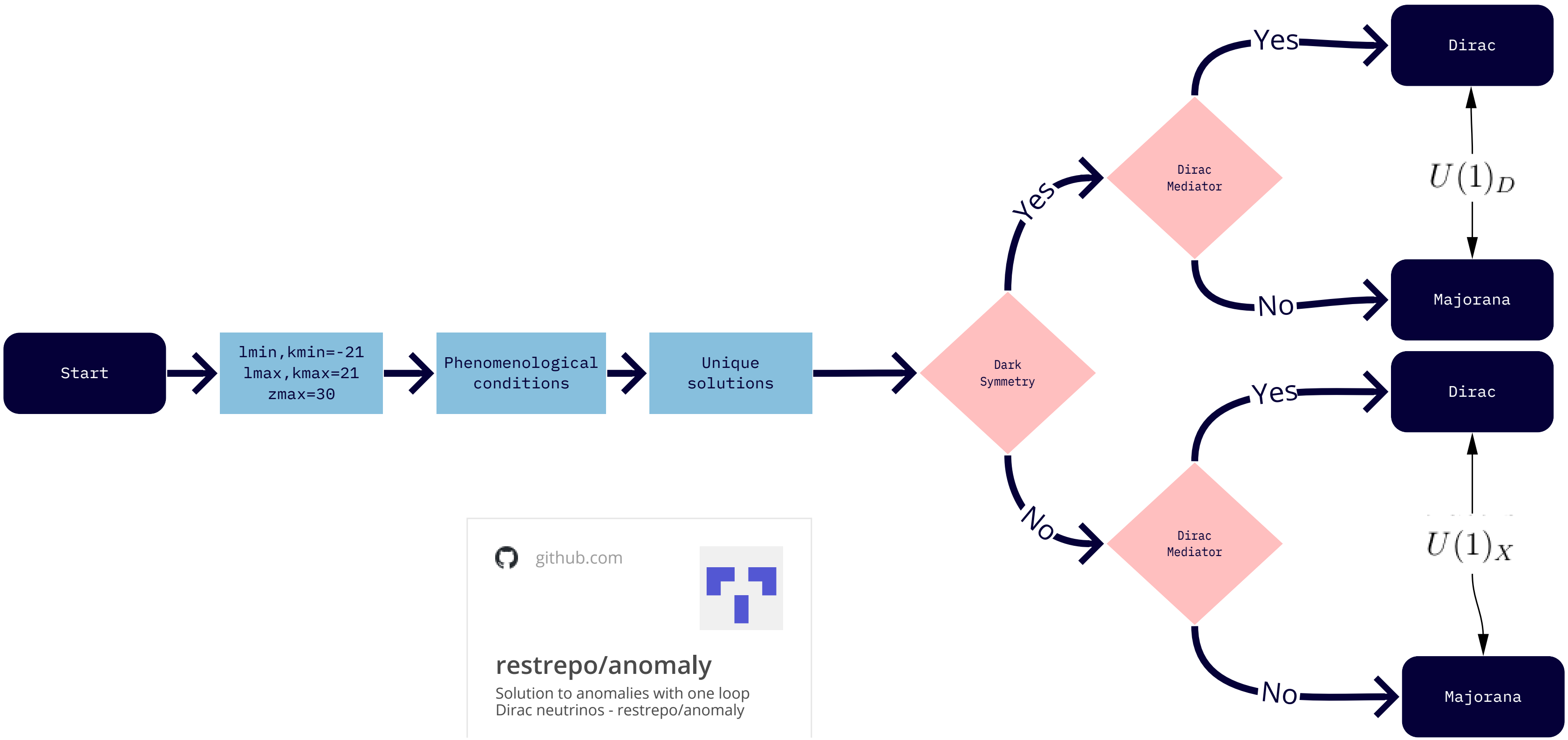
T1-2-B-D



$$D_M = (\nu, \nu, \cdots, r, \cdots),$$

$$X_M = (\nu, \nu, \cdots, m, m, m, r, \cdots),$$

T1-2-B-M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Dirac
Majorana
Dirac
Majorana
$U(1)_D$
$U(1)_X$
github.com
restrepo/anomaly
Solution to anomalies with one loop Dirac neutrinos - restrepo/anomaly

D-5

N	ℓ	k	Charges	GCD	Ref.	$U(1)_D$ Dirac	$U(1)_D$ Maj.	$U(1)_X$ Dirac	$U(1)_X$ Maj.
			Solutions			ν_R			
6	$(-1,1)$	$(-2,0)$	$(1,-2,-3,5,5,-6)$	1	[29]	$(\mathbf{5})$			
7	$(-1,1)$	$(-1,0,-1)$	$(1,2,2,-3,-3,-3,4)$	1	[29]	(-3)			
7	$(1,-1)$	$(-2,-5,-4)$	$(1,1,-3,-4,6,6,-7)$	1		$(1,6)$	(6)		
7	$(-1,1)$	$(-1,-2,-1)$	$(1,3,-4,5,-6,-6,7)$	1		(-6)	(-6)		
8	$(-1,-5,-3)$	$(-6,-4,-7)$	$(1,1,2,3,-4,-4,-5,6)$	1		$(-4,1)$	(-4)		
8	$(-1,2,-2)$	$(-7,4,0)$	$(1,2,2,2,-3,-5,-6,7)$	1		(2)			
8	$(1,2,1)$	$(-5,-10,-11)$	$(1,2,2,4,-5,-5,-7,8)$	1		(-5)			
8	$(1,-3,-2)$	$(-4,-9,-5,-3)$	$(1,3,3,3,-5,-7,-7,9)$	1				(-7)	
8	$(2,-1,0)$	$(-5,-9,-1,0)$	$(1,2,3,5,-6,-6,-9,10)$	1		(-6)	(-6)		
8	$(-1,0,-1)$	$(-2,1,-1)$	$(2,-5,-5,-5,7,8,8,-10)$	1	[22]	(8)		(8)	
8	$(0,-1,0)$	$(-1,-5,-1,1)$	$(1,1,1,-5,-7,11,11,-13)$	2				(11)	
8	$(-4,-1,0)$	$(-1,0,-8,8)$	$(3,-4,-5,8,8,-11,-12,13)$	1		(8)	(8)		
9	$(-2,0,2)$	$(-1,1,0,-1)$	$(1,1,-4,-5,9,9,9,-10,-10)$	1	[28]	$(\mathbf{9})$			(1)
9	$(-4,-5,3)$	$(-2,0,-1,-2)$	$(3,3,-4,5,5,-6,-8,-8,10)$	1		$(-8,3,5)$			
9	$(5,0,1)$	$(-1,-2,0,2)$	$(1,1,-5,-7,12,14,14,-15,-15)$	3		$(-15,1,14)$	(14)		
9	$(1,4,-1)$	$(-2,-5,-4,8)$	$(1,1,1,2,5,-6,-6,-6,8)$	1		$(-6,1)$			
9	$(-1,0,1)$	$(-1,1,-2,-1)$	$(1,-3,-3,-3,-5,8,8,8,-11)$	1		$(-3,8)$			
9	$(-7,-5,3)$	$(-6,-4,-5,2)$	$(1,-2,-2,-4,7,-9,11,11,-13)$	1		$(-2,11)$	(-2)		
9	$(3,-2,3)$	$(-2,-1,-2,4)$	$(4,4,4,-5,-9,-10,-10,11,11)$	1				$(-10,11)$	
9	$(-2,-6,5)$	$(-5,-1,-3,-6)$	$(1,1,2,2,3,-5,-6,-6,8)$	1		(2)	(-6)		
9	$(-2,3,2)$	$(-2,-9,-5,9)$	$(1,-2,3,4,6,-7,-7,-7,9)$	1		$(\mathbf{-7})$			
9	$(-3,-1,5)$	$(-9,3,-4,-1)$	$(1,2,-3,4,-5,-6,8,8,-9)$	1		(8)			
9	$(-8,-7,5)$	$(-9,3,-4,-2)$	$(1,-2,-2,-2,5,-7,8,9,-10)$	1		$(\mathbf{-2})$	$(\mathbf{-2})$		
9	$(-4,-1,-4)$	$(-3,-5,1,-4)$	$(2,-3,4,4,4,-6,-7,-7,9)$	1		(4)		(-7)	
9	$(-3,1,-2)$	$(-4,-3,-6,-3)$	$(2,-3,-3,-3,-5,7,7,8,-10)$	1		(-3)			
9	$(-3,6,5)$	$(-1,-6,2,-7)$	$(2,-3,-3,-3,-6,7,7,11,-12)$	2				(7)	
9	$(-4,2,-3)$	$(-2,-5,5,-6)$	$(1,2,-6,-6,-6,8,9,9,-11)$	2		(9)		$(\mathbf{9})$	
9	$(1,-1,2)$	$(-2,-1,0,-2)$	$(2,-3,4,6,6,-7,-10,-11,13)$	1		(6)	(6)		
9	$(-2,-1,-3)$	$(-1,-4,-3,-4)$	$(4,4,6,6,-7,-7,-7,-12,13)$	1				(6)	
9	$(-4,1,2)$	$(-1,-4,2,1)$	$(1,-2,-2,-3,-3,-3,14,20,-22)$	2			(-2)		
9	$(1,4,7)$	$(-1,6,4,-5)$	$(1,-2,-2,5,-7,-7,14,18,-20)$	4			(-2)		
9	$(8,-1,0)$	$(-1,-6,-3,-6)$	$(1,9,-12,-21,-21,24,24,24,-28)$	18			(24)	(-21)	

[22]arXiv:1812.05523 (DR,...) [28] arXiv:2008.08573 [Wong] [29] arXiv:2101.12138 (Ma)

D-5

			Solutions			ν_R $U(1)_D$		$U(1)_X$	
N	ℓ	k	Charges	GCD	Ref.	Dirac	Maj.	Dirac	Maj.
6	$(-1,1)$			1	[29]	**(5)**			
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7	$(1,-1)$			1		$(1,6)$	(6)		
7	$(-1,1)$			1		(-6)	(-6)		
8	$(-1,-5,-3)$			1		$(-4,1)$	(-4)		
8	$(-1,2,-2)$			1		(2)			
8	$(1,2,1)$			1		(-5)			
8	$(1,-3,-2)$			1				(-7)	
8	$(2,-1,0)$			1		(-6)	(-6)		
8	$(-1,0,-1)$			1	[22]	(8)		(8)	
8	$(0,-1,0)$			2				(11)	
8	$(-4,-1,0)$		)	1		(8)	(8)		
9	$(-2,0,2)$		0)	1	[28]	**(9)**			(1)
9	$(-4,-5,3)$		)	1		$(-8,3,5)$			
9	$(5,0,1)$		$-15)$	3		$(-15,1,14)$	(14)		
9	$(1,4,-1)$			1		$(-6,1)$			
9	$(-1,0,1)$		1)	1		$(-3,8)$			
9	$(-7,-5,3)$	$(-6,-4,-5,2)$	$(1,-2,-2,-4,7,-9,11,11,-13)$	1		$(-2,11)$	(-2)		
9	$(3,-2,3)$	$(-2,-1,-2,4)$	$(4,4,4,-5,-9,-10,-10,11,11)$	1				$(-10,11)$	
9	$(-2,-6,5)$	$(-5,-1,-3,-6)$	$(1,1,2,2,3,-5,-6,-6,8)$	1		(2)	(-6)		
9	$(-2,3,2)$	$(-2,-9,-5,9)$	$(1,-2,3,4,6,-7,-7,-7,9)$	1		**(−7)**			
9	$(-3,-1,5)$	$(-9,3,-4,-1)$	$(1,2,-3,4,-5,-6,8,8,-9)$	1		(8)			
9	$(-8,-7,5)$	$(-9,3,-4,-2)$	$(1,-2,-2,-2,5,-7,8,9,-10)$	1		**(−2)**	**(−2)**		
9	$(-4,-1,-4)$	$(-3,-5,1,-4)$	$(2,-3,4,4,4,-6,-7,-7,9)$	1		(4)		(-7)	
9	$(-3,1,-2)$	$(-4,-3,-6,-3)$	$(2,-3,-3,-3,-5,7,7,8,-10)$	1		(-3)			
9	$(-3,6,5)$	$(-1,-6,2,-7)$	$(2,-3,-3,-3,-6,7,7,11,-12)$	2				(7)	
9	$(-4,2,-3)$	$(-2,-5,5,-6)$	$(1,2,-6,-6,-6,8,9,9,-11)$	2		(9)		**(9)**	
9	$(1,-1,2)$	$(-2,-1,0,-2)$	$(2,-3,4,6,6,-7,-10,-11,13)$	1		(6)	(6)		
9	$(-2,-1,-3)$	$(-1,-4,-3,-4)$	$(4,4,6,6,-7,-7,-7,-12,13)$	1				(6)	
9	$(-4,1,2)$	$(-1,-4,2,1)$	$(1,-2,-2,-3,-3,-3,14,20,-22)$	2			(-2)		
9	$(1,4,7)$	$(-1,6,4,-5)$	$(1,-2,-2,5,-7,-7,14,18,-20)$	4			(-2)		
9	$(8,-1,0)$	$(-1,-6,-3,-6)$	$(1,9,-12,-21,-21,24,24,24,-28)$	18			(24)	(-21)	

$\nu_{R_i} \to -7$ $S \to 7$

$(2,-9) \to \psi_{D1}$

$(3,4) \to \psi_{D2}$

$(1,6) \to \psi_{D3}$

$\langle S \rangle$

Z_7

3 multicomponent
Dirac-fermion DM

[22]arXiv:1812.05523 (DR,...)   [28] arXiv:2008.08573 (Wong)   [29] arXiv:2101.12138 (Ma)

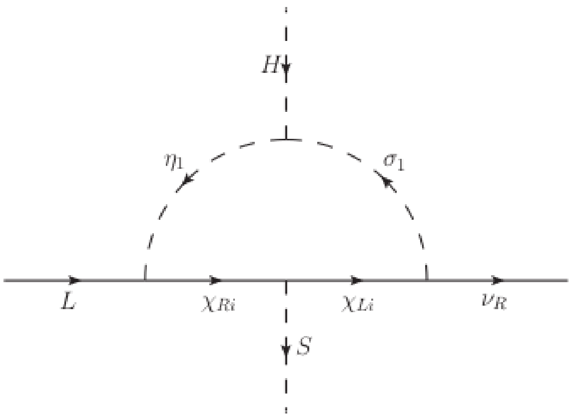
D-6

			Solutions			ν_R			
						$U(1)_D$		$U(1)_X$	
N	ℓ	k	Charges	GCD	Refs.	Dirac	Maj.	Dirac	Maj.
6	(−1, −2)	(−1, 2)	(1, 1, 1, −4, −4, 5)	1	[25, 29]		(−4)		
6	(1, −2)	(−4, 1)	(1, −4, −4, 9, 9, −11)	3			(−4)		
6	(2, 1)	(−2, −1, 0)	(1, −4, −8, 14, 14, −17)	1		(14)			
7	(1, −2, 1)	(−9, 6, 3)	(1, 1, −4, −4, 7, 8, −9)	1			(−4)		
7	(3, 1)	(−1, −5, 7)	(2, 2, −4, 7, −8, −8, 9)	1		(2)	(−8)		
7	(−3, −1)	(−2, −3, 1)	(3, 3, 3, −5, −5, −7, 8)	1				**(−5)**	
7	(−3, −4)	(−5, −7, −4)	(4, 4, 5, −7, −8, −9, 11)	2		(4)			
8	(1, 2, −2)	(−7, 3, 0)	(1, −7, −7, 17, 17, 19, −20, −20)	6		(−20)			
8	(1, 2, 1)	(−5, −10, −11)	(1, 2, 2, 4, −5, −5, −7, 8)	1		(2)			
8	(3, 2, −2)	(−4, −3, 4, 5)	(1, 1, −4, −4, −4, 12, 15, −17)	4			(−4)		
8	(−1, 2, 4)	(−4, 2, −3, 0)	(4, 4, 7, 14, −16, −16, −22, 25)	4		(4)	(−16)		
8	(−10, −5, −15)	(−10, −12, 12)	(5, 5, 5, −17, −27, −27, 28, 28)	100				(−27, 28)	
8	(3, 1, −3)	(−12, −14, −4)	(1, −3, −3, 5, −11, 12, 12, −13)	1		(12)	(12)		
8	(−2, 0, −1)	(−4, −3, −2)	(1, 2, 2, −8, −8, 12, 15, −16)	2		(2)	(−8)		
8	(−2, −5, −4)	(−3, −5, −2, 0)	(2, −3, 7, −8, −8, 11, 14, −15)	2		(−8)	(−8)		
8	(0, −9, 4)	(−4, −6, −7, 4)	(1, −2, −4, −4, −4, 15, 22, −24)	2			(−4)		
8	(−1, 0, −1)	(−9, 1, −1)	(3, 3, 3, −7, 17, −23, −23, 27)	4				(−23)	
8	(0, 1, 0)	(−1, −4, 3, −4)	(1, −5, −11, 15, −16, 20, 20, −24)	2		(20)	(20)		
9	(3, −4, 5)	(−4, −3, 1, −3)	(1, −3, 8, 8, 8, −12, −12, −17, 19)	4				**(−12)**	**(−12)**
9	(−2, 6, −4)	(−8, −7, 6, 3)	(3, 3, 3, 5, −16, 22, −23, −23, 26)	20				(−23)	(−23)
9	(−9, 2, 3)	(−1, −7, 6, −9)	(1, −4, 5, 5, −9, −9, −9, 10, 10)	3					(5)
9	(1, 4, −1)	(−2, −5, −4, 8)	(1, 1, 1, 2, 5, −6, −6, −6, 8)	1		(−6)			
9	(−9, 6, 7)	(−2, 4, 3, 1)	(1, 1, 1, 4, −9, −10, −10, 11, 11)	3		(−10)		(−10, 11)	
9	(−3, −2, −4)	(−1, −9, −7, 4)	(3, 3, 3, −4, −4, 8, −11, −11, 13)	2		(−4)		(−11, −4)	
9	(−3, 0, −1)	(−4, −1, −6, −4)	(2, −3, −3, −8, −9, 12, 12, 14, −17)	3		(12)	(12)		
9	(4, 6, 4)	(−3, −4, −3, 5)	(3, 4, −10, −10, −10, 12, 12, 13, −14)	2		(12)		(12)	
9	(2, 7, −4)	(−5, −6, 3, −6)	(1, 1, 1, −4, −4, −11, 18, 26, −28)	50			(−4)		
9	(−3, −6, 2)	(−5, 1, 7, −8)	(3, −4, −4, −9, −13, 16, 16, 16, −21)	90			(16)	(−4)	
9	(5, −3, 7)	(−1, 3, 2, −4)	(5, 7, 7, −8, −15, −15, −15, 17, 17)	22				(7, 17)	
9	(−6, −3, 5)	(−4, −2, −6, 8)	(4, 7, −8, 9, −16, −16, −16, 18, 18)	4		(18)	(−16)	(18)	
9	(−2, 1, 2)	(−6, −8, 7, 6)	(4, 4, 4, 5, −6, −6, −6, −10, 11)	2				(4, −6)	
9	(−9, 2, −3)	(−2, −8, 5, 2)	(1, −2, −2, −4, −4, −4, 17, 27, −29)	81			(−4)		
9	(−5, −4, 0)	(−2, −1, −4, 4)	(1, −4, −4, −4, 12, −14, 15, 18, −20)	2			(−4)		
9	(2, 4, −2)	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]		

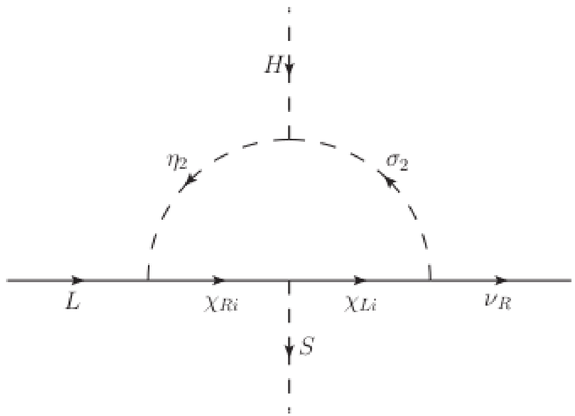
[25] arXiv:1904.07407 (...,Saad) [29] arXiv:2101.12138 (Ma)

Field	$\nu_{R\alpha}$	χ_{R1}	$(\chi_{L1})^\dagger$	η_1	σ_1	χ_{R2}	$(\chi_{L2})^\dagger$	η_2	σ_2	S
`T1-3-E-D-(I-II)`	5	-2	-3	-2	-2	1	-6	1	1	-5
`T1-3-E-D-(I-I)`	5	-2	-3	-2	-2	1	-6	-2	-2	-5
`T1-3-E-D-(II-II)`	5	-2	-3	1	1	1	-6	1	1	-5

Table 3: Possible charge assignments to obtain a light Dirac neutrino mass matrix of rank 2, for the first solution of Table 1, i.e., $(1, -2, -3, 5, 5, -6)$. In the row `T1-3-E-D-(I-I)` [`T1-3-E-D-(II-II)`] the second [first] heavy Dirac fermion, from $(\chi_{L2})^\dagger \chi_{R2} S^*$ [$(\chi_{L1})^\dagger \chi_{R1} S^*$], does not participate directly in the neutrino-loop.



`T1-3-E-D-I` ($i = 1$) or `T1-3-E-D-II` ($i = 2$)



T1-3-E-D-I ($i = 1$) or T1-3-E-D-II ($i = 2$)

Conclusions

Dark symmetry for all $\longrightarrow$ simple Diophantine equations

Find the full set of solutions for any phenomenological problem

Extend to multiplets or non-universal D-charges