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Gravitino dark matter and observable gravity waves

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Shifted μ hybrid inflation in SUSY $SU(4)_c \times SU(2)_L \times SU(2)_R$ model
(or 4-2-2 model)

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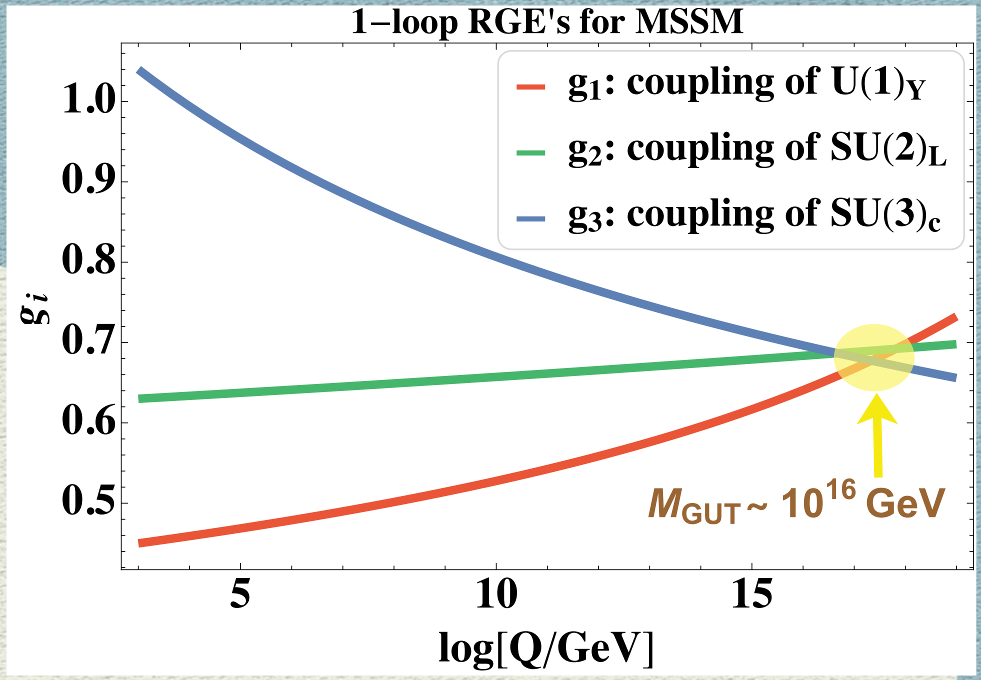
BSM-2021, Cairo Egypt

Grand unified theories (GUTs) are gateway to new physics BSM



- unification of SM / MSSM gauge couplings
- unification of matter/quark-lepton multiplets
- $b - \tau$ Yukawa unification in realistic models
- electric charge quantisation, magnetic monopoles
- prediction for proton decay
- emergent see-saw physics, neutrino oscillations
- baryogengesis / leptogenesis
- inflation / gravity waves, $\delta\rho/\rho$ and cosmic strings

well-motivated particle physics models

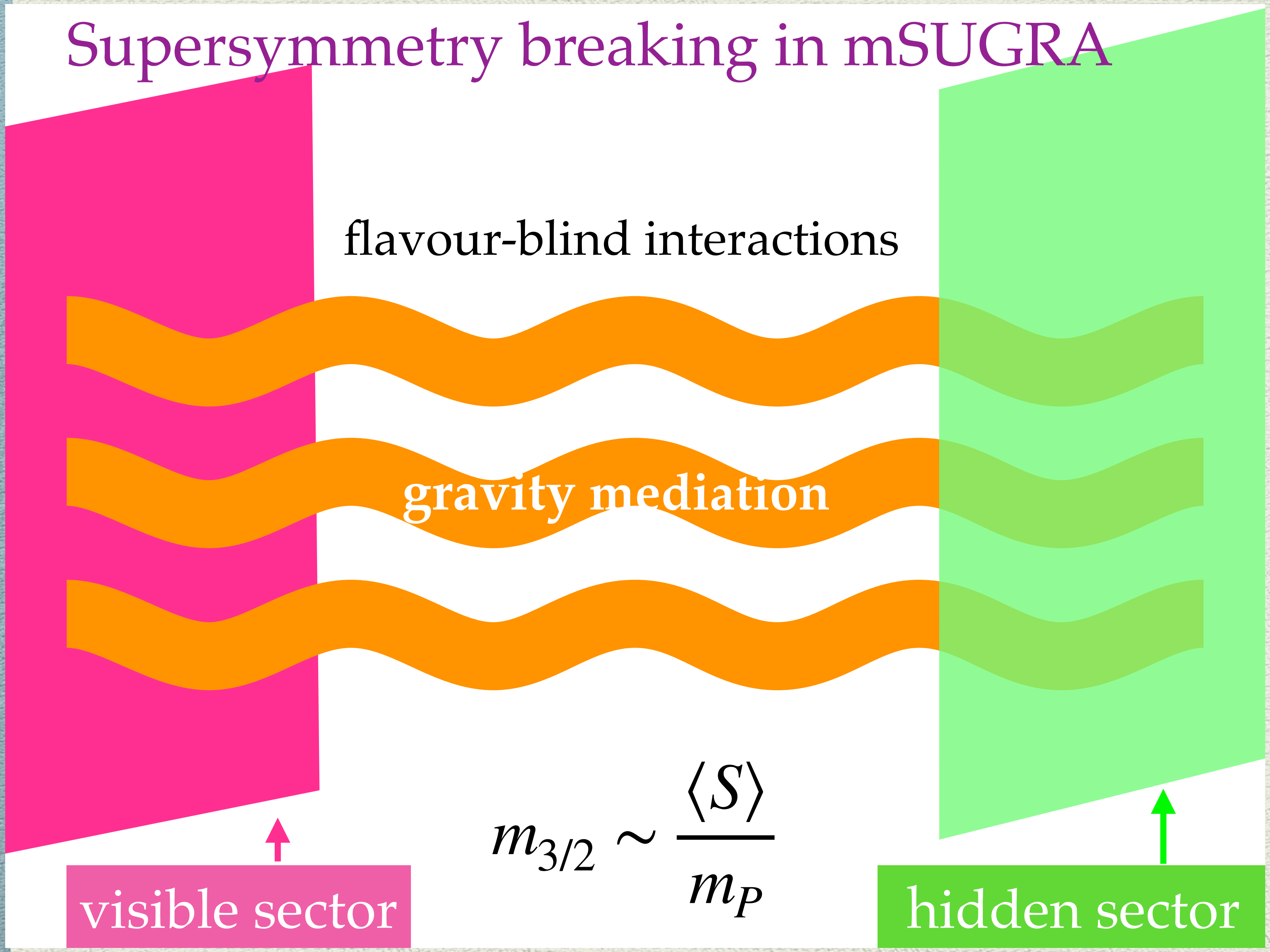
Minimal supersymmetric standard model (MSSM)

superfield		spin 0	spin 1/2	$SU(3)_c \times SU(2)_L \times U(1)_Y$
squarks, quarks	$\widehat{Q}$	$\widetilde{q}_L \equiv (\widetilde{u}_L \;\; \widetilde{d}_L)$	$q_L \equiv (u_L \;\; d_L)$	$(\mathbf{3},\, \mathbf{2},\, \frac{1}{6})$
(×3 families)	$\widehat{U}^c$	$\widetilde{u}^*_R$	u^c_R	$(\overline{\mathbf{3}},\, \mathbf{1},\, -\frac{2}{3})$
	$\widehat{D}^c$	$\widetilde{d}^*_R$	d^c_R	$(\overline{\mathbf{3}},\, \mathbf{1},\, \frac{1}{3})$
sleptons, leptons	$\widehat{L}$	$(\widetilde{\nu} \;\; \widetilde{e}_L)$	$l \equiv (\nu \;\; e_L)$	$(\mathbf{1},\, \mathbf{2},\, -\frac{1}{2})$
(×3 families)	$\widehat{E}^c$	$\widetilde{e}^*_R$	e^c_R	$(\mathbf{1},\, \mathbf{1},\, 1)$
Higgs, Higgsinos	$\widehat{H}_u$	$(H^+_u \;\; H^0_u)$	$(\widetilde{H}^+_u \;\; \widetilde{H}^0_u)$	$(\mathbf{1},\, \mathbf{2},\, +\frac{1}{2})$
	$\widehat{H}_d$	$(H^0_d \;\; H^-_d)$	$(\widetilde{H}^0_d \;\; \widetilde{H}^-_d)$	$(\mathbf{1},\, \mathbf{2},\, -\frac{1}{2})$

Non-gauge (matter) degrees of freedom of MSSM. The SM fields, quarks, leptons, and Higgs are shown in green and their respective superpartners squarks, sleptons, and higgsinos are shown in red.

prefix \`s' → scalar superpartner, suffix \`ino' → fermionic superpartners

Minimal supergravity model (mSUGRA): Planck-scale-mediated SUSY breaking (PMSB) or gravity-mediated scenario



If SUSY is broken in the hidden sector by a vev$\langle S \rangle$

then the soft-terms in the visible sector are $m_{\text{soft}} \sim \langle S/m_P \rangle$

μ-problem of MSSM

- $W_{MSSM} \supset \widehat{U}^c y_u \widehat{Q}\,\widehat{H}_u - \widehat{D}^c y_d \widehat{Q}\,\widehat{H}_d - \widehat{E}^c y_e \widehat{L}\,\widehat{H}_u + \mu \widehat{H}_u \widehat{H}_d$
- μ and soft-SUSY-breaking terms ~ EW scale<<Planck scale
- forbid the direct MSSM μ term, invoke it later as a coupling of some scalar field S to Higgs $\lambda S H_u H_d$
- μ is linked to mechanism of SUSY breaking. The vev is determined by minimising a potential that depends on soft-SUSY breaking terms $< S > \propto m_{3/2}$
- explain why $m_{soft} << m_P$ then we can explain why μ is of the same order $\mu = \frac{\lambda}{\kappa} m_{3/2} \equiv \gamma m_{3/2}$

4-2-2

Superfields	$4_c \times 2_L \times 2_R$	$3_c \times 2_L \times 1_Y$	$q(R)$
F_i	$(4,\ 2,\ 1)$	$Q_{ia}(3,\ 2,\ 1/6)$ $L_i(1,\ 2,\ -1/2)$	1
F_i^c	$(\bar{4},\ 1,\ 2)$	$u_{ia}^c(\bar{3},\ 1,\ -2/3)$ $d_{ia}^c(\bar{3},\ 1,\ 1/3)$ $\nu_i^c\ (1,\ 1,\ 0)$ $e_i^c\ (1,\ 1,\ 1)$	1
H^c	$(\bar{4},\ 1,\ 2)$	$u_{Ha}^c(\bar{3},\ 1,\ -2/3)$ $d_{Ha}^c(\bar{3},\ 1,\ 1/3)$ $\nu_H^c\ (1,\ 1,\ 0)$ $e_H^c\ (1,\ 1,\ 1)$	0
$\overline{H^c}$	$(4,\ 1,\ 2)$	$\overline{u_{Ha}^c}(3,\ 1,\ 2/3)$ $\overline{d_{Ha}^c}(3,\ 1,\ -1/3)$ $\overline{\nu_H^c}\ (1,\ 1,\ 0)$ $\overline{e_H^c}\ (1,\ 1,\ -1)$	0
S	$(1,\ 1,\ 1)$	$S(1,\ 1,\ 0)$	2
G	$(6,\ 1,\ 1)$	$g_a(3,\ 1,\ -1/3)$ $g_a^c(\bar{3},\ 1,\ 1/3)$	2
h	$(1,\ 2,\ 2)$	$h_u\ (1,\ 2,\ 1/2)$ $h_d\ (1,\ 2,\ -1/2)$	0

maximal subgroup of $SO(10)$

ν_R

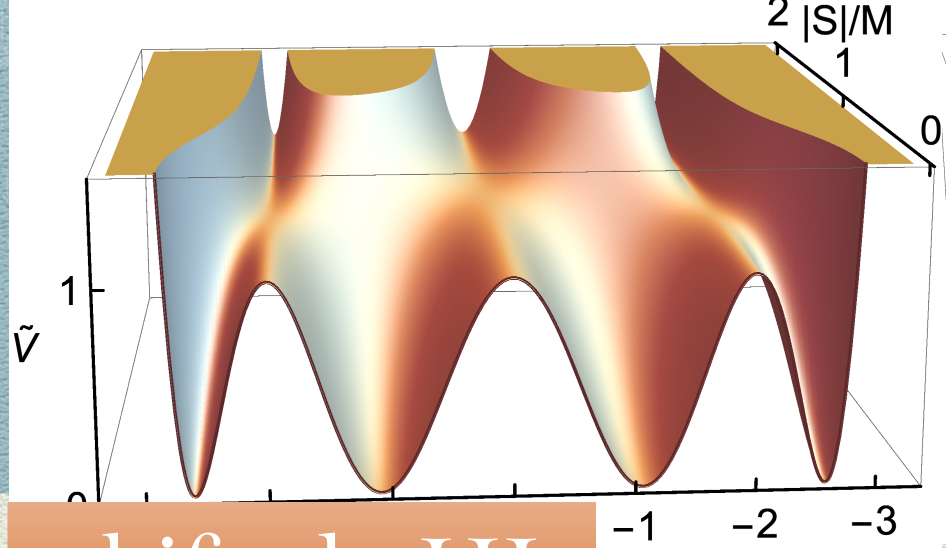
charge quantisation

B-L as a local symmetry

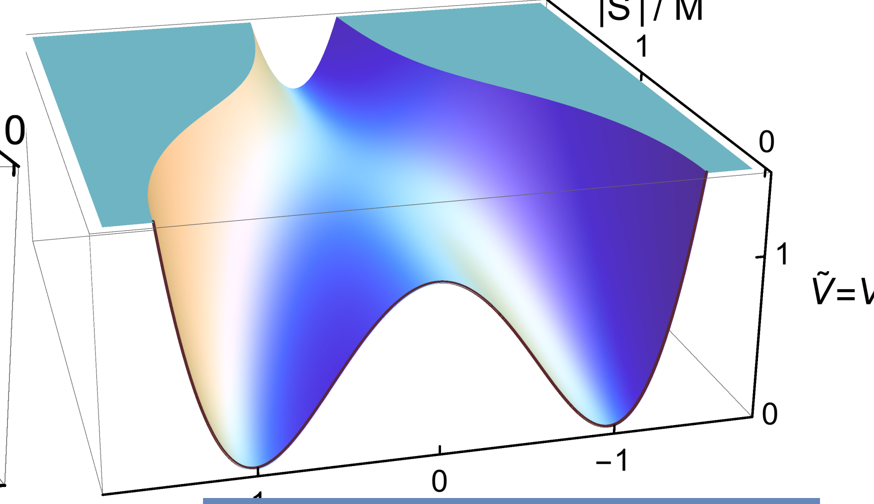
cold & hot DM

string inspired

Shifted μHI with minimal Kähler potential



shifted μHI



standard μHI

Superpotential (standard): $W_{\rm st} = \kappa S(\Phi\overline{\Phi} - M^2) + \lambda S H_u H_d,$

Kähler potential: $K_c = |S|^2 + |\Phi|^2 + |\overline{\Phi}|^2 + |H_u|^2 + |H_d|^2$

Superpotential(shifted): $W_{\rm sh} \supset \kappa S(\overline{H}^c H^c - M^2) + \lambda S \mathfrak{H}^2 - \xi \dfrac{\kappa S(\overline{H}^c H^c)^2}{M^2}$

Scalar potential:

$$V(x) \simeq \kappa^2 M^4 \left(1 + \underbrace{\mathcal{N}\frac{k^2}{8\pi^2}F_\kappa(x) + \frac{\lambda^2}{4\pi^2}F_\lambda(y)}_{\text{radiative}} + \underbrace{\frac{1}{2}\left(\frac{M}{m_P}\right)x^4}_{\text{SUGRA}} + \underbrace{a\frac{m_{3/2}}{\kappa M}x + \left(\frac{m_S}{\kappa M}\right)^2 x^2}_{\text{soft}} \right)$$

gauge fermion of supergravity

spin-3/2 super-partner of graviton (spin-2)

Gravitino problem [1]

- the interacts `gravitationally' → decays late; or
- if gravitino is lightest supersymmetric particle (LSP) ; then next-to-LSP decays into gravitino very late

disastrous effect on BBN→bound on reheat temperature

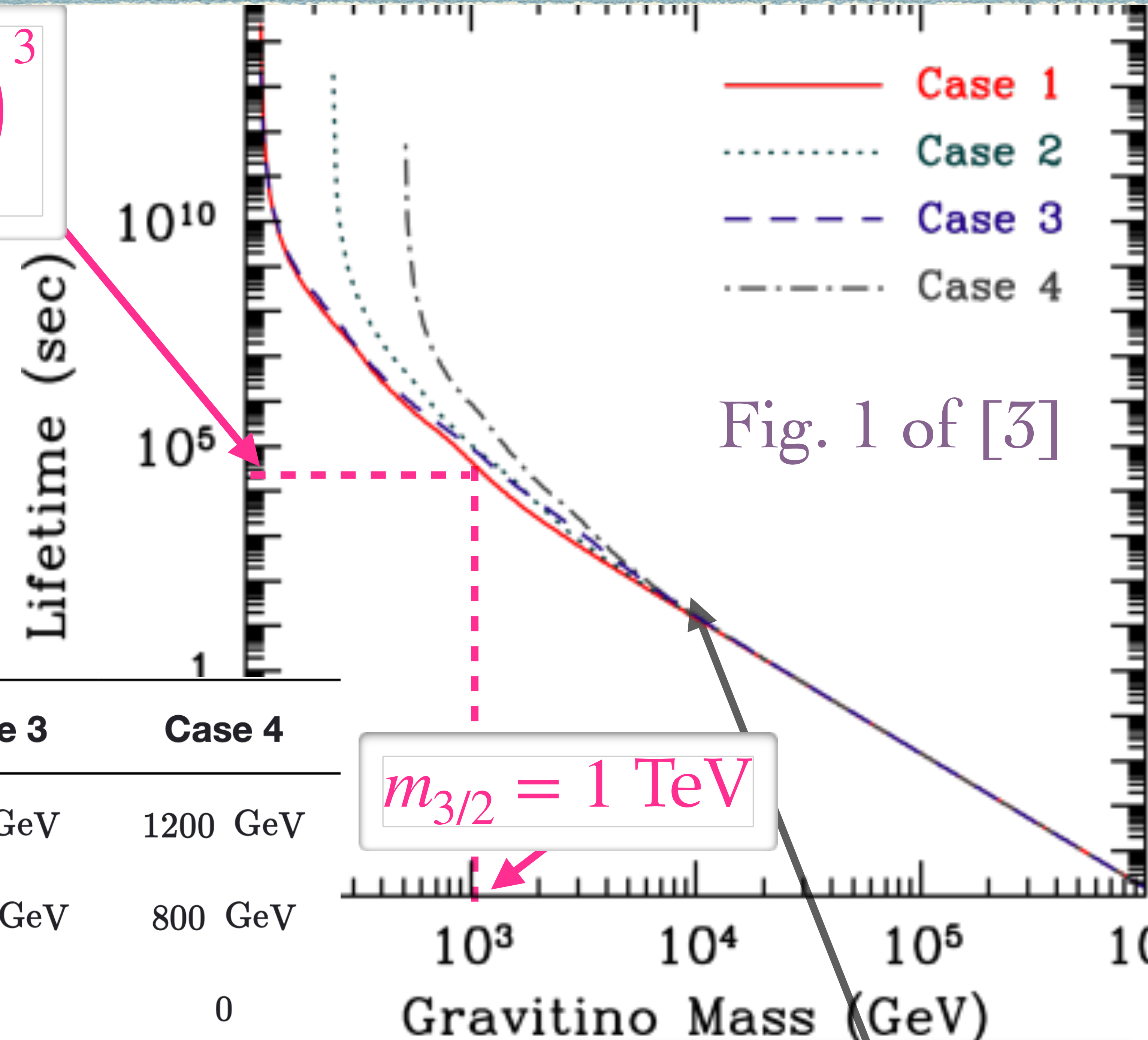
[2]

I. **stable LSP gravitino**

II. **unstable long-lived gravitino** $\Longrightarrow m_{3/2} < 25$ TeV

III. **unstable short-lived gravitino** $\Longrightarrow m_{3/2} > 25$ TeV

$$\tau_{3/2} \simeq 1.6 \times 10^4 \left(\frac{1\ \text{TeV}}{m_{3/2}} \right)^3$$



mSUGRA

	Case 1	Case 2	Case 3	Case 4
$m_{1/2}$	300 GeV	600 GeV	300 GeV	1200 GeV
m_0	141 GeV	218 GeV	2397 GeV	800 GeV
A_0	0	0	0	0
$\tan\beta$	30	30	30	45
μ_H	389 GeV	726 GeV	231 GeV	−1315 GeV
$m_{\chi_1^0}$	117 GeV	244 GeV	116 GeV	509 GeV
$\Omega_{\mathrm{LSP}}^{(\mathrm{thermal})} h^2$	0.111	0.110	0.106	0.111

life time insensitive to mass of final state particles

BBN constraints

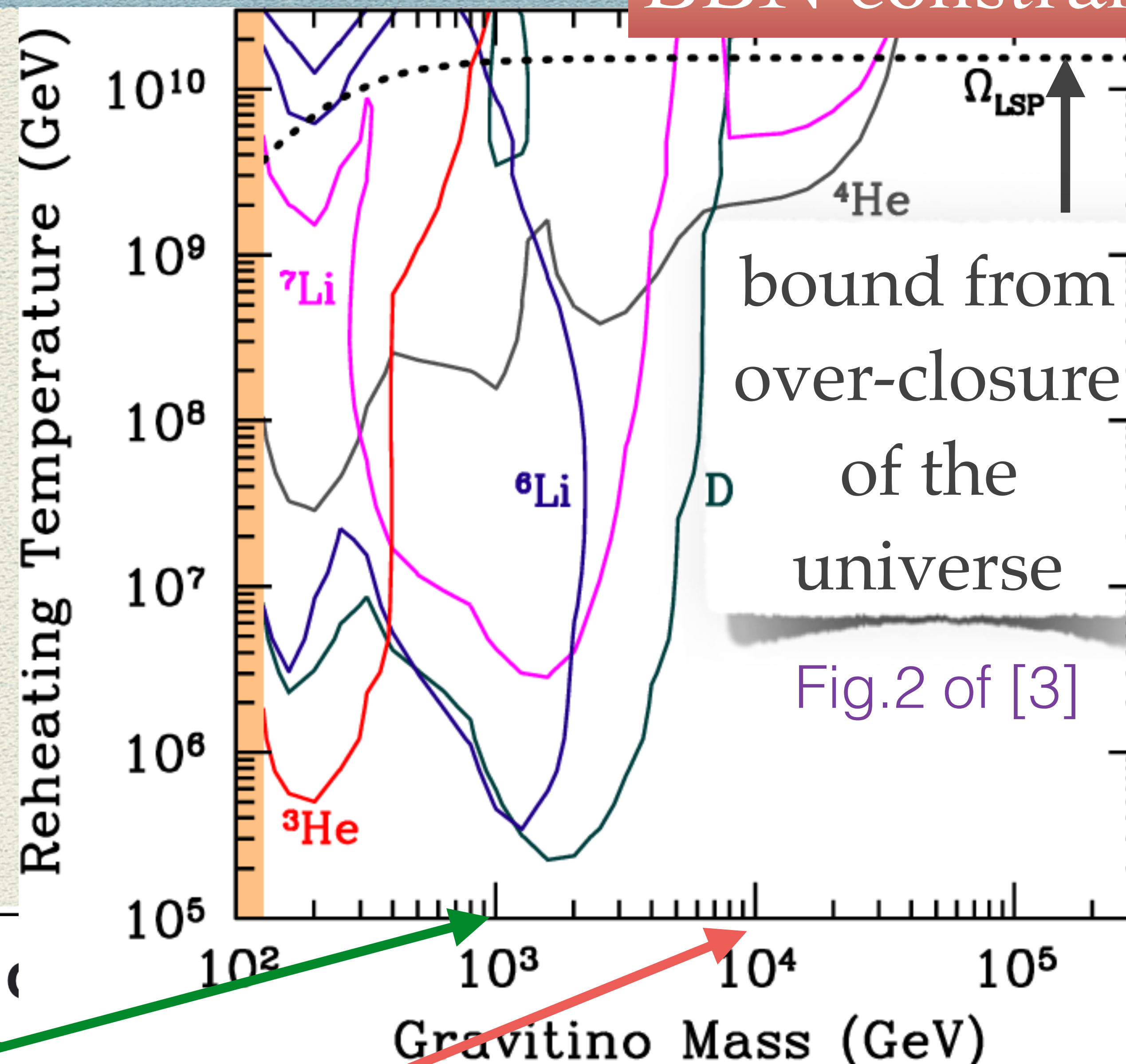
BBN limits:

★ $T_r \lesssim 3 \times (10^5 - 10^6)$ GeV

for $m_{3/2} \sim 1$ TeV

★ $T_r \lesssim 2 \times 10^9$ GeV

$m_{3/2} \sim 10$ TeV



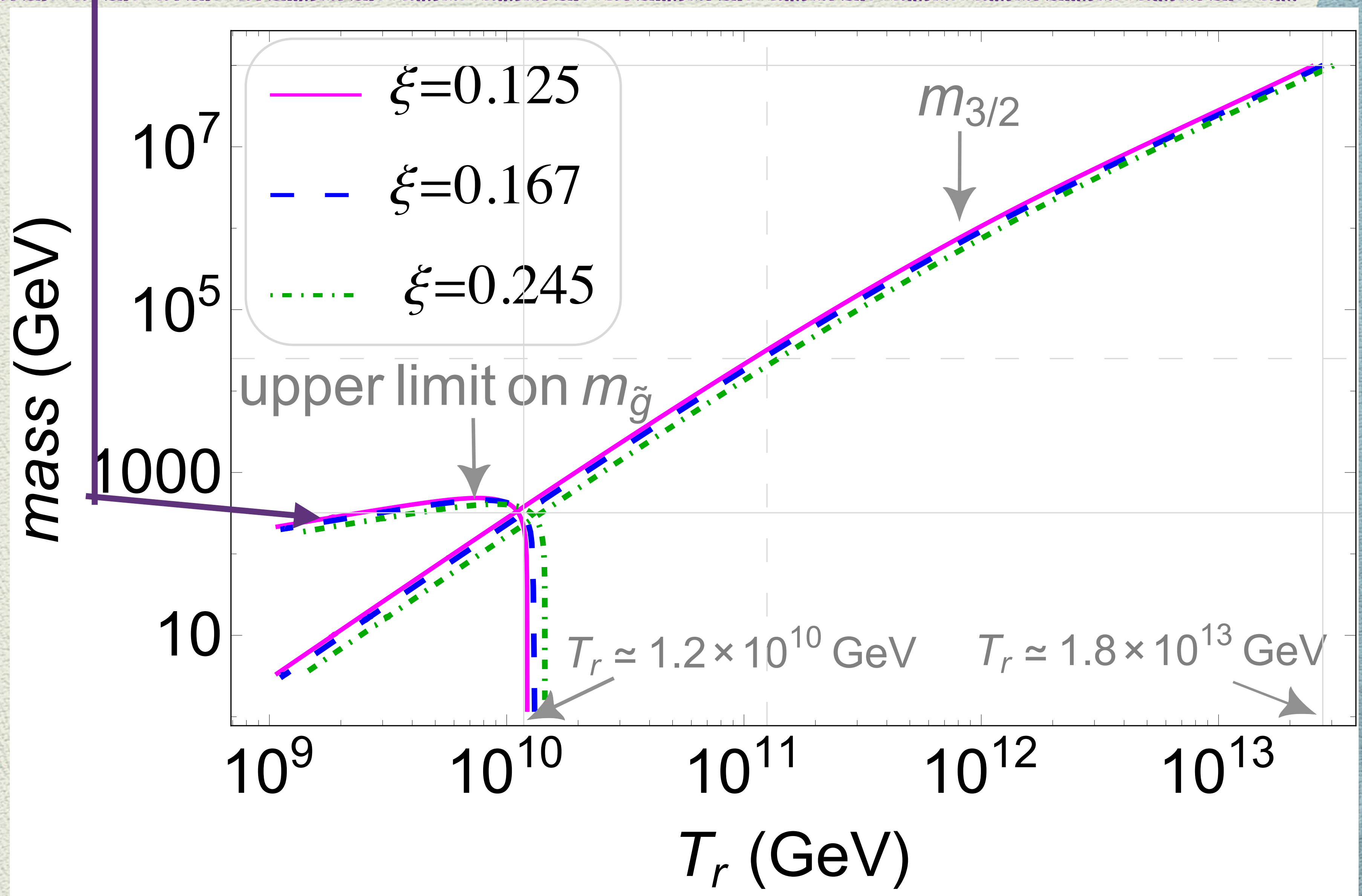
$m_{3/2}$	Case 1	Case 2		
300 GeV	1×10^6 (^3He)	4×10^5 (^3He)	1×10^6 (^3He)	–
1 TeV	5×10^5 (^6Li)	9×10^5 (^6Li)	3×10^5 (^6Li)	3×10^6 (^6Li)
3 TeV	5×10^5 (D)	4×10^5 (D)	2×10^5 (D)	5×10^5 (D)
10 TeV	2×10^9 (^4He)	2×10^9 (^4He)	2×10^9 (^4He)	2×10^9 (^4He)
30 TeV	9×10^9 (^4He)	8×10^9 (^4He)	7×10^9 (^4He)	8×10^9 (^4He)

Shifted μHI with minimal Kähler potential

$$\Omega_{DM}h^2 = 0.12$$ [5]

$$\Omega_{3/2}h^2 = 0.08\left(\frac{T_r}{10^{10}\mathrm{GeV}}\right)\left(\frac{m_{3/2}}{1\ \mathrm{TeV}}\right)\left(1+\frac{m_{\tilde{g}}^2}{3m_{3/2}^2}\right)$$ [4]

- The intersection point $m_{3/2} \simeq 325$ GeV
- Maximum value of the gluino mass in the region where $m_{3/2}$ is smaller than the upper limit on $m_{\tilde{g}}$ is $m_{\tilde{g}} \sim 500$ GeV, which is lower than the lower LHC bound on the gluino ($m_{\tilde{g}} > 1$ TeV).
- Gravitino LSP scenario is inconsistent



Results of Shifted μHI with minimal Kähler potential

I. A stable LSP gravitino, ✗

II. an unstable long-lived gravitino, ✗

III(a). an unstable short-lived gravitino ✗

III(b). Unstable short-lived gravitino; with LSP neutralino in thermal equilibrium ✓ [2]

$$m_{3/2} \gtrsim 10^8 \text{ GeV}\left(\frac{m_{\tilde{\chi}_1^0}}{2 \text{ TeV}}\right)^{2/3}$$

split supersymmetry

Shifted μ-HI with nonminimal Kähler potential

- $K = K_c + \kappa_S \frac{|S|^4}{4m_P^2} + \kappa_{SS}\frac{|S|^6}{6m_P^4} + \ldots.$

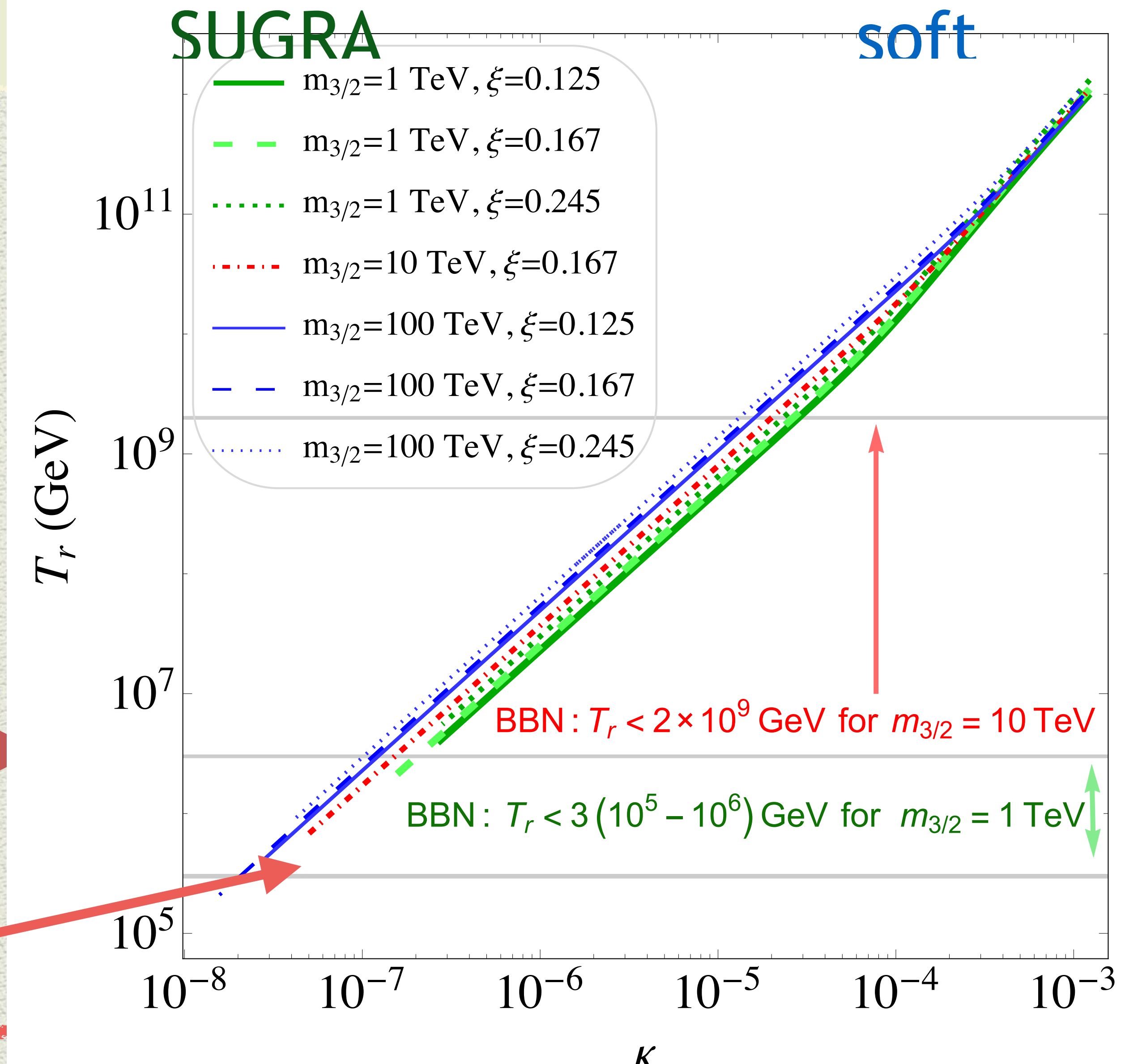
$$\gamma_S = 1 + 2\kappa_S^2 - \frac{7\kappa_S}{2} - 3\kappa_{SS}$$

$$V(x) \simeq \kappa^2 M^4\left(1 + \mathcal{N}\frac{k^2}{8\pi^2}F_\kappa(x) + \frac{\lambda^2}{4\pi^2}F_\lambda(y) + \frac{\gamma_S}{2}\left(\frac{M}{m_P}\right)x^4 + \kappa_S\left(\frac{M}{m_P}\right)^2 x^2 + a\frac{m_{3/2}}{\kappa M}x + \left(\frac{m_S}{\kappa M}\right)^2 x^2\right)$$

radiative — SUGRA — soft

modified SUGRA corrections

nonminimal Kähler potential decreases T_r



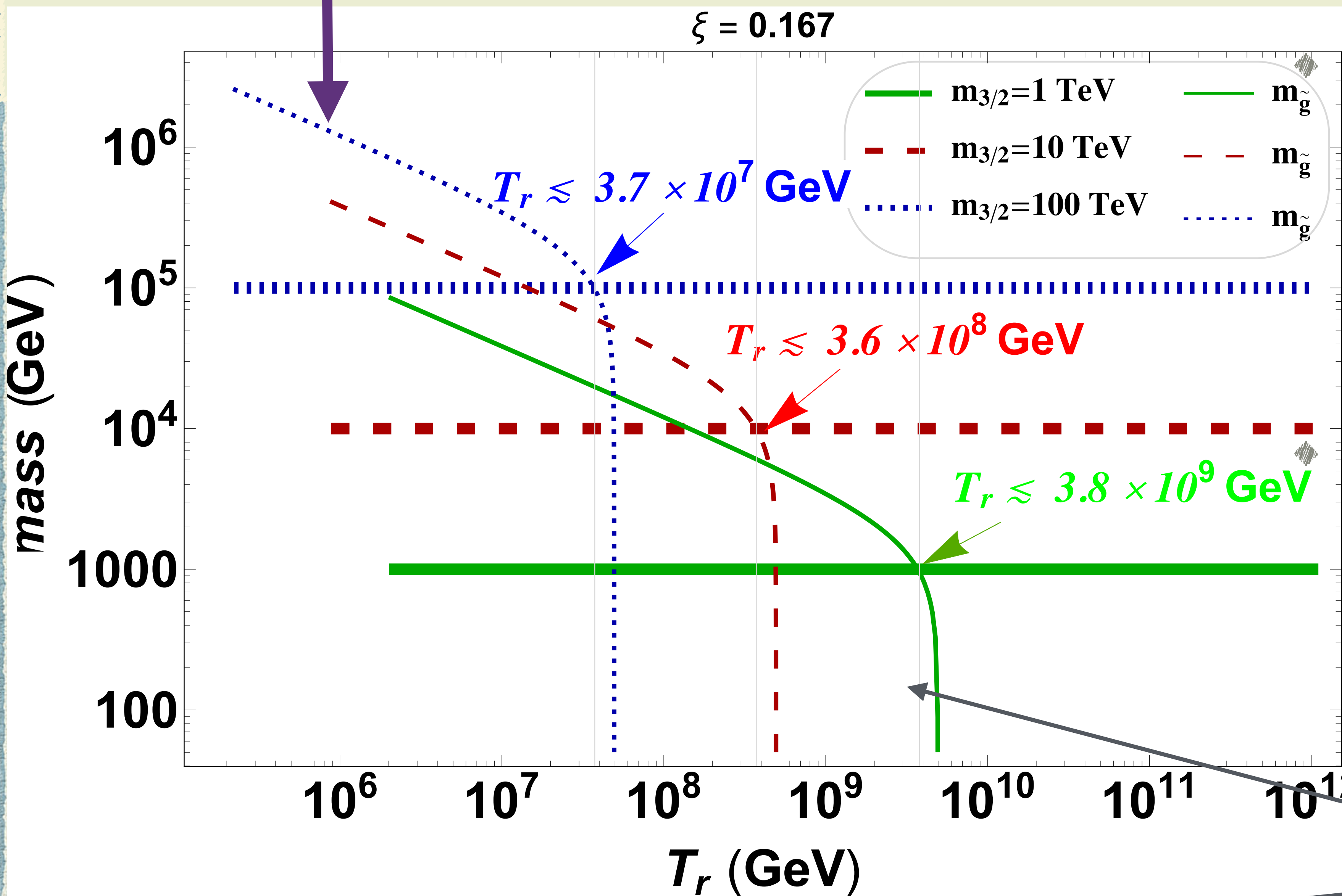
$T_r \gtrsim 2\times10^6,\ 7\times10^5,$ or 2×10^5 GeV for $m_{3/2} = 1,\ 10,$ or 100 TeV.

I. Stable LSP gravitino

nonminimal Kähler

$$\Omega_{3/2}h^2 = 0.08\left(\frac{T_r}{10^{10}\text{GeV}}\right)\left(\frac{m_{3/2}}{1\ \text{TeV}}\right)\left(1+\frac{m_{\tilde{g}}^2}{3m_{3/2}^2}\right)$$ [4]

$$\Omega_{DM}h^2 = 0.12$$ [5]



These upper bounds on T_r are consistent with the lower bounds on slide 13

Our assumption for a gravitino LSP holds for T_r values below these intersection points.

grey vertical lines

$$T_r \lesssim 3.7\times(10^9,\ 10^8,\ 10^7)\ \text{GeV for } m_{3/2} = 1,\ 10,\ 100\ \text{TeV}$$

LSP gravitino scenario can be consistently realised nonminimal Kähler

$m_{3/2} < 25$ TeV $\quad \tau_{3/2} \gtrsim 1s$

II. Unstable long-lived gravitino

BBN constraints

- Gravitino decay after BBN$\Longrightarrow$ adverse effect on light nuclei abundance$\Longrightarrow$ ruin success of BBN
- from inflationary constraints $T_r < 2 \times 10^6$ GeV ★ and 7×10^5 GeV ★ for 1 TeV and 10 TeV gravitino masses see ★ ★ on slide 13
- compare with BBN bounds see ★ ★ on slide 10
- This scenario, is marginally ruled out for 1 TeV gravitino ✗
- but sits comfortably within BBN for 10 TeV gravitino mass ✓

$m_{3/2} > 25$ TeV $\quad \tau_{3/2} \lesssim 1s$

nonminimal Kähler

III. Unstable short-lived gravitino

LSP neutralino constraints

$m_{\tilde{\chi}_1^0} \gtrsim 18$ GeV [6]

- gravitino decays into LSP neutralino
$$\Omega_{\tilde{\chi}_1^0} h^2 \simeq 2.8 \times 10^{11} \times Y_{3/2} \left(\frac{m_{\tilde{\chi}_1^0}}{1 \text{ TeV}} \right)$$

- where gravitino yield: $Y_{3/2} \simeq 2.3 \times 10^{-12} \left(\frac{T_r}{10^{10} \text{ GeV}} \right)$ [2]

- LSP neutralino density should not exceed the observed DM relic density $\Omega_{DM} h^2 = 0.12$ [5]

- upper bound on LSP neutralino
$m_{\tilde{\chi}_1^0} \lesssim (18 - 10^6)$ GeV for 10^{11} GeV $\gtrsim T_r \gtrsim 6 \times 10^5$ GeV

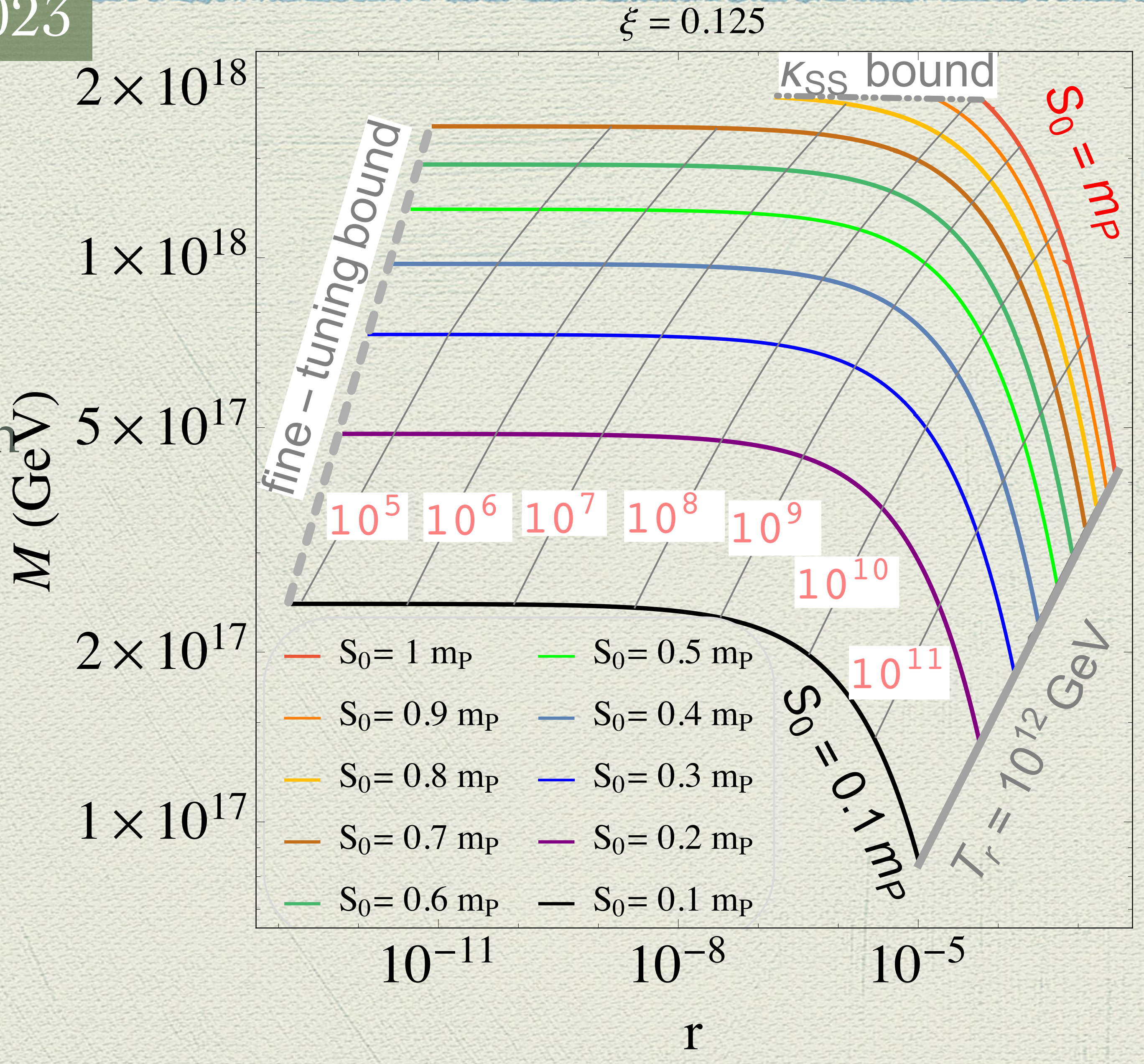
- nonLSP $m_{3/2} \sim 100$ TeV holds for 10^5 GeV $\lesssim T_r \lesssim 10^{11}$ GeV ✓

PGW are gravitational waves observed cosmic microwave background. Tensor-to-scalar ratio $r \to$ GW

Future missions include:

2023

- **P**rimordial **I**nflation **E**xplorer: **PIXIE**, which aims to measure $r < 10^{-3}$ at 5-σ, [7]
- **Lite**(light) satellite for the study of **B**-mode polarization and **I**nflation from cosmic background **R**adiation **D**etection **(LiteBIRD),** will provide a precision of $\delta r < 0.001$ 2025 [8]
- **P**olarized **R**adiation **I**maging and **S**pectroscopy **M**ission **(PRISM)** will measure $r \sim 5 \times 10^{-4}$ [9]
- **CORE**, which forecasts to lower the detection limit for the tensor-to-scalar ratio down to the 10^{-3} level [10]



upper bound range $r < 10^{-6} - 10^{-3}$

Conclusion

- Minimal Kähler potential scheme → intermediate scale gravitino mass $m_{3/2} \sim 10^8$ GeV with the gravitino decaying before the freeze-out of the LSP neutralino and with $T_r \sim 10^{13}$ 13GeV ⟹split SUSY.

- Nonminimal Kähler case → successful inflation with reheat temperatures as low as 10^5 GeV⟹resolution of the gravitino problem and compatible with a stable LSP (gravitino dark matter) and low-scale (~TeV) SUSY.

- Provided a framework that predicts the presence of PGW with the tensor-to-scalar ratio r in the observable range $10^{-4} - 10^{-3}$.

References

[1] PLB **138** (1984) 265; PLB **145 (1984)** 181,

[2] Proc.Sci.PLANCK **121**(2015);PRD **96**, 063527 (2017),

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[4] NP **B606** (2001) 518; NP **B790** (2008) 336E,

[5] Astrophys. Space Sci. **364** (2019) 69,

[6] PLB **562** (2003) 18,

[7] J. Cosmol. Astropart. Phys. 07 (2011) 025,

[8] J. Low. Temp. Phys. **176** (2014) 733,

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[10] J. Cosmol. Astropart. Phys. 04 (2018) 016.

—Thank you!

Back up slides

Significance of 'a' term

Solves cosmological problem : 'a' plays brings n_s within bounds

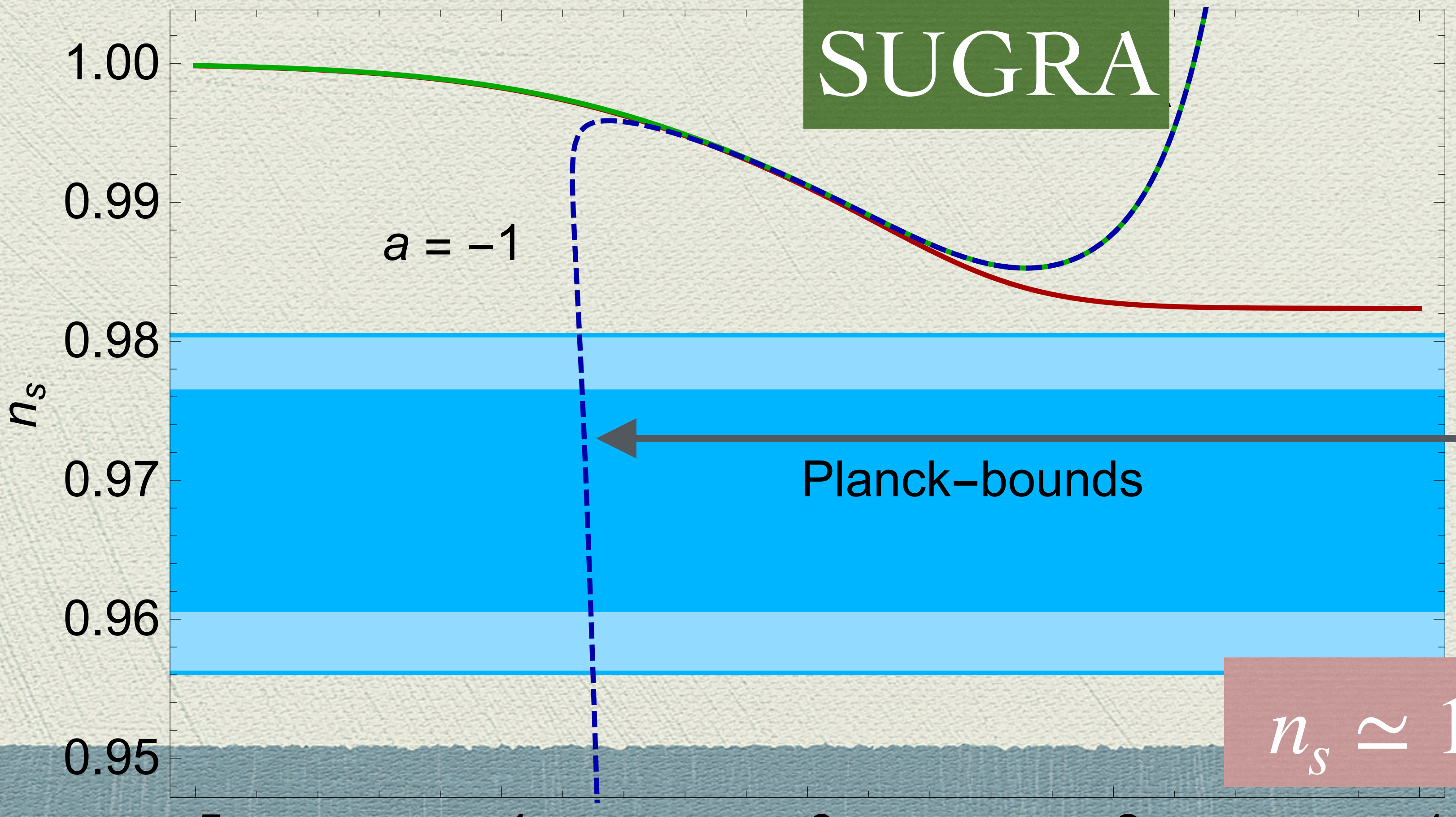
Solves particle physics problem : inflaton acquires nonzero vev due to soft SUSY breaking terms.

$$V(x) \simeq \kappa^2 M^4 \left(1 + \mathcal{N} \frac{k^2}{8\pi^2} F_\kappa(x) + \frac{\lambda^2}{4\pi^2} F_\lambda(y) + \frac{1}{2}\left(\frac{M}{m_P}\right) x^4 + a \frac{m_{3/2}}{\kappa M} x + \left(\frac{m_S}{\kappa M}\right)^2 x^2 \right)$$

radiative

SUGRA

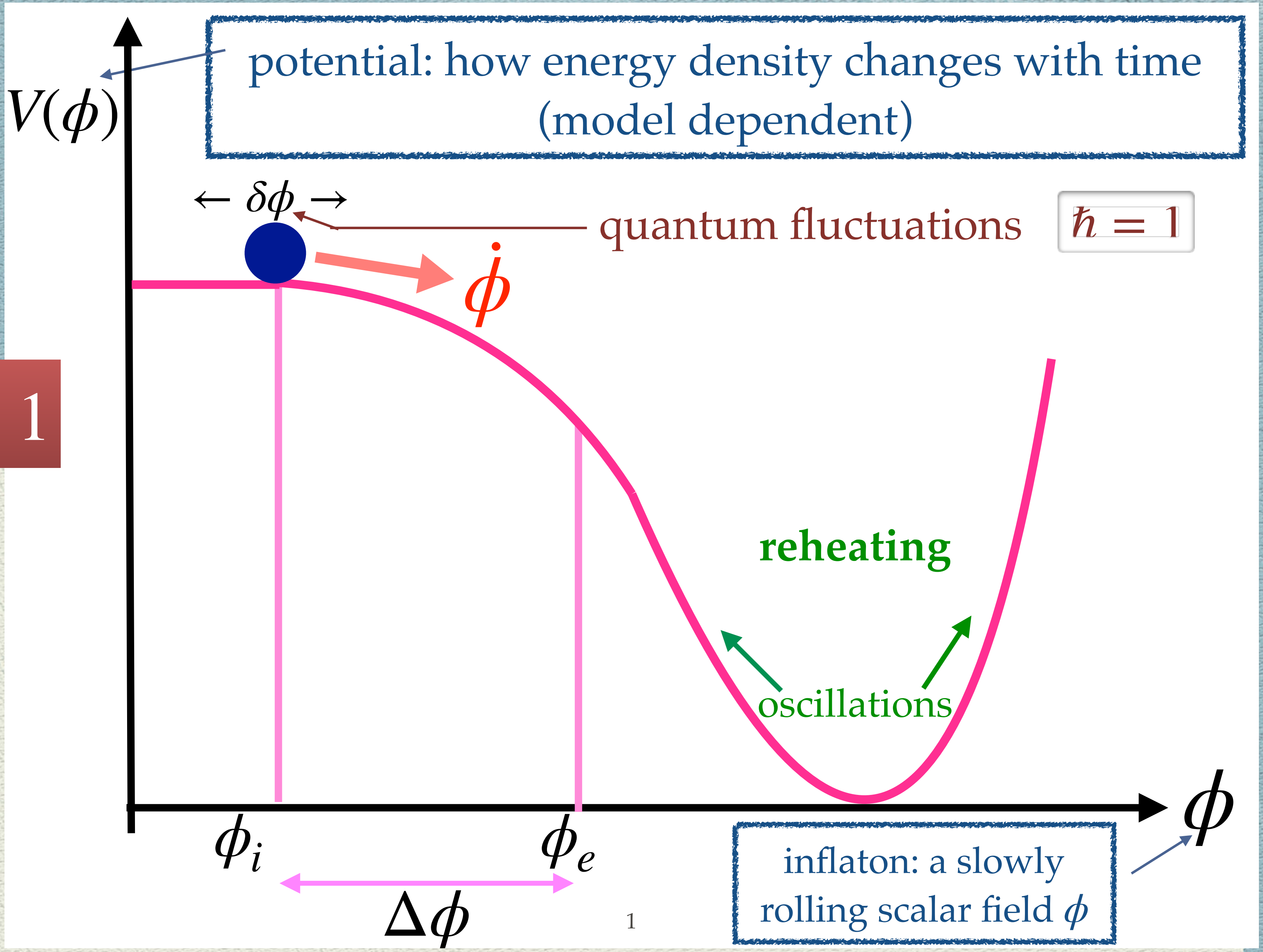
soft



$n_s \simeq 1 - 6\epsilon + 2\eta$

4. Physics of inflation

$\epsilon_v, |\eta_v| << 1$



- action is $S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$

- $$w \equiv \frac{p_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}$$

model dependence

- $\epsilon < 1 \implies \dfrac{d(aH)^{-1}}{dt} < 0 \rightleftarrows \ddot{a} > 0,\ w < \dfrac{1}{3}$

- $dN = Hdt = d\ln a$

$\max\left[\epsilon_v,\ |\eta_v|\right] = 1$

- $\epsilon \equiv -\dfrac{\dot{H}}{H^2} = -\dfrac{d\ln H}{dN}$

$\eta \equiv -\dfrac{\ddot{\phi}}{H\dot{\phi}} = \epsilon - \dfrac{1}{2}\dfrac{d\ln\epsilon}{dN}$

- $\epsilon_v \equiv \dfrac{M_P^2}{2}\left(\dfrac{V_{,\phi}}{V}\right)^2 \eta_v \equiv M_P^2\left(\dfrac{V_{,\phi\phi}}{V}\right)^2 \approx \eta$