Nonlinear Behaviour of Dark Matter Axions

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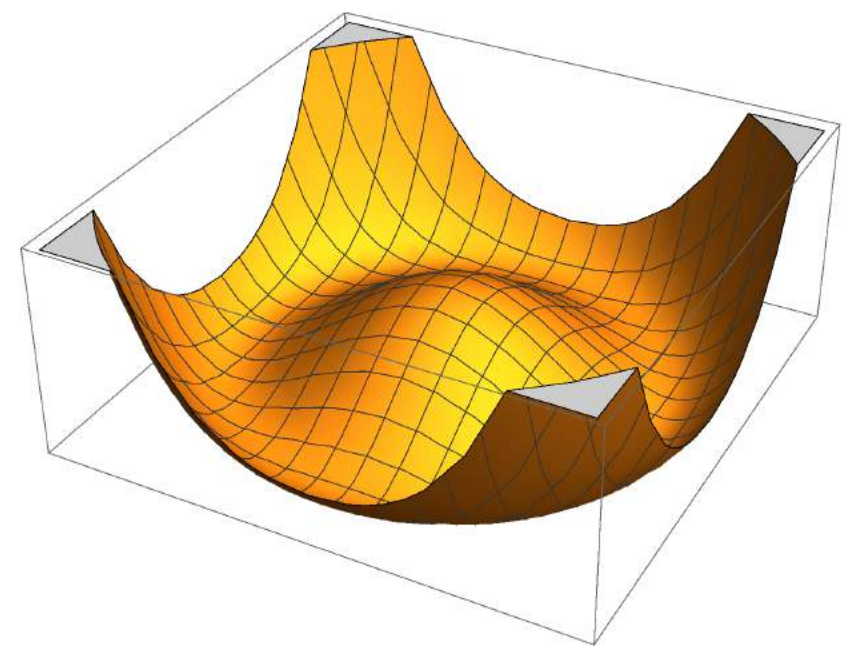
QCD-Axion

$$\Delta\mathcal{L}_{qcd} \sim \theta\, \mathbf{E}^a \cdot \mathbf{B}^a$$

$$|\theta| \lesssim 10^{-10}$$

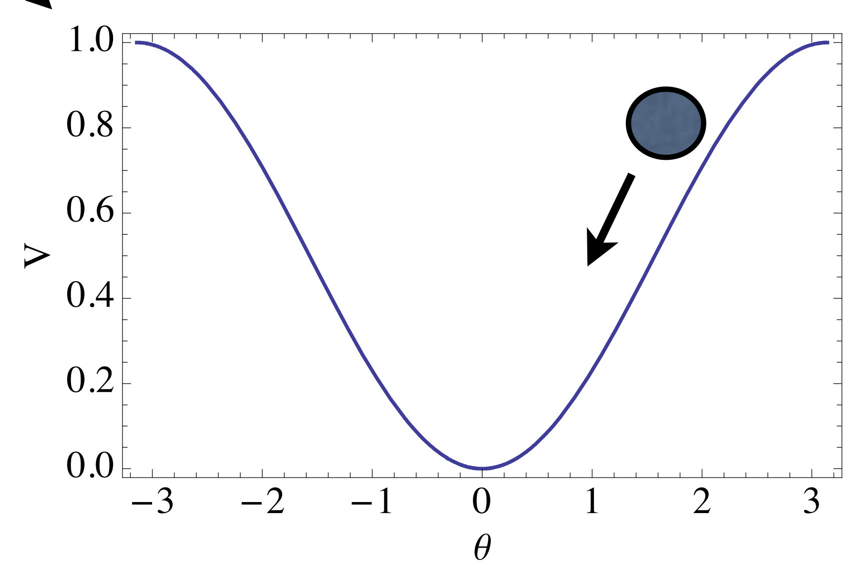
(Peccei, Quinn, Weinberg, Wilczek
Kim, Shifman, Vainshtein, Zakharov
Dine, Fischler, Srednicki, Zhitnitsky)

$$\theta \to \phi/f_a$$



$$\Delta\mathcal{L}_a \sim \frac{\phi}{f_a}\mathbf{E}^a \cdot \mathbf{B}^a + \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

$$V(\phi) = \frac{1}{2}m_a^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \ldots$$



Axion mass:

$$m_a \sim \frac{\Lambda_{qcd}^2}{f_a}$$

(Attractive) Self-Coupling:

$$\lambda \sim -\frac{\Lambda_{qcd}^4}{f_a^4}$$

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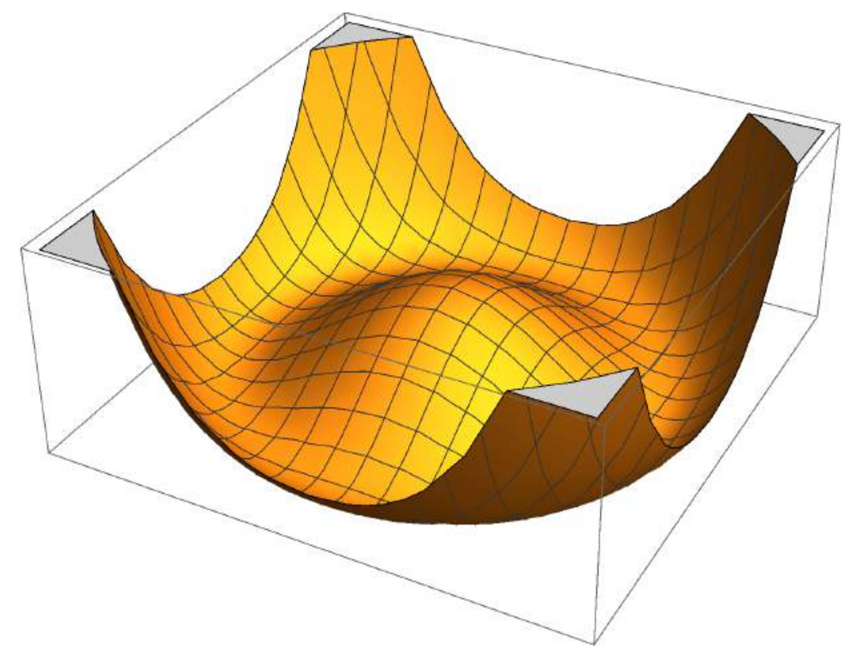

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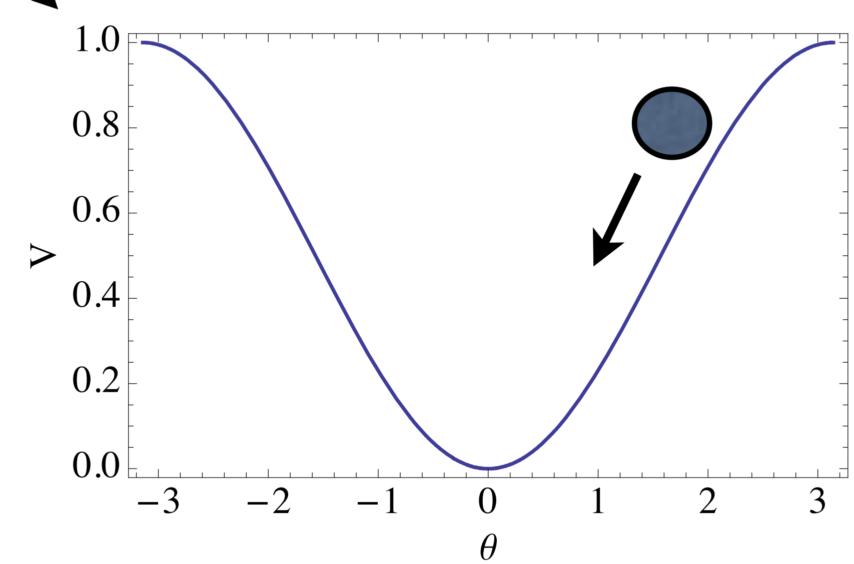
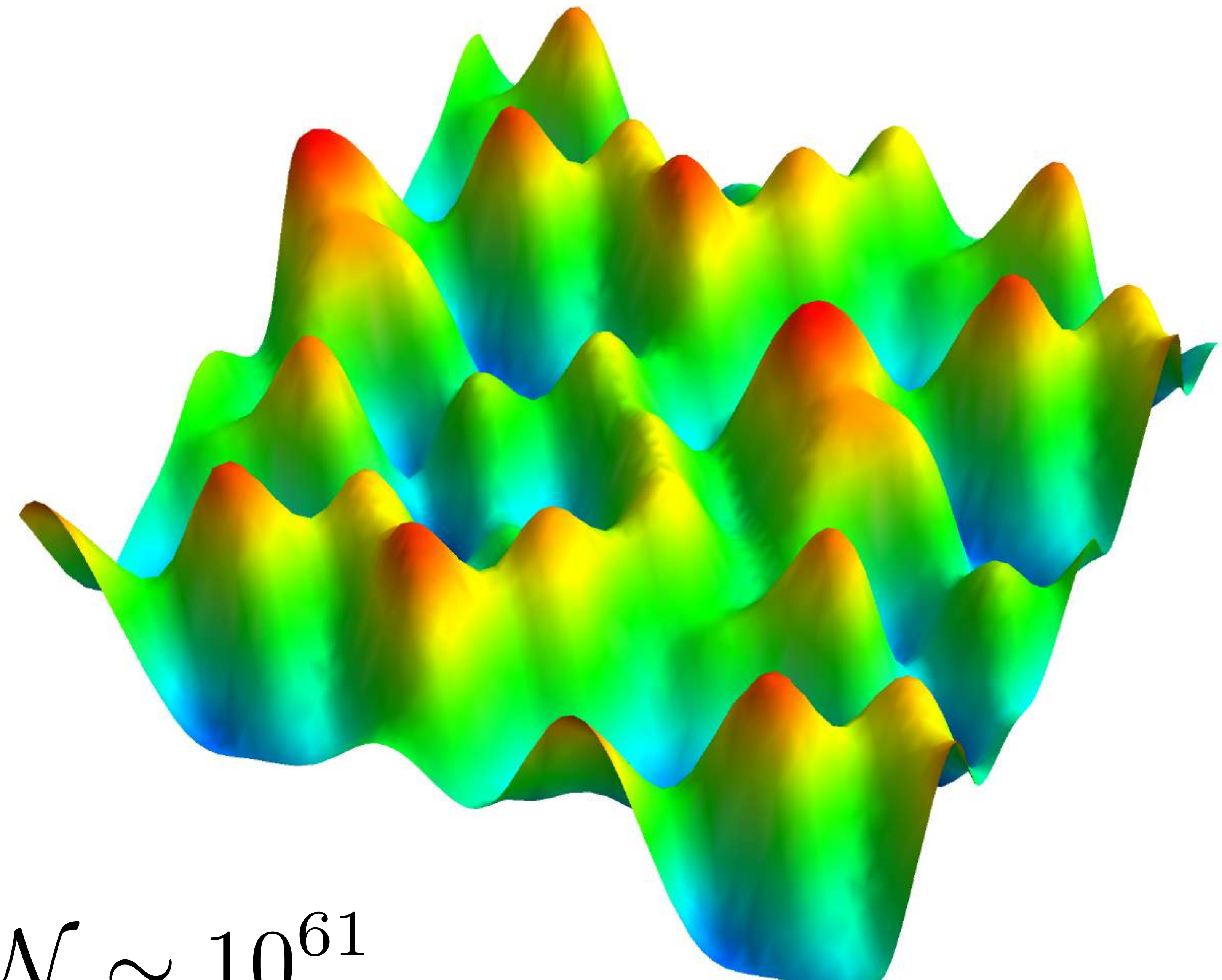


Abundance

$$\Omega_a \sim \langle\theta_i^2\rangle \left(\frac{10^{-6}\,\mathrm{eV}}{m_a}\right)^{7/6}$$

Related issues for string-axions,
ALPs, light bosonic DM

Early Universe; Axion Initial Distribution



$\mathcal{N} \sim 10^{61}$

Consider Non-Relativistic Behavior

$$\phi(\mathbf{x},t) = \frac{1}{\sqrt{2\,m}}\left(e^{-imt}\psi(\mathbf{x},t) + e^{imt}\psi^*(\mathbf{x},t)\right)$$

Hamiltonian

$$\hat{H} = \hat{H}_{\mathbf{kin}} + \hat{H}_{\mathbf{int}} + \hat{H}_{\mathbf{grav}}$$

$$\hat{H}_{\mathrm{kin}} = \int d^3x \frac{1}{2m} \nabla\hat{\psi}^\dagger \cdot \nabla\hat{\psi}$$

$$\hat{H}_{\mathrm{int}} = \int d^3x \frac{\lambda}{16m^2} \hat{\psi}^\dagger\hat{\psi}^\dagger\hat{\psi}\hat{\psi}$$

$$\hat{H}_{\mathrm{grav}} = -\frac{Gm^2}{2}\int d^3x \int d^3x' \frac{\hat{\psi}^\dagger(\mathbf{x})\hat{\psi}^\dagger(\mathbf{x}')\hat{\psi}(\mathbf{x})\hat{\psi}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}$$

Number Density

$$\hat{n}(\mathbf{x}) = \hat{\psi}^\dagger(\mathbf{x})\hat{\psi}(\mathbf{x})$$

(For corrections: Namjoo, Guth, Kaiser 2017,
Eby, Mukaida, Takimoto, Wijewardhana, Yamada 2018)

Dynamical Time Scales

Equation of Motion

$$i\,\dot{\psi} = -\frac{1}{2m}\nabla^2\psi + \frac{\lambda}{8m^2}|\psi|^2\psi - Gm^2\psi\int d^3x'\,\frac{|\psi(\mathbf{x}')|^2}{|\mathbf{x}-\mathbf{x}'|}$$

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Equation of Motion

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Occupancy number change rate $\qquad \Gamma_k \equiv \frac{\dot{\mathcal{N}}_k}{\mathcal{N}_k} \sim \frac{G\,m^2\,n_{ave}}{k^2}$

(See Sikivie, Yang 2009)

Relaxation rate $\qquad \Gamma_{rel} \sim \frac{G^2\,n^2\,m^5}{k^6} \qquad (\sim n\,\sigma\,v\,\mathcal{N})$

(See Levkov, Panin, Tkachev 2018)

Equilibrium with high occupancy suggests BEC

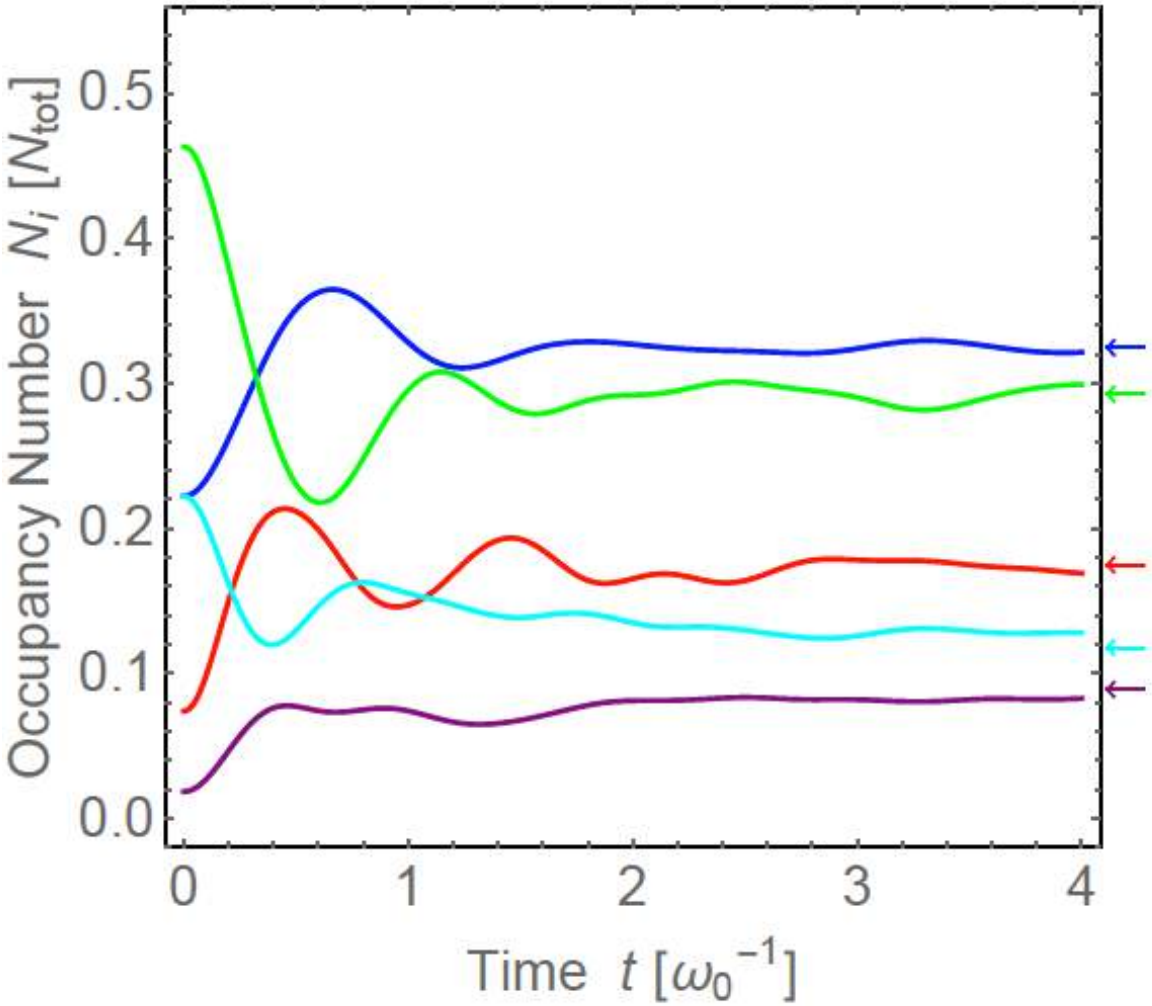
Axion BEC Literature

- Sikivie, Yang (2009)
- Erken, Sikivie, Tam, Yang (2011)
- Chavanis (2012)
- Banik, Sikivie (2013)
- Davidson, Elmer (2013)
- Saikawa, Yamaguchi (2013)
- Noumi, Saikawa, Sato, Yamaguchi (2014)
- Vega, Sanchez (2014)
- Li, Rindler-Daller, Shapiro (2014)
- Berges, Haeckel (2014)
- Guth, Hertzberg, Prescod-Weinstein (2014)
- Banik, Christopherson, Sikivie, Todarello (2015)
- Davidson (2015)
- Eby, Suranyi, Wijewardhana (2014, 2015, 2016, 2017, 2018, 2019, 2020) [w/Leembruggen, Ma, Street, Vaz]
- ……. and others ……

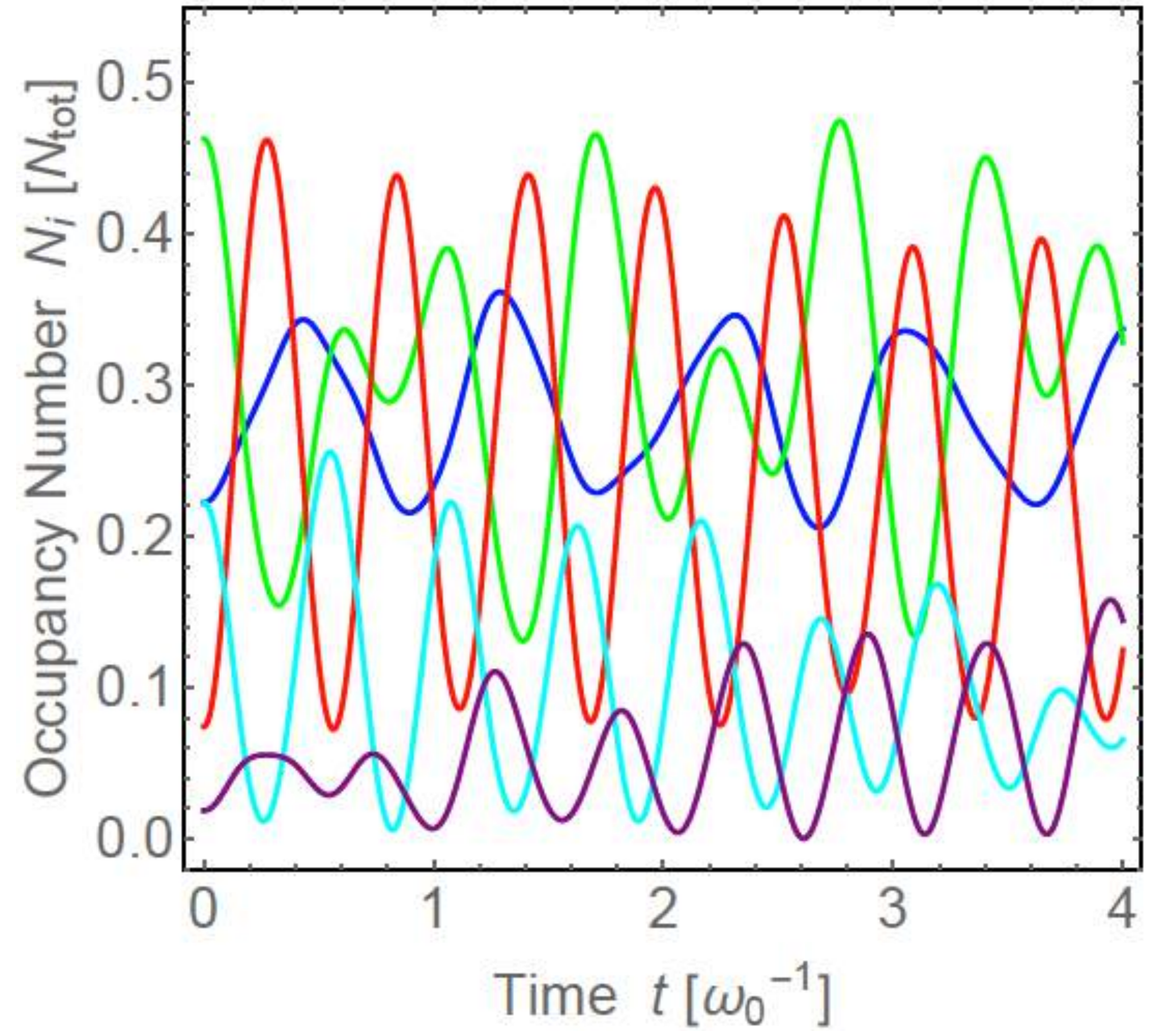
Classical vs Quantum with Interactions

Quantum vs Classical??

Quantum



Classical (fixed initial conditions)



Sikivie, Todarello, 1607.00949

Correct Classical Treatment

Initial classical state $a_i = \sqrt{N_i}\, e^{I\theta_i}, \quad \theta_i \in [0, 2\pi)$

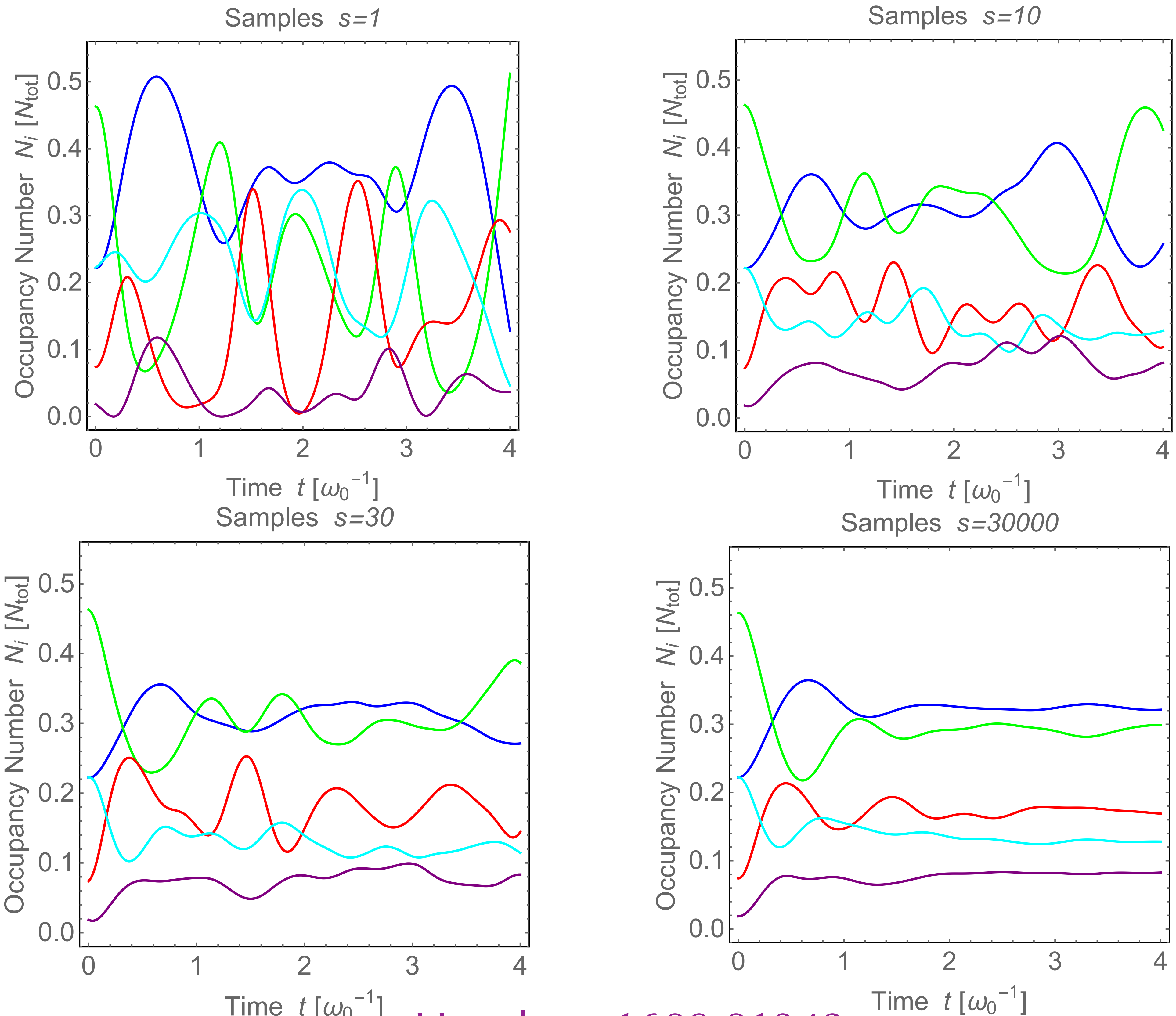
Ensemble average over random initial phases

Meaningful comparison

Connects to uncertainty in branch of wavefunction

Hertzberg 1609.01342

Correct Classical Treatment



(matches quantum)

Hertzberg 1609.01342

Implication for Correlation Functions

At high occupancy:

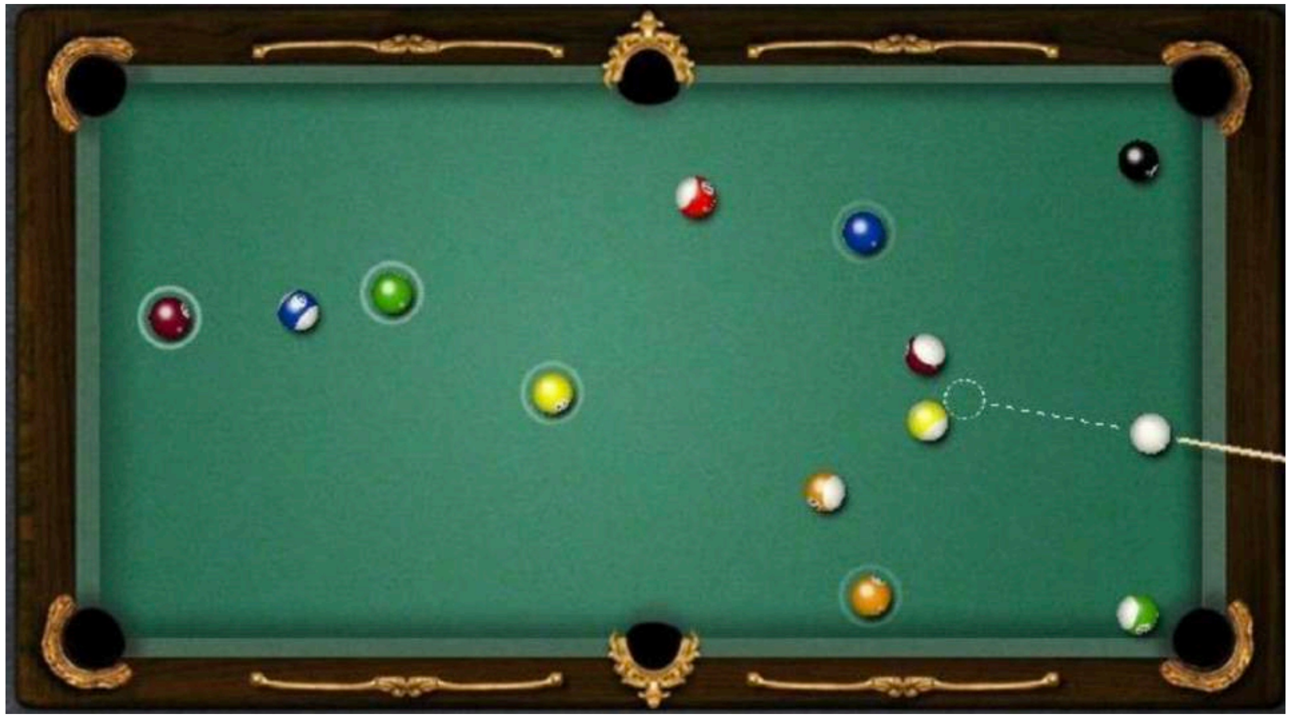
$$\langle \{N_i\} | \hat{\psi}^\dagger(\mathbf{x},t)\,\hat{\psi}(\mathbf{y},t) | \{N_i\} \rangle \approx \langle \psi^*(\mathbf{x},t)\,\psi(\mathbf{y},t) \rangle_{ens}$$

Hertzberg 1609.01342

Implication for Axion Simulations

Basic correlation functions of quantum fields can be mimicked by classical averaging, at high occupancy, despite the macroscopic spreading of wave-functions in these chaotic systems

Note: this is not some trivial consequence of Ehrenfest theorem…



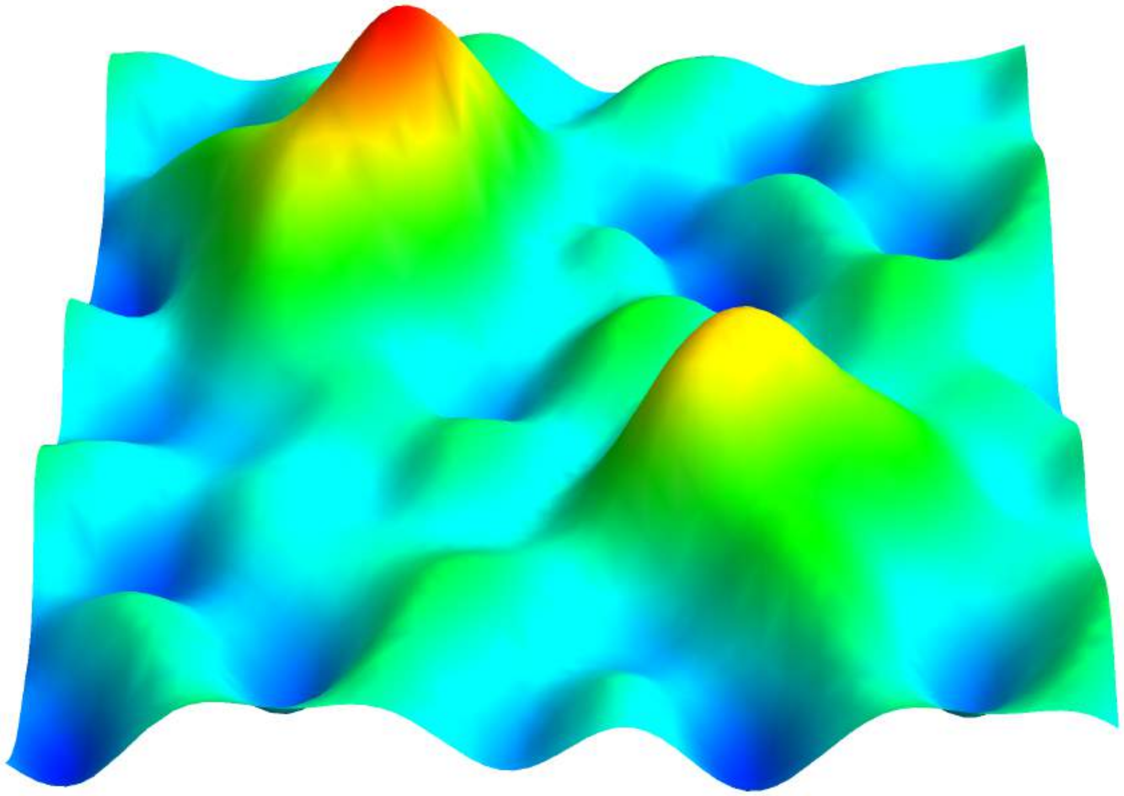
Hertzberg 1609.01342

(Return to residual quantumness at end of talk)

Implication for Axion Dark Matter

Statistically, axions should be well described by classical field theory

What is the BEC?



Miniclusters —> Axion stars

May be numerous in galaxies

Guth, Hertzberg, Prescod-Weinstein 2014

Hogan, Rees 1988; Kolb, Tkachev 1993, 1994, 1995; Barranco, Bernal 2001; Fairbairn, Marsh, Quevillon, Rozier 2017; Kitajima, Soda, Urakawa 2018; Eby, Leembruggen, Ma, Street, Suranyi, Vaz, Wijewardhana 2014-2020

Also can form in halos of PBHs: Hertzberg, Sciappacasse, Yanagida 2020

Axion Stars in Detail

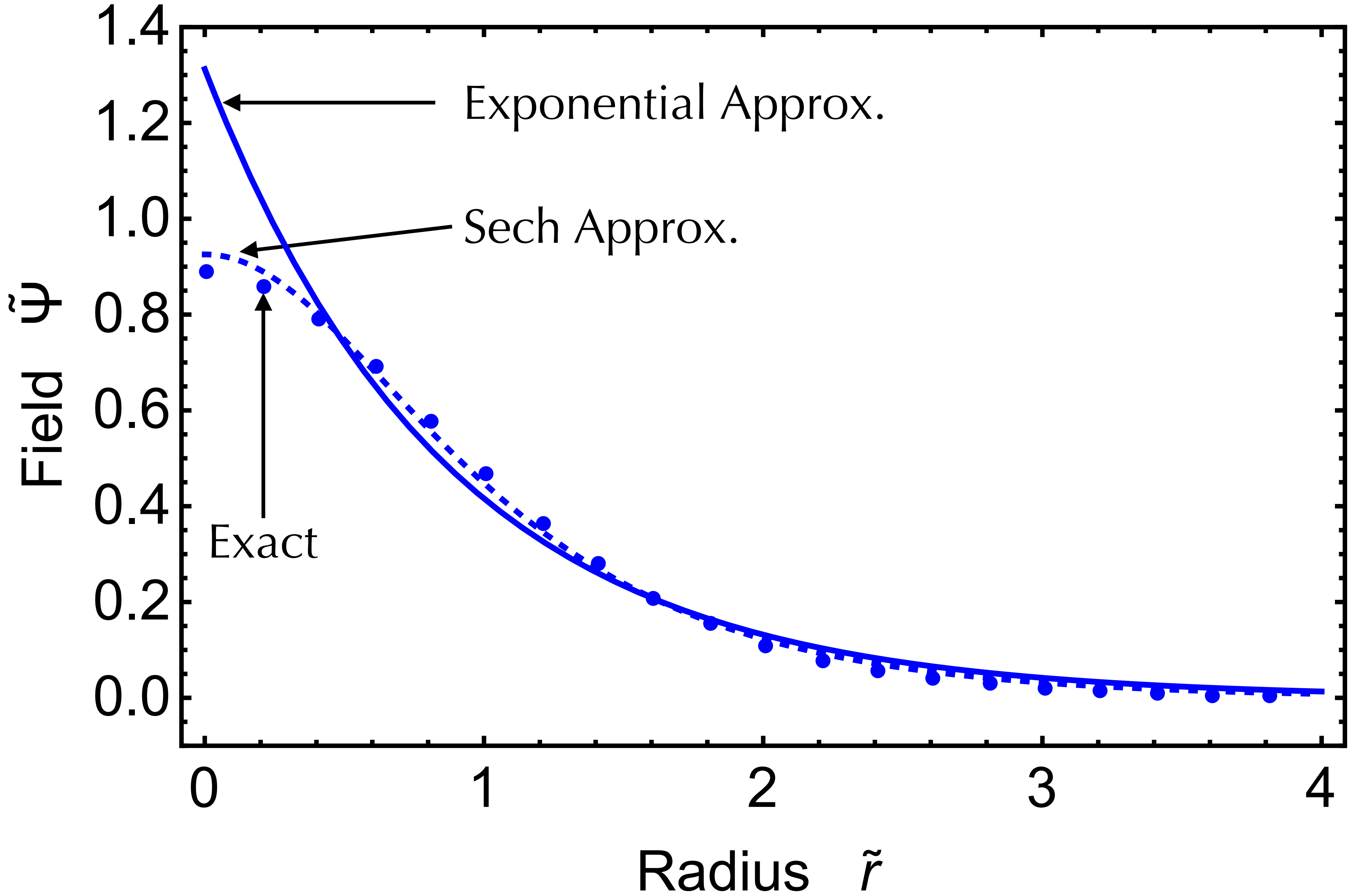
Return to Non-Relativistic Classical Field Theory

Equation of Motion

$$i\,\dot{\psi} = -\frac{1}{2m}\nabla^2\psi + \frac{\lambda}{8m^2}|\psi|^2\psi - Gm^2\psi\int d^3x'\,\frac{|\psi(\mathbf{x}')|^2}{|\mathbf{x}-\mathbf{x}'|}$$

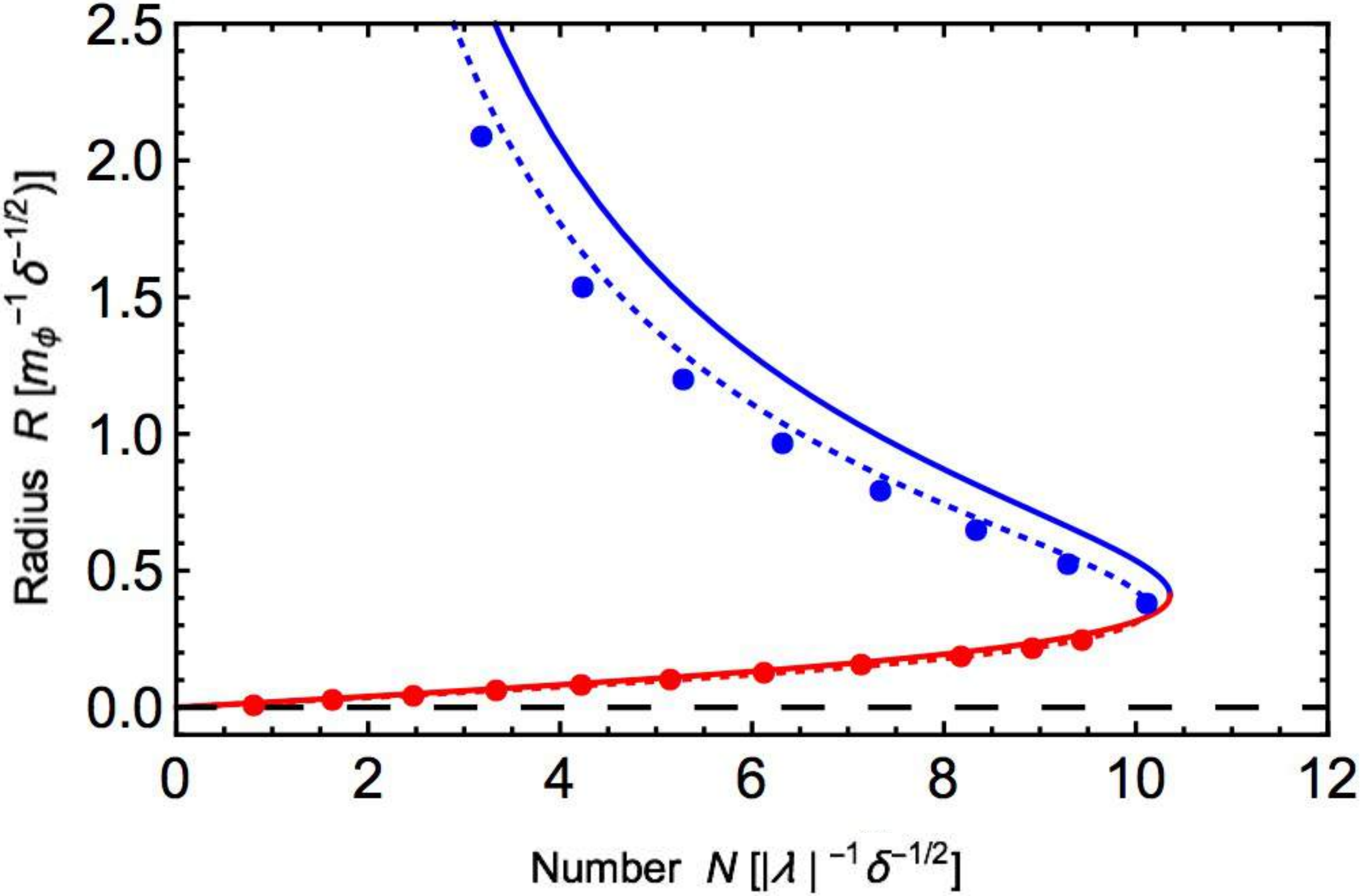
$$(\lambda < 0)$$

Star Solutions (BEC) at fixed N



Schiappacasse, Hertzberg 1710.04729

Two Branches of Solutions



See Chavanis, Delfini 2011 and others…
Schiappacasse, Hertzberg 1710.04729

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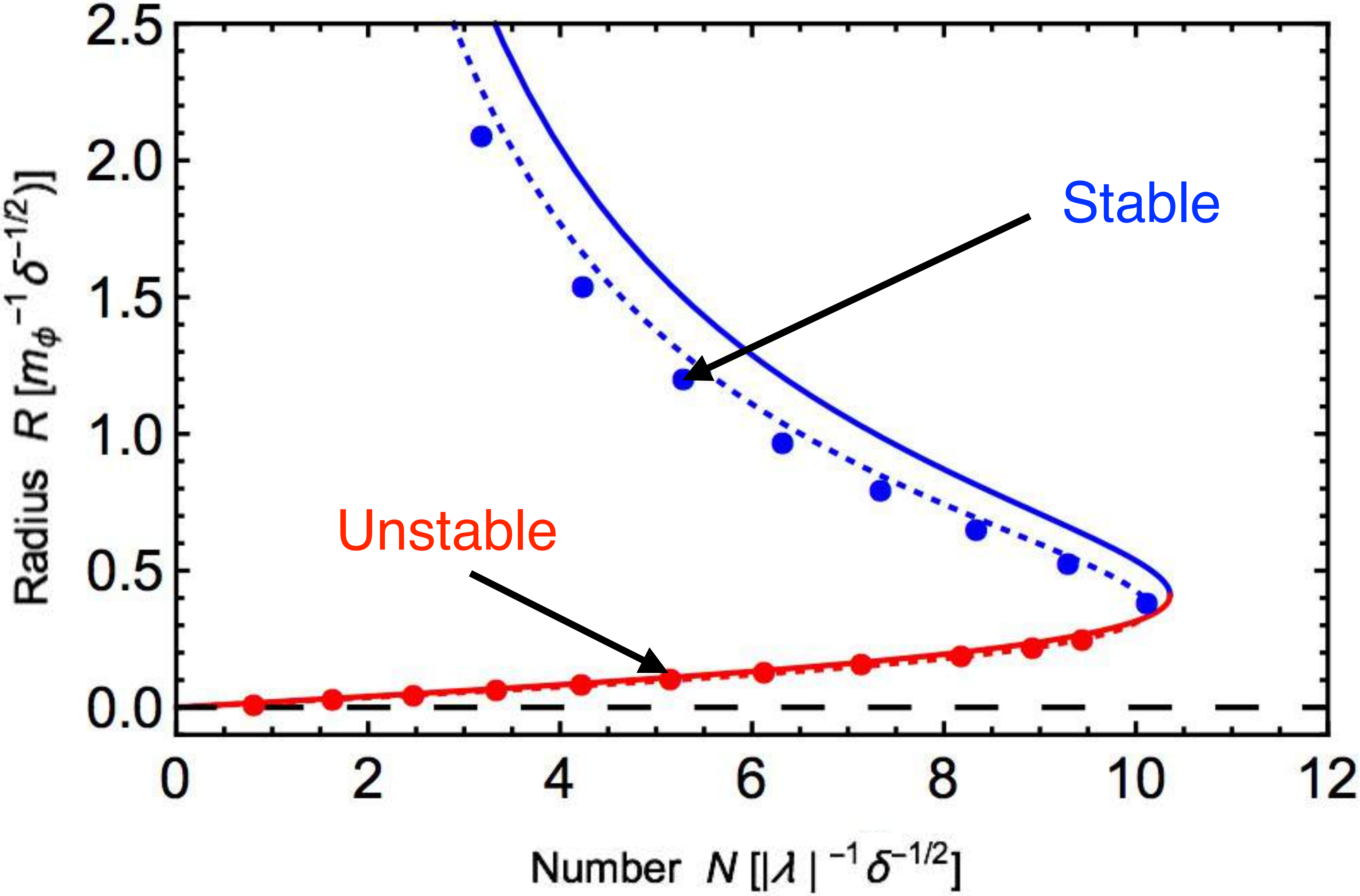

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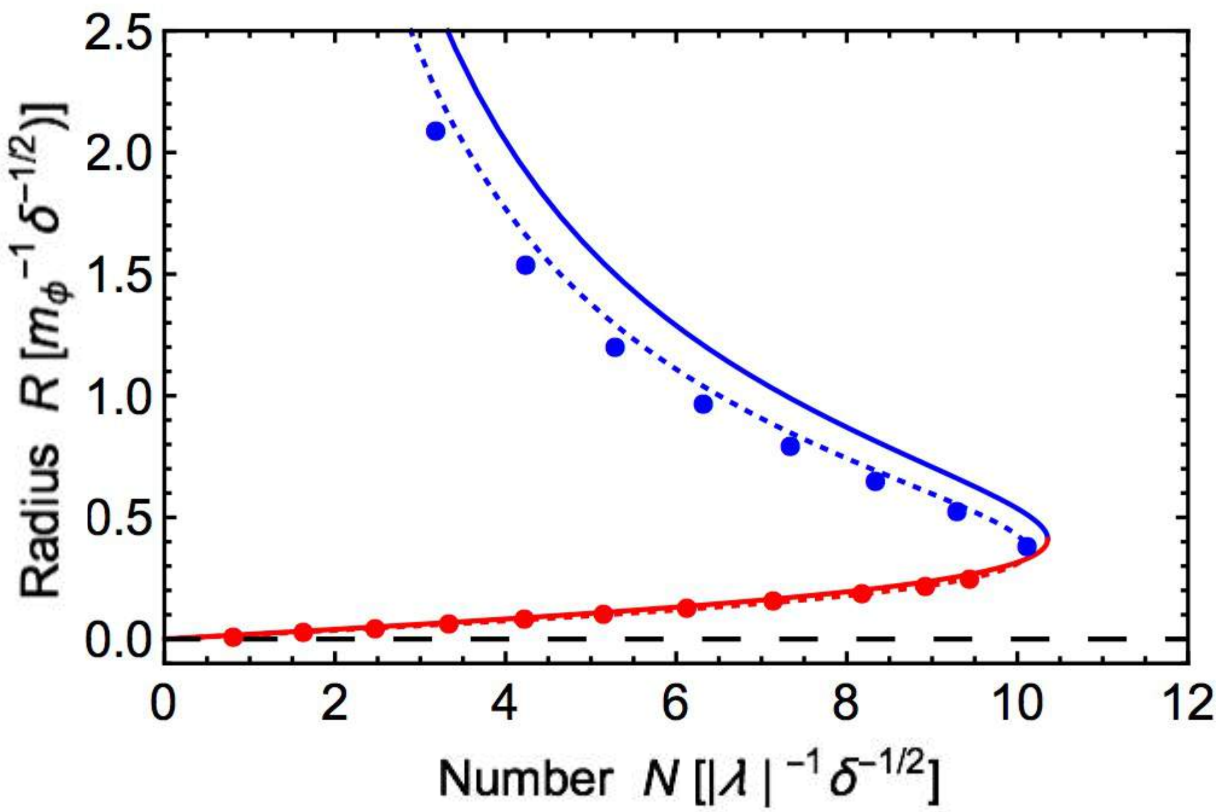


$$N_{max} = \frac{f_a}{m^2\sqrt{G}}\tilde{N}_{max} \sim 8\times 10^{59}\,(\tilde{m}^{-2}\tilde{f}_a)\,,$$

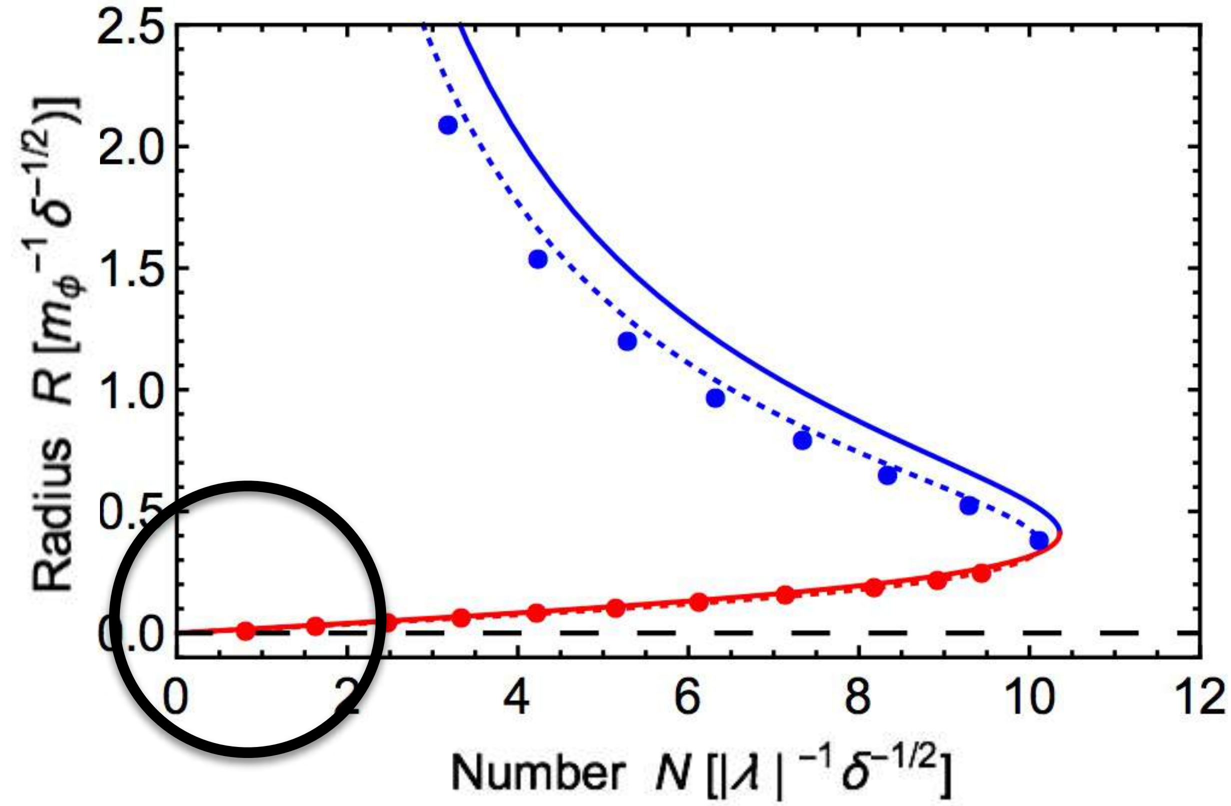
$$M_{max} = N_{max}\,m \qquad \sim 1.4\times 10^{19}\,\mathrm{kg}\,(\tilde{m}^{-1}\tilde{f}_a)\,,$$

$$R_{90,min} = \frac{a\,(\tilde{R}_{90}/\tilde{R})}{bN_{max}Gm^3} \sim 130\,\mathrm{km}\,(\tilde{m}^{-1}\tilde{f}_a^{-1})\,,$$

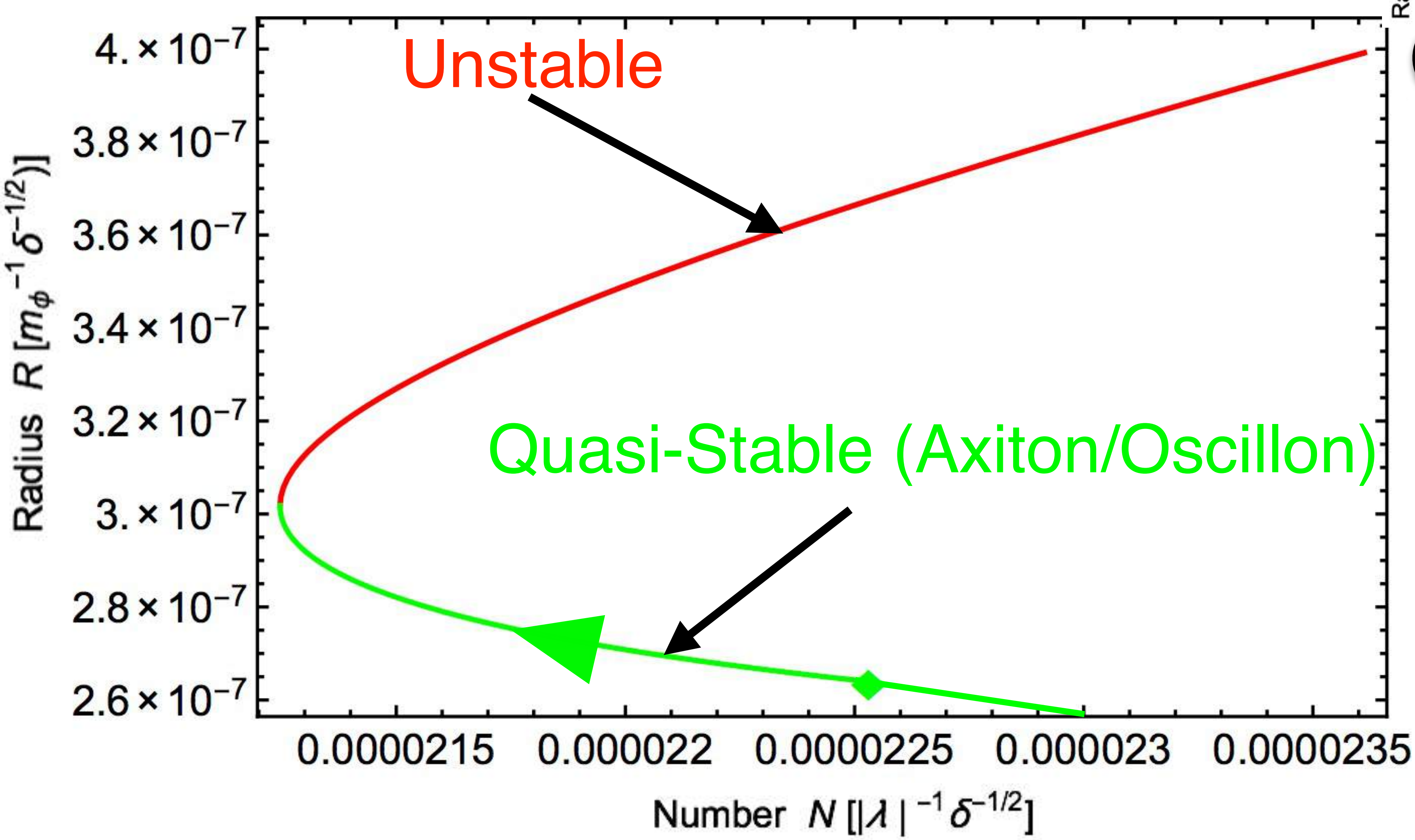
where $\tilde{f}_a \equiv f_a/(6\times 10^{11}\,\mathrm{GeV})$ and $\tilde{m} \equiv m/(10^{-5}\,\mathrm{eV})$.

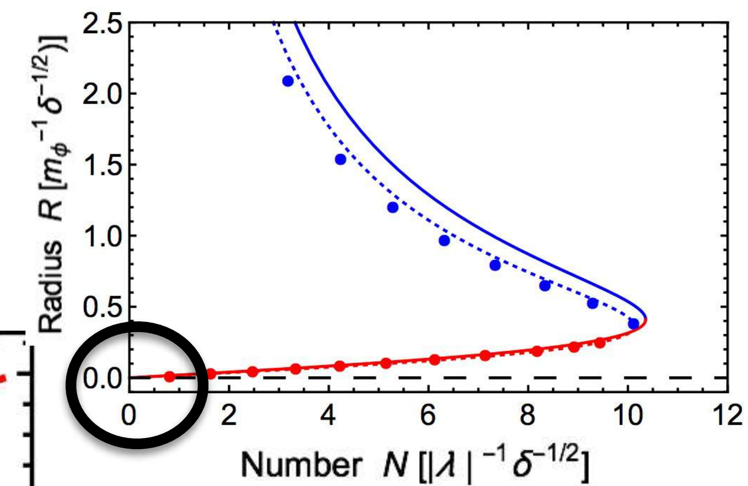
See Chavanis, Delfini 2011 and others…
Schiappacasse, Hertzberg 1710.04729

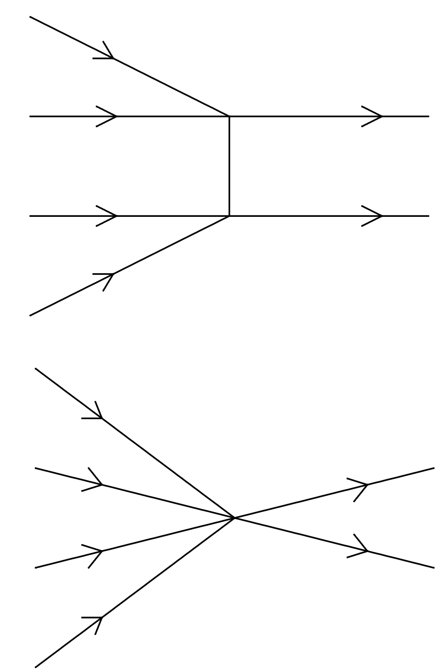
Relativistic Branch (Axiton)



Relativistic Branch (Axiton/Oscillon)







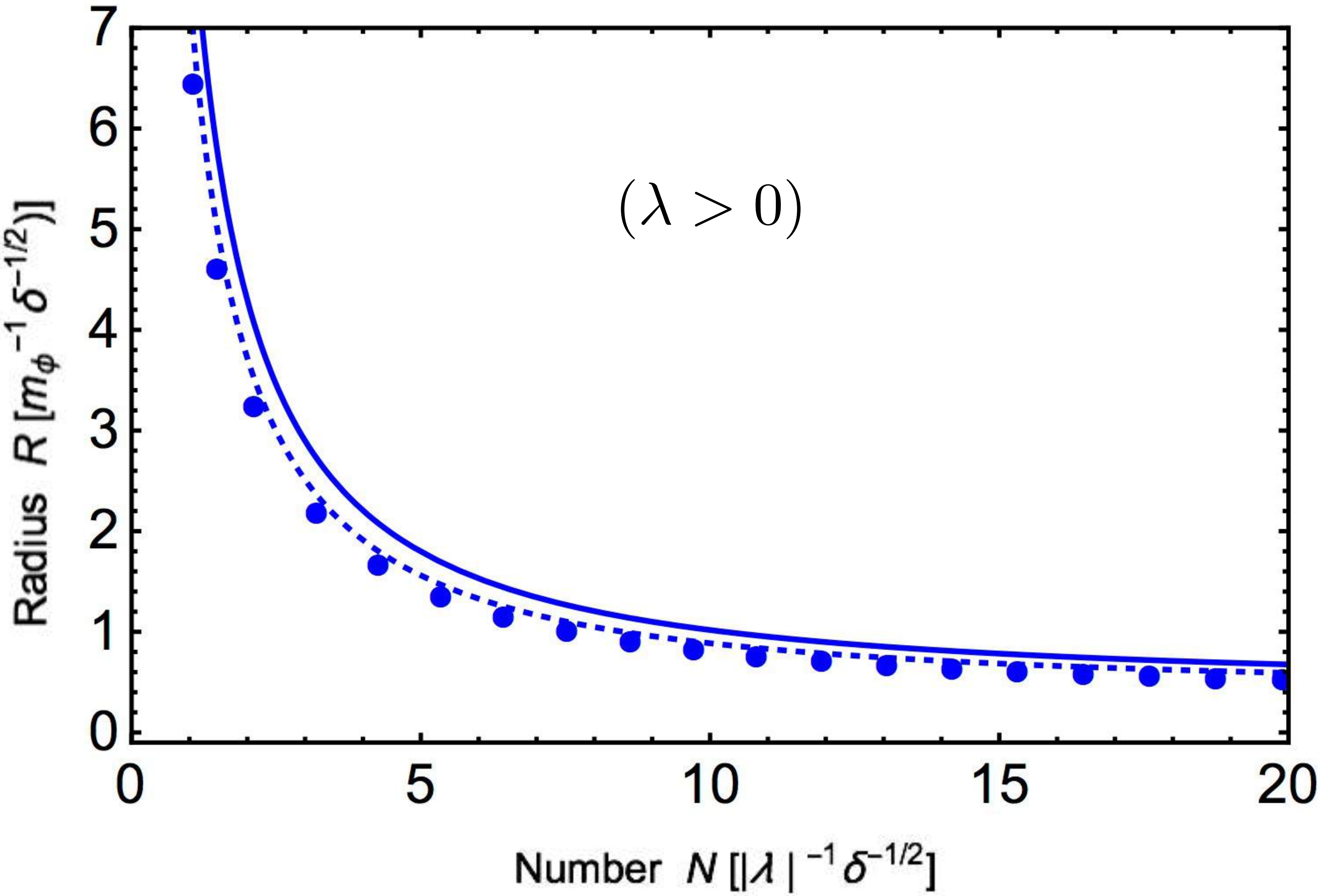
Kolb, Tkachev astro-ph/9311037; Fodor, Fogacs, Horvath, Mezei 0903.0953; Hertzberg 1003.3459; Eby, Suranyi, Wijewardhana 1512.01709; Schiappacasse, Hertzberg 1710.04729; Visinelli, Baum, Redondo, Freese, Wilczek 1710.08910; …

Repulsive Self Interactions

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4, \quad (\lambda > 0)$$

(see; Fan 2016)

Repulsive Self Interaction (Axion-Like Particle)



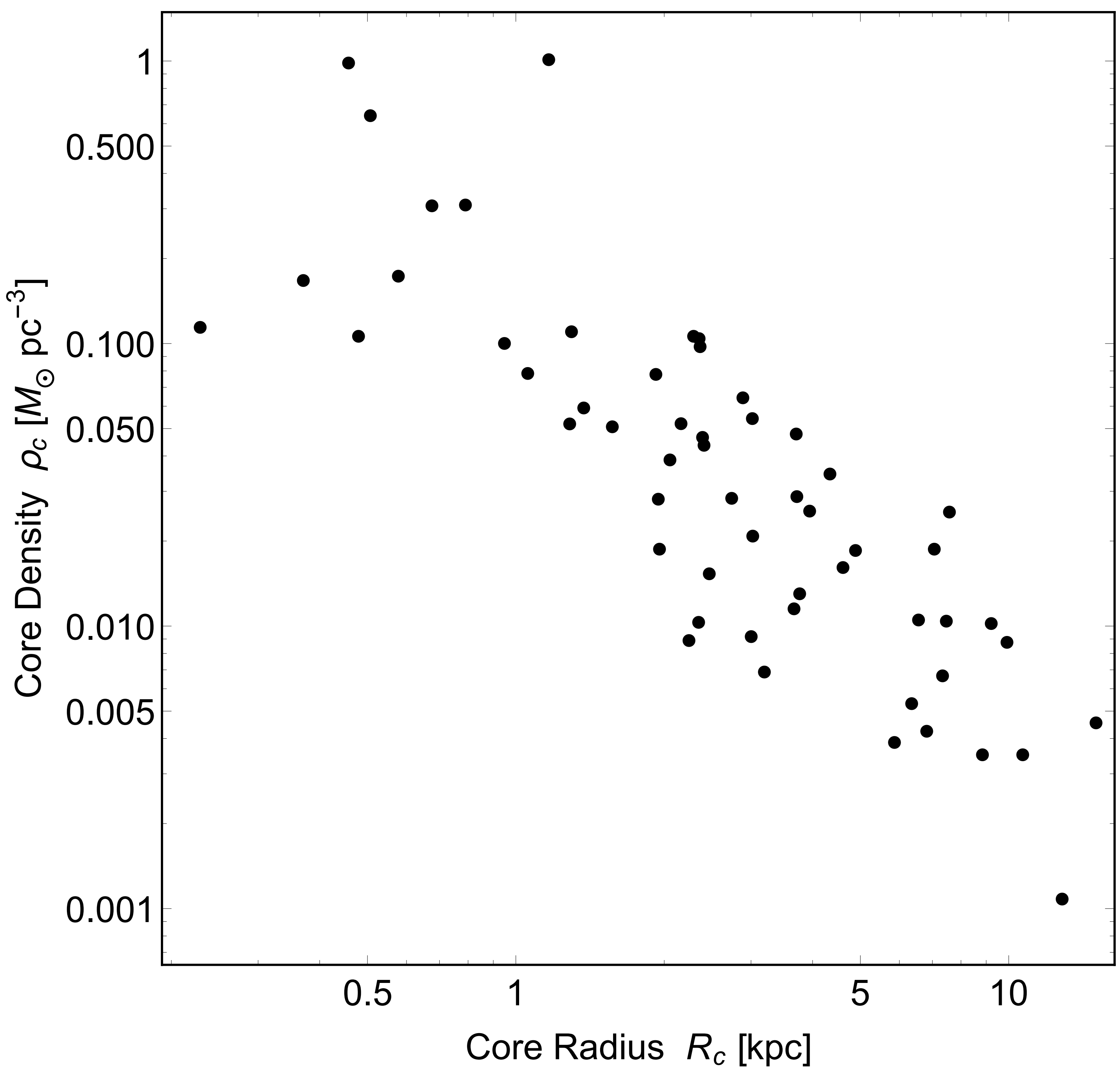
See Colpi, Shapiro, Wasserman 1986; Chavanis, Delfini 2011 and others…
Schiappacasse, Hertzberg 1710.04729; Hertzberg, Rompineve, Yang 2010.07927

Implications for Fuzzy Dark Matter

Can it explain galactic cores? $m_a \sim 10^{-21}\,\mathrm{eV}$

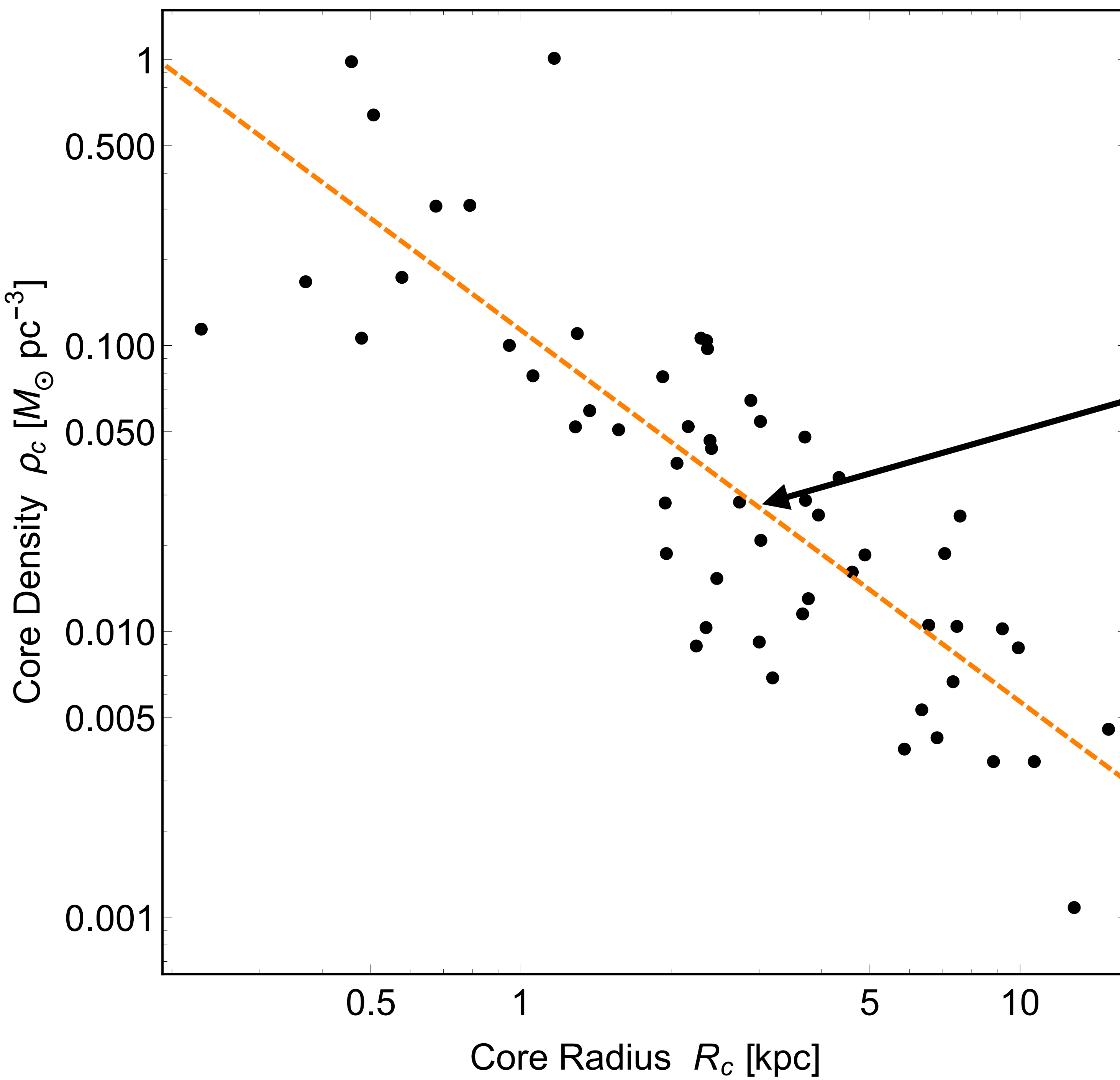
Hu, Barkana, Gruzinov 2000, ….

Core Density Vs Core Radius (Data)



Data from: Rodriguez, del Popolo, Marra, de Oliveira 2017

Core Density Vs Core Radius (Data)

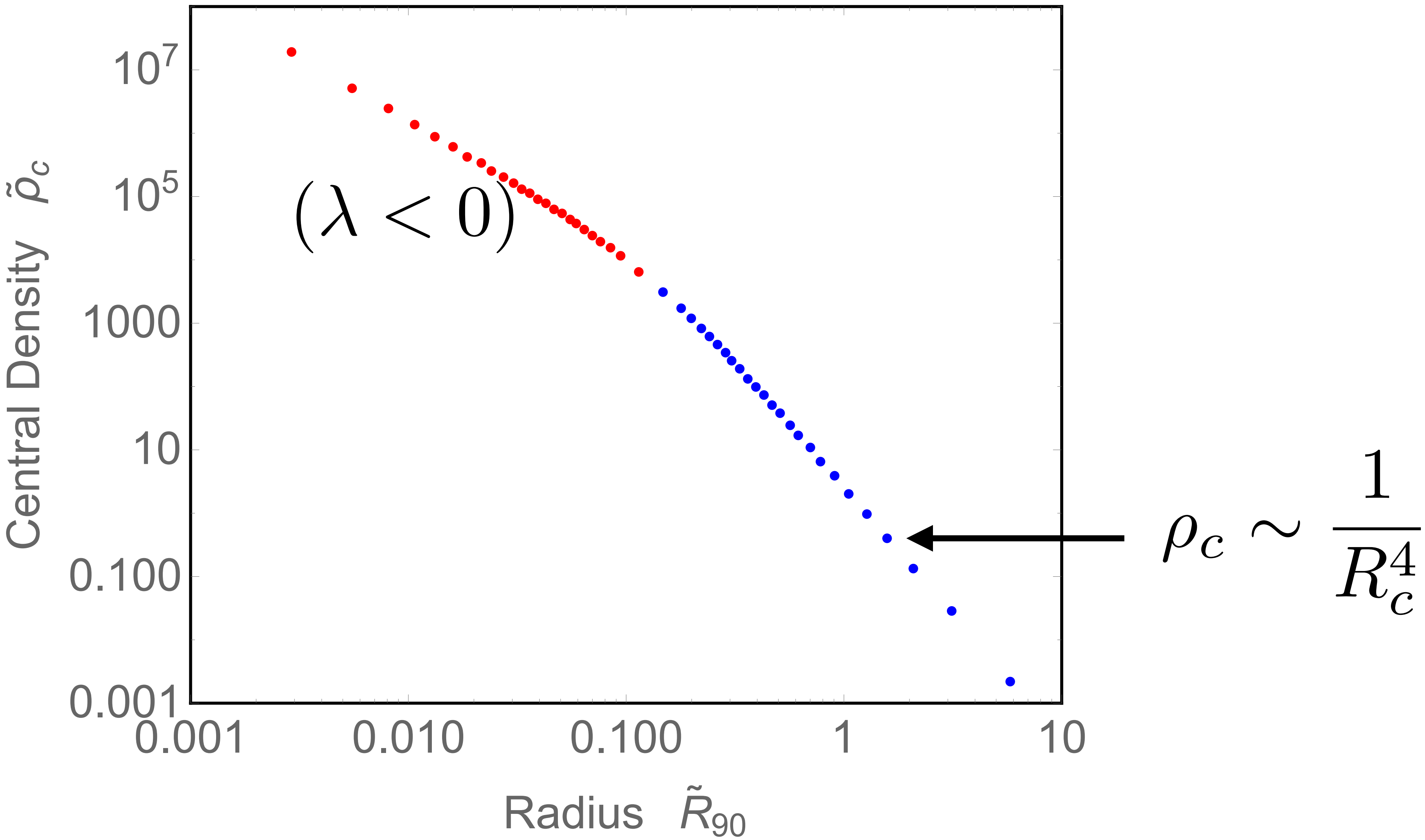


$$\rho_c \sim \frac{1}{R_c^\beta}$$

$$\beta \approx 1$$

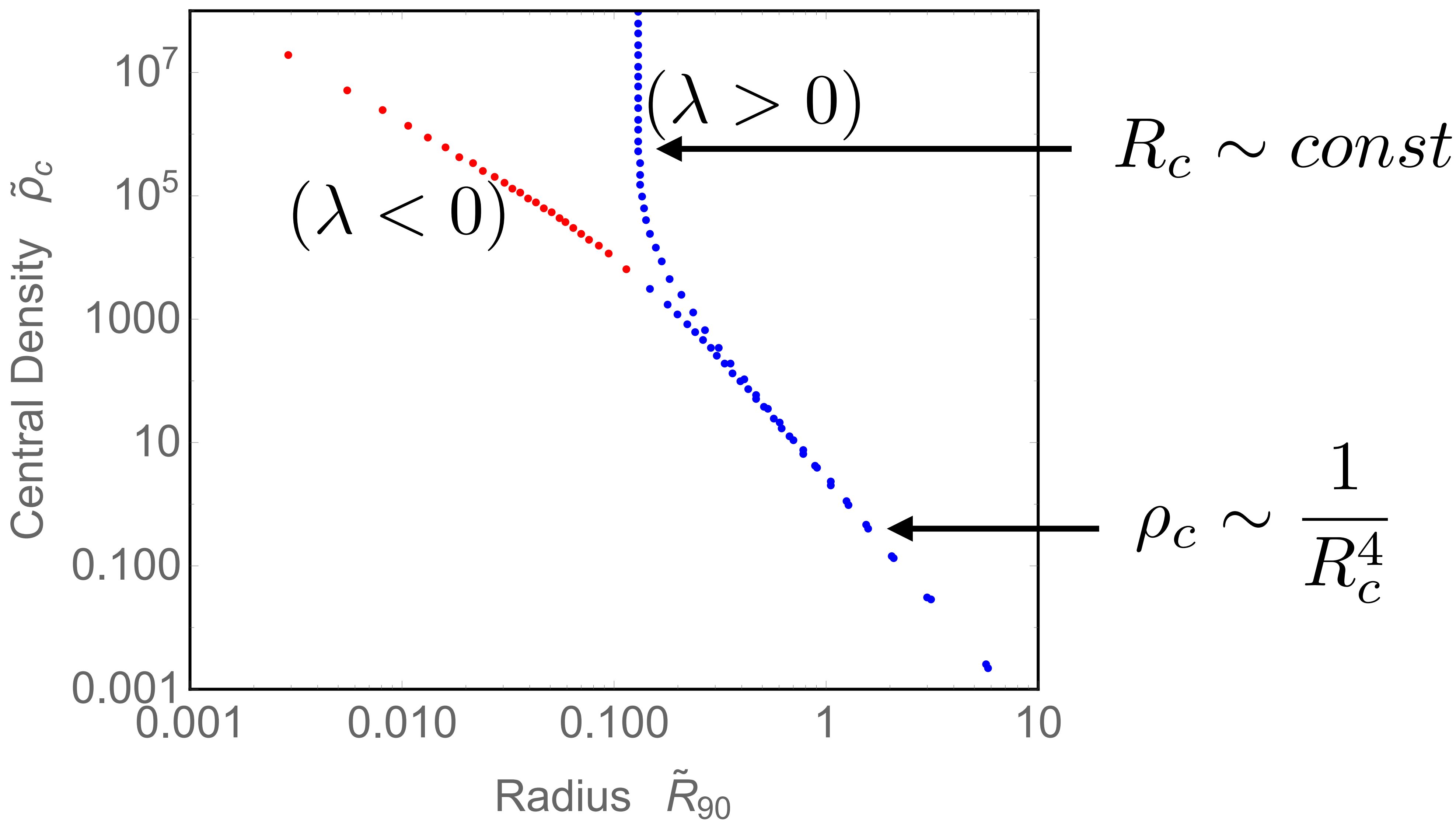
Data from: Rodriguez, del Popolo, Marra, de Oliveira 2017

Core Density Vs Core Radius (Light Scalar in BEC)



Deng, Hertzberg, Namjoo, Masoumi 1804.05921

Core Density Vs Core Radius (Light Scalar in BEC)



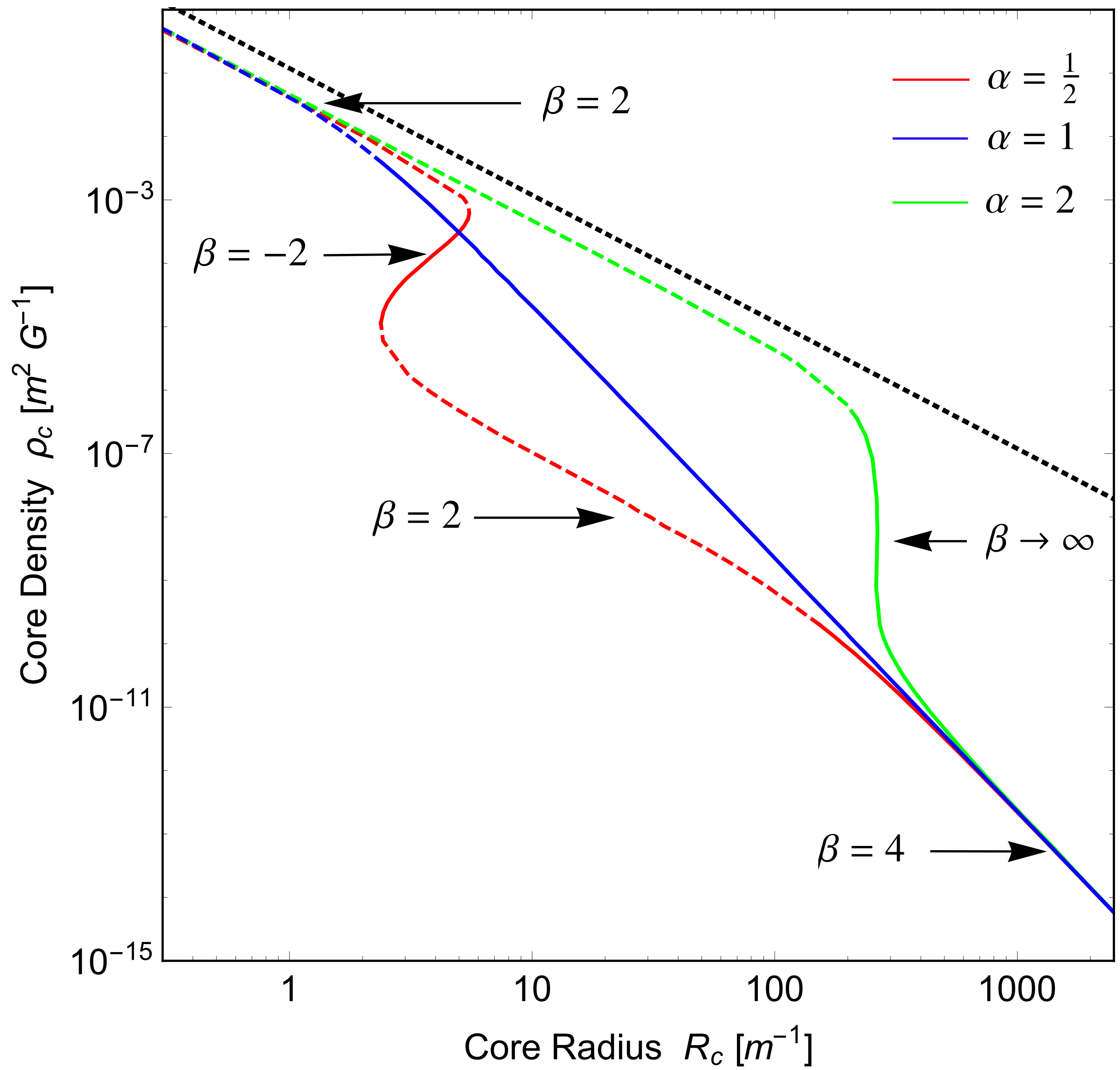
Deng, Hertzberg, Namjoo, Masoumi 1804.05921

Core Density Vs Core Radius (Light Scalar in BEC)

Extension to general potentials, polytropes, full relativistic

$$V(\phi) \propto ((1+\phi^2/F^2)^\alpha - 1)$$

Solid = Stable
Dashed = Unstable



$$\rho_c \sim \frac{1}{R_c^\beta}$$

Never obtain $\beta \sim 1$ and stable

Deng, Hertzberg, Namjoo, Masoumi 1804.05921

Axion Star Resonance into Photons

Consider Axion to Photon Coupling

Photon Lagrangian

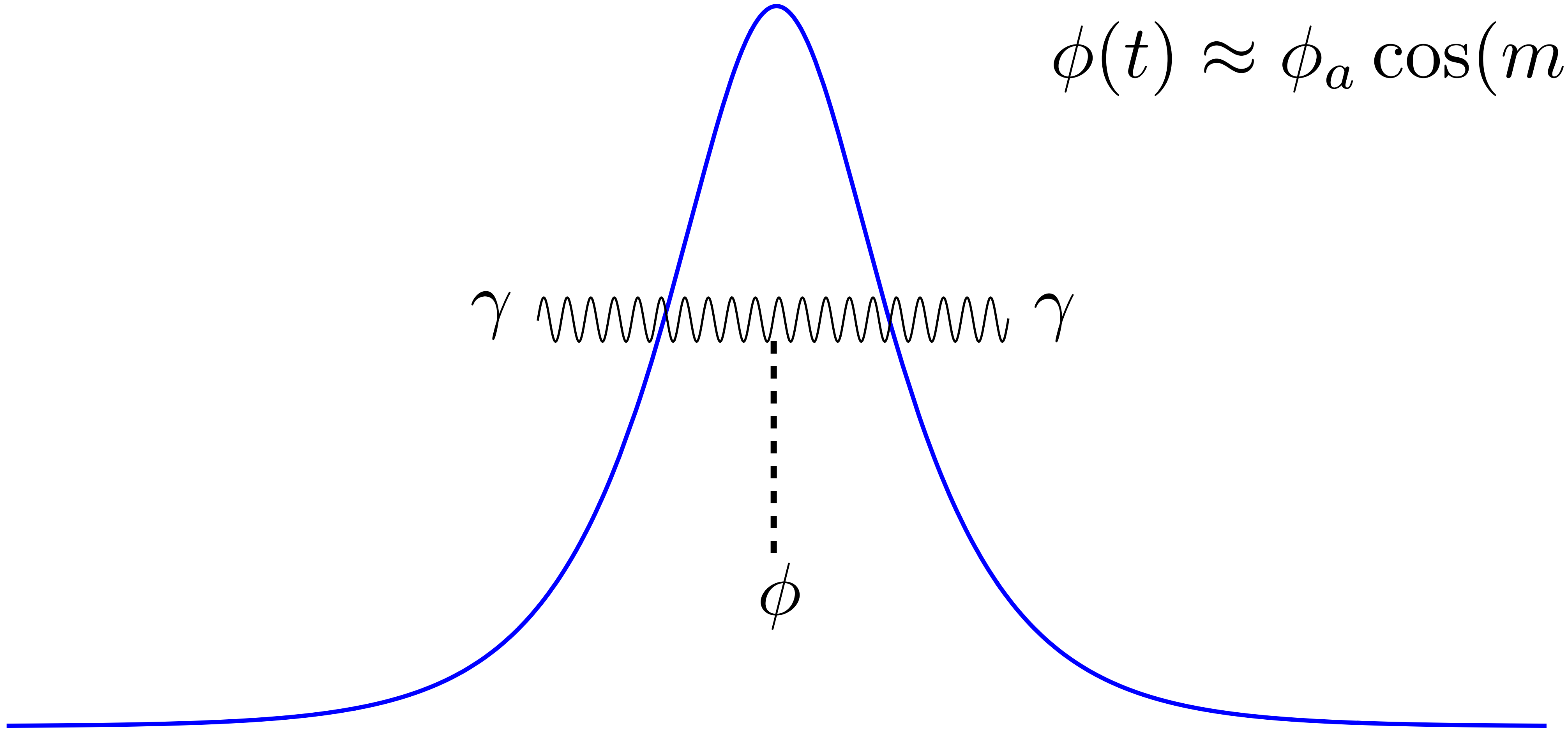
$$\mathcal{L}_\gamma = \frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2) + g_{a\gamma}\,\phi\,\mathbf{E}\cdot\mathbf{B}$$

(Sikivie 1983; Adshead, Giblin, Scully, Sfakianakis 2015, 2016; Masaki, Aoki, Soda 2017)

Equation of motion

$$\ddot{\mathbf{A}} - \nabla^2\mathbf{A} + g_{a\gamma}\,\partial_t\phi\,\nabla\times\mathbf{A} = 0$$

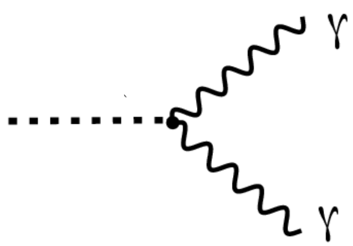
$$\phi(t) \approx \phi_a \cos(m_\phi t)$$

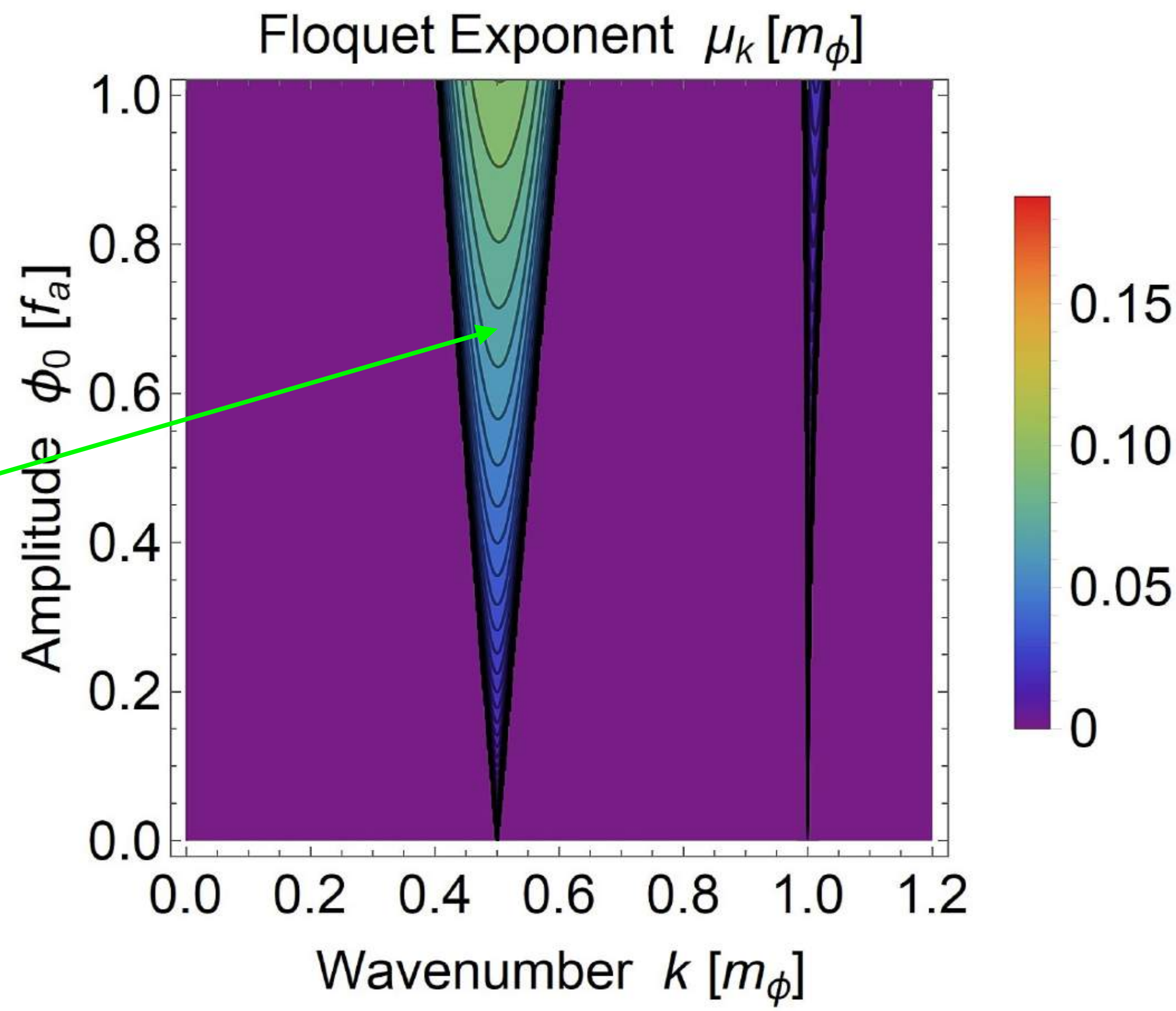


Homogeneous Axion Field

Mathieu Equation

$$\ddot{\mathbf{A}}^T_{\mathbf{k}} + k^2 \mathbf{A}^T_{\mathbf{k}} + g_{a\gamma}\, k\, \partial_t \phi(t) \mathbf{A}^T_{\mathbf{k}} = 0$$





Parametric resonance
always present

$$k \approx \frac{m_a}{2}$$

$$\mu^*_H \approx \frac{1}{4} g_{a\gamma}\, m_\phi\, \phi_a$$

e.g., Yoshimura 1996;

- maybe relevant to Hubble tension for ultralight axions: Gonzalez, Hertzberg, Rompineve 2006.13959

Comment on Photon Plasma Mass

In plasma, the photon acquires an effective mass $\omega_p^2 = \dfrac{4\pi\alpha\, n_e}{m_e}$

In VERY EARLY universe, this is huge; preventing resonance

Clumps in halo: $$\omega_p^2 \approx \frac{n_e}{0.03\,\mathrm{cm}^{-3}} \left(6 \times 10^{-12}\,\mathrm{eV}\right)^2$$

Negligibly small; allowing for resonance

Inhomogeneous (Spherical) Axion Star

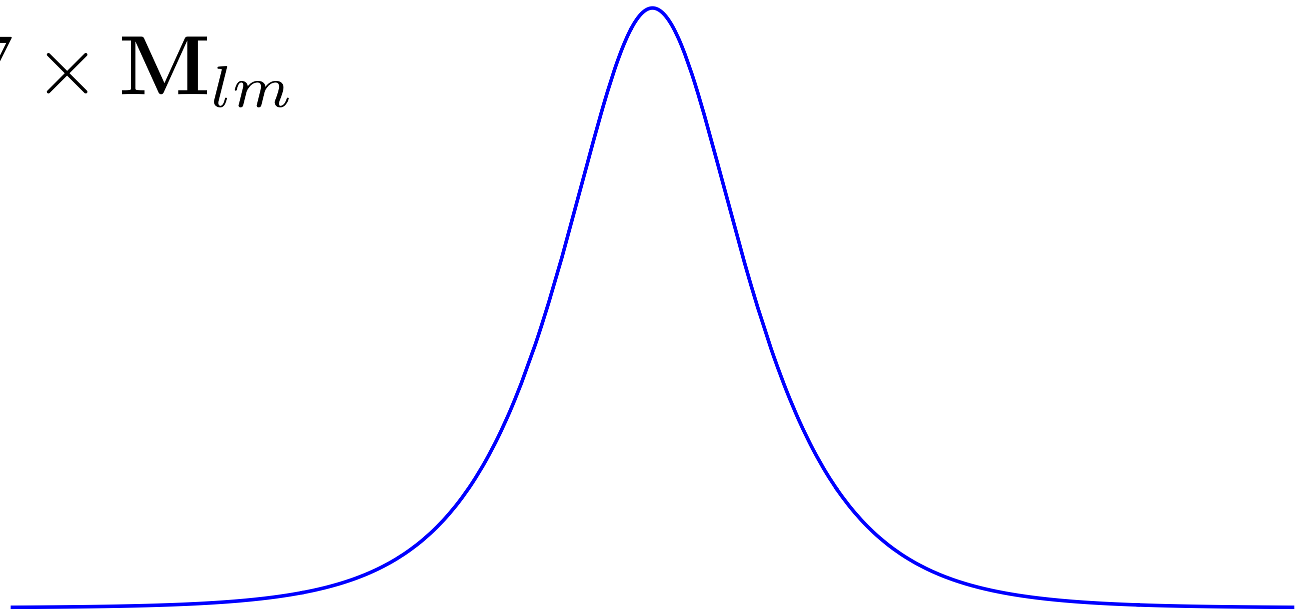
Decomposition into vector spherical harmonics

$$\mathbf{A}(\mathbf{r},t) = \sum_{lm} \int \frac{d^3k}{(2\pi)^3} \left[a_{lm}(k,t)\mathbf{N}_{lm}(k,\mathbf{r}) + b_{lm}(k,t)\mathbf{M}_{lm}(k,\mathbf{r})\right]$$

$$\mathbf{M}_{lm}(k,\mathbf{r}) = i\frac{j_l(kr)}{\sqrt{l(l+1)}}\nabla \times \left[Y_{lm}(\theta,\varphi)\mathbf{r}\right]$$

where

$$\mathbf{N}_{lm}(k,\mathbf{r}) = \frac{i}{k}\nabla \times \mathbf{M}_{lm}$$

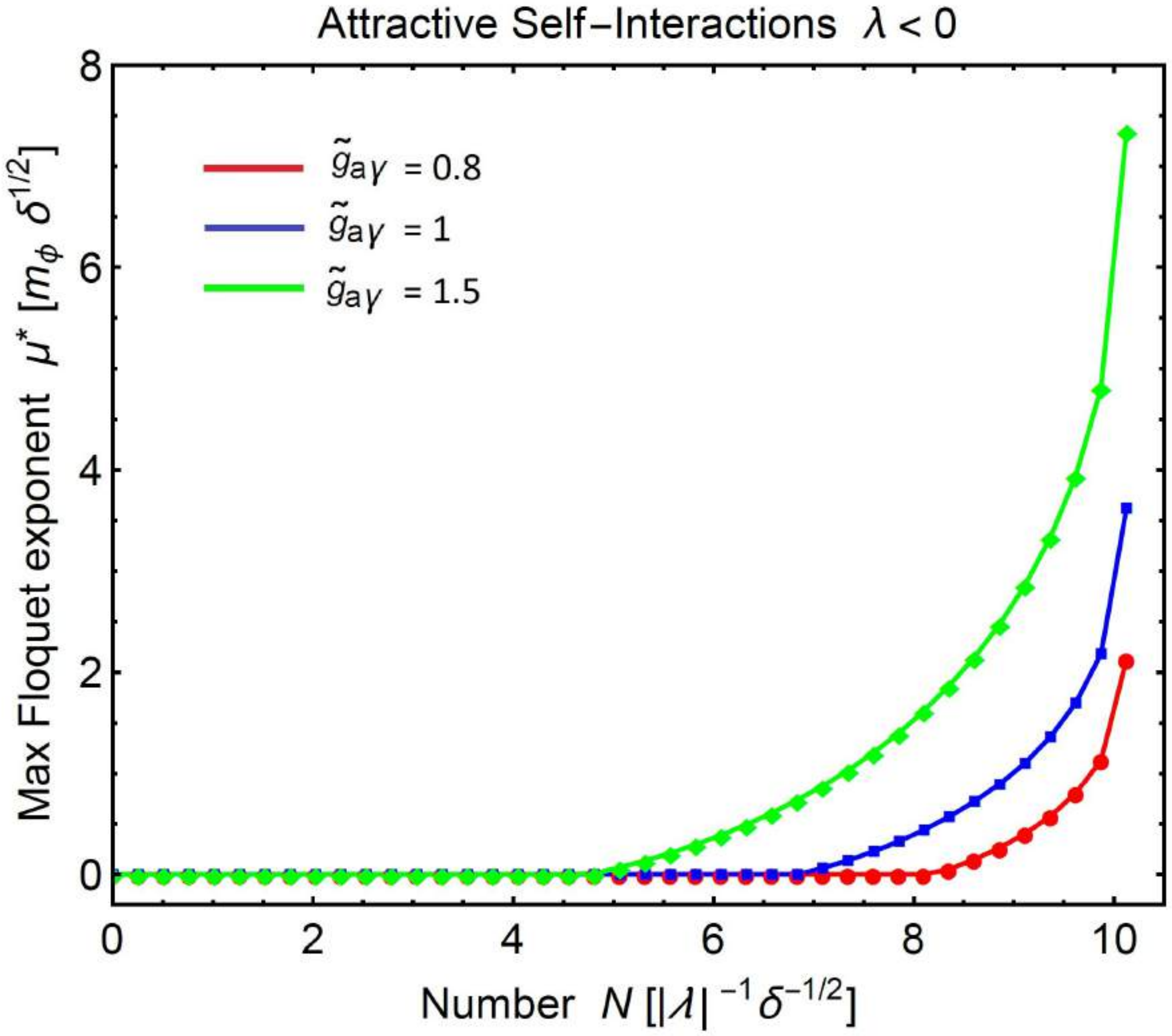


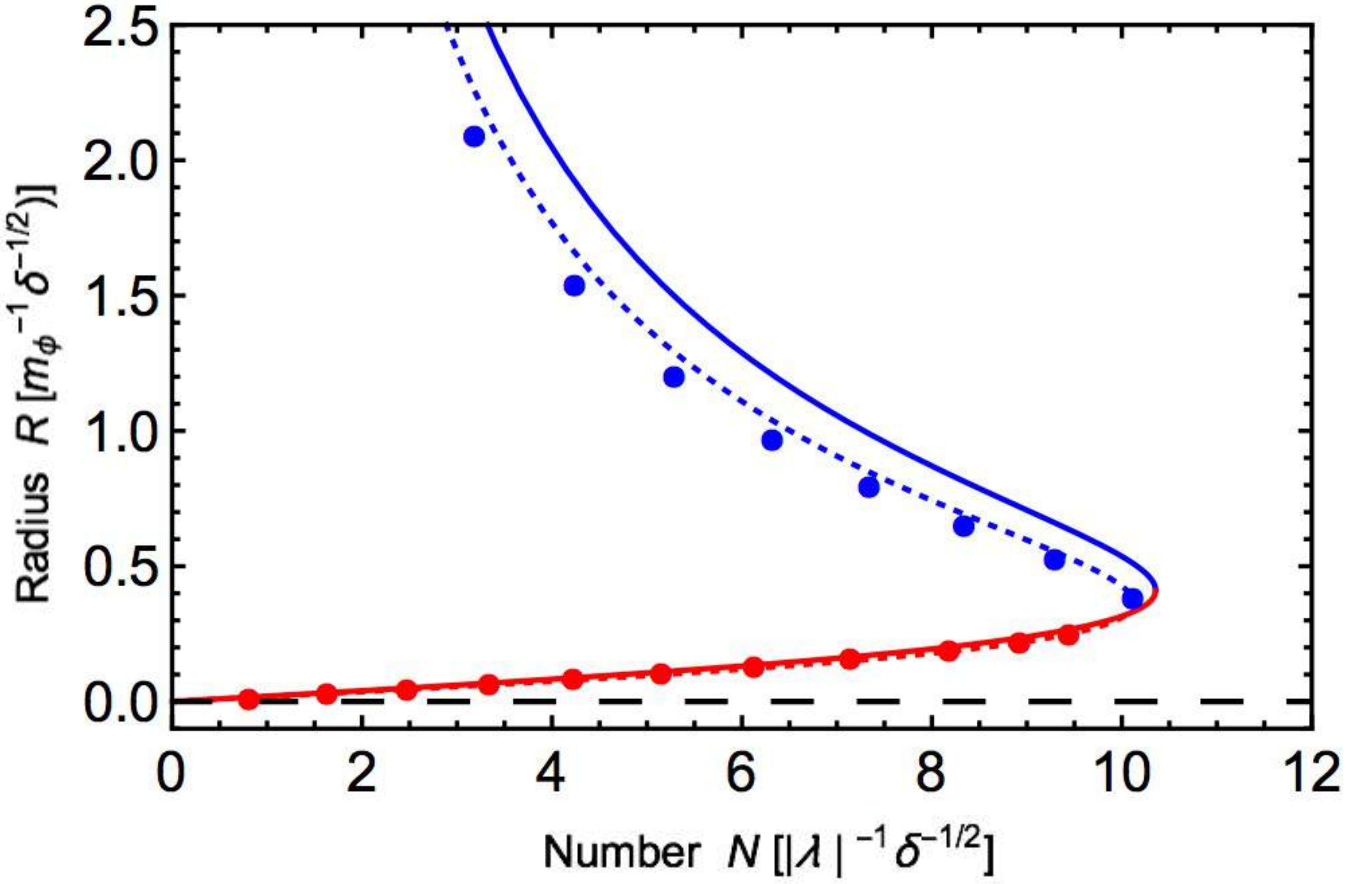
Hertzberg, Schiappacasse 1805.00430

Inhomogeneous (Spherical) Axion Star

Instability channel $\quad l = 1, m = 0, \quad b_{10} = -i\, a_{10}$

$$\ddot{a}_{10}(k,t) + k^2 a_{10}(k,t) + g_{a\gamma} k \int \frac{dk'}{(2\pi)} \partial_t \tilde{\phi}(k-k') a_{10}(k',t) = 0$$





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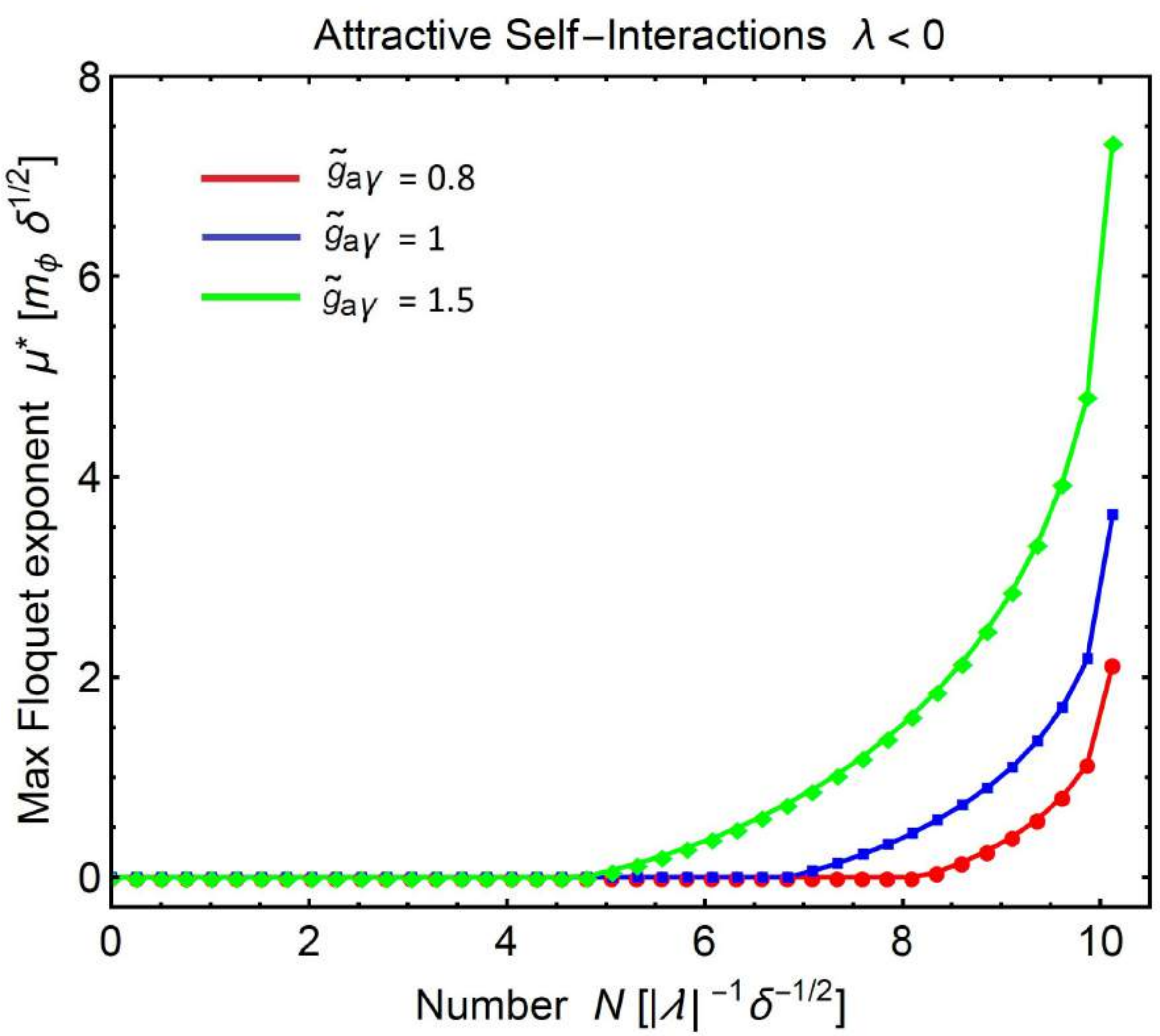

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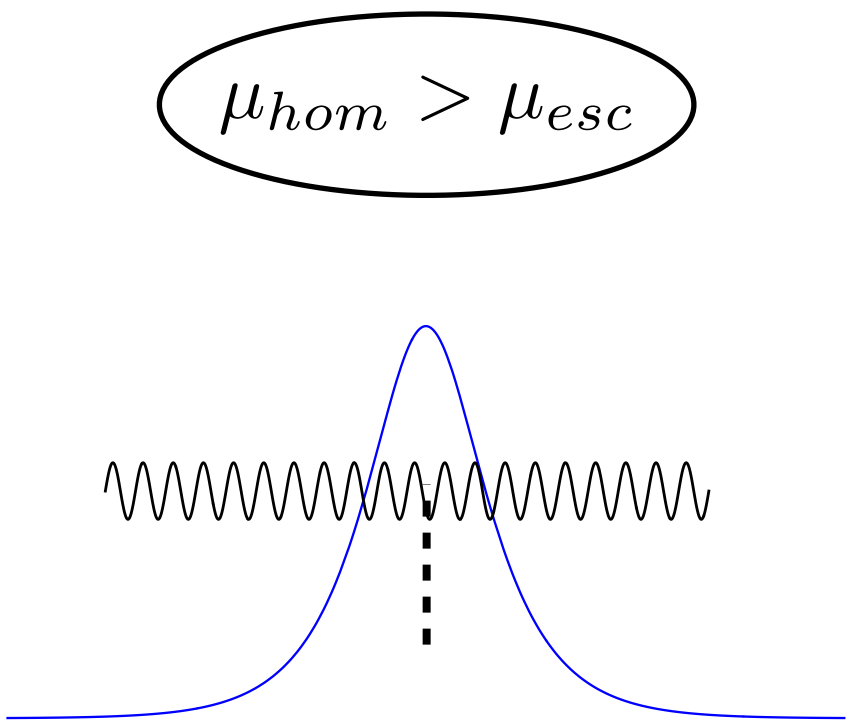


Tkachev 1986, 1987, 2015;
Hertzberg 2010; Kawasaki, Yamada 2014

Resonance Condition (Spherical) Axion Star

$$\boxed{g_{a\gamma} > \frac{0.3}{f_a}} \qquad (\lambda < 0)$$

No resonance for standard QCD axion-photon coupling $\quad g_{a\gamma} \sim \dfrac{\alpha}{f_a}$

Allowed for models with enhanced couplings, decay to hidden sectors, or repulsive interactions

(e.g., Daido, Takahashi, Yokozaki 2018)

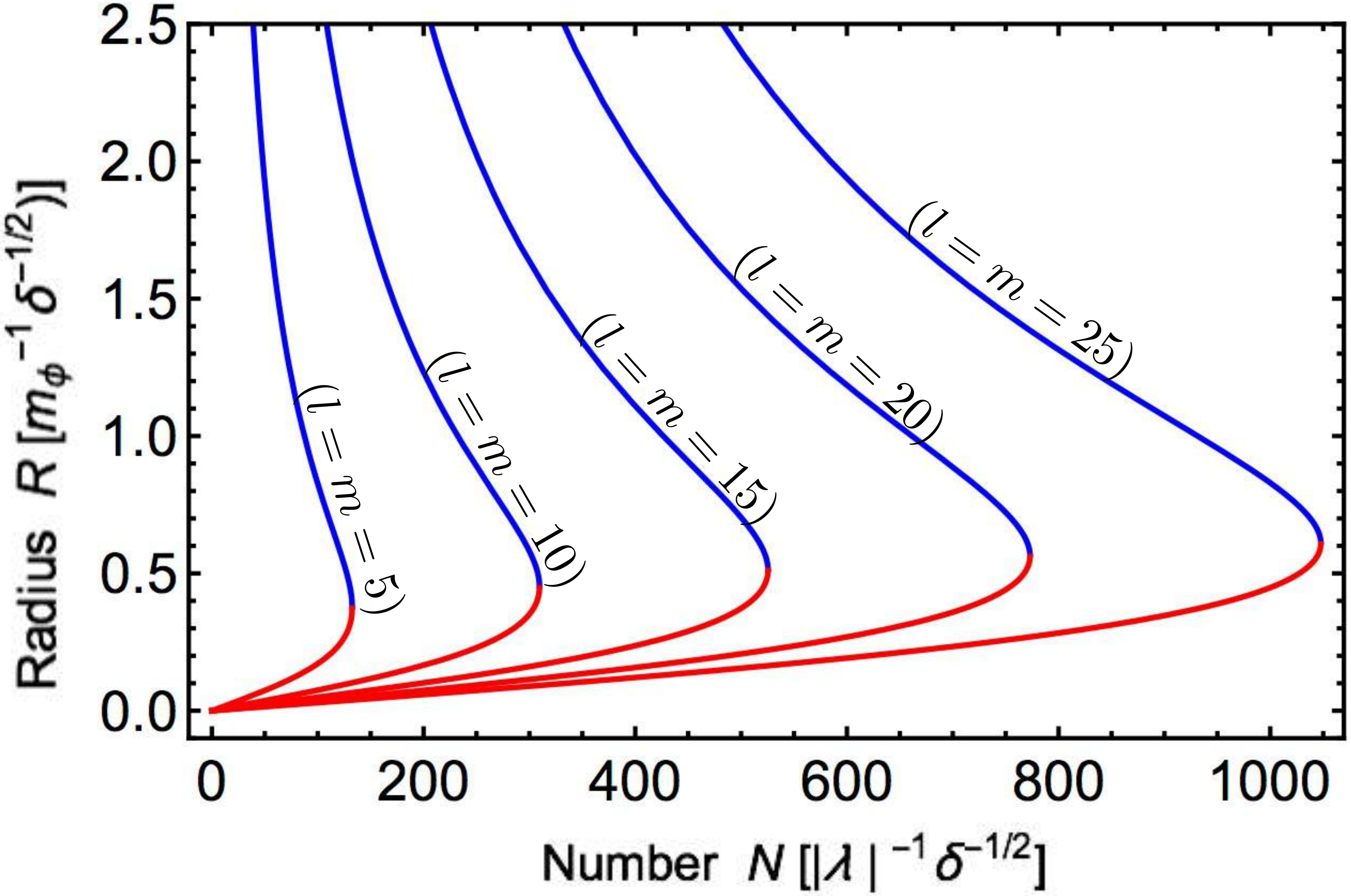
(e.g., Farina, Pappadopulo, Rompineve, Tesi 2016)

(e.g., Fan 2016)

Hertzberg, Schiappacasse 1805.00430

Including Angular Momentum

Two Branches of Solutions (with Angular Momentum)

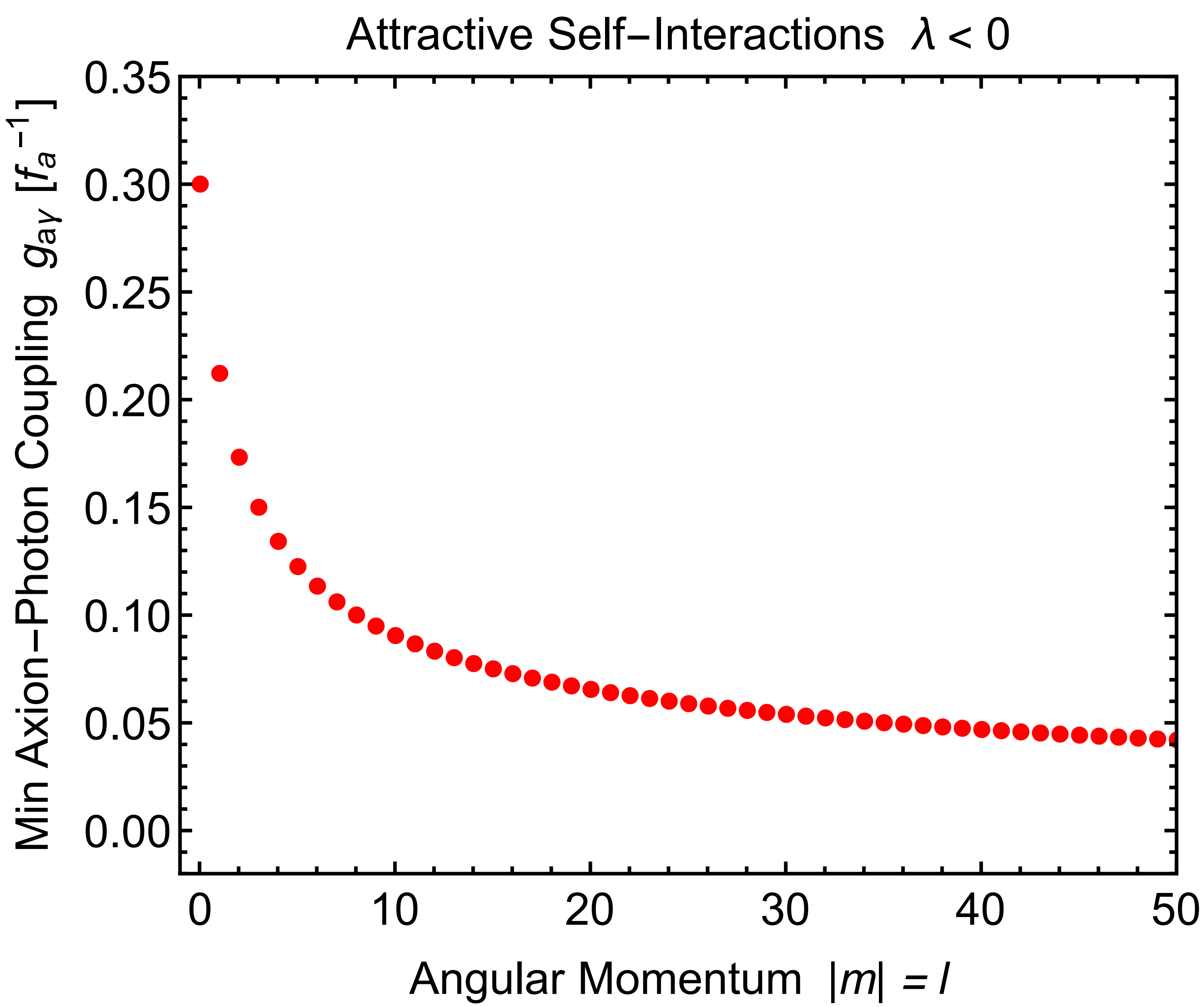


High angular momentum allows higher amplitude at core, which helps for resonance into photons

Hertzberg, Schiappacasse 1804.07255

Resonance Condition (Non-Spherical) Axion Star

$$\boxed{g_{a\gamma} > \frac{0.3}{f_a\sqrt{l+1}}} \qquad (\lambda < 0)$$

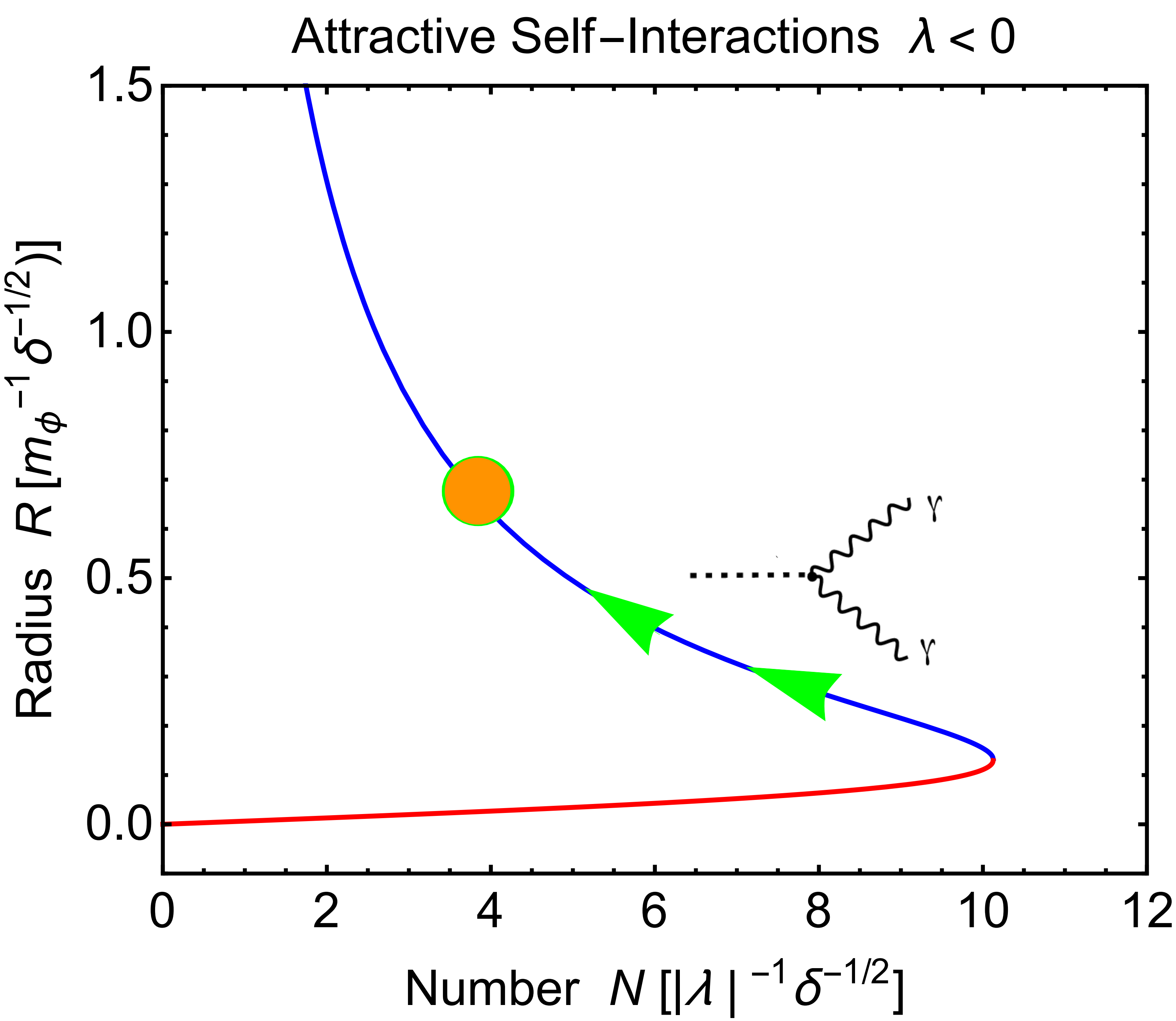


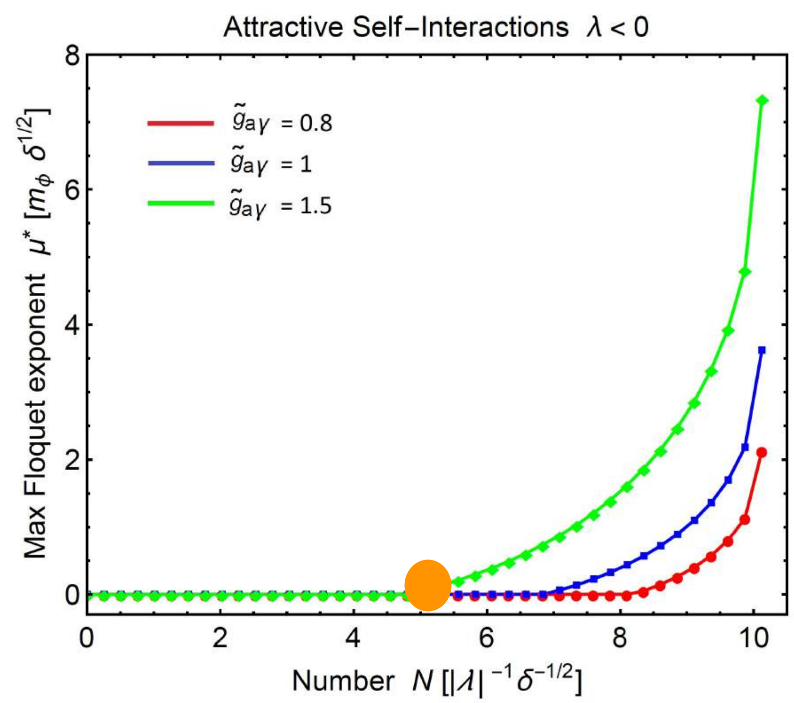
Resonance allowed for standard QCD axion-photon couplings, with high angular momentum

Hertzberg, Schiappacasse 1805.00430

Astrophysical Consequences

Lasing Stars Early Universe





(i) Mass Pile Up

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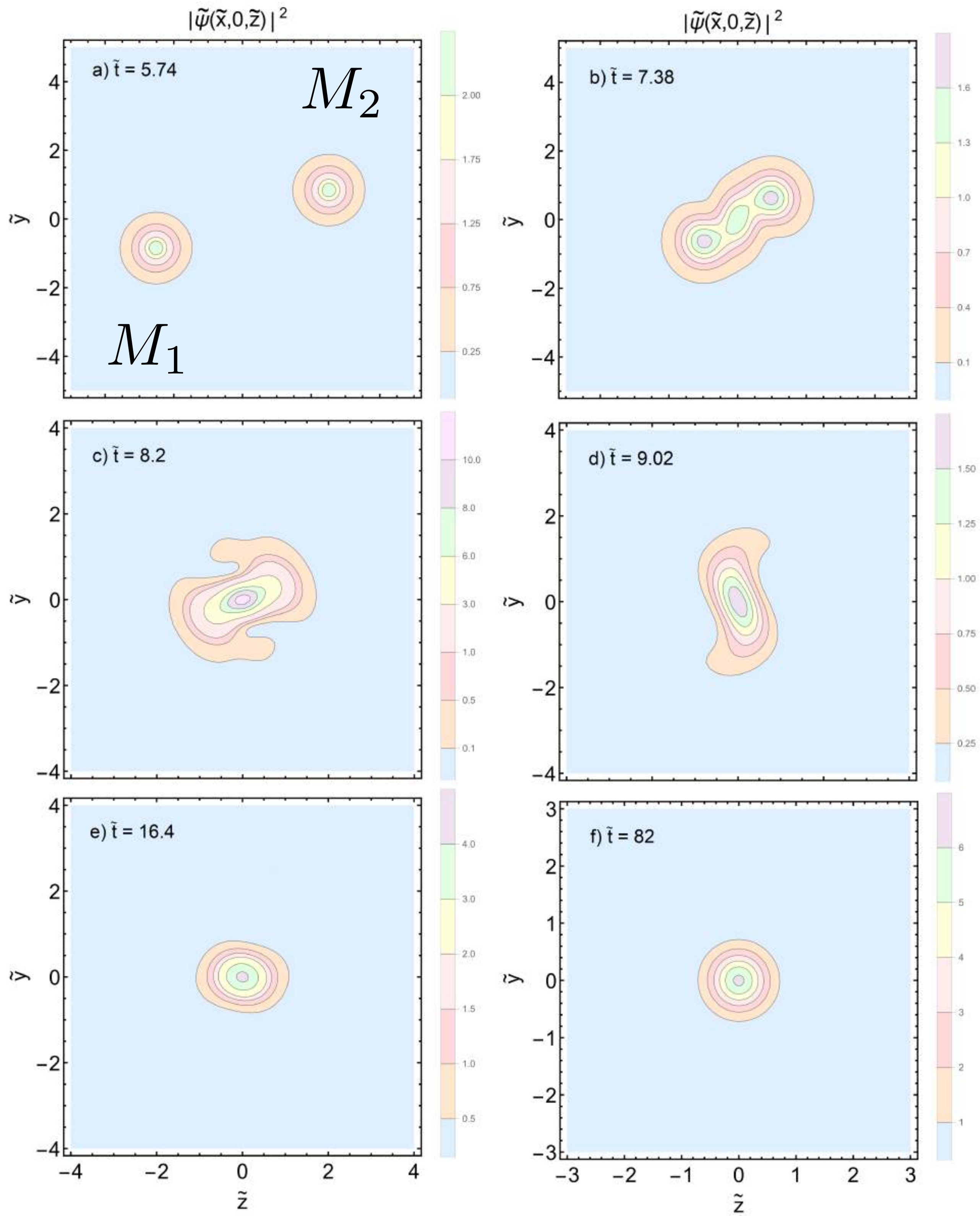
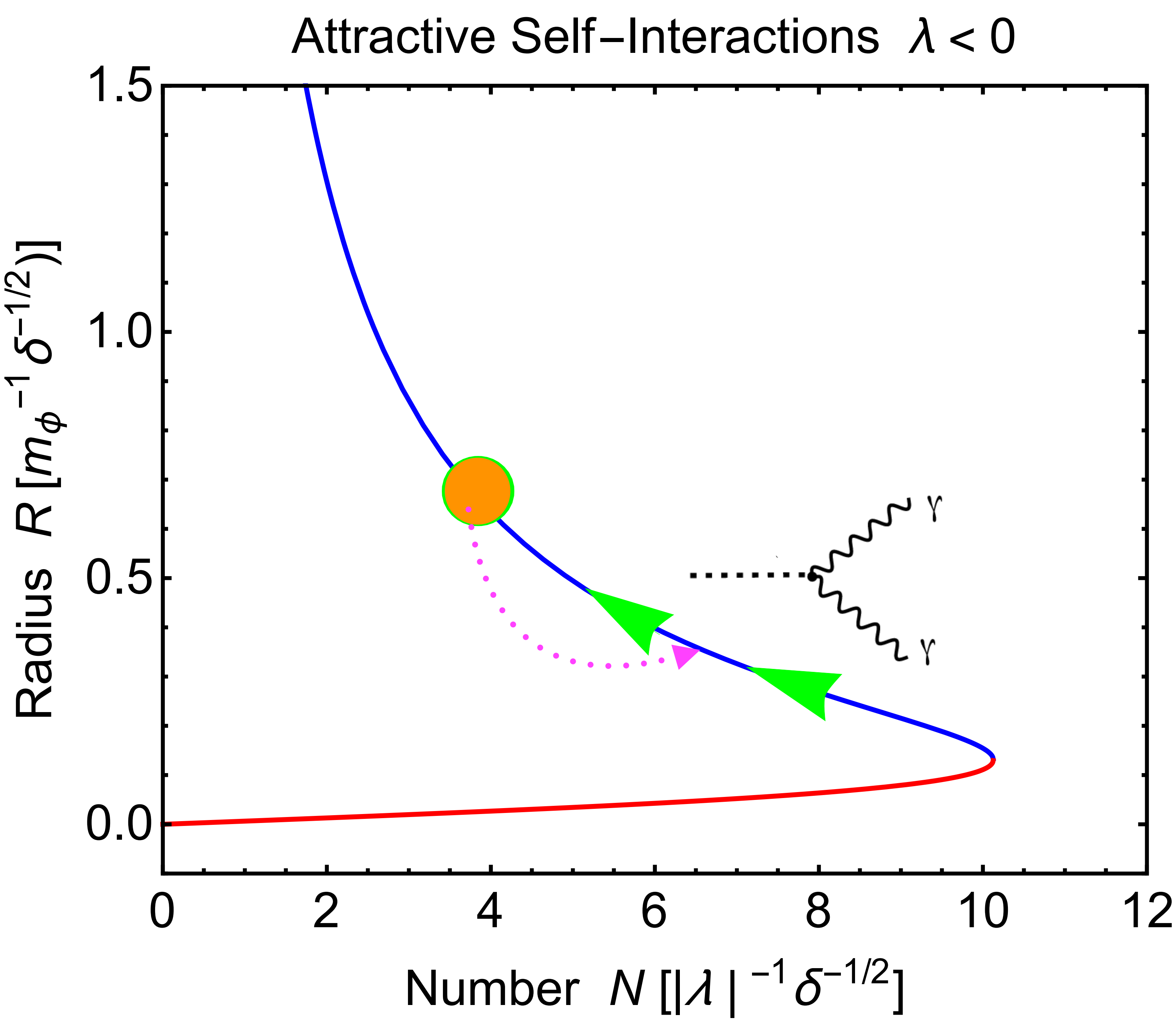
Axion Star Mergers

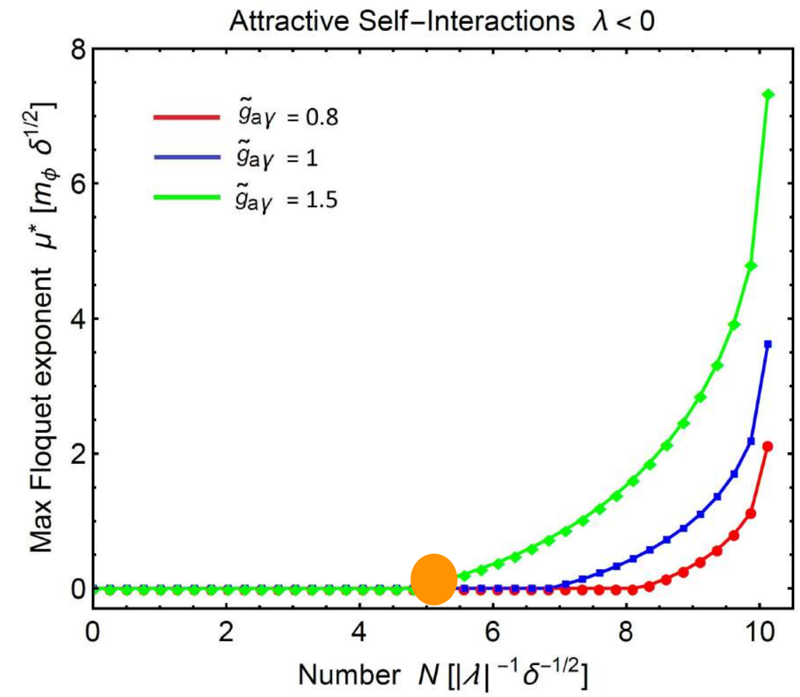


$$M_{merged} > \max\{M_1,\, M_2\}$$

Hertzberg, Li, Schiappacasse 2005.02405

Lasing Stars Late Universe





(i) Mass Pile Up

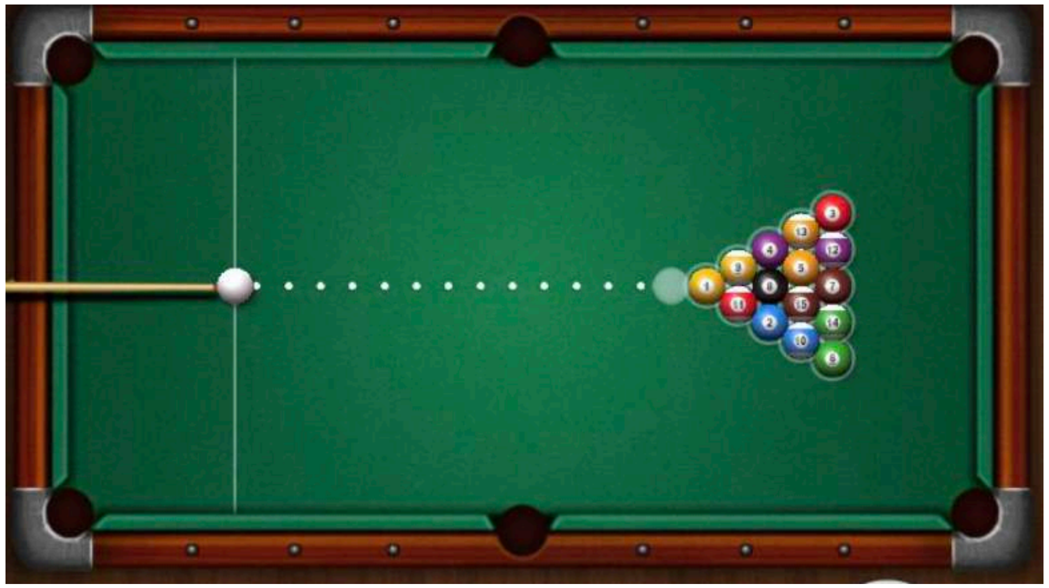
(ii) Late Time Mergers;
Radio-wave Bursts

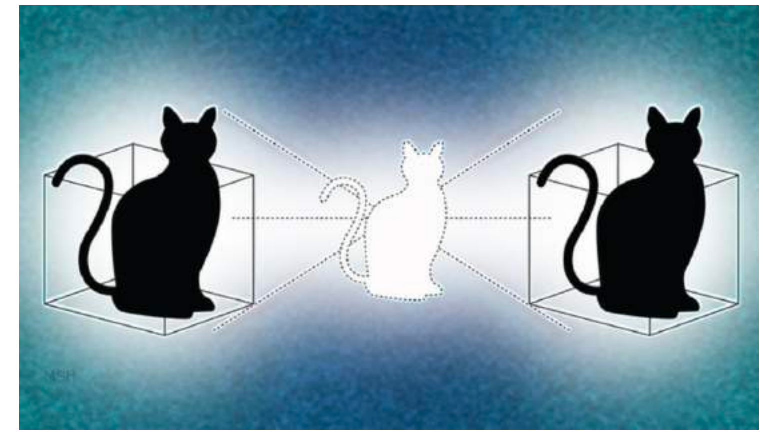
$$\lambda_{EM} = \frac{2\pi}{k} \approx \frac{4\pi}{m_a} = \mathcal{O}(1)\, meters$$

Hertzberg, Li, Schiappacasse 2005.02405

(also Tkachev 2015)

Recall that non-linear dynamics can launch states into Schrodinger cat-like states

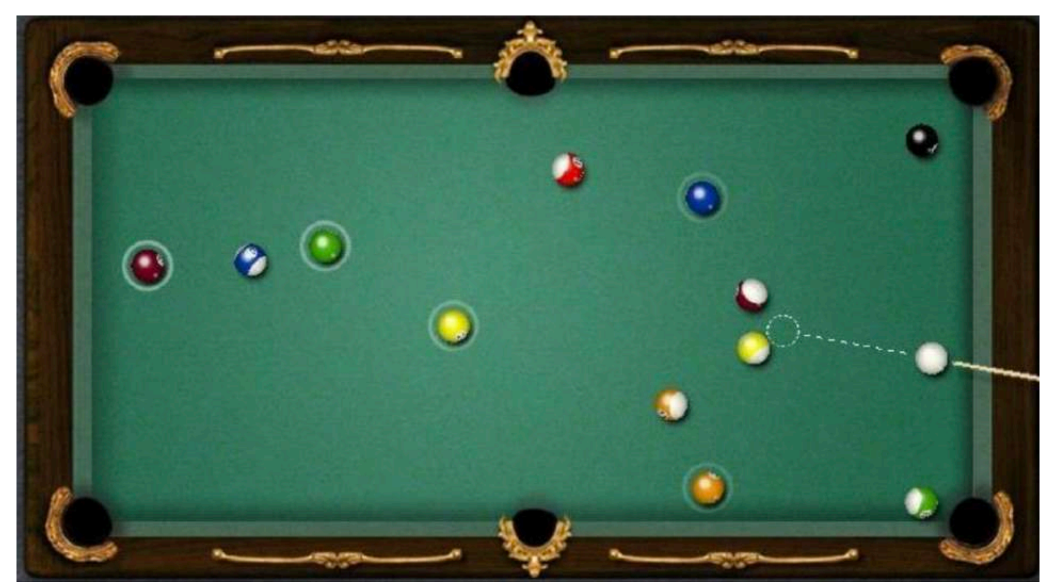




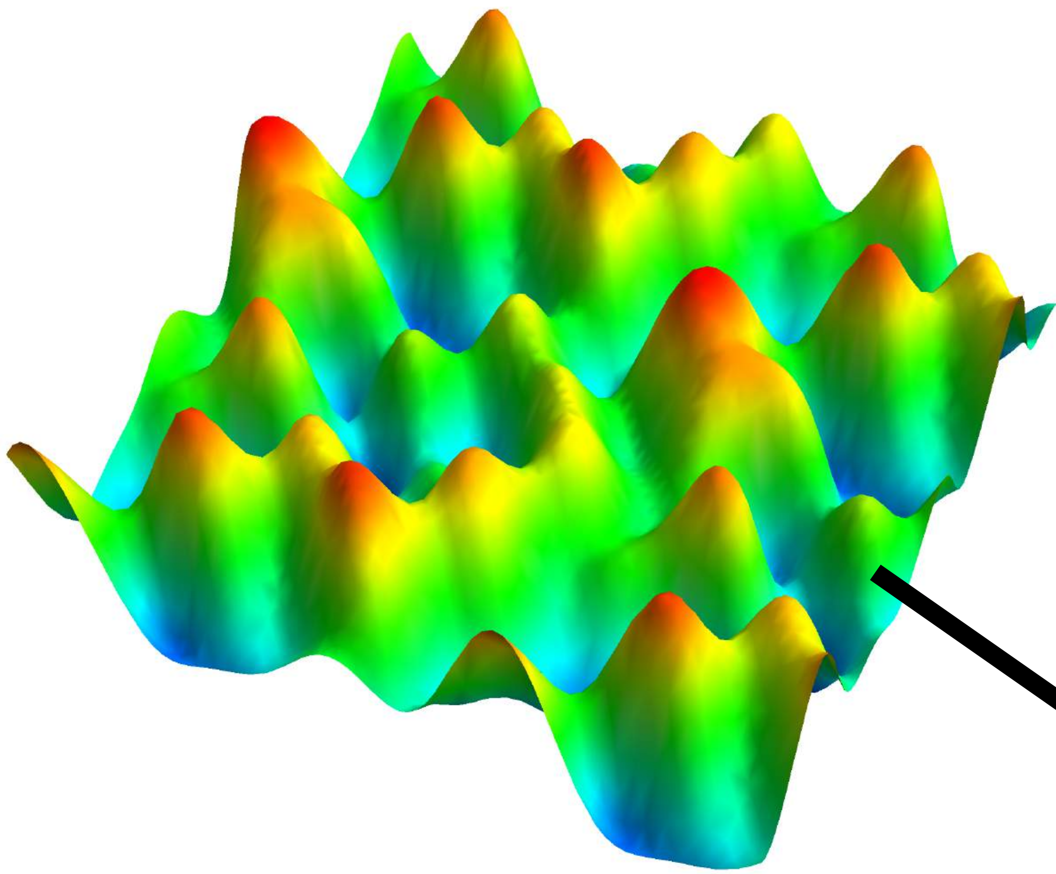
Quantumness destroyed due to DECOHERENCE

Schrodinger Cat Billiards

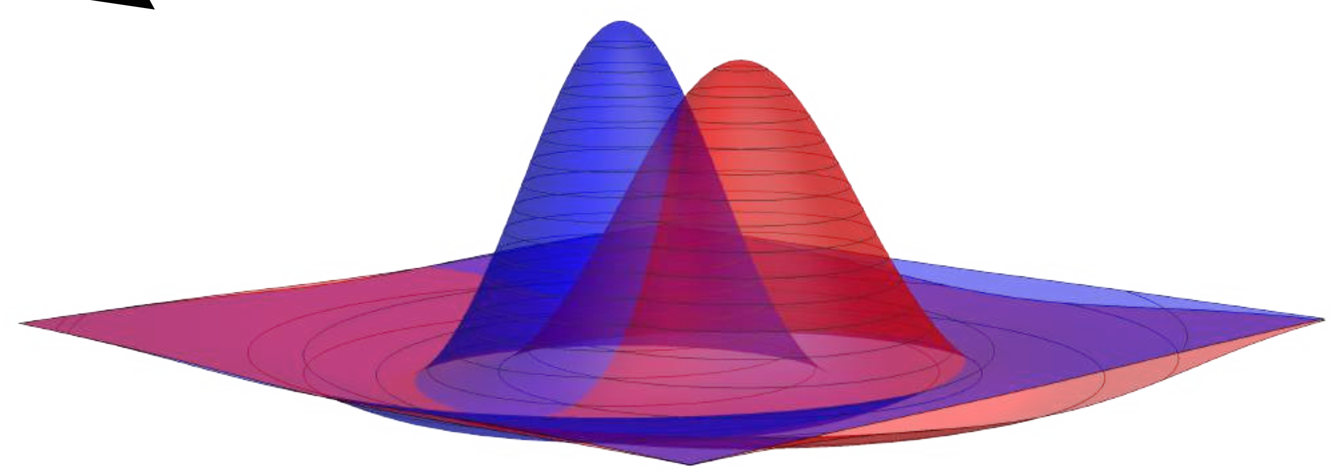
+



Recall that non-linear dynamics can launch states into Schrodinger cat-like states



(Axion) Dark Matter Schrodinger Cat

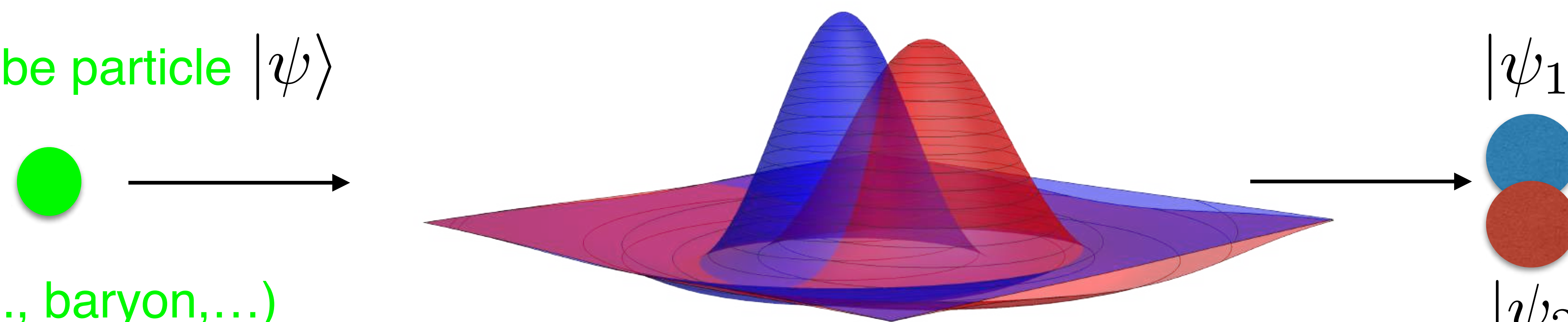


Quantumness destroyed due to DECOHERENCE???

Less clear because dark matter has tiny (non-gravitational) interactions

Could Dark Matter Schrodinger Cats Survive?

Entanglement from Gravitational Scattering



$$|\Psi_i\rangle = (|A_1\rangle + |A_2\rangle)|\psi\rangle$$

Product State

$$|\Psi_f\rangle = |A_1\rangle|\psi_1\rangle + |A_2\rangle|\psi_2\rangle$$

Entangled State

Allali, Hertzberg 2005.12287

Trace Out Probe Particle

$$\rho = |\Psi_f\rangle\langle\Psi_f|$$ Full Density Matrix

$$\rho_{red} = \mathrm{Tr}_p[\rho]$$ Reduced Density Matrix

$$= |A_1\rangle\langle A_1| + |A_2\rangle\langle A_2| + \langle\psi_1|\psi_2\rangle|A_2\rangle\langle A_1| + \langle\psi_2|\psi_1\rangle|A_1\rangle\langle A_2|$$

Allali, Hertzberg 2005.12287

Trace Out Probe Particle

$$\rho = |\Psi_f\rangle\langle\Psi_f|$$

Full Density Matrix

$$\rho_{red} = \mathrm{Tr}_p[\rho]$$

Reduced Density Matrix

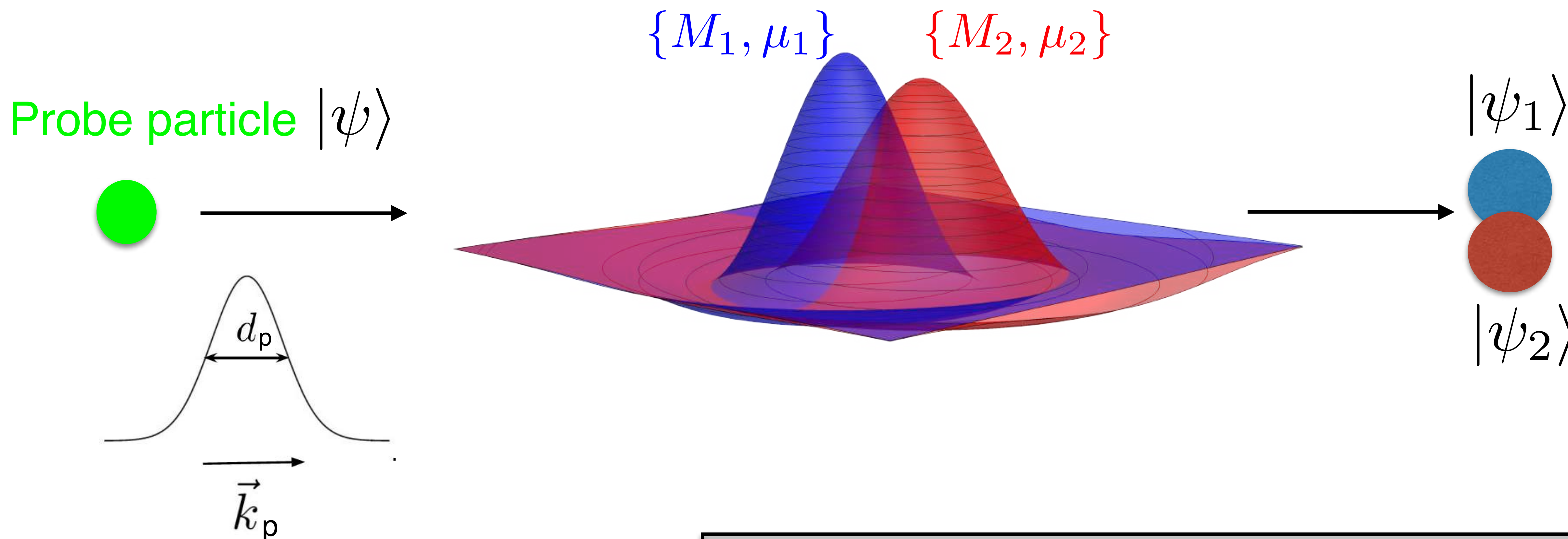
$$= |A_1\rangle\langle A_1| + |A_2\rangle\langle A_2| + \boxed{\langle\psi_1|\psi_2\rangle}|A_2\rangle\langle A_1| + \boxed{\langle\psi_2|\psi_1\rangle}|A_1\rangle\langle A_2|$$

Off diagonal elements;
controlling true quantum effects

Allali, Hertzberg 2005.12287

Result for Overlap of Probe Particle States

We evolve a Gaussian wave packet using perturbation theory



$$|\langle\psi_1|\psi_2\rangle|^2 = 1 - 2\Delta$$

$$\boxed{\Delta_0 = \frac{2G^2m^4}{\hbar^4k^2d^2}\left[\frac{M_1^2}{\mu_1^2}\chi_{11} + \frac{M_2^2}{\mu_2^2}\chi_{22} - 2\frac{M_1M_2}{\mu_1\mu_2}\chi_{12}\right]}$$

Allali, Hertzberg 2005.12287

Decoherence Rate from N-Probe Particles

Off diagonal element of density matrix

$$\prod_{n=1}^{N} |\langle\psi_1|\psi_2\rangle|_n = \prod_{n=1}^{N}(1-\Delta_b) \sim e^{-\sum_{n=1}^{N}\Delta_b}$$

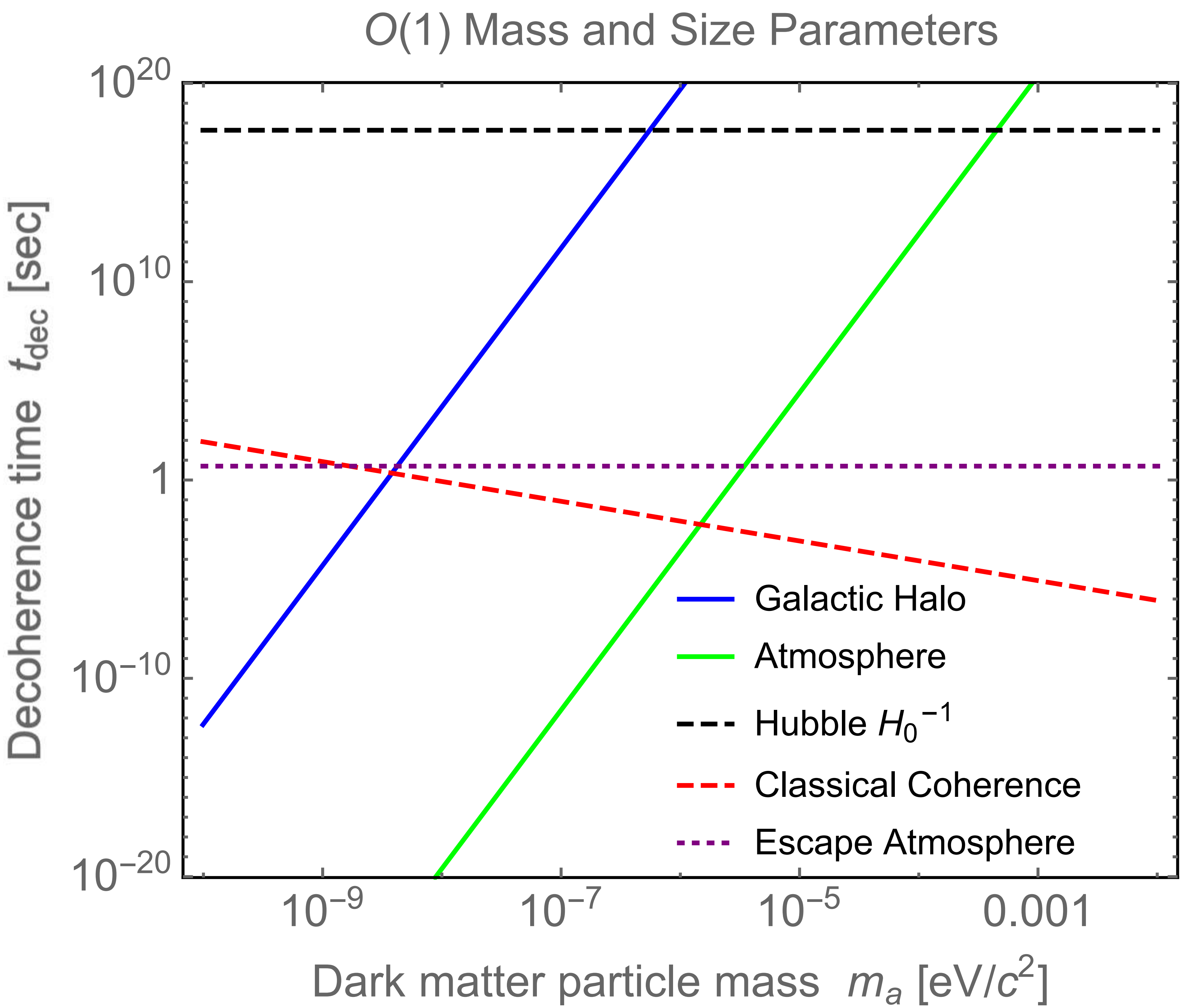
Decoherence rate

$$\Gamma_{\rm dec} = n\,v\int d^2b\,\Delta_b$$

$$\boxed{\Gamma_{\rm dec} = \frac{4\pi G^2 m^4 n\,v}{\hbar^4 k^2}\left[\frac{M_1^2}{\mu_1^2}\chi_{11} + \frac{M_2^2}{\mu_2^2}\chi_{22} - 2\frac{M_1 M_2}{\mu_1\mu_2}\chi_{12}\right]}$$

Allali, Hertzberg 2005.12287

Application to Diffuse Axions



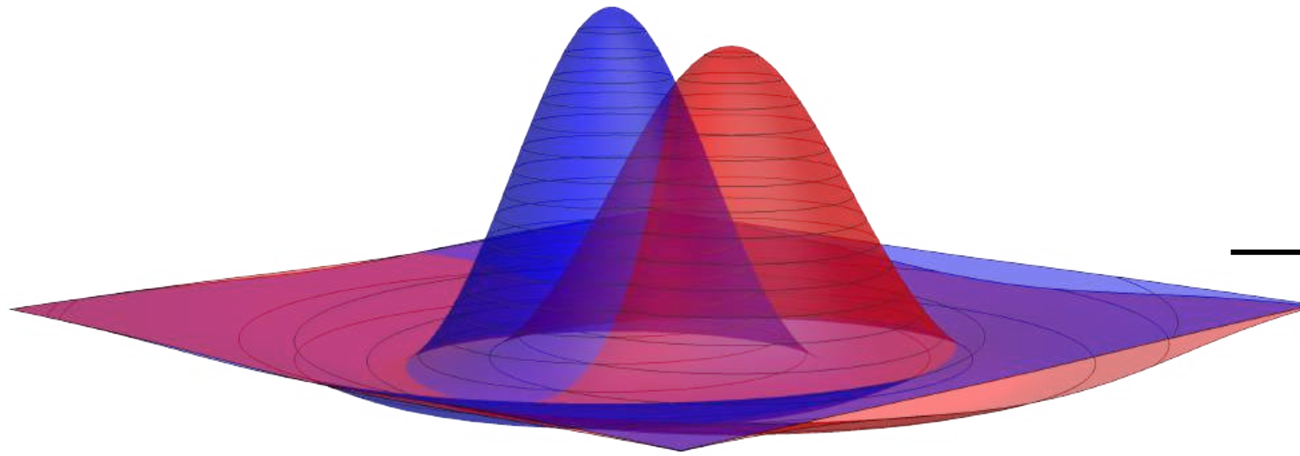
Allali, Hertzberg 2005.12287

Application to Boson Stars

Boson star

Probe particle $|\psi\rangle$

(e.g., baryon,…)



$|\psi_1\rangle$

$|\psi_2\rangle$

Boson stars are much denser

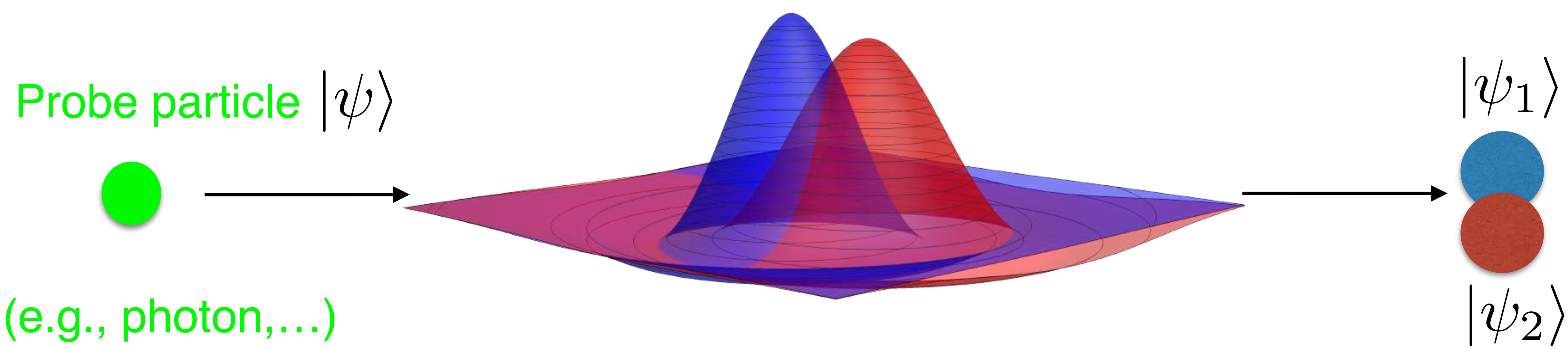
Decoherence Rate

$$\boxed{\Gamma_{\rm dec} \gtrsim \frac{\hbar^2 m_p \rho_p}{v_p m_a^4} \sim 10^{21}\,\text{sec}^{-1}\left(\frac{1\,\text{eV}}{m_a\,c^2}\right)^4}$$

Extremely rapid decoherence —> Very classical

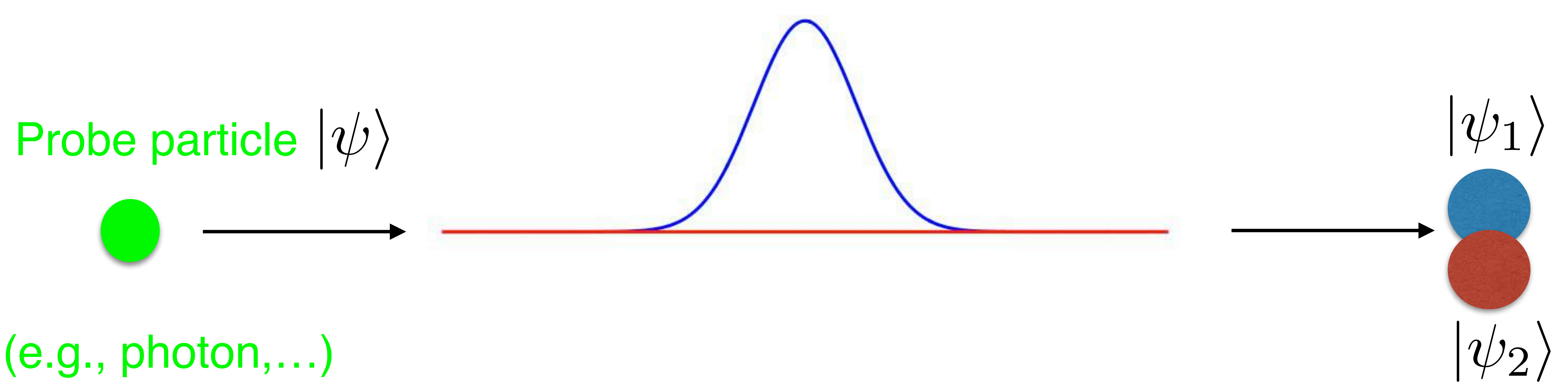
Allali, Hertzberg 2005.12287

General Relativistic Extension



Allali, Hertzberg 2012.12903

General Relativistic Extension



Allali, Hertzberg 2012.12903

General Relativistic Extension

Starting from QFT, can derive RSE for 1-particle sub-space: (ignoring spin)

$$(i\partial_t - \sqrt{-\nabla^2 + m^2})\psi(\mathbf{x}, t) = \left(\Phi(\mathbf{x}, t)\sqrt{-\nabla^2 + m^2} - \frac{\Psi(\mathbf{x}, t)\nabla^2}{\sqrt{-\nabla^2 + m^2}}\right)\psi(\mathbf{x}, t)$$

Decoherence Rate
for superposition of
different phases

$$\boxed{\Gamma_{dec} \propto \exp\left[-\frac{m_a^2 E_p^2}{\mu^2 k^2}\right] \sim \exp\left[-\frac{1}{v_a^2 v_p^2}\right]}$$

Exponentially suppressed for non-relativistic axions or probes

So the phase is rather robust against decoherence - may be relevant to direct detection

(Although may decohere near black hole horizons)

Allali, Hertzberg 2012.12903

Thank you