The recent speed up of the Universe:

A Phenomelogical Approach

Beyond Standard Model: From Theory to Experiment 2021
@ Cairo (Egypt)

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March 31st, 2021

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Phys.Dark Univ. 26 (2019) 100391 • e-Print: 1905.07304

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e-Print: 2011.08222 [gr-qc]

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Introduction

Introduction-1-: A brief sketch of the universe

- The universe is homogeneous and isotropic on large scales (cosmological principle)
- The matter content of the universe:
    - Standard matter
    - Dark matter
    - Something that induce the late-time acceleration of the Universe
- The acceleration of the universe is backed by several measurments: $H(z)$, SneIa, BAO, CMB, LSS (matter power spectrum, growth function)...

Introduction-2-

- The effective equation of state of whatever is driving the current speed up of the universe is roughly -1. For example, for a wCDM model with w constant and $k = 0$, Planck (TT, TE, EE+lensing) + ext(BAO,H0,JLA) results implies w is very close to -1
- Such an acceleration could be due
  - A new component of the energy budget of the universe: dark energy; i.e. it could be Λ, quintessence or of a phantom(-like/effective) nature
  - A change on the behaviour of gravity on the largest scale. No new component on the budget of the universe but rather simply GR modifies its behaviour, within a metric, Palatini (affine metric) ....

Late-time acceleration of the Universe within GR: dark energy with a constant EoS

Constant equation of state for DE: background-1-

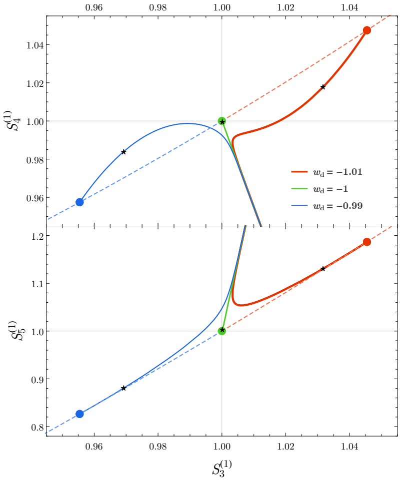
- Cosmic acceleration:

$$\frac{\ddot{a}}{a} = -\frac{4\pi \mathrm{G}}{3}(\rho_{\mathrm{m}} + \rho_{\mathrm{de}} + 3p_{\mathrm{de}})$$

- Observation indicates that for $w_{\mathrm{de}} \sim -1$ where $w_{\mathrm{de}} = p_{\mathrm{de}}/\rho_{\mathrm{de}}$.
- Therefore, as soon as DE starts dominating the Universe starts accelerating, i.e. $\ddot{a} > 0$.
- Simplest cases ΛCDM or wCDM.

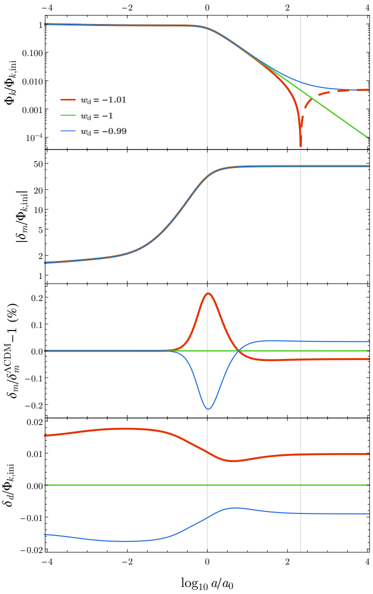
Constant equation of state for DE: background-2-

- State finders approach (Sahni, Saini and Starobinsky JETP Lett. [arXiv:astro-ph/0201498])
  - Scale factor: $\frac{a(t)}{a_0} = 1 + \sum_{n=1}^{\infty} \frac{A_n(t_0)}{n!} \left[H_0\,(t - t_0)\right]^n$, where $A_n := a^{(n)}/(a\,H^n)$, $n \in \mathbb{N}$.
  - State finders parameters: $S_3^{(1)} = A_3$, $S_4^{(1)} = A_4 + 3\,(1 - A_2)$, $S_5^{(1)} = A_5 - 2\,(4 - 3A_2)\,(1 - A_2)$
  - $\Omega_\mathrm{m} = 0.309$, $\Omega_\mathrm{d} = 0.691$ and $H_0 = 67.74$ km s^{-1} Mpc^{-1} (according to Planck).



Albarran, B.L. and Morais, EPJC 2018 [arXiv:1706.01484]

Constant equation of state for DE: perturbations-1-



- Example of the evolution of the perturbations: $k = 10^{-3}$ Mpc^{-1}
- ΛCDM model: Φ_k vanishes asymptotically
- Phantom model: Φ_k also evolves towards a constant in the far future but a change of sign occurs roughly at $\log_{10} a/a_0 \simeq 2.33$, corresponding to 8.84×10^{10} years in the future. A dashed line indicates negative values of Φ_k
- Quintessence model: Φ_k evolves towards a constant in the far future without changing sign

Albarran, B.L. and Morais, EPJC 2018 [arXiv:1706.01484]

Constant equation of state for DE: perturbations-2-

- What about $f\sigma_8$ for the three different DE models?

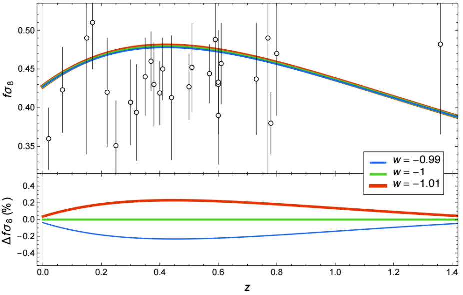


Figure 2: (Top panel) evolution of $f\sigma_8$ for low red-shift $z \in (0, 1.4)$ for three dark energy models: (blue) $w = -0.99$, (green) $w = -1$ and (red) $w = -1.01$. White circles and vertical bars indicate the available data points and corresponding error bars (cf. Table I of [13]). (Bottom panel) evolution of the relative differences of $f\sigma_8$ for each model with regard to ΛCDM ($w = -1$). $\Delta f\sigma_8$ is positive in the phantom case and negative in the quintessence case. For all the models, it was considered that σ_8 evolves linearly with δ_m and that $\sigma_8 = 0.816$ at the present time [7].

$$f \equiv \frac{d\left(\ln \delta_{\rm m}\right)}{d\left(\ln a\right)}, \qquad \sigma_8\left(z,\, k_{\sigma_8}\right) = \sigma_8\left(0,\, k_{\sigma_8}\right) \frac{\delta_{\rm m}\left(z,\, k_{\sigma_8}\right)}{\delta_{\rm m}\left(0,\, k_{\sigma_8}\right)}$$

$k_{\sigma_8} = 0.125$ hMpc^{-1}, $\sigma_8(0, k_{\sigma_8}) = 0.820$ (Planck)

Albarran, B.L. and Morais [arXiv:1706.01484]

Late-time acceleration of the Universe within GR and with a phantom fluid

Late-time acceleration of the Universe within GR and with a phantom fluid

The models

The models

- We are going to focus on the genuinely phantom matter. i.e. when the Equation of State satisfies $w < -1$.
- The phantom matter violates the Null energy condition. In consequence, the rest of the energy conditions are violated.
    - Null energy condition $\Rightarrow$ $p + \rho \geqslant 0$.
    - Weak energy condition $\Rightarrow p + \rho \geqslant 0$, $\rho \geqslant 0$.
    - Dominant energy condition $\Rightarrow$ $\rho \geqslant |p|$.
    - Strong energy condition $\Rightarrow$ $p + \rho \geqslant 0$, $3p + \rho \geqslant 0$.
- For example, a suitable way to write the Equation of State of a phantom fluid is

$$p = -\rho - C\rho^{\alpha},$$

where C is a positive constant and α is a real number. We are going to focus on the cases $\alpha = 1, 1/2, 0$.

Genuine phantom models: BR, LR and LSBR

- The DE content can be described for example with a perfect fluid or a scalar field

Event	EoS for a perfect fluid	Potential for a scalar field
BR	$p_d = w_\mathrm{d} \rho_d$	$V(\phi) = C_{br} e^{\lambda\phi}$
LR	$p_\mathrm{d} = -\rho - B\sqrt{\rho_\mathrm{d}}$	$V(\phi) = C_{lr}\phi^4 + D_{lr}\phi^2$
LSBR	$p_\mathrm{d} = -\rho_\mathrm{d} - A/3$	$V(\phi) = C_{ls}\phi^2 + D_{ls}$

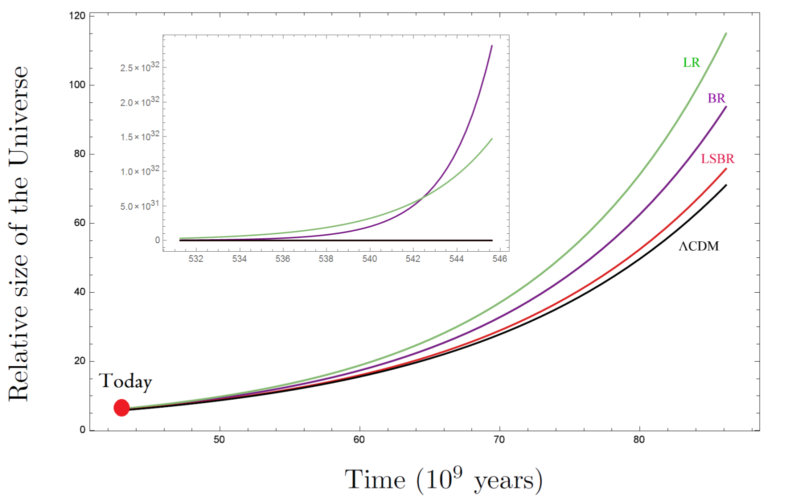
Where $w_d < -1$, the parameters A and B are positive and C_{br}, C_{lr}, D_{lr}, C_{ls} and D_{lr} are constants.

- The lower is the power on ϕ of $V(\phi)$, the smoother is the abrupt event.

(1) A.A. Starobinsky. astro-ph 9912054; R.R. Cadwell astro-ph 9908168; Cadwell *et al.* astro-ph/0301273
(2) H. Štefančić. astro-ph 0411630; S. Nojiri, S. Odintsov and S. Tsujikawa. hep-th/0501025
(3) M. Bouhmadi-López , A. Errahmani, P. Martín-Moruno, T. Ouali and Y. Tavakoli. arXiv:1407.2446
(4) M. P. Dąbrowski, C. Kiefer and B. Sandhöfer. hep-th/0605229

Phantom energy: Should we be afraid?

- Evolution of the scale factor for different models vs cosmic time.



Late-time singularities

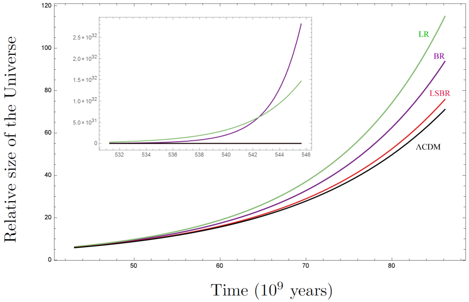
DE might induce a future cosmic singularity

Some of the cosmological parameters:

- t → Cosmic time
- a → Scale factor (relative size)
- H → Hubble parameter (growth rate)
- $\dot{H}$ → Time derivative of H

Singularity	t	a	H	$\dot{H}$	$\ddot{H}, \dddot{H}$ ...
Big Bang	0	0	∞	∞	∞
De Sitter $_{(\Lambda CDM)}$	∞	∞	H_{ds}	0	0
Big Rip	t_s	∞	∞	∞	∞
LR	∞	∞	∞	∞	∞
LSBR	∞	∞	∞	$\dot{H}_s$	0
Big Freeze	t_s	a_s	∞	∞	∞
Sudden. S.	t_s	a_s	H_s	∞	∞
Type IV	t_s	a_s	H_s	$\dot{H}_s$	∞

Asymptotic evolution of the scale factor



Bouhmadi-López, Kiefer, Martín-Moruno, arXiv:1904.01836 [gr-qc] (review published in GRG)

Late-time acceleration of the Universe within GR and with a phantom fluid

Observational data and constraints

SNeIa data-1-

- The Pantheon compilation: 1048 SNeIa dataset $0.01 < z < 2.26$
- The distance modulus for supernovae is given by

$$\mu_{th} = 5 \log_{10} \frac{d_L}{Mpc} + 25$$

- The luminosity distance reads $d_L = (c/H_0)\, D_L$ where H_0 is the Hubble constant, c is the speed of light and $D_L = (1+z) \int_0^z \frac{dz}{E(z)}$, where $E(z) = H(z)/H_0$
- The observed apparent magnitude for the Pantheon compilation is given by $m_{obs} = \mu_{obs} + M$, where μ_{obs} is the observed distance modulus and M is the absolute magnitude.

SNeIa data-2-

- To estimate the cosmological parameters, we compute χ^2

$$\chi^2_{\mathrm{SN}} = (\mu_{\mathrm{obs}} - \mu_{\mathrm{th}})^{\mathrm{T}}.\mathrm{C}^{-1}_{\mathrm{Pantheon}}.(\mu_{\mathrm{obs}} - \mu_{\mathrm{th}}),$$

where $(\mu_{obs} - \mu_{th})$ is the difference vector between the model expectations and the observed magnitudes, $\mathrm{C}_{\mathrm{Pantheon}}$ is the covariance matrix of Pantheon data which is given by the sum of a statistical part and systematic part $\mathrm{C}_{\mathrm{Pantheon}} = \mathrm{C}_{\mathrm{stat}} + \mathrm{C}_{\mathrm{sys}}$.

- In order to get rid of the nuisance parameter M, we perform an analytical marginalization over it, by defining a new chi-square

$$\chi^2_{\mathrm{SN}} = \mathrm{A} + \ln \frac{\mathrm{C}}{2\pi} - \frac{\mathrm{B}^2}{\mathrm{C}},$$

where

$$\mathrm{A} = (\mu_{\mathrm{obs}} - \mu_{\mathrm{th}})^{\mathrm{T}}.\mathrm{C}^{-1}_{\mathrm{Pantheon}}.(\mu_{\mathrm{obs}} - \mu_{\mathrm{th}})$$

,

$$\mathrm{B} = (\mu_{\mathrm{obs}} - \mu_{\mathrm{th}})^{\mathrm{T}}.\mathrm{C}^{-1}_{\mathrm{Pantheon}}.1, \qquad \mathrm{C} = 1^{\mathrm{T}}.\mathrm{C}^{-1}_{\mathrm{Pantheon}}.1$$

being 1 the 1048×1048 identity matrix.

CMB data-1-

- The power spectrum of CMB affects crucially the physics, from the decoupling epoch till today.
- Effects are mainly quantified by the acoustic scale l_a and the shift parameter R Komatsu et 2008

$$R \equiv \sqrt{\Omega_{\rm m} H_0^2}(1+z_{\rm CMB}) D_A(z_{\rm CMB}),$$

$$l_a \equiv (1+z_{\rm CMB}) \frac{\pi D_A(z_{\rm CMB})}{r_s(z_{\rm CMB})}.$$

- The angular diameter distance of photons in a flat FLRW universe

$$D_A(z) = \frac{1}{H_0(1+z)} \int_0^z \frac{dz'}{E(z')},$$

- The comoving sound horizon

$$r_s(z) = \frac{1}{H_0} \int_0^a \frac{da'}{a' E(a') \sqrt{3(1+\bar{R}_b) a'}},$$

and $R_b = 31500 \Omega_{\rm b} h^2 (T_{\rm CMB}/2.7K)^{-4}$, with $T_{\rm CMB} = 2.275K$

CMB data-2-

- The CMB contribution to the total χ^2 is

$$\chi^2_{\rm CMB} = \mathsf{X}^{\mathsf{T}}_{\rm CMB}.\mathsf{C}^{-1}_{\rm CMB}.\mathsf{X}_{\rm CMB},$$

where $X_{\rm CMB}$ is the CMB parameters vector based on Planck 2018 release Zhai et al 2018

$$\mathsf{X}_{\rm CMB} = \begin{pmatrix} R - 1.74963 \\ l_a - 301.80845 \\ \Omega_{\rm b} h^2 - 0.02237 \end{pmatrix}.$$

BAO data and $H(z)$ data

- The BAO peaks present in the matter power spectrum can be used to determine the Hubble parameter $H(z)$ and the angular diameter distance $D_A(z)$ Eisenstein et 2005

$$D_V(z) \equiv \left[(1+z)^2 D_A^2(z) \frac{z}{H(z)}\right]^{1/3},$$

- Then once again we calculate

$$\chi^2_{\mathrm{BAO}} = \mathsf{X}^{\mathsf{T}}_{\mathrm{BAO}}.\mathsf{C}^{-1}_{\mathrm{BAO}}.\mathsf{X}_{\mathrm{BAO}},$$

where $\mathsf{X}_{\mathrm{BAO}}$ is the difference vector between the theoretical predictions and observations.

- For $H(z)$ data we can consider

$$\chi^2_{H(z)} = \sum_{i=1}^{36} \left[\frac{H_{obs,i} - H(z_i)}{\sigma_{H,i}}\right]^2,$$

where $H_{obs,i}$ is the observational value of the Hubble parameter and $H(z)$ is the theoretical prediction of the Hubble parameter.

Model fitted

- BR model: $p_{\rm d} = w_{\rm d}\rho_{\rm d}$

$$E(a)^2 = \Omega_{\rm r} a^{-4} + \Omega_{\rm m} a^{-3} + \Omega_{\rm d} a^{-3(1+w_{\rm d})}.$$

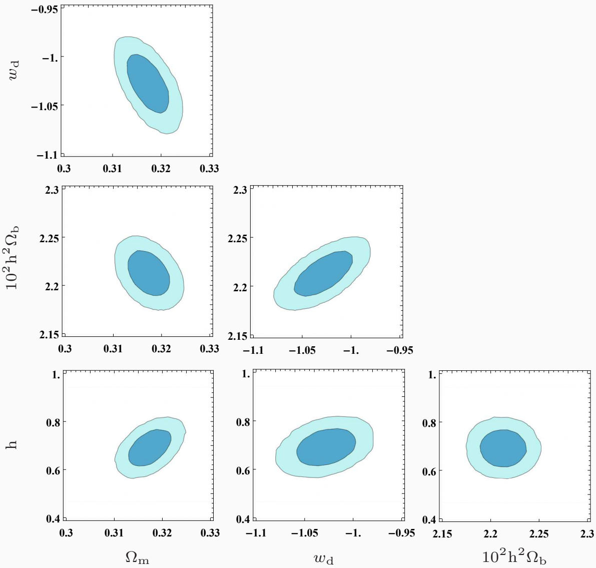
- LR model: $p_{\rm d} = -\left(\rho_{\rm d} + \beta\rho_{\rm d}^{\frac{1}{2}}\right)$

$$E^2(a) = \Omega_{\rm r} a^{-4} + \Omega_{\rm m} a^{-3} + \Omega_{\rm d}\left(1 + \frac{3}{2}\sqrt{\frac{\Omega_{\rm lr}}{\Omega_{\rm d}}}\ln(a)\right)^2.$$

- LSBR model: $p_{\rm d} = -\left(\rho_{\rm d} + \frac{\alpha}{3}\right)$

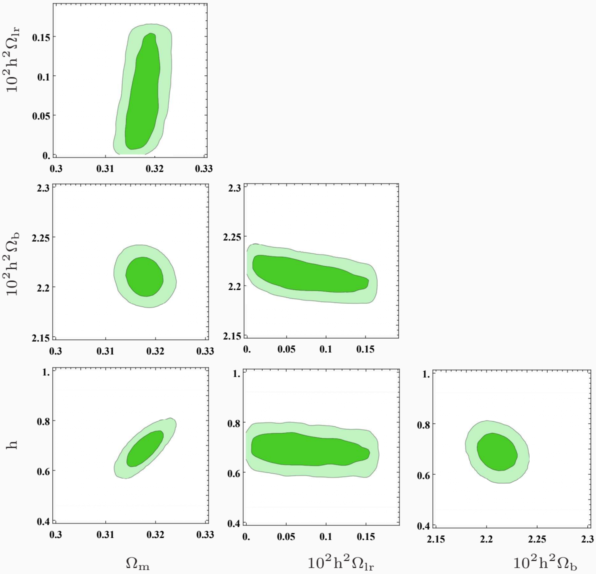
$$E^2(a) = \Omega_{\rm r} a^{-4} + \Omega_{\rm m} a^{-3} + \Omega_{\rm d}\left(1 - \frac{\Omega_{\rm lsbr}}{\Omega_{\rm d}}\ln(a)\right).$$

BR Model



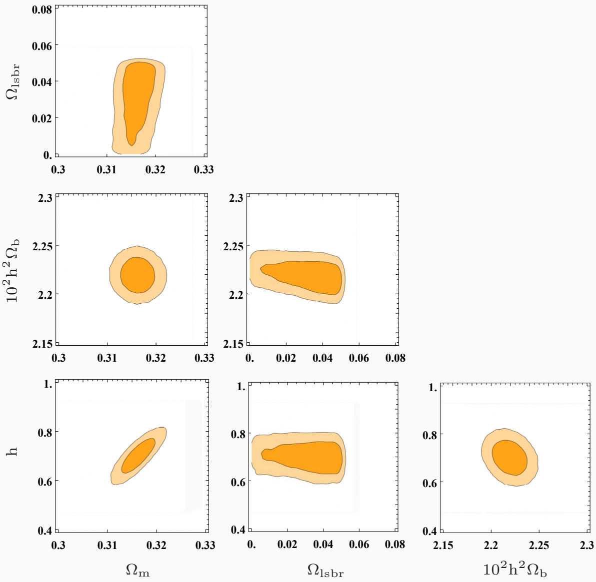
Bouali, Albarran, Bouhmadi-López and Ouali, arXiv:1905.07304 [astro-ph.CO]. Published in Physics of the Dark Universe.

LR Model



Bouali, Albarran, Bouhmadi-López and Ouali, arXiv:1905.07304 [astro-ph.CO]. Published in Physics of the Dark Universe.

LSBR Model



Bouali, Albarran, Bouhmadi-López and Ouali, arXiv:1905.07304 [astro-ph.CO]. Published in Physics of the Dark Universe.

Comparison with LCDM

Model	Par	Best fit	Mean	χ^2_{tot}	$\chi^{2\ \text{red}}_{\text{tot}}$	AIC_c	ΔAIC_c
ΛCDM	Ω_{m}	$0.318349^{+0.00248001}_{-0.00248001}$	$0.31834^{+0.00248987}_{-0.00248987}$	1047.42	0.957422	1053.441953	0
	h	$0.69814^{+0.0480814}_{-0.0480814}$	$0.698602^{+0.0481787}_{-0.0481787}$				
	$\Omega_{\text{b}}h^2$	$0.022218^{+0.000120872}_{-0.000120872}$	$0.0222202^{+0.000122619}_{-0.000122619}$				
BR	Ω_{m}	$0.317173^{+0.00318473}_{-0.00318473}$	$0.317327^{+0.0031808}_{-0.0031808}$	1047.51	0.958380	1055.54663	2.104677
	w_{br}	$-1.02758^{+0.0240102}_{-0.0240102}$	$-1.02874^{+0.0239306}_{-0.0239306}$				
	h	$0.691013^{+0.0507771}_{-0.0507771}$	$0.691523^{+0.0507536}_{-0.0507536}$				
	$\Omega_{\text{b}}h^2$	$0.0221218^{+0.000170789}_{-0.000170789}$	$0.022123^{+0.000170538}_{-0.000170538}$				
LR	Ω_{m}	$0.317198^{+0.00276851}_{-0.00276851}$	$0.317705^{+0.00280131}_{-0.00280131}$	1047.53	0.958398	1055.56663	2.124677
	Ω_{lr}	$0.000445721^{+0.000416159}_{-0.000416159}$	$0.000763824^{+0.000416359}_{-0.000416359}$				
	h	$0.694604^{+0.0494111}_{-0.0494111}$	$0.688584^{+0.0493315}_{-0.0493315}$				
	$\Omega_{\text{b}}h^2$	$0.0221295^{+0.000130585}_{-0.000130585}$	$0.0221028^{+0.000132755}_{-0.000132755}$				
LSBR	Ω_{m}	$0.317115^{+0.00253975}_{-0.00253975}$	$0.316144^{+0.00253899}_{-0.00253899}$	1047.56	0.958426	1055.59663	2.154677
	Ω_{lsbr}	$0.0500261^{+0.0130141}_{-0.0130141}$	$0.0299424^{+0.0133398}_{-0.0133398}$				
	h	$0.695705^{+0.0481201}_{-0.0481201}$	$0.701962^{+0.0481465}_{-0.0481465}$				
	$\Omega_{\text{b}}h^2$	$0.022138^{+0.000121724}_{-0.000121724}$	$0.0221928^{+0.000121811}_{-0.000121811}$				

Table III. Summary of the best fit and the mean values of the cosmological parameters.

Late-time acceleration of the Universe within GR and with a phantom fluid

A perturbative approach: GR and phantom fluids

Our approach

- We start considering that the late-time acceleration of the universe is described by a dark energy component effectively encapsulated within a perfect fluid with energy density ρ_d and pressure p_d. On this setup, we consider two simple scenario:
  - A constant equation of state for DE
  - A DE in an effective and genuinely phantom DE universe. The reason of this second choice will become clear after considering the first case.
- Of course, on top of this we invoke a dark matter component.
- Given that to get the matter power spectrum, we start our numerical integration since the radiation dominated epoch, we will consider as radiation as well on our model.

Cosmological perturbations: GR and for the late Universe-1

- We worked on the Newtonian gauge and carried the first order perturbations considering DM, DE and radiation on GR. Radiation was included because our numerical integrations start from well inside the radiation dominated epoch (to get the matter power spectrum)
- We assumed initial adiabatic conditions for the different fractional energy density perturbations
- The total fractional energy density is fixed by Planck measurments; i.e. through A_s and n_s
- The speed of sound for DE:
  - The pressure perturbation of DE reads: $\delta p_d = c_{sd}^2 \delta\rho_d - 3\mathcal{H}\left(1+w_d\right)\left(c_{sd}^2 - c_{ad}^2\right)\rho_d v_d$, where $c_{sd}^2 = \left.\frac{\delta p_d}{\delta \rho_d}\right|_{r.f.}$ and $c_{aA}^2 = \frac{p_d'}{\rho_d'}$
  - Given that c_{sd}^2 is negative, we can end up with a problem (this is not intrisic to phantom matter as it can happen for example with fluids with a negative constant equation of state larger than -1)
  - We choose $c_{sd}^2 = 1$ as a phenomenological parameter

Cosmological perturbations: GR and for the late Universe-2

- Evolution equations for the different components

$$
\begin{aligned}
(\delta_{\rm r})_x &= \frac{4}{3}\left(\frac{k^2}{\mathcal{H}} v_{\rm r} + 3\Psi_x\right) ,\\
(v_{\rm r})_x &= -\frac{1}{\mathcal{H}}\left(\frac{1}{4}\delta_{\rm r} + \Psi\right) ,\\
(\delta_{\rm m})_x &= \left(\frac{k^2}{\mathcal{H}} v_{\rm r} + 3\Psi_x\right) ,\\
(v_{\rm m})_x &= -\left(v_{\rm m} + \frac{\Psi}{\mathcal{H}}\right) ,\\
(\delta_{\rm d})_x &= (1+w_{\rm d})\left\{\left[\frac{k^2}{\mathcal{H}} + 9\mathcal{H}\left(c_{s{\rm d}}^2 - c_{a{\rm d}}^2\right)\right] v_{\rm d} + 3\Psi_x\right\} + 3\left(w_{\rm d} - c_{s{\rm d}}^2\right)\delta_{\rm d} ,\\
(v_{\rm d})_x &= -\frac{1}{\mathcal{H}}\left(\frac{c_{s{\rm d}}^2}{1+w_{\rm d}}\delta_{\rm d} + \Psi\right) + \left(3c_{s{\rm d}}^2 - 1\right) v_{\rm d} .
\end{aligned}
$$

Cosmological perturbations: GR and for the late Universe-3

- Perturbed Einstein equations

$$
\begin{aligned}
\Psi_x + \Psi\left(1 + \frac{k^2}{3\mathcal{H}^2}\right) &= -\frac{1}{2}\delta\,, \\
\Psi_x + \Psi &= -\frac{3}{2}\mathcal{H}v\left(1+w\right)\,, \\
\Psi_{xx} + \left[3 - \frac{1}{2}\left(1+3w\right)\right]\Psi_x - 3w\Psi &= \frac{3}{2}\frac{\delta p}{\rho}\,.
\end{aligned}
$$

where

$$\rho = \sum_A \delta\rho_A\,, \delta p = \sum_A \delta p_A\,, \delta = \sum_A \frac{\rho_A}{\rho}\delta_A = \sum_A \Omega_A\delta_A$$

and

$$v = \sum_A \frac{1+w_A}{1+w}\Omega_A v_A\,.$$

Cosmological perturbations: GR and for the late Universe-4

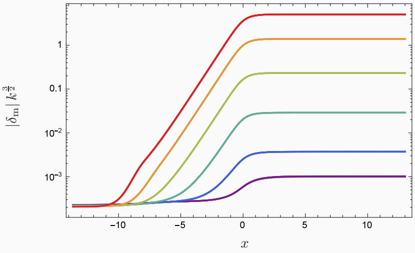
- Adiabatic conditions:

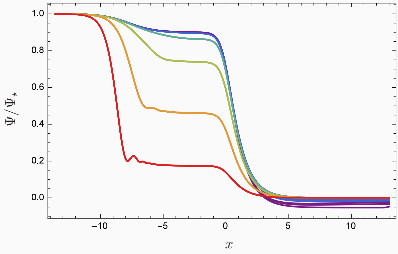
$$\frac{3}{4}\delta_{\rm r,ini} = \delta_{\rm m,ini} = \frac{\delta_{\rm d,ini}}{1+w_{\rm d,ini}} \approx \frac{3}{4}\delta_{\rm ini}$$

$$v_{\rm r,ini} = v_{\rm m,ini} = v_{\rm d,ini} \approx \frac{\delta_{\rm ini}}{4\mathcal{H}_{\rm ini}}$$

- Initial conditions for δ are fixed through the amplitude and spectral index of the primordial inflationary power spectrum: $A_s = 2.143 \times 10^{-9}$, $n_s = 0.9681$ and $k_* = 0.05$ Mpc^{-1} (Planck values): $\Phi_{\rm ini} = \frac{2\pi}{3}\sqrt{2A_s}\left(\frac{k}{k_*}\right)^{n_s-1} k^{-3/2}$
- Well inside the radiation era: $\Phi_{\rm ini} \approx -\frac{1}{2}\delta_{\rm tot,ini}$ and $\Phi_{\rm ini} \approx -2\mathcal{H}_{\rm ini} v_{\rm tot,ini}$
- We choose $c_{sd}^2 = 1$ as a phenomenological parameter
- The parameters of the models will be fixed through the fitting we did previously.

Results: DM perturbations and the gravitational potential



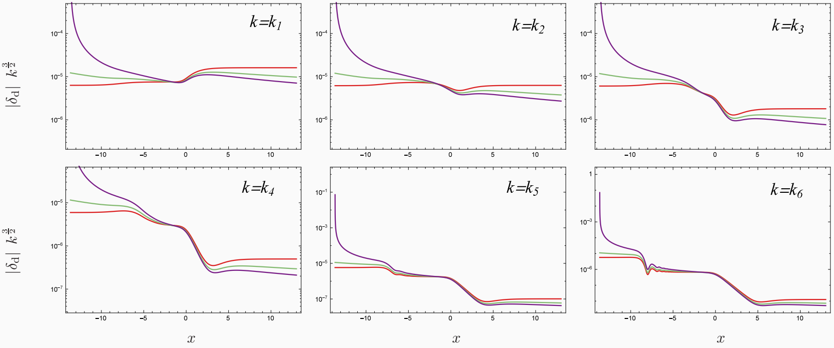


$k_1 = 3.33 \times 10^{-4}\mathrm{h\ Mpc}^{-1}$, $k_4 = 1.02 \times 10^{-2}\mathrm{h\ Mpc}^{-1}$,

$k_2 = 1.04 \times 10^{-4}\mathrm{h\ Mpc}^{-1}$, $k_5 = 3.19 \times 10^{-2}\mathrm{h\ Mpc}^{-1}$,

$k_3 = 3.26 \times 10^{-3}\mathrm{h\ Mpc}^{-1}$, $k_6 = 1.00 \times 10^{-1}\mathrm{h\ Mpc}^{-1}$.

Results: DE perturbations

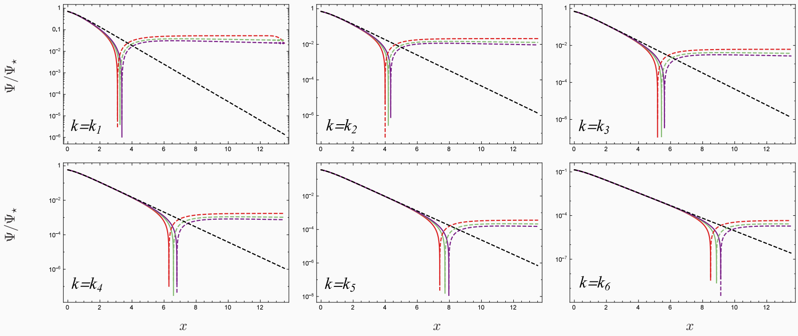


$k_1 = 3.33 \times 10^{-4} \mathrm{h\ Mpc}^{-1}$, $k_4 = 1.02 \times 10^{-2} \mathrm{h\ Mpc}^{-1}$,

$k_2 = 1.04 \times 10^{-4} \mathrm{h\ Mpc}^{-1}$, $k_5 = 3.19 \times 10^{-2} \mathrm{h\ Mpc}^{-1}$,

$k_3 = 3.26 \times 10^{-3} \mathrm{h\ Mpc}^{-1}$, $k_6 = 1.00 \times 10^{-1} \mathrm{h\ Mpc}^{-1}$.

Results: a closer look at the gravitational potential



$k_1 = 3.33 \times 10^{-4} \mathrm{h\ Mpc}^{-1}$,

$k_2 = 1.04 \times 10^{-4} \mathrm{h\ Mpc}^{-1}$,

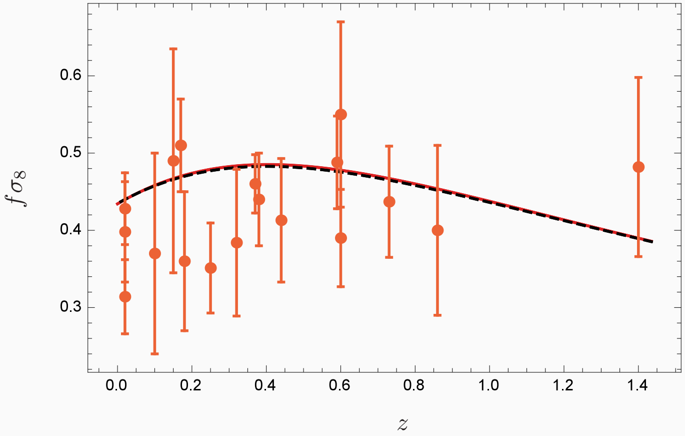
$k_3 = 3.26 \times 10^{-3} \mathrm{h\ Mpc}^{-1}$,

$k_4 = 1.02 \times 10^{-2} \mathrm{h\ Mpc}^{-1}$,

$k_5 = 3.19 \times 10^{-2} \mathrm{h\ Mpc}^{-1}$,

$k_6 = 1.00 \times 10^{-1} \mathrm{h\ Mpc}^{-1}$.

Results: The evolution of $f\sigma_8$ (growth rate)-1-

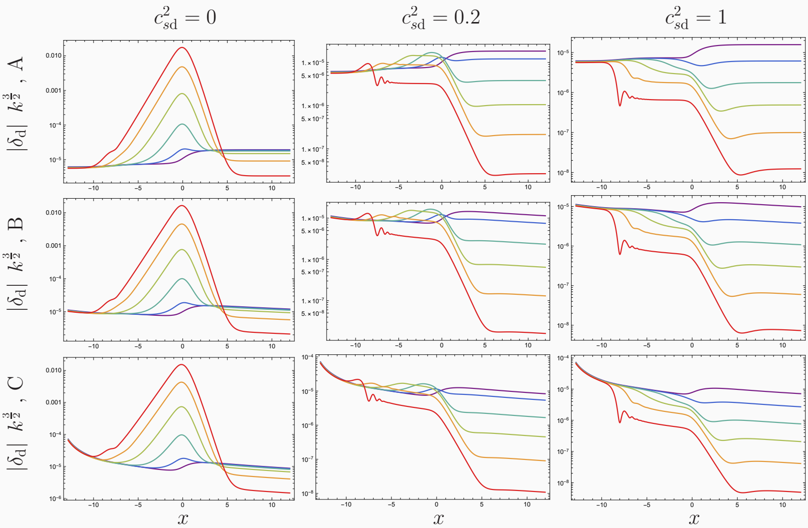


The evolution of $f\sigma_8$ for the 3 models.

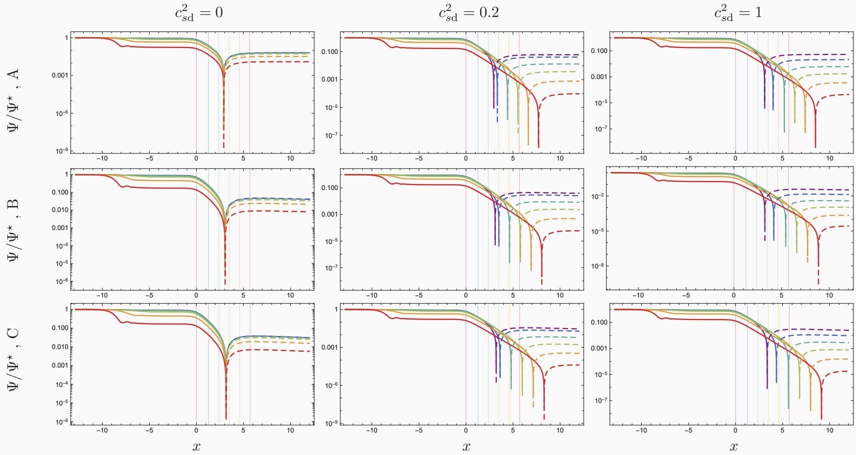
$$f \equiv \frac{d\left(\ln \delta_{\rm m}\right)}{d\left(\ln a\right)}, \qquad \sigma_8\left(z,\ k_{\sigma_8}\right) = \sigma_8\left(0,\ k_{\sigma_8}\right) \frac{\delta_{\rm m}\left(z,\ k_{\sigma_8}\right)}{\delta_{\rm m}\left(0,\ k_{\sigma_8}\right)}$$

$k_{\sigma_8} = 0.125$ hMpc^{-1}, $\sigma_8(0,\ k_{\sigma_8}) = 0.820$ (Planck)

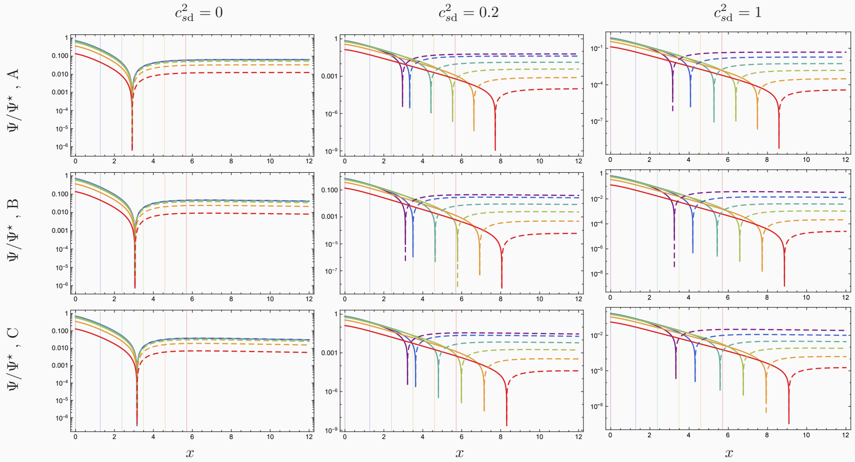
Effect of the speed of sound, C_{sd}^2, on DE perturbations



Effect of the speed of sound, C_{sd}^2, on the gravitational potential-1-



Effect of the speed of sound, C_{sd}^2, on the gravitational potential-2-



Conclusions

Conclusions

- We have followed a phenomenological approach to describe the late-time acceleration of the universe.
- It can be perfectly described by a constant equation of state.
- We have also shown that the late-time acceleration of the Universe can be described through a phantom DE component
- Finally, we have looked for the observational fit and the perturbations