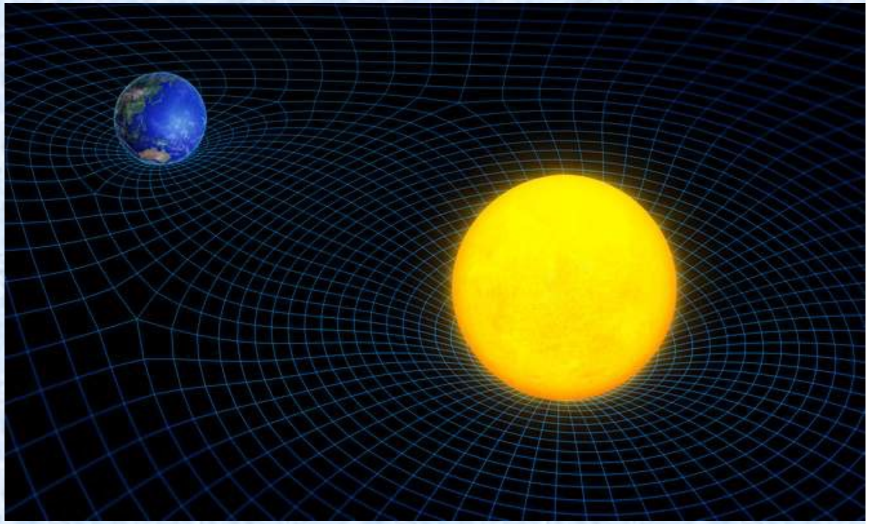
Foundation and Cosmological Applications of Non-Local Gravity



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Beyond Standard Model: From Theory to Experiment, March 31 , 2021

Outline

- *Locality* and *Non-Locality*
- Non-Local Theories of Gravity
- The Noether Symmetry Approach
- Non-Local Theories with scalar curvature
- Non-Local *Gauss-Bonnet* Gravity
- Non-Local Teleparallel Gravity
- Spherical Symmetry and astrophysical tests
- Conclusions and perspectives

Locality and Non-locality

Kinematics

It refers to the *STATES*

Classical Theories: local

CM → *Points of a tangent/cotangent bundle*

CFT → *Tensor fields over a manifold*

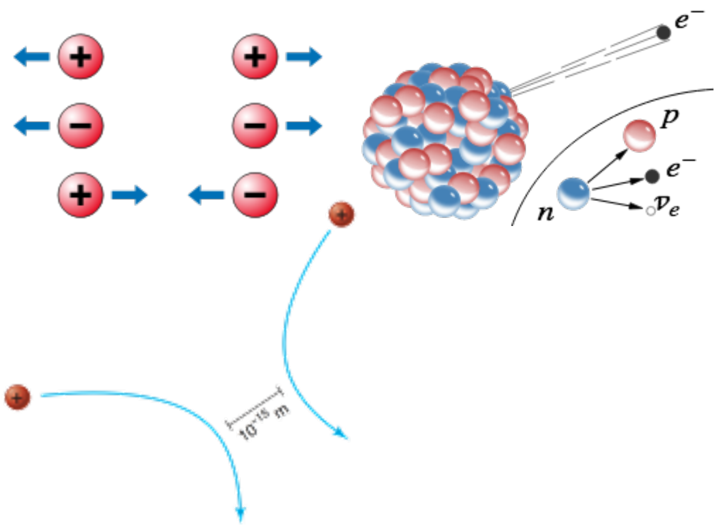
VS

Dynamics

It refers to the *INTERACTIONS*

Quantum Theories: non-local

- *Born interpretation of* Ψ
- *Heisenberg uncertainty principle*



Non-Locality in Physics

- **Fundamental interactions are Non-Local. It can be shown by considering the one-loop effective action**

Euler-Heisenberg Lagrangian

$$\mathcal{L}_{EH} = -\frac{1}{4}\mathcal{F}^2 - \frac{e^2}{32\pi^2}\int_0^\infty \frac{ds}{s}\, e^{i\varepsilon s} e^{-m^2 s}\left[\frac{\mathrm{Re}\cosh(esX)}{\mathrm{Im}\cosh(esX)} F_{\mu\nu}F^{\mu\nu} - \frac{4}{e^2 s^2} - \frac{2}{3}\mathcal{F}^2\right]$$

$$\mathcal{F} = \tfrac{1}{2}\left(|\mathbf{E}|^2 - |\mathbf{B}|^2\right),\ X = \mathcal{F} + i\mathbf{E}\cdot\mathbf{B}$$

Non-Locality in Physics

Another example

Yukawa Lagrangian

$$\mathcal{L}_Y = i\bar{\psi}\partial\!\!\!/\psi - \frac{1}{2}\phi(\Box + m^2)\phi + \lambda\,\phi\bar{\psi}\psi$$

The related effective action is

$$\mathcal{L}_{eff} = i\bar{\psi}\partial\!\!\!/\psi + \frac{\lambda^2}{2}\bar{\psi}\psi(\Box + m^2)^{-1}\bar{\psi}\psi$$

The non-locality is in the operator

$$(\Box + m^2)^{-1}$$

Local Action vs Non-local Action

Local Action

it is a functional of only local fields, *i.e.* algebraic functions of fields or their derivatives evaluated at a single point

It is the paradigm of all fundamental field theories, both classical and quantum

Non-local Action

it is a functional of non-local fields (at least one), *i.e.* functions of fields evaluated at more than one point or transcendental functions of fields or their derivatives

It describes an effective theory

We must find a link between GR and QM

GR must be changed ✓

QFT must be changed ✗

Shortcomings in GR

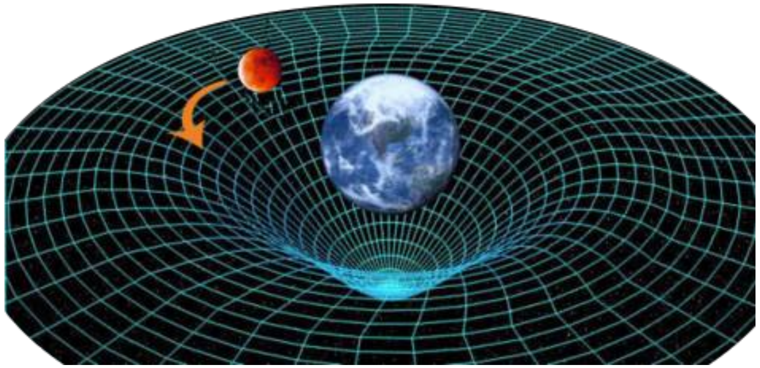
Large Scales

- Universe accelerated expansion
- Inflation
- Galaxy Rotation Curve
- Mass-Radius Diagram of Neutron Stars
- Fine-tuning of cosmological parameters

Small Scales

- Renormalizability
- GR cannot be quantized
- GR cannot be treated under the same standard of the other interactions
- Discrepancy between theoretical and experimental value of Λ
- Classical spacetime singularities

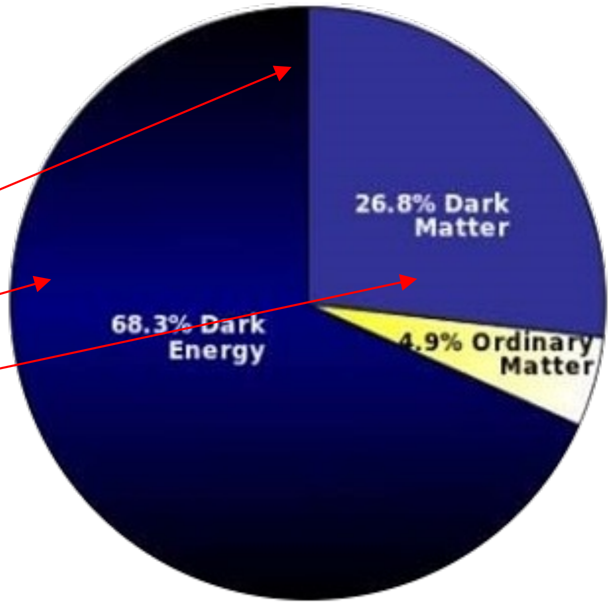
No theory is capable of solving these problems at once so far



No evidence for DM and DE at fundamental level

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$



Can Dark Side Issue be solved by Non-locality?

Local Extended Theories of Gravity (ETG)

Purely Metric Formalism

- Scalar-tensor Theories

$$S_{BD} = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[\phi R - \frac{\omega}{\phi} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi \right] + S^{(m)}$$

- Higher-order Theories

$$S_{Starobinsky} = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} \left[R + \alpha R^2 \right] + S^{(m)}$$

→ *"F (R)−gravity"*

$$S_{Stelle} = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} \left[R + \alpha R^2 + \beta R^{\mu\nu} R_{\mu\nu} \right] + S^{(m)}$$

- Higher-order-scalar-tensor Theories

$$S = \int d^4x \sqrt{-g} \left[F(R, R, \Box^2 R, \ldots, \Box^k R, \phi) - \frac{\varepsilon}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi \right] + 2\kappa S^{(m)}$$

Non-local ETGs

- Infinite Derivative Theories of Gravity (**IDG**s)

$$S \propto F_i(\Box_s)\, R$$

$$F_i(\Box_s) = \sum_{n=0}^{\infty} f_{i,n}\, \Box_s^{n}$$

- Integral Kernel Theories of Gravity (**IKG**s)

$$S \propto F(R, \Box^{-1} R)$$

$$S \propto F(T, \Box^{-1} T)$$

$$S \propto F(G, \Box^{-1} G)$$

They could be very useful to address astrophysical and cosmological scale and, eventually, infrared dynamics

Infinite Derivative Theories of Gravity (**IDG**s)

We can start from the infinite-derivative Lorentz-invariant action depending on a scalar field

$$S = \frac{1}{2}\int d^4x d^4y\, \phi(x)\mathcal{K}(x-y)\phi(y) - \int d^4x V(\phi)$$

Prototype of Non-Locality:
a general operator depending on
the distance (x-y)

Starting from *S* and ***performing***:

1. A Fourier transformation
2. The reparameterization $\mathcal{K}(x-y) = F(\Box)\delta^{(4)}(x-y)$ with $F(\Box) = e^{-\gamma(\Box)}\prod_{i=1}^{N}(\Box - m_i^2)$

We get

$$\frac{1}{2}\int d^4x d^4y\, \phi(x)\mathcal{K}(x-y)\phi(y) \sim \frac{1}{2}\int d^4x \phi(x)F(\Box)\phi(x)$$

Infinite Derivative Theories of Gravity (**IDG**s)

The most general gravitational action in 4D, quadratic in curvature and ghost-free, has to contain infinite covariant derivatives:

$$S = \kappa \int d^4x \sqrt{-g}\,\left[R + \alpha\left(R F_1(\Box_s) R + R_{\mu\nu} F_2(\Box_s)\, R^{\mu\nu} + R_{\mu\nu\rho\sigma} F_3(\Box_s) R^{\mu\nu\rho\sigma}\right)\right] + S^{(m)}$$

- $\kappa \equiv (16\pi G_N)^{-1}$, $\alpha \equiv (M_s)^{-2}$, $[M_s] = lenght$
- $\Box_s \equiv \Box / M_s^2$, $\Box \equiv g^{\mu\nu} \nabla_\mu \nabla_\nu$
- $F_i(\Box_s)$ *transcendental and analytic* ⟶ $F_i(\Box_s) = \sum_{n=0}^{\infty} f_{i,n}\, \Box_s^n$

(T. Biswas, E. Gerwick, T. Koivisto, and A. Mazumdar. “Towards singularity and ghost free theories of gravity”. In: Phys. Rev. Lett. **108** (2012), p. 031101)

Infinite Derivative Theories of Gravity (**IDG**s)

Super-renormalizable and Unitary theories

$$\mathcal{S} = \kappa \int d^4x \sqrt{-g} \left(R - G_{\mu\nu} \frac{e^{H(-\Box_s)} - 1}{\Box} R^{\mu\nu} \right)$$

$$\mathcal{S} = \kappa \int d^4x \sqrt{-g} \left[R - G_{\mu\nu} \frac{V_2^{-1} - 1}{\Box} R^{\mu\nu} + \frac{1}{2} R \frac{V_0^{-1} - V_2^{-1}}{\Box} R \right]$$

$$V_2^{-1} \equiv e^{H_2(-\Box_s)} p^{(n_2)}(-\Box_s) \,, \quad V_0^{-1} - V_2^{-1} \equiv \frac{1}{3} \left[e^{H_0(-\Box_s)} (1 + \Box_s) - e^{H_2(-\Box_s)} \right]$$

admits regular blackhole solutions

"maximal" UV-completion of $S_{Starobinsky}$

(L. Modesto. "Super-renormalizable Quantum Gravity". In: Phys. Rev. **D86** (2012), p. 044005; F. Briscese, L. Modesto, and S. Tsujikawa. "Super-renormalizable or finite completion of the Starobinsky theory". In: Phys. Rev. **D89**.2(2014), p. 024029.)

Integral Kernel Theories of Gravity (**IKG**s)

- ***Involve non-local operator of the form*** $\Box^{-1}$
- ***Firstly considered by Deser and Woodard in cosmology***

S. Deser and R. P. Woodard. "Nonlocal Cosmology". In: Phys. Rev. Lett. ***99*** *(2007), p. 111301*

They start from

$$S = \frac{1}{16\pi G_N}\int d^4x\,\sqrt{-g}\,R\left[1 + F\left(\Box^{-1} R\right)\right] + S^{(m)}$$

- $\Box \equiv g^{\mu\nu}\,\nabla_\mu \nabla_\nu = \frac{1}{\sqrt{-g}}\,\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\right)$
- $\left(\Box^{-1} R\right)(x) \equiv \int d^4x'\,\sqrt{-g}\,G(x,x')\,R(x')\quad with\quad G(x,x')\ "retarded"\ Green$

$\Box^{-1}$ *could explain the current late-time accelerated cosmic expansion without invoking any Dark Energy:*

$$g_{\mu\nu}^{FLRW} = diag(1, -a^2(t), -a^2(t), -a^2(t))$$

$$(\Box^{-1}R)(t) = \int_{t_i}^{t} dt'\,\frac{1}{a^3(t')}\int_{t_i}^{t'} dt''a^3(t'')\,R(t'')$$

$t_i = t_{eq} \sim 10^5 y$

$t = t_0 \sim 10^{10} y$

$a(t) \sim t^s$

$s = 2/3$

$$\Rightarrow \quad (\Box^{-1}R)(t_0)\big|_{s=2/3} \sim 14{,}0$$

Large number required by the current cosmic acceleration avoiding the fine-tuning of parameters

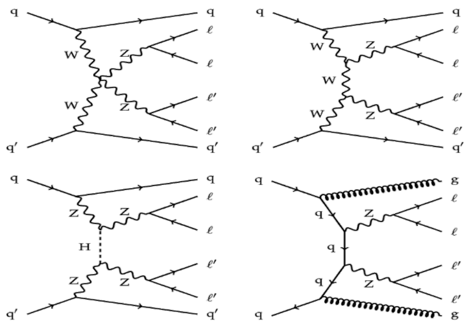
$$F(R, \Box^{-1} R) \qquad F\left(T, \Box^{-1} T, B, \Box^{-1} B\right) \qquad F(G, \Box^{-1} G)$$

Classification

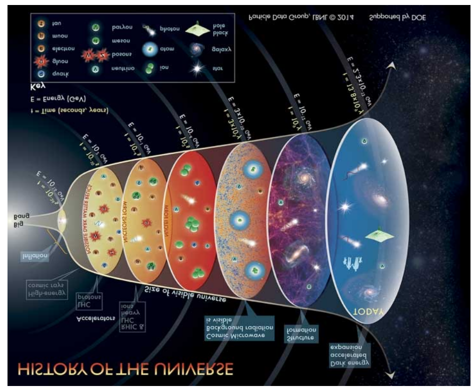
- Higher-order **IKG** (in the metric, affine, teleparallel formalism)
- Non-local extension of $F(R) - gravity$

Motivations

- It could account for UV and IR quantum corrections →



- It could reproduce both UV and IR cosmic evolution →



Purposes

- Cosmography, Dark Energy
- Physically motivated cosmological models
- Reproducing cosmic history from UV to IF scales

Method → *Noether Symmetry Approach*

Noether Point Symmetries

$$\bar{t} = \bar{t}(t, q; \varepsilon) \simeq t + \varepsilon \xi(t, q)$$
$$\bar{q}^i = \bar{q}^i(t, q; \varepsilon) \simeq q^i + \varepsilon \eta^i(t, q)$$

→ *1-parameter (ε) group of point transformations*

$$\boldsymbol{X} = \xi(t,q)\frac{\partial}{\partial t} + \eta^i(t,q)\frac{\partial}{\partial q^i}$$

→ *infinitesimal group generator*

$$\boldsymbol{X}^{[1]} = \boldsymbol{X} + \eta^{[1]i}\frac{\partial}{\partial \dot{q}^i} = \boldsymbol{X} + \left(\dot{\eta}^i - \dot{\xi}\dot{q}^i\right)\frac{\partial}{\partial \dot{q}^i}$$

→ *''first prolongation'' of the infinitesimal generator*

Noether Theorem. *If and only if it exists a function* $g(t, q(t))$ *such that*

$$\boldsymbol{X}^{[1]}L + \dot{\xi}L = \dot{g},$$

then the one-parameter group of point transformations generated by $\boldsymbol{X}$ *is a one-parameter group of Noether point symmetries for the dynamical system described by the Lagrangian* L.

The associated first integral of motion is:

$$I(t, q, \dot{q}) = \xi\left(\dot{q}\frac{\partial L}{\partial \dot{q}^i} - L\right) - \eta^i\frac{\partial L}{\partial \dot{q}^i} + g$$

Noether Symmetry Approach

The recipe:

1. Consider a point-like (cosmological) Lagrangian

2. Write the ansatz for $\boldsymbol{X}$ ed $\boldsymbol{X}^{[1]}$

3. Derive the Noether point symmetry existence condition

$$\boldsymbol{X}^{[1]}L + \dot{\xi}L = \dot{g}$$

to obtain a polynomial depending on $\xi(t,q)$, $\eta^i(t,q)$, $\dot{g}(t,q)$ and products of the Lagrangian velocities $(e.g.\ \dot{\eta}^i\ \dot{\eta}^j\ \dot{\xi}\ \ldots)$

3. We obtain a system of PDEs for $\xi, \eta^i, \dot{g}$

The system contains the unknown function $F(R,\phi)$, so that it can provide, in principle, the explicit form for $F(R,\phi)$ related to the existence of symmetries. In other words, the existence of symmetries gives physically motivated Lagrangians.

1)

$$F(R, \square^{-1} R)$$

Cosmological Lagrangian

$$\mathcal{S} = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\,F\left(R, \Box^{-1}R\right)$$

$$S = \frac{1}{16\pi G_N}\int d^4x\,\sqrt{-g}\;F(R,\phi)$$

formal localization

$$\phi \equiv \Box^{-1}R \quad\longrightarrow\quad R \equiv \Box\phi$$

$$g_{\mu\nu}^{FLRW} \Rightarrow \begin{cases} R = -6\left[\dfrac{\ddot a}{a} + \left(\dfrac{\dot a}{a}\right)^2\right] \\ R = \Box\,\phi = \ddot\phi + 3H\dot\phi \end{cases}$$

$$S = \kappa\int dt\, a^3\left\{F(R,\phi) - \epsilon(R - \ddot\phi - 3H\dot\phi) - \left(\frac{\partial F(R,\phi)}{\partial R} - \epsilon\right)\left[R + 6\left(\frac{\ddot a}{a} + \left(\frac{\dot a}{a}\right)^2\right)\right]\right\}$$

$$L = a^3F - a^3\dot\phi\dot\epsilon - a^3R\,\partial_R F + 6a\dot a^2\partial_R F - 6a\dot a^2\epsilon + 6a^2\dot a\dot R\,\partial_{RR}F + 6a^2\dot a\dot\phi\,\partial_{R\phi}F - 6a^2\dot a\dot\epsilon$$

Minisuperspace

$$\boldsymbol{q}(t) = \{a(t), R(t), \phi(t), \epsilon(t)\}$$

New scalar field

Selection of the models by symmetries

Noether Vector

$$X^{[1]} = \alpha\frac{\partial}{\partial a} + \beta\frac{\partial}{\partial R} + \gamma\frac{\partial}{\partial \phi} + \delta\frac{\partial}{\partial \epsilon} + (\dot{\alpha} - \dot{\xi}\dot{a})\frac{\partial}{\partial \dot{a}} + (\dot{\beta} - \dot{\xi}\dot{R})\frac{\partial}{\partial \dot{R}} + (\dot{\gamma} - \dot{\xi}\dot{\phi})\frac{\partial}{\partial \dot{\phi}} + (\dot{\delta} - \dot{\xi}\dot{\epsilon})\frac{\partial}{\partial \dot{\epsilon}}$$

$$L = a^3F - a^3\dot{\phi}\dot{\epsilon} - a^3R\,\partial_R F + 6a\dot{a}^2\partial_R F - 6a\dot{a}^2\epsilon + 6a^2\dot{a}\dot{R}\,\partial_{RR}F + 6a^2\dot{a}\dot{\phi}\,\partial_{R\phi}F - 6a^2\dot{a}\dot{\epsilon}$$

System of 28 PDE

2 classes of solutions: same generator, different functions

$$\mathcal{X} = (\xi_0 t + \xi_1)\partial_t + \frac{\xi_0}{3}(2n-1)\partial_a - 2\xi_0 R\partial_R + \frac{2\xi_0(1-\ell)}{n}\partial_\phi + (2\xi_0(1-n)\epsilon + \delta_1)\partial_\epsilon$$

$$\begin{cases} f_I(R,\phi) = \dfrac{\delta_1}{2\xi_0(n-1)}R + [2\xi_0 R]^n\mathcal{F}\left(\phi + \dfrac{(1-n)}{\ell}\log[2\xi_0 R]\right) \\ f_{II}(R,\phi) = \dfrac{\delta_1}{2\xi_0(n-1)}R + G(R)\,e^{k\phi} \end{cases}$$

Arbitrary functions

Selection of the model: First Case

Simple choice

$$\mathcal{F}_1\left(\phi + \frac{(1-n)}{\ell}\log[2\xi_0 R]\right) \equiv \phi + \frac{(1-n)}{\ell}\log[2\xi_0 R] + q$$

The first function becomes

$$f_1(R,\phi) = \frac{\delta_1}{2\xi_0(n-1)}R + (2\xi_0 R)^n(q+\phi) + (2\xi_0 R)^n \frac{(1-n)}{\ell}\log[2\xi_0 R]$$

Example: n=2 ⟹ **NON-LOCAL EXTENSION** of $S_{Starobinsky}$

$$f_1(R,\phi)\Big|_{n=2} = \frac{\delta_1}{2\xi_0(n-1)}R + 4\,\xi_0^2 R^2(q+\phi) - \frac{4\xi_0^2}{\ell}R^2\,\log[2\xi_0 R]$$

Cosmological Solutions for the First Case

Replacing $$f_1(R,\phi) = \frac{\delta_1}{2\xi_0(n-1)} R + (2\xi_0 R)^n (q+\phi) + (2\xi_0 R)^n \frac{(1-n)}{\ell} \log[2\xi_0 R]$$

Into the system of E-L equations, we get *three* different cosmological solutions

I:
$$a(t) = a_0\, e^{\Lambda t} \quad R(t) = -12\,\Lambda^2 \quad \phi(t) = -\frac{1}{3}(40+3q) - 4\Lambda t$$
$$\epsilon(t) = 576(2\xi_0)^3 \Lambda^5 t - \frac{C_3 e^{-3\Lambda t}}{3\Lambda} + \frac{\delta_1}{2\xi_0(n-1)},$$

II:
$$a(t) = a_0\, t^{-10} \quad R(t) = -1260\, t^{-2} \quad \phi(t) = C_2 + \frac{1260}{31}\log(t)$$
$$\epsilon(t) = \frac{\delta_1}{2\xi_0(n-1)} + \frac{C_3}{31} t^{31} + 14288400(2\xi_0)^3\, t^{-4}$$

Both constrain the function to be

$\phi \equiv \Box^{-1} R$

$$f_1(R,\phi) = \frac{\delta_1}{2\xi_0(n-1)} R + (\phi+q)(2\xi_0 R)^3 - \frac{16\xi_0^3}{\ell} R^3 \log[2\xi_0 R]$$

Cosmological Solutions for the First Case

$$f_1(R,\phi) = \frac{\delta_1}{2\xi_0(n-1)}R + (2\xi_0 R)^n(q+\phi) + (2\xi_0 R)^n \frac{(1-n)}{\ell}\log[2\xi_0 R]$$

Third solution of the first function:

$$a(t) = a_0\, t^{\frac{1}{2}} \quad R(t) = 0 \quad \phi(t) = C_2 \quad \epsilon(t) = \frac{\delta_1}{2\xi_0(n-1)} - \frac{2C_3}{\sqrt{t}}$$

Constraining the function to be

$$f_1(R,\phi) = \frac{\delta_1}{2\xi_0(n-1)}R + \phi \quad \rightarrow \quad \phi \equiv \Box^{-1}R$$

which is nothing else but GR minimally coupled to a scalar field

Let us now consider the second selected function

Cosmological Solutions for the Second Case

$$f_{II}(R,\phi) = \frac{\delta_1}{2\xi_0(n-1)}R + G(R)\,e^{k\phi}$$

$$a(t) = a_0 e^{mt} \quad \phi(t) = -4h_0 mt \quad R(t) = -12m^2$$

$$\epsilon(t) = \frac{e^{-3mt}\left[\frac{3^{1+n}4^n e^{(3-4h_0k)mt} f_0(-m^2)^n}{h_0(3-4h0k)}\right]}{(12m^2)}$$

Where:

1. **We chose $G(R) = R^n$**

2. **The system of E-L equations provides the constraint $\delta_1 = 0$**

2)

$$F(G, \square^{-1} h(G))$$

Why considering the Gauss-Bonnet term?

1. **The Gauss-Bonnet term is a topological surface term and reduces dynamics**

2. **The Gauss-Bonnet Term naturally emerges in gauge theories of gravity**

3. **In homogeneous cosmology, it turns out that** $f(\mathcal{G}) \doteq \sqrt{\mathcal{G}} \longrightarrow R$

Gauss-Bonnet Invariant: $\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$

Let us start from

$$S = \int \sqrt{-g} f(\mathcal{G}, \Box^{-1} h(\mathcal{G})) d^4x$$

So that we may have

$$h(\mathcal{G}) \doteq \sqrt{\mathcal{G}} \longrightarrow R$$

Cosmological Lagrangian

$$S=\int\sqrt{-g}f(\mathcal{G},\Box^{-1}h(\mathcal{G}))d^4x$$

$$S=\int\sqrt{-g}\left\{f(\mathcal{G},\phi)+\epsilon(t)(\Box\phi-h(\mathcal{G}))\right\}d^4x$$

formal localization

$$\Box^{-1}h(\mathcal{G}):=\phi(t)\longrightarrow h(\mathcal{G})=\Box\phi(t)$$

$$g_{\mu\nu}^{FLRW}\Rightarrow\mathcal{G}=24\frac{\dot{a}^2\ddot{a}}{a^3}$$

$$S=2\pi^2\int\left\{a^3f(\mathcal{G},\phi)+a^3\epsilon(t)\left[\ddot{\phi}+3\frac{\dot{a}}{a}\dot{\phi}-h(\mathcal{G})\right]-\lambda\left(\mathcal{G}-24\frac{\dot{a}^2\ddot{a}}{a^3}\right)\right\}dt$$

$$\mathcal{L}=a^3\left[f(\mathcal{G},\phi)-\mathcal{G}f_{\mathcal{G}}(\mathcal{G},\phi)-\epsilon h(\mathcal{G})+\epsilon\mathcal{G}h_{\mathcal{G}}(\mathcal{G})\right]-a^3\dot{\phi}\dot{\epsilon}-8\dot{a}^3\dot{\mathcal{G}}f_{\mathcal{G}\mathcal{G}}(\mathcal{G},\phi)+$$
$$+8\epsilon\dot{a}^3\dot{\mathcal{G}}h_{\mathcal{G}\mathcal{G}}(\mathcal{G})+8\dot{a}^3\dot{\epsilon}h_{\mathcal{G}}(\mathcal{G})-8\dot{a}^3\dot{\phi}f_{\mathcal{G}\phi}(\mathcal{G},\phi)$$

Minisuperspace $\boldsymbol{q}(t)=\{a(t),G(t),\phi(t),\varepsilon(t)\}$

New scalar field

Selection of the models by symmetries

Generator

$$\mathcal{X} = \xi(t,a,\phi,\mathcal{G},\epsilon)\partial_t + \alpha(t,a,\phi,\mathcal{G},\epsilon)\partial_a + \beta(t,a,\phi,\mathcal{G},\epsilon)\partial_\phi + \\ + \gamma(t,a,\phi,\mathcal{G},\epsilon)\partial_\mathcal{G} + \delta(t,a,\phi,\mathcal{G},\epsilon)\partial_\epsilon$$

$$\mathcal{L} = a^3\left[f(\mathcal{G},\phi) - \mathcal{G}f_\mathcal{G}(\mathcal{G},\phi) - \epsilon h(\mathcal{G}) + \epsilon\mathcal{G}h_\mathcal{G}(\mathcal{G})\right] - a^3\dot{\phi}\dot{\epsilon} - 8\dot{a}^3\dot{\mathcal{G}}f_{\mathcal{G}\mathcal{G}}(\mathcal{G},\phi) + \\ + 8\epsilon\dot{a}^3\dot{\mathcal{G}}h_{\mathcal{G}\mathcal{G}}(\mathcal{G}) + 8\dot{a}^3\dot{\epsilon}h_\mathcal{G}(\mathcal{G}) - 8\dot{a}^3\dot{\phi}f_{\mathcal{G}\phi}(\mathcal{G},\phi)$$

3 classes of solutions: with different functions ← **System of 37 PDE**

I:

$$\mathcal{X} = (\xi_0 t + \xi_1)\partial_t + \alpha_0 a\partial_a + (\beta_0\phi + \beta_1)\partial_\phi - 4\xi_0\mathcal{G}\partial_\mathcal{G} + \delta_0\epsilon\partial\epsilon$$

$$h(\mathcal{G}) = h_0\mathcal{G}^{\frac{1}{2}+\frac{n}{k}} \qquad f(\mathcal{G},\phi) = f_0\mathcal{G}^n + f_1\mathcal{G} + f_2\left(\beta_0\phi + \beta_1\right)^k$$

II:

$$\mathcal{X} = (\xi_0 t + \xi_1)\partial_t + \alpha_0 a\partial_a + (\beta_0\phi + \beta_1)\partial_\phi - 4\xi_0\mathcal{G}\partial_\mathcal{G} + \delta_0\epsilon\partial\epsilon$$

$$h_0\mathcal{G}^z \qquad f(\mathcal{G},\phi) = f_0\mathcal{G}^n(\beta_0\phi + \beta_1)^k$$

III:

$$\mathcal{X} = (\xi_0 t + \xi_1)\partial_t + \alpha_0 a\partial_a + \beta_1\partial_\phi - 4\xi_0\mathcal{G}\partial_\mathcal{G} + \delta_0\epsilon\partial\epsilon$$

$$h(\mathcal{G}) = h_0\sqrt{\mathcal{G}} \qquad f(\mathcal{G},\phi) = f_0\mathcal{G}^n e^{k\phi} \qquad k \equiv \frac{\delta_0 + 4n\xi_0}{\beta_1}$$

Cosmological Solutions

I: $$f(\mathcal{G},\phi) = f_0\mathcal{G}^n + f_1\mathcal{G} + f_2\left(\beta_0\phi+\beta_1\right)^k$$

$$a(t) = a_0 e^{qt} \quad \mathcal{G}(t)\sim \text{const} \quad \phi(t)\sim t \quad \epsilon(t)\sim t \quad k=1,\ n=\frac{1}{2}$$

$$f(\mathcal{G},\Box^{-1}h(\mathcal{G})) = f_0\sqrt{\mathcal{G}} + f_1\mathcal{G} + f_2\Box^{-1}\mathcal{G} + f_3$$

Note that $\sqrt{\mathcal{G}}\sim R$ is selected by Noether simmetry + EL equations!!

Parameters are constrained by E-L equations

II: $$f(\mathcal{G},\phi) = f_0\mathcal{G}^n(\beta_0\phi+\beta_1)^k$$

$$a(t)\sim t^{\frac{2}{3}(2n+2kz-k)} \quad \mathcal{G}(t)\sim t^{-4} \quad \phi(t)\sim t^{2-4z} \quad \epsilon(t)\sim t^{2k(1-2z)}$$

$$f(\mathcal{G},\Box^{-1}h(\mathcal{G})) = f_2\mathcal{G}^n(\Box^{-1}\mathcal{G}^z)^k$$

Cosmological Solutions

Note that solution I with $f_3 = 0$ is equivalent to $S = \int \sqrt{-g}\,(R + f_2 \Box^{-1} \mathcal{G})\, d^4x$

$$f(\mathcal{G}, \Box^{-1} h(\mathcal{G})) = f_0 \sqrt{\mathcal{G}} + f_1 \mathcal{G} + f_2 \Box^{-1} \mathcal{G} + f_3$$

Due to the topological nature of $\mathcal{G}$

Solution III)

$$f(\mathcal{G}, \phi) = f_0 \mathcal{G}^n e^{k\phi} \qquad k \equiv \frac{\delta_0 + 4n\xi_0}{\beta_1}$$

leads to

Case I

$$a(t) = a_0 e^{q\,t} \quad \phi(t) = \sqrt{\frac{8}{3}} q\, t \quad \epsilon(t) \sim e^{\sqrt{\frac{8}{3}} k q\, t}$$

$$f(\mathcal{G}, \Box^{-1}\sqrt{\mathcal{G}}) = f_0 \mathcal{G}^{\frac{12\sqrt{6}}{4k - \sqrt{6}}} e^{k\phi}$$

Case II

$$a(t) = a_0 t^q \quad \phi(t) = \frac{2\sqrt{6q^3(q-1)}\, \ln[(1-3q)t]}{3q-1}$$

$$\mathcal{G}(t) = \frac{24 q^3 (q-1)}{t^4} \qquad \epsilon(t) \sim t^{2 - 4n + \frac{2k\sqrt{6q^3(q-1)}}{3q-1}}$$

What happens if we add the Ricci Scalar to the action?

Instead of considering $S = \int \sqrt{-g} f(\mathcal{G}, \Box^{-1} h(\mathcal{G})) d^4x$

We now consider $S = \int \sqrt{-g} \left[\chi R + f(\mathcal{G}, \Box^{-1} h(\mathcal{G}) \right] d^4x$

With the same localization procedure we get the Lagrangian

$$\mathcal{L} = a^3 \left[f(\mathcal{G}, \phi) - \mathcal{G} f_{\mathcal{G}}(\mathcal{G}, \phi) - \epsilon h(\mathcal{G}) + \epsilon \mathcal{G} h_{\mathcal{G}}(\mathcal{G}) \right] - a^3 \dot{\phi} \dot{\epsilon} - 8 \dot{a}^3 \dot{\mathcal{G}} f_{\mathcal{G}\mathcal{G}}(\mathcal{G}, \phi) + \\ + 8 \dot{a}^3 \dot{\epsilon} h_{\mathcal{G}}(\mathcal{G}) + 8 \epsilon \dot{a}^3 \dot{\mathcal{G}} h_{\mathcal{G}\mathcal{G}}(\mathcal{G}) - 8 \dot{a}^3 \dot{\phi} f_{\mathcal{G}\phi}(\mathcal{G}, \phi) + 6 \chi a \dot{a}^2$$

and the Noether identity provides five solutions

$$\left\{\begin{array}{l}
\mathcal{X} = (3\alpha_0 t + \xi_1)\partial_t + \alpha_0 a\partial_a + (\beta_0\phi + \beta_1)\partial_\phi - 12\alpha_0\mathcal{G}\partial_\mathcal{G} + \delta_0\epsilon\partial\epsilon \\
h(\mathcal{G}) = h_0\mathcal{G}^{\frac{1}{2}+\frac{1}{2k}} \quad f(\mathcal{G},\phi) = f_0\mathcal{G}^{\frac{1}{2}} + f_1\mathcal{G} + f_2\left(\beta_0\phi + \beta_1\right)^k \\
\\
\mathcal{X} = (3\alpha_0 t + \xi_1)\partial_t + \alpha_0 a\partial_a + (\beta_0\phi + \beta_1)\partial_\phi - 12\alpha_0\mathcal{G}\partial_\mathcal{G} + (\delta_0\epsilon + \delta_1)\partial\epsilon \\
h(\mathcal{G}) = h_0\mathcal{G} \quad f(\mathcal{G},\phi) = f_0\mathcal{G}^{\frac{1}{2}} + f_1\mathcal{G} + f_2(\beta_0\phi + \beta_1) \\
\\
\mathcal{X} = (3\alpha_0 t + \xi_1)\partial_t + \alpha_0 a\partial_a + (\beta_0\phi + \beta_1)\partial_\phi - 12\alpha_0\mathcal{G}\partial_\mathcal{G} + \delta_0\epsilon\partial\epsilon \\
h_0\mathcal{G}^{\frac{1-2n}{2k}} \quad f(\mathcal{G},\phi) = f_0\mathcal{G}^n(\beta_0\phi + \beta_1)^k \\
\\
\mathcal{X} = (3\alpha_0 t + \xi_1)\partial_t + \alpha_0 a\partial_a + (\beta_0\phi + \beta_1)\partial_\phi - 12\alpha_0\mathcal{G}\partial_\mathcal{G} + (\delta_0\epsilon + \delta_1)\partial\epsilon \\
h(\mathcal{G}) = h_0\mathcal{G} \quad f(\mathcal{G},\phi) = f_0\mathcal{G}^n(\beta_0\phi + \beta_1)^{1-2n} \\
\\
\mathcal{X} = (3\alpha_0 t + \xi_1)\partial_t + \alpha_0 a\partial_a + \beta_1\partial_\phi - 12\alpha_0\mathcal{G}\partial_\mathcal{G} \\
h(\mathcal{G}) = h_0\sqrt{\mathcal{G}} \quad f(\mathcal{G},\phi) = f_0\mathcal{G}^n e^{k\phi}
\end{array}\right.$$

R is now redundant

R complicates dynamics

No cosmological solutions are compatible with Noether Symmetries this time

3)

$$F(T, B, \Box^{-1} T, \Box^{-1} B)$$

S. Bahamonde, S. Capozziello, K.F. Dialektopoulos "Constraining generalized non-local cosmology from Noether symmetries". In: EPJC **77** (2017), p. 722

Non-Local Gravity in the Teleparallel picture

Thanks to the relation $R = -T + B$ **we write the Non-Local version**

$$\Box^{-1}R = -\Box^{-1}T + \Box^{-1}B$$

...and we start from the action

$$S = \int h\left[-T + (\tau T + \chi B) f(\Box^{-1}T, \Box^{-1}B)\right] d^4x$$

- h → **determinant of tetrad fields**
- τ, χ → **real constants**

→ $\tau = 1, \chi = -1$ → R

Localization procedure

$$\Box^{-1}T \equiv \phi(t) \rightarrow T = \Box\phi(t) \quad \Big| \quad \Box^{-1}B \equiv \varphi(t) \rightarrow B = \Box\varphi(t)$$

New scalar fields

$$S = \int h\left[-T + (\tau T + \chi B) f(\phi, \varphi) + \epsilon(\Box\phi - T) + \zeta(\Box\varphi - B)\right] d^4x$$

$$\begin{cases} \Box\epsilon = (\tau T + \chi B) f_\phi(\phi, \varphi) \\ \Box\zeta = (\tau T + \chi B) f_\varphi(\phi, \varphi) \end{cases}$$

→ **Klein-Gordon equations**

Cosmological Lagrangian

$$h = diag(1, -a(t), -a(t), -a(t))$$

$$B \equiv \frac{2}{h}\partial_\mu (hT^{\nu\mu}{}_\nu) = -12\frac{\dot{a}^2}{a^2} - 6\frac{\ddot{a}}{a}$$

$$T = -6\frac{\dot{a}^2}{a^2}$$

$$\mathcal{L} = 6a^2\dot{a}\left[\chi f_\phi(\phi,\varphi)\dot{\phi} + \chi f_\varphi(\phi,\varphi)\dot{\varphi} - \dot{\zeta}\right] + \\ + 6a\dot{a}^2\left[\epsilon + 1 - \tau f(\phi,\varphi)\right] - a^3\dot{\zeta}\dot{\varphi} - a^3\dot{\epsilon}\dot{\phi}$$

With minisuperspace

$$\mathcal{S} = \{a, \phi, \varphi, \epsilon, \zeta\}$$

The generator of the symmetry

$$\mathcal{X} = \xi(t)\partial_t + \alpha(a,\phi,\varphi,\epsilon,\zeta,t)\partial_a + \beta(a,\phi,\varphi,\epsilon,\zeta,t)\partial_\phi + \\ + \gamma(a,\phi,\varphi,\epsilon,\zeta,t)\partial_\varphi + \delta(a,\phi,\varphi,\epsilon,\zeta,t)\partial_\epsilon + \theta(a,\phi,\varphi,\epsilon,\zeta,t)\partial_\zeta$$

Can be selected via Noether's approach

↓

Provides a system of 43 PDE

Solutions

$$\begin{cases} \mathcal{X}_1 = (\xi_0 t + \xi_1)\,\partial_t + \frac{\alpha_0}{3} a\partial_a + [\beta_0 + \beta_1(6\ln a + \phi)]\,\partial_\phi + \\ \qquad + [\gamma_0 + \gamma_1(6\ln a + \varphi) + \gamma_2]\,\partial_\varphi + \delta_0\epsilon\partial_\epsilon + [(\delta_0 - \gamma_1)\,\zeta - \beta_1\epsilon + \theta_0]\,\partial_\zeta \\ f(\phi,\varphi) = \frac{1}{\tau} + f_0 \exp\{n\,(\beta_1\varphi - \gamma_1\phi)\} \\ \\ \mathcal{X}_2 = (\xi_0 t + \xi_1)\,\partial_t + \frac{\alpha_0}{3} a\partial_a + [\beta_0 + \beta_1(6\ln a + \varphi)]\,\partial_\phi \\ \qquad + \gamma_0\partial_\varphi + (\delta_0 + \delta_1\epsilon)\,\partial_\epsilon + (\delta_1\zeta - \beta_1\epsilon + \theta_0)\,\partial_\zeta \\ f(\phi,\varphi) = \frac{1}{\tau}\left(1 - \frac{\delta_0}{\delta_1}\right) + f_0 e^{\frac{\delta_1}{\gamma_0}\varphi} \\ \\ \mathcal{X}_3 = (\xi_0 t + \xi_1)\,\partial_t - \frac{\alpha_0}{3} a\partial_a + \beta_0\partial_\phi + (\delta_0 + \delta_1\epsilon)\,\partial_\epsilon \\ f(\phi) = f_0 e^{\frac{\beta_0}{\delta_1}\phi} - \frac{\beta_0}{\delta_0} + 1 \\ \\ \mathcal{X}_4 = (\xi_0 t + \xi_1)\,\partial_t - \frac{\alpha_0}{3} a\partial_a + \beta_0\partial_\phi + \delta_0\partial_\epsilon \\ f(\phi) = f_1 + \frac{\beta_0}{\delta_0}\phi. \end{cases}$$

$\mathcal{X}_2$ → **A particular case of 1 With $\gamma_1 = 0$**

$\mathcal{X}_4$ → **No cosmological solutions**

Cosmological Solutions

Cosmological solutions related to the first and third generator

$$f(\phi,\varphi)=\frac{1}{\tau}+f_0\exp\left\{n\left(\beta_1\varphi-\gamma_1\phi\right)\right\}$$

$$a(t)=a_0e^{H_0t},\qquad \phi(t)=-2H_0t,\qquad \varphi(t)=-6H_0t$$

$$\epsilon(t)=\epsilon_0e^{-3H_0t(1+c_1)}-\epsilon_1e^{-3H_0t},\qquad \zeta(t)=\zeta_0e^{-3H_0t(1+c_1)}-\zeta_1e^{-3H_0t}$$

$$a(t)=a_0t^p,\qquad \phi(t)=\frac{6p^2\ln(t-3pt)}{1-3p},\qquad \varphi(t)=-6p\ln t$$

$$\epsilon(t)=\epsilon_0t^{2-3p}+\epsilon_1t^{1-3p}\qquad \zeta(t)=\zeta_0t^{2-3p}+\zeta_1t^{1-3p}$$

$$f(\phi)=f_0e^{\frac{\beta_0}{\delta_1}\phi}-\frac{\beta_0}{\delta_0}+1$$

$$\Downarrow$$

$$a(t)=e^{H_0t},\quad \phi(t)=-2H_0t,\quad \epsilon(t)=e^{-3H_0t}\left[f_0\left(3H_0t+1\right)-\frac{\epsilon_1}{3H_0}\right]-1$$

$$f(\phi,\varphi)=f_0e^{\frac{\beta_0}{\delta_1}\phi}$$ $\Longrightarrow$ **Constrained by EL equations**

$$f(\phi)=f_0e^{\frac{\beta_0}{\delta_1}\phi}-\frac{\beta_0}{\delta_0}+1$$

$$\Downarrow$$

$$a(t)=t^p\quad \phi(t)=\frac{6p^2\log(t-3pt)}{1-3p}\quad \epsilon(t)=f_0(1-3p)^{3(1-p)}t^{2-3p}\frac{\epsilon_0t^{1-3p}}{1-3p}-1$$

$$f(\phi)=f_0e^{\frac{\left(9p^2-9p+2\right)\phi}{6p^2}}$$ $\Longrightarrow$ **Constrained by EL equations**

Spherical Symmetry and astrophisical considerations

K.F. Dialektopoulos, D. Borka, S. Capozziello, V. Borka Jovanovic, P. Jovanovic “Constraining non-local gravity by S2 star orbits”. In: Phys. Rev. D **99** (2019), p. 044053

Objectives

- **Selecting the form of the Non-Local action containing symmetries**

- **Performing the post-Newtonian limit**

- **Constraining the free parameters by S2 star orbit around SgrA***

- **Estimate the reduced χ^2 and constrain characteristic lengths related to NLG**

Non-Local Gravity in Spherical Symmetry

We focus our attention in a spherically symmetric spacetime of the form

$$ds^2 = e^{\nu(r,t)}dt^2 - e^{\lambda(r,t)}dr^2 - r^2 d\Omega^2$$

With the aim to perform again the Noether Symmetry Approach

$$\phi \equiv \Box^{-1} R \longrightarrow S = \frac{1}{2\kappa^2}\int \sqrt{-g}\left\{R[1+f(\phi)] + \varepsilon(r,t)(\Box\phi - R)\right\}d^4x$$

New scalar field depending on both r and t

A particular form of modified Non-Local action:
Deser and Woodard action

$$\begin{aligned}\mathcal{L}(r,\nu,\lambda) = e^{-\frac{1}{2}(\lambda+\nu)}\Big[&-e^{\nu}r^2\nu_r\phi_r f_\phi(\phi) + e^{\lambda}r^2\lambda_t\phi_t f_\phi(\phi) +\\ &-2e^{\nu}f(\phi)\left(e^{\lambda}+r\lambda_r-1\right) - 2e^{\lambda+\nu} + 2e^{\nu} + e^{\nu}r^2\varepsilon_r\phi_r + e^{\nu}r^2\nu_r\varepsilon_r +\\ &-e^{\lambda}r^2\varepsilon_t\phi_t - e^{\lambda}r^2\lambda_t\varepsilon_t + 2e^{\nu}\varepsilon\left(e^{\lambda}+r\lambda_r-1\right) - 2e^{\nu}r\lambda_r\Big]\end{aligned}$$

Minisuperspace containing $r, t, \nu, \lambda, \phi, \varepsilon$

Solution and Selection

Noether Symmetry Approach selects

$$\begin{cases} \mathcal{X} = (\xi_0 t + \xi^t(r))\partial_t - 2\xi_0\partial_\nu + (\gamma_0 + 2\xi_0)\partial_\phi + \delta_0(\gamma_0 + 2\xi_0)\partial_\varepsilon \\ f(\phi) = \delta_0\phi + f_1 \\ \\ \mathcal{X} = (\xi_0 t + \xi^r(r))\partial_t - \frac{\xi_1}{2} r\partial_r - (2\xi_0 + \xi_1)\partial_\nu + \gamma_0\partial_\phi + \xi_1(\varepsilon - \delta_0 - 1)\partial_\varepsilon \\ f(\phi) = \delta_0 + f_1 e^{\frac{\gamma_0}{\xi_1}\phi} \end{cases}$$

$\xi^\mu = (\xi^t, \xi^r, 0, 0)$

1) We restrict the interval to a subclass of spacetimes where the Birkhoff theorem holds

$$ds^2 = A(r)dt^2 - B(r)\,dr^2 - r^2 d\Omega^2$$

2) We consider the sixth-order approximation of the metric

$$g_{00} \sim \mathcal{O}(6),\; g_{0i} \sim \mathcal{O}(5) \text{ and } g_{ij} \sim \mathcal{O}(4)$$

Post Newtonian Limit

The approximation $g_{00} \sim \mathcal{O}(6), g_{0i} \sim \mathcal{O}(5)$ and $g_{ij} \sim \mathcal{O}(4)$

Potentials

Leads to

$$\begin{cases} A(r) = 1 + \frac{1}{c^2}\Phi(r)^{(2)} + \frac{1}{c^4}\Phi(r)^{(4)} + \frac{1}{c^6}\Phi(r)^{(6)} + \mathcal{O}(8) \\ B(r) = 1 + \frac{1}{c^2}\Psi(r)^{(2)} + \frac{1}{c^4}\Psi(r)^{(4)} + \mathcal{O}(6) \\ \phi(r) = \phi_0 + \frac{1}{c^2}\phi(r)^{(2)} + \frac{1}{c^4}\phi(r)^{(4)} + \frac{1}{c^6}\phi(r)^{(6)} + \mathcal{O}(8) \\ \varepsilon(r) = \varepsilon_0 + \frac{1}{c^2}\varepsilon(r)^{(2)} + \frac{1}{c^4}\varepsilon(r)^{(4)} + \frac{1}{c^6}\varepsilon(r)^{(6)} + \mathcal{O}(8) \end{cases}$$

Constants

The above functions can be replaced into the field equations

$$[1+f(\phi)-\varepsilon]G_{\mu\nu} = (\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box)\ f(\phi) - \frac{1}{2}g_{\mu\nu}D_\alpha\varepsilon D^\alpha\phi + D_\mu\varepsilon D_\nu\phi$$

Corrected Newtonian potentials

Replacing the second function selected by Noether's approach

$$f(\phi) = \delta_0 + f_1 e^{\frac{\gamma_0}{\xi_1}\phi}$$

Into the field equations, with the approximations

$$\begin{cases} A(r) = 1 + \frac{1}{c^2}\Phi(r)^{(2)} + \frac{1}{c^4}\Phi(r)^{(4)} + \frac{1}{c^6}\Phi(r)^{(6)} + \mathcal{O}(8) \\ B(r) = 1 + \frac{1}{c^2}\Psi(r)^{(2)} + \frac{1}{c^4}\Psi(r)^{(4)} + \mathcal{O}(6) \\ \phi(r) = \phi_0 + \frac{1}{c^2}\phi(r)^{(2)} + \frac{1}{c^4}\phi(r)^{(4)} + \frac{1}{c^6}\phi(r)^{(6)} + \mathcal{O}(8) \\ \varepsilon(r) = \varepsilon_0 + \frac{1}{c^2}\varepsilon(r)^{(2)} + \frac{1}{c^4}\varepsilon(r)^{(4)} + \frac{1}{c^6}\varepsilon(r)^{(6)} + \mathcal{O}(8) \end{cases}$$

We obtain →

Order of the potential ←

$$A(r) = 1 - \frac{2G_N M\phi_c}{c^2 r} + \frac{G_N^2 M^2}{c^4 r^2}\left[\frac{14}{9}\phi_c^2 + \frac{18r_\varepsilon - 11r_\phi}{6r_\varepsilon r_\phi}r\right] + - \frac{G_N^3 M^3}{c^6 r^3}\left[\frac{50r_\varepsilon - 7r_\phi}{12r_\varepsilon r_\phi}\phi_c r + \frac{16\phi_c^3}{27} - \frac{r^2\left(2r_\varepsilon^2 - r_\phi^2\right)}{r_\varepsilon^2 r_\phi^2}\right]$$

$$B(r) = 1 + \frac{2G_N M\phi_c}{3c^2 r} + \frac{G_N^2 M^2}{c^4 r^2}\left[\frac{2\phi_c^2}{9} + \left(\frac{3}{2r_\varepsilon} - \frac{1}{r_\phi}\right)r\right]$$

$$\phi(r) = \frac{4G_N M\phi_c}{3c^2 r} - \frac{G_N^2 M^2}{c^4 r^2}\left[\left(\frac{11}{6r_\varepsilon} + \frac{1}{r_\phi}\right)r - \frac{2\phi_c^2}{9}\right] + - \frac{G_N^3 M^3}{c^6 r^3}\left[\frac{r^2}{r_\phi^2} - \left(\frac{25}{12r_\varepsilon} - \frac{7}{6r_\phi}\right)\phi_c r - \frac{4\phi_c^3}{81}\right]$$

$$\varepsilon(r) = 1 + \frac{G_N^2 M^2}{c^4 r^2}\left[\frac{2\phi_c^2}{3} - \left(\frac{13}{6r_\varepsilon} - \frac{1}{r_\phi}\right)r\right] + + \frac{G_N^3 M^3}{c^6 r^3}\left[\frac{20\phi_c^3}{27} - \left(\frac{1}{r_\varepsilon^2} - \frac{1}{r_\phi^2}\right)r^2 - \left(\frac{131}{36r_\varepsilon} + \frac{1}{6r_\phi}\right)\phi_c r\right]$$

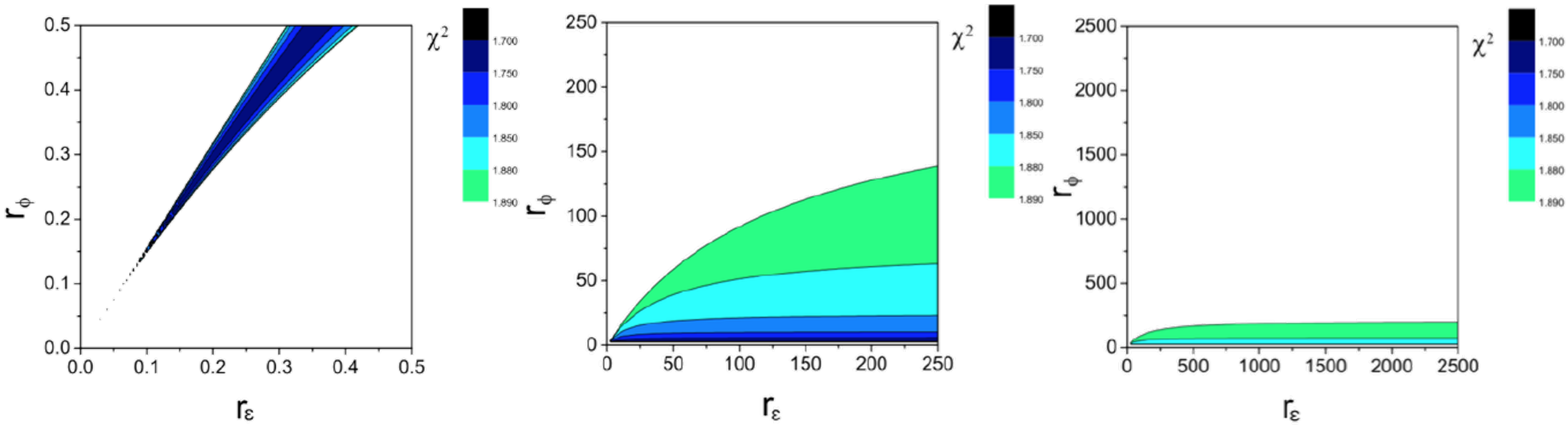
$$\Phi^{(2)}(r) = -\frac{2G_N M}{r}\phi_c$$

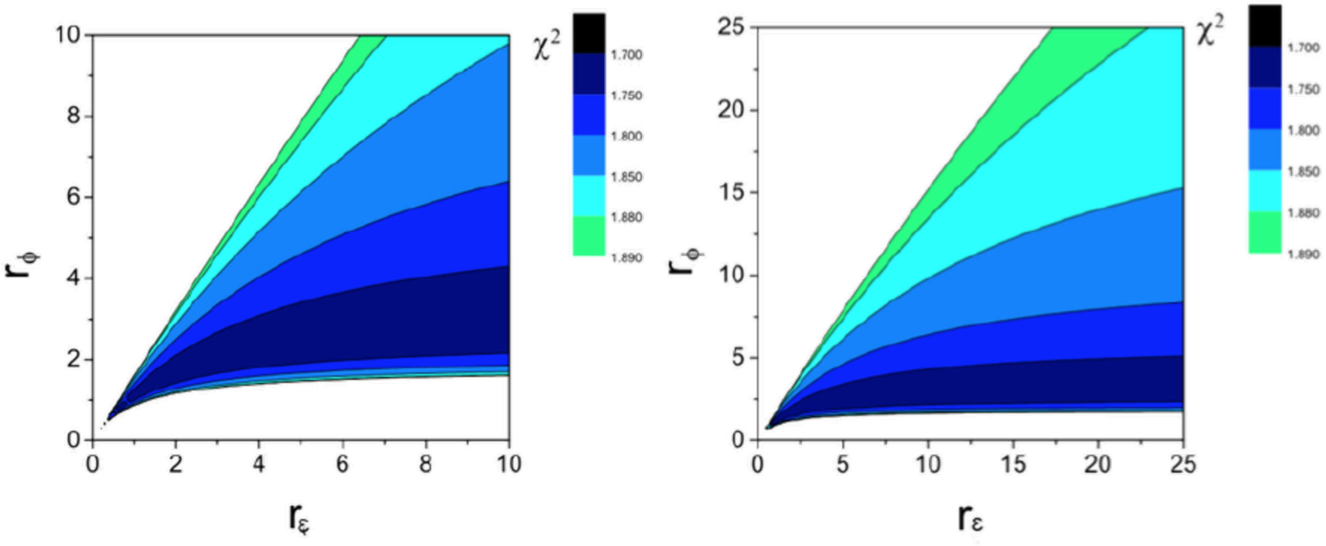
$$\Phi^{(4)}(r) = \frac{G_N^2 M^2}{r^2}\left[\frac{14}{9}\phi_c^2 + \frac{18r_\varepsilon - 11r_\phi}{6r_\varepsilon r_\phi}r\right]$$

$$\Phi^{(6)}(r) = \frac{G_N^3 M^3}{r^3}\left[\frac{7r_\phi - 50r_\varepsilon}{12r_\varepsilon r_\phi}\phi_c r - \frac{16\phi_c^3}{27} + \frac{2r_\varepsilon^2 - r_\phi^2}{r_\varepsilon^2 r_\phi^2}r^2\right]$$

Solution of the Perturbation

Two new length appears: r_ε and r_ϕ, searching for those by simulated orbits giving *at least* the same χ^2 as the Keplerian orbit $(\chi^2 \sim 1.89)$



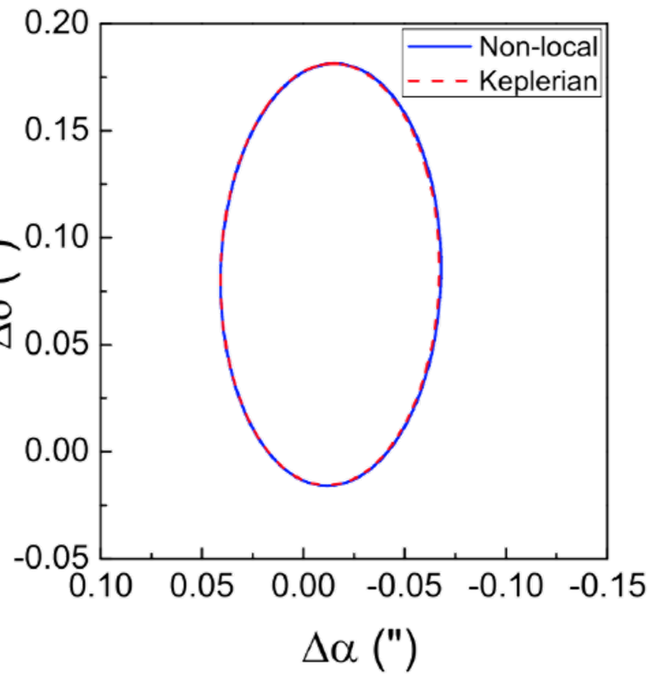


Maps of the reduced χ^2 in different regions, representing the lengths in AU giving at least the same χ^2 as the Keplerian orbit

After fixing the right parameters minimizing the χ^2 we plot the orbit

Comparisons between the Keplerian orbit of S2 star (red dashed line) and the orbit predicted by Non-Local gravity (blue solid line) with parameter values that minimize the χ^2 :

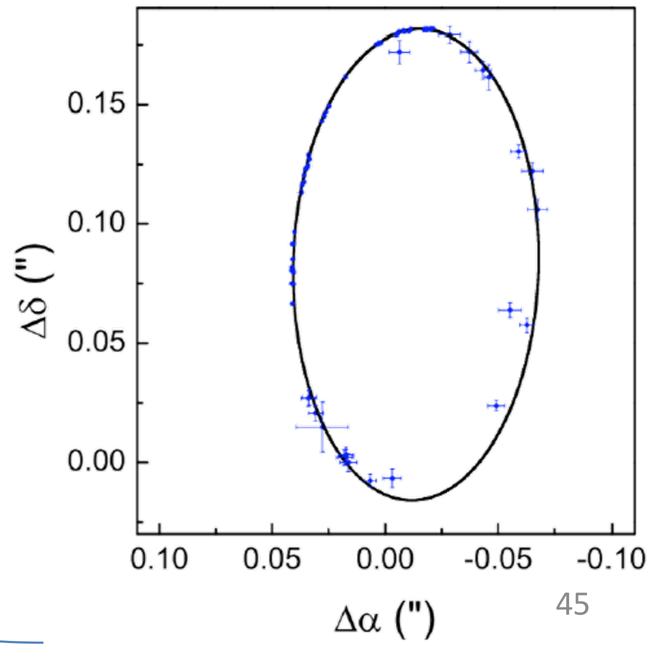
$r_\phi \sim 1.2\, AU\ and\ r_\varepsilon \sim 1.1\, AU.$



$\Delta\alpha\ and\ \Delta\delta$

coordinates of S2 star

Same comparison but with the error bars. Same value for the characteristic lengths.



Conclusions

- $F(R, \Box^{-1} R)$, $F\left(G, \Box^{-1} G\right)$, $F(T, \Box^{-1} T, B, \Box^{-1} B)$ could reproduce, in principle, **both UV and IR cosmic evolution**

using the ''Noether Symmetry Approach'', it is possible:

- *to select physically relevant cosmological models*
- *to derive exact cosmological solutions*
- *to constraint solutions by means of experimental observations*

Perspectives

I. Theoretical perspectives:

- Search for approximate cosmological solutions
- Study of renormalizability and unitarity
- Application of the Noether Symmetry Approach in general spherically symmetric background

II. Experimental perspectives:

- Observational constraining of the models free parameters *via* cosmological data, *e.g.* $\mathrm{SNe\ Ia} + \mathrm{BAO} + \mathrm{CC} + H_0$
- Constraining at astrophysical scales too, *e.g.* by S2 star orbit observations NTT/VLT

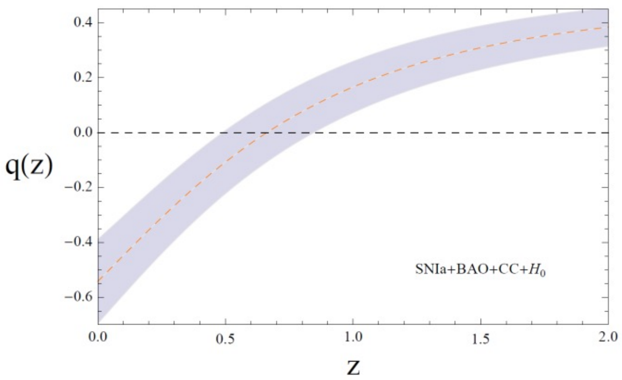
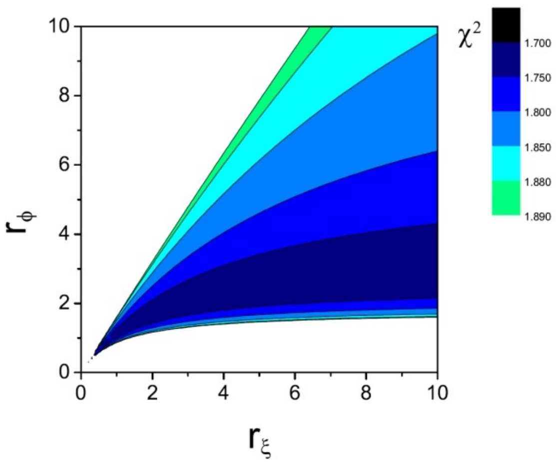
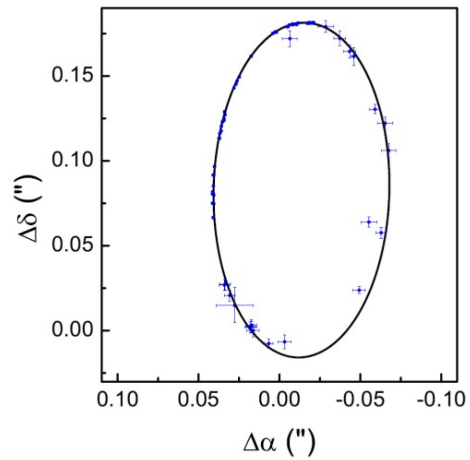


Figure from: *S. Bahamonde, S. Capozziello, M. Faiza, R. C. Nunes. ''Nonlocal Teleparallel Cosmology''. In: Eur. Phys. J.* ***C**77.9 (2017), p.628*)





Figures from: *K. F.. Dialektopoulos, D. Borka, S. Capozziello, V. Borka Jovanović, P. Jovanović. ''Constraining Non-local Gravity by S2 star orbits''. In: Phys. Rev.* ***D99**.4 (2019) p. 044053*