Neutron stars as strongly gravitating objects

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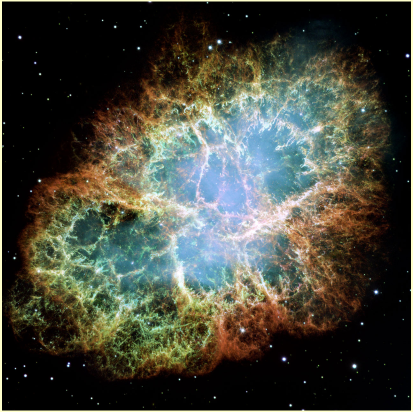
Beyond Standard Model: From Theory to Experiment
(BSM- 2021)
https://indico.cern.ch/event/959266/

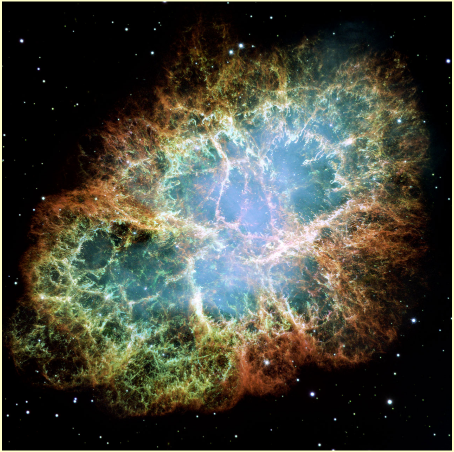
Crab nebula

- A supernova remnant
- Born in July 4, 1054
- ‘guest star’ recordings of Chinese
- Death of a massive star
- NS are born in SN explosions (Baade & Zwicky 1934)



The Crab pulsar

- Period: $P = 33\,\mathrm{ms}$
- Period derivative:
  $\dot{P} = 4.22 \times 10^{-13}\,\mathrm{s/s}$
- Age:
  $2021 - 1054 = 967\,\mathrm{yr}$
- Spin-down power:
  $L = -I\Omega\dot{\Omega} = 10^5 L_\odot$



NS phenomenology

NS are the central engines of many diverse phenomena:

- Radio pulsars (rotationally powered, isolated NS)
- X-ray pulsars (accreting NS in binaries)
- Magnetars (isolated NS with extreme magnetic fields of $B \sim 10^{15}\,\mathrm{Gauss}$ causing crustal break.)
- Gamma-ray bursts (GRBs):
  - mergers of neutron stars (short-GRBs: $\Delta t < 2\,\mathrm{s}$),
  - formation of magnetars by core-collapse (long-GRBs: $\Delta t > 2\,\mathrm{s}$)
- Fast radio bursts (FRBs): recently traced back to the magnetosphere of magnetars.

Basic parameters of NS

Some basic parameters of NS:

- $M \simeq 1.4 - 2.5\, M_{\odot}$
- $R \simeq 10 - 12\,\mathrm{km}$
- $\bar{\rho} \gtrsim 10^{15}\,\mathrm{g\,cm^{-3}}$ (An order of magnitude larger than nuclear density)
- Compactness $\frac{2GM}{c^2 R} \simeq 0.4$ (Almost a black hole!)
- They can rotate with frequencies as high as $\nu \sim 1000\,\mathrm{Hz}$
- $B \sim 10^{12}$ Gauss (typical pulsars) ($\sim 10^8$ times the B in MRI device.)
- $B \sim 10^{15}$ Gauss (magnetars) (Above the QED limit $B_{\mathrm{c}} = m_e^2 c^3/\hbar e = 4.4 \times 10^{13}\,\mathrm{G}$)

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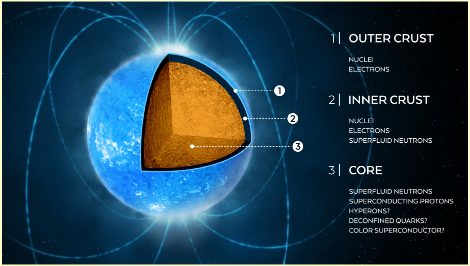
NS stars are the most compact objects whose surface is not hidden behind an event horizon!

Neutron stars and fundamental physics

- Understanding the ground state of dense matter (EoS?)
- Testing Einstein's GR in strong gravity



Watts+2016

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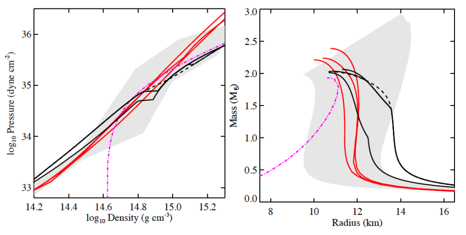

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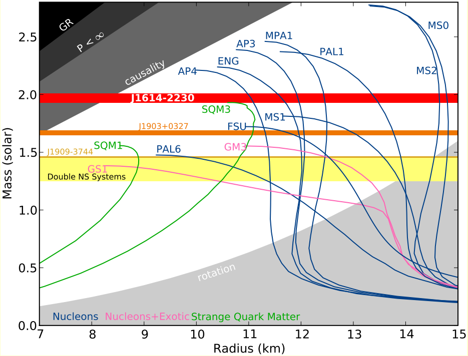

Demorest+2010

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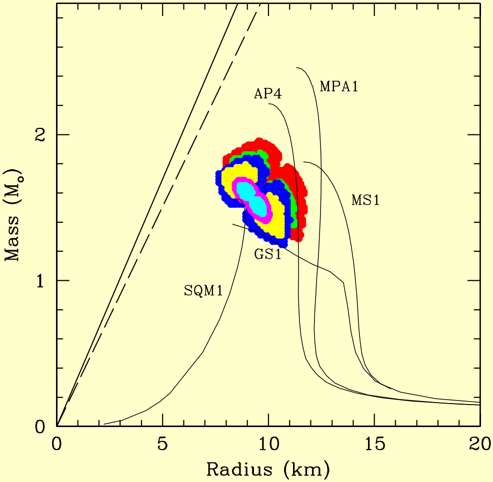

Feryal Özel, Gordon Baym & Tolga Güver 2010

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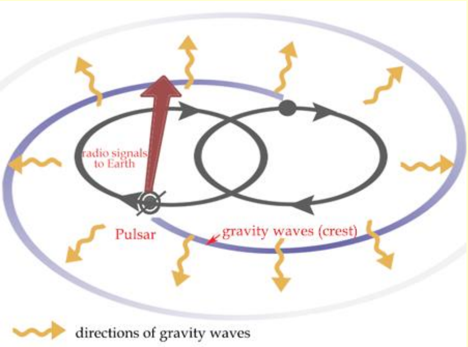

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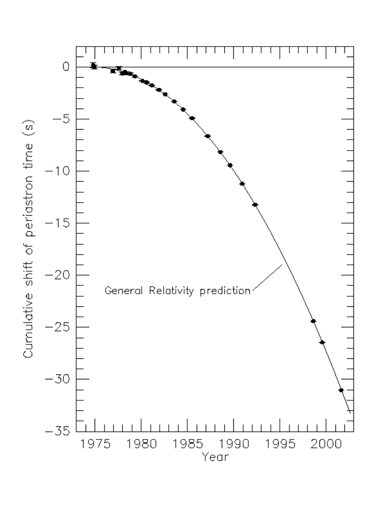

Hulse & Taylor 1975

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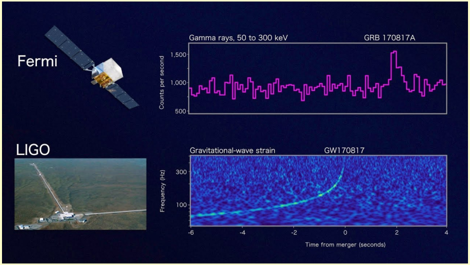


Relativistic stars

- Consider the spherically symmetric metric

$$\mathrm{d}s^2 = \mathrm{e}^{2\nu(r)}\,\mathrm{d}t^2 - \mathrm{e}^{2\lambda}\,\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)$$

- Non-vanishing components of Ricci tensor

$$R_{00} = \left(-\nu'' + \lambda'\nu' - \nu'^2 - 2\frac{\nu'}{r}\right)\mathrm{e}^{2(\nu-\lambda)}$$

$$R_{11} = \nu'' - \lambda'\nu' + \nu'^2 - 2\frac{\lambda'}{r}$$

$$R_{22} = (1 + r\nu' - r\lambda')\mathrm{e}^{-2\lambda} - 1$$

$$R_{33} = R_{22}\sin^2\theta$$

- The Ricci scalar:

$$R = \left(-2\nu'' + 2\lambda'\nu' - 2\nu'^2 - \frac{2}{r^2} + 4\frac{\lambda'}{r} - 4\frac{\nu'}{r}\right)\mathrm{e}^{-2\lambda} + \frac{2}{r^2}$$

GR in spherical symmetry

Energy-momentum tensor for perfect fluid

$$T_{\mu\nu} = (\epsilon + P)u_\mu u_\nu - P g_{\mu\nu}, \qquad u_\mu = \frac{\mathrm{d}x_\mu}{\mathrm{d}\tau} \equiv (\mathrm{e}^{-\nu}, 0, 0, 0)$$

Plugging these into Einstein's field equations

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}, \qquad G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

one gets (Tolman 1939, Oppenheimer & Volkoff 1939)

$$\left(\frac{1}{r^2} - 2\frac{\lambda'}{r}\right)\mathrm{e}^{-2\lambda} - \frac{1}{r^2} = 8\pi G \epsilon(r)$$

$$\left(\frac{1}{r^2} + 2\frac{\nu'}{r}\right)\mathrm{e}^{-2\lambda} - \frac{1}{r^2} = -8\pi G P(r)$$

$$\left(\nu'' + \nu'^2 - \lambda'\nu' + \frac{\nu' - \lambda'}{r}\right)\mathrm{e}^{-2\lambda} = 8\pi G P(r)$$

Hydrostatic equilibrium equations

After a bit of manupilations one gets the hydrostatic equilibrium equations:

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{Gm\rho}{r^2}\left(1 + \frac{P}{\rho c^2}\right)\left(1 + \frac{4\pi r^3 P}{mc^2}\right)\left(1 - \frac{2Gm}{rc^2}\right)^{-1}$$

and

$$\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 \rho$$

(Oppenheimer & Volkoff 1939)

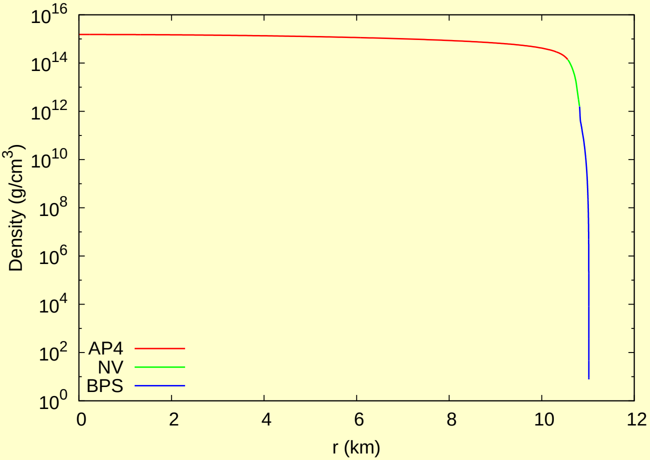
EoS, $P = P(\rho)$

Boundary conditions:

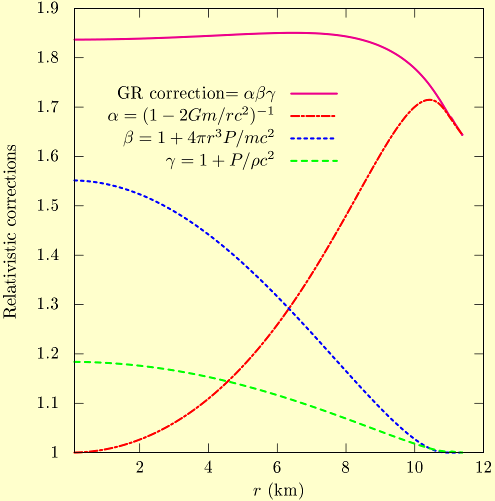
$\rho(0) = \rho_{\mathrm{c}}$, $m(0) = 0$,

$P(R_*) = 0$ and $m(R_*) = M_*$

Density distribution within the star



Relativistic corrections



Ekşi, TJP (2016) 40: 127-138, arXiv:1511.04305

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{Gm\rho}{r^2} \times \left(1 + \frac{P}{\rho c^2}\right) \times \left(1 + \frac{4\pi r^3 P}{mc^2}\right) \times \left(1 - \frac{2Gm}{rc^2}\right)^{-1}$$

GR and maximum mass

Recall

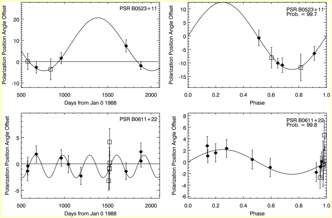
$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{Gm\rho}{r^2}\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi r^3 P}{mc^2}\right)\left(1-\frac{2Gm}{rc^2}\right)^{-1}$$

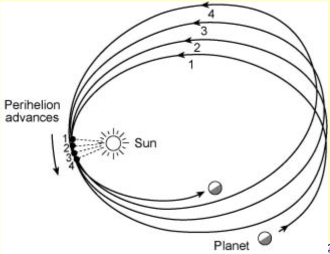
$$\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2\rho$$

- All corrections > 1
- All kinds of energy contribute to gravity in GR.
- Increasing pressure not only balance gravity but enhances the internal gravity (!)
- Beyond a critical mass, contribution of pressure to gravity overwhelms the resistence of the pressure against gravity.
- There is thus a *maximum mass* of relativistic stars in GR.

Why GR?

- Einstein's GR passed from all solar system tests with great successs.
- GR fits the Hulse-Taylor pulsar data.
- GR presents us the fundamental framework for understanding the expanding universe.





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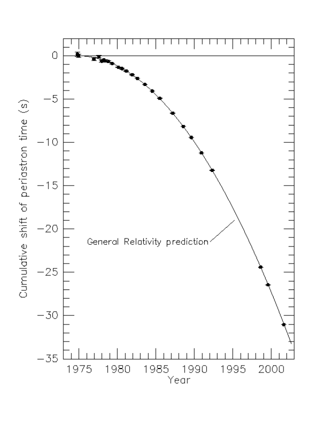

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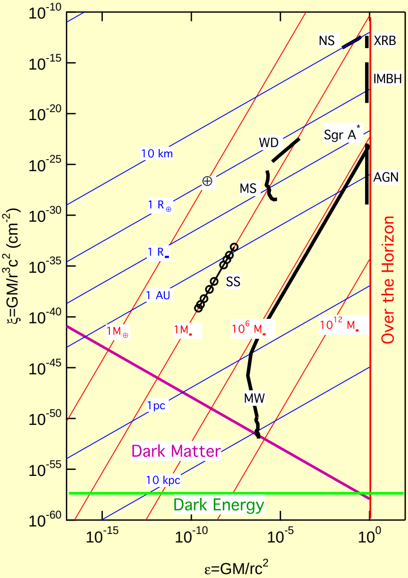
-You're right Edwin, it is really expanding!

What is strong gravity?



Psaltis 2008

- In GR, the strength of the gravitational field is measured by the gravitational potential

$$\epsilon \equiv \frac{GM}{rc^2} \qquad \text{compactness}$$

- In a general Lagrangian theory with an additional scale, the strength of the field would be measured by the curvature

$$\xi \equiv \frac{GM}{r^3c^2} \qquad \text{curvature}$$

Neutron star tests

A gravity test with *neutron stars* ($M = 1.4 M_\odot$ and $R = 10\,\mathrm{km}$) would probe a compactness

$$\epsilon \simeq \frac{GM}{Rc^2} \simeq 0.2 \sim 10^5 \epsilon_\odot \ ,$$

a spacetime curvature of

$$\xi = \frac{GM}{R^3 c^2} \simeq 4 \times 10^{-13}\ \mathrm{cm}^{-2} \sim 10^{15} \xi_\odot \ .$$

Note that the precision of the Solar Systems tests is 10^{-5}.

Curvature

The tidal force is the only sign of gravity that cannot be cast aside by a coordinate transformation. The tidal force on a body moving along a geodesic leads to

1. a distortion of the shape of the body
2. a change in the volume of the body

Curvatures are different on the basis of information they convey:

- The Riemann curvature tensor, $R_{\mu\nu\rho\sigma}$, captures both (1) and (2).
- The Ricci curvature, $R_{\mu\nu}$, the trace component of the Riemann tensor, conveys only (2).
- The Weyl tensor $C_{\mu\nu\rho\sigma}$, the traceless component of the Riemann tensor, conveys only (1).

Curvature scalars in a spherically symmetric metric

$$R = \kappa(\rho c^2 - 3P), \qquad \kappa \equiv \frac{8\pi G}{c^4}$$

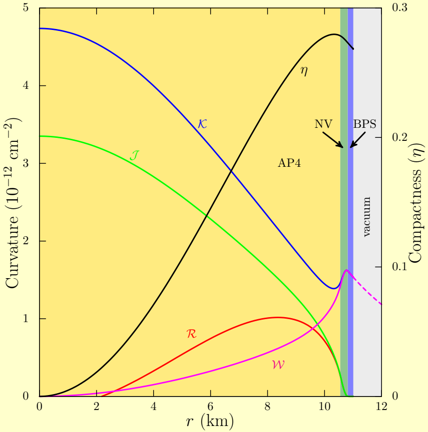
$$J^2 \equiv R_{\mu\nu} R^{\mu\nu} = \kappa^2 \left[(\rho c^2)^2 + 3P^2 \right]$$

$$K^2 \equiv R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}$$

$$= \kappa^2 \left[3(\rho c^2)^2 + 3P^2 + 2P\rho c^2 \right] - \frac{16\kappa G m \rho}{r^3} + \frac{48 G^2 m^2}{r^6 c^4}$$

$$W^2 \equiv C^{\mu\nu\rho\sigma} C_{\mu\nu\rho\sigma} = \frac{4}{3} \left(\frac{6Gm}{c^2 r^3} - \kappa \rho c^2 \right)^2$$

What does an MR measurement constrain?



Ekşi, Güngör, Türkoğlu, 2014, PRD

$$\eta \equiv \frac{2Gm(r)}{rc^2}$$

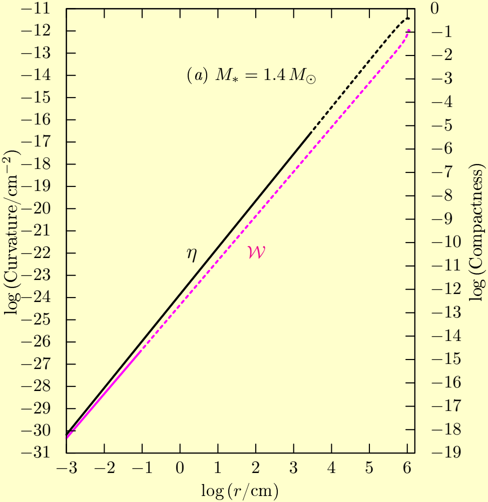
$$R = \kappa(\rho c^2 - 3P)$$

$$J^2 \equiv R_{\mu\nu}R^{\mu\nu}$$

$$K^2 \equiv R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$$

$$W^2 \equiv C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma}$$

An untested gravity regime!



How much can we deviate from GR?

Constrain α in a gravity model

$$S = \frac{c^4}{16\pi G}\int \mathrm{d}^4x\,\sqrt{-g}(R+\alpha R^2)$$

by using neutron stars.

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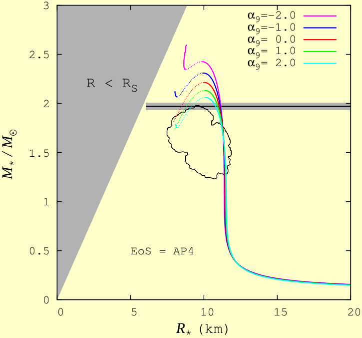
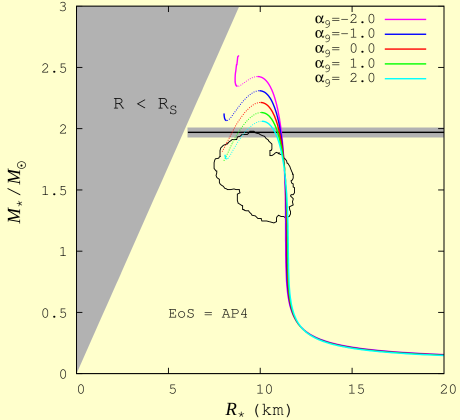
Arapoğlu, Deliduman, KYE (2011)



$$\alpha_9 = \alpha/10^9\,\mathrm{cm}^2$$

$$f(R) = R + \alpha R^2$$



- We have found, in all cases, that

$$|\alpha| < 10^{10}\,\mathrm{cm}^2$$

Arapoğlu, Deliduman, KYE (2011)

- Gravity Probe B data implies

$$\alpha \lesssim 5 \times 10^{15}\,\mathrm{cm}^2$$

Näf & Jetzer 2010

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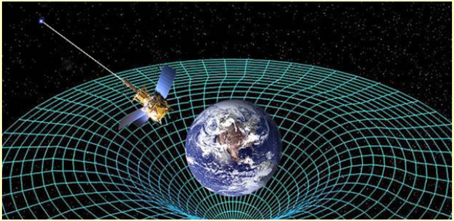

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Neutron stars are really good for constraining gravity even if we do not know the EoS!

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  Näf & Jetzer 2010

Handling the perturbative terms

- $f(R)$ gravities any other than linear in R, i.e. GR, have high order derivatives in the field equations!
- A similar situation exists in fluid mechanics when one introduces viscosity:

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\frac{\boldsymbol{\nabla} p}{\rho} + \nu \nabla^2 \boldsymbol{u}$$

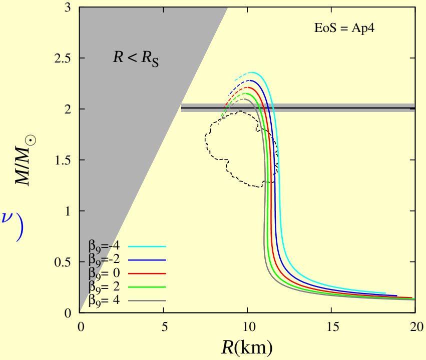
- A small-viscosity flow does not asmptotically converge to a zero-viscosity flow since the former requires two boundary conditions.
- In fluid mechanics such issues are resolved by employing *matched asymptotic expansion* and the concept of *boundary layer*.
- Uniform neutron stars analytically handled with the method of matched asymptotic expansions Arapoğlu, Çıkıntoğlu, KYE (2017)

Yet another gravity model

Constrain β in a gravity model

$$S = \frac{c^4}{16\pi G}\int d^4x\,\sqrt{-g}\,(R+\beta R_{\mu\nu}R^{\mu\nu})$$

Deliduman, KYE, Keleş, 2012



$$\beta_9 = \frac{\beta}{10^9\,\mathrm{cm}^2}$$

Neutron stars in energy-momentum squared gravity

- Add a self-contraction of the energy-momentum tensor to the Einstein-Hilbert action Katırcı & Kavuk 2014: $F(R, T_{\mu\nu}T^{\mu\nu})$
- Structure and $M\&R$-relations of neutron stars in EMSG
- Constraints on α in $F(R, T_{\mu\nu}T^{\mu\nu}) = R + \alpha T_{\mu\nu}T^{\mu\nu}$.

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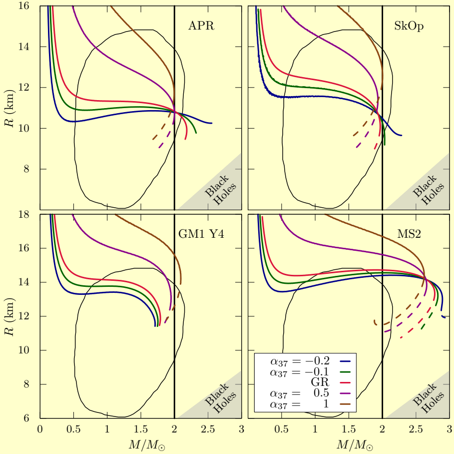
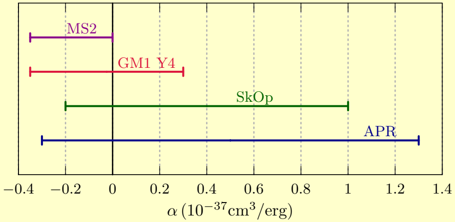


Akarsu, Barrow, Çıkıntoğlu, KYE, Katırcı, 2018

Neutron stars in energy-momentum squared gravity

- Add a self-contraction of the energy-momentum tensor to the Einstein-Hilbert action
  Katırcı & Kavuk 2014: $F(R, T_{\mu\nu}T^{\mu\nu})$
- Structure and $M\&R$-relations of neutron stars in EMSG
- Constraints on α in $F(R, T_{\mu\nu}T^{\mu\nu}) = R + \alpha T_{\mu\nu}T^{\mu\nu}$.



Akarsu, Barrow, Çıkıntoğlu, KYE, Katırcı, 2018

Conclusion

- Gravity in the bulk of a neutron star is in a non-explored regime.
- The gravity regime inside the NS is less constrained than the EoS.
- As one goes deeper inside the NS, the mass contained decreases, and gravity decreases. But the curvature keeps increasing since the size also becomes smaller.
- This strong gravity of neutron stars allow for constraining the parameters of modified theories of gravity even though the EoS is unknown.

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