Thank you for the invitation.

SWAMPLAND CONJECTURES

AND

CYCLIC COSMOLOGY

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REFERENCE

C. Coriano and P.H. Frampton,
Swampland Conjectures and Cosmological Expansion.

arXiv:2010:02939[hep-th].

References are not included in the talk.

1 Introduction

An interesting question is whether string theory is the correct theory of quantum gravity. Popular books often state that the motivation for string theory is to reconcile general relativity with quantum mechanics, especially at microscopic distances. This is not the whole story. String theory was invented originally to describe strong nuclear interactions and was, and remains, quite successful in that goal. String theory is an impressive mathematical framework which has inspired significant progress in pure mathematics.

String theory is over fifty years old and it might be expected that it could be even more decades before there exist relevant experimental or observational data on quantum gravity that could enable an informed response to our initial question: is string theory the correct theory of quantum gravity?

But if the answer to this question is negative, we may be able to decide much sooner. Extraordinarily strong swampland conjectures have been proposed, so strong that preclusion of agreement with existing data is feasible. We could consider either experiments testing particle theory at colliders or astronomical observations which confront theoretical cosmology. In the present article, we favour theoretical cosmology as the better testing ground, particularly the future of the universe. This may seem paradoxical because no observations are possible, but that is our contention.

It is of broad interest to understand whether the present expansion of the universe will last for an infinite time as in the ΛCDM model with constant Λ or whether the present expansion will end at a future finite time, to be followed by a contraction era as in an infinitely cyclic cosmology which can provide a more satisfactory explanation of how time never began at a finite past time. Although the future behaviour is not directly observable, its mathematical description can lead to testable predictions at the present time concerning *e.g.* the equation of state of the dark energy.

This question will be studied in the present talk within the most developed cyclic cosmology invented in 2007 in which the surprisingly accurate estimate according to a calculation by one of us is that the end of the present expansion era will occur at a turnaround time, $t_T = 1.3$Ty, when expansion ends and contraction begins, derived by the matching of expansion and contraction scale factors necessary for a consistent cyclic cosmology to be of infinite duration in both past and future.

Although the swampland conjectures (SCs) concerning string theory were first enunciated in 2005 and 2006 by Ooguri and Vafa the implications of these SCs for cosmology were first carefully considered only in 2018. Implications of the SCs for particle theory have also been discussed but in the present talk we shall focus on predictions of the SCs for cyclic cosmology.

The SCs are tied to the assumption that string theory, in which we include M-theory and F-theory, is the correct theory of quantum gravity and were suggested based on that optimistic assumption. In the present talk, we adopt the SCs as being correct and study what they can tell us about the future of the universe.

It is worth mentioning that the minimal standard model of particle theory with only one scalar doublet violates the SCs and that the standard ΛCDM cosmological model with Λ constant violates the SCs. Thus the SCs are very powerful and do not respect long-held prejudices. This is what makes the SCs so remarkably interesting that they merit further study. An additional scalar field ϕ must be added to accommodate the two very successful theories of particle theory and cosmology.

The swampland conjectures which have been deemed necessary, to ensure that a low-energy effective field theory have an ultra-violet completion within string theory and so belong to the string landscape rather than the very much bigger swampland, are that the scalar field ϕ and its potential $V(\phi)$ satisfy two swampland conjectures, SC1 and SC2 as follows:

SC1: The range traversed by ϕ in field space is bounded by $\Delta \sim O(1)$ in reduced Planck units.

SC2: $\left(\frac{|\nabla_\phi V|}{V}\right) \geq c$ *or* $min(\nabla_i \nabla_j V) \leq -c^{'}$ *with* $c, c^{'} > 0$ *and both* $O(1)$ *in reduced Planck units, and where* $min(\nabla_i \nabla_j V)$ *is the minimum eigenvalue of the Hessian* $\nabla_i \nabla_j V$ *in an orthonormal frame.*

These conjectures are not rigorously proven in string theory but are supported by all string theory examples so far studied assiduously and hence a working hypothesis is to assume that they are necessary as well as sufficient for a successful UV completion. The conjectures ut supra are sometimes called the "range" constraint and the "slope" constraint, respectively, but we shall denote them ut infra simply by SC1 and SC2.

Assuming SC1 and SC2 of string theory is not consistent with the simplest single-field theories of inflation. This may disappoint the inflation community but does lend support for cyclic cosmology in which a cosmic bounce from contraction to expansion could provide an explanation, alternative to inflation, for the flatness and other issues. We now discuss dark energy represented by a quintessence scalar field ϕ.

2 Quintessence

We start from the action

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{Pl}^2R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] \tag{1}$$

in which M_{Pl} *is the Planck mass,* ϕ *is a scalar field and* S_m *(added) is the matter action.*

We assume a FLRW metric with scale factor $a(t)$ *normalised to* $a(t_0) = 1$ *at the present time and Hubble parameter* $H(t) = \dot{a}(t)/a(t)$ *which has the present value* $H(t_0) = H_0$*. We define the density, pressure of dark energy as* ρ_ϕ*,* p_ϕ *so that the dark energy equation of state* ω *is*

$$\omega = \left(\frac{p_\phi}{\rho_\phi}\right) = \left(\frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}\right), \tag{2}$$

since $p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$ *and* $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$*.*

The continuity equation is

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = 0. \qquad (3)$$

Differentiating this and defining $V_{,\phi} = dV/d\phi$ *we find that*

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0. \qquad (4)$$

The two equations of motion arising from the action in Eq.(1) are

$$3M_{Pl}^2 H^2 = \left(\frac{\dot{\phi}^2}{2}\right) + V(\phi) + \rho_m, \qquad (5)$$

$$2M_{Pl}^2 \dot{H} = -\left[\dot{\phi}^2 + (1+\omega_m)\rho_m\right], \qquad (6)$$

where ω_m *is the equation of state* $\omega_m = p_m/\rho_m$ *corresponding to the matter described by* S_m *in the action.*

To calculate the time evolution of the scalar field, we shall find it useful to employ variables x, y *first introduced by Copeland, Liddle and Wands.*

$$x = \left(\frac{\dot{\phi}}{\sqrt{6} M_{Pl} H} \right); \qquad y = \left(\frac{\sqrt{V(\phi)}}{\sqrt{3} M_{Pl} H} \right). \tag{7}$$

The fraction of the critical density contributed by the dark energy is

$$\Omega_\phi = \left(\frac{\rho_\phi}{3 M_{Pl}^2 H^2} \right) = \left(\frac{\frac{1}{2}\dot{\phi}^2 + V(\phi)}{3 M_{Pl}^2 H^2} \right) = x^2 + y^2. \tag{8}$$

The matter density is

$$\Omega_m = \left(\frac{\rho_m}{3 M_{Pl}^2 H^2} \right) = 1 - \Omega_\phi = 1 - x^2 - y^2. \tag{9}$$

Noting that

$$x^2 - y^2 = \left(\frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{3 M_{Pl}^2 H^2} \right) = \left(\frac{p_\phi}{3 M_{Pl}^2 H^2} \right), \tag{10}$$

we find for the equation of state ω *of the dark energy*

$$\omega = \frac{p_\phi}{\rho_\phi} = \left(\frac{x^2 - y^2}{x^2 + y^2}\right). \qquad (11)$$

Eqs.(8) and (11) show how x, y *relate to physical quantities.*

The next step is to calculate how x, y *evolve with cosmic time for which we use, as a convenient variable, the logarithm of the FLRW scale factor* $N = \ln a$*. Considering first* x*, we have from Eq.(7)*

$$\frac{dx}{dN} = \frac{1}{\sqrt{6}M_{Pl}}\left(\frac{1}{H}\frac{d\dot{\phi}}{dN} - \frac{\dot{\phi}}{H^2}\frac{dH}{dN}\right). \qquad (12)$$

In the first term of Eq.(12) we use $d\dot{\phi}/dN = H^{-1}\ddot{\phi}$*, and define*

$$\lambda \equiv -M_{Pl}\frac{V_{,\phi}{}'}{V} \qquad (13)$$

then use Eq.(4) to rewrite this term as $-3x + \lambda y^2\sqrt{6}/2$.

In the second term of Eq.(12), we use $dH/dN = \dot{H}/H$ *and* $\dot{H}/H^2 = 3x^2 + \frac{3}{2}(1+\omega_m)(1-x^2-y^2)$, *to arrive at a simplified expression.*

Let us consider the time evolution of y *defined in Eq.(7)*

$$\frac{dy}{dN} = \frac{1}{3M_{Pl}H^2}\left(H\frac{d}{dN}\sqrt{V(\phi)} - \sqrt{V(\phi)}\frac{dH}{dN}\right). \tag{14}$$

The first term in Eq.(14) is readily shown to equal $-\frac{\sqrt{6}}{2}\lambda xy$, *using Eqs. (7) and(13). In the second term of Eq.(14), we use* $dH/dN = \dot{H}/H$ *to arrive at*

$$\frac{dy}{dN} = \frac{\sqrt{6}}{2}\lambda xy + \frac{3}{2}y\left[(1-\omega_m)x^2 + (1+\omega_m)(1-y^2)\right] \tag{15}$$

If we consider high redshift, observations of structure formation require that $\Omega_\phi = x^2 + y^2 \to 0$ *and therefore in the xy-plane the time evolution locus begins in the past at the origin* $(x, y) = (0, 0)$*. It evolves in time according to coupled equations and requires numerical analysis.*

What concerns us here is the extrapolation of the (x, y) *locus into the future. The crucial point is that, although at the present time* $t = t_0 = 13.8 Gy$ *the swampland conjectures are valid, SC1 will be violated at a finite time* t_T *in the future. We define* E_T *as the number of e-foldings before* $t = t_T$ *i.e.* $(t_T - t_0) = E_T H^{-1}$.

E_T can be estimated as

$$E_T \simeq \left(\frac{3\Delta}{2c\Omega_\phi^0}\right) \tag{16}$$

where Δ and c are $O(1)$ constants appearing in SC1 and SC2 respectively and $\Omega_\phi = 0.7$.

In summary, from Eq. (16), the swampland conjectures predict that

$$E_T = O(1), \tag{17}$$

implying that only a few e-foldings are permitted in the future cosmological expansion before the turnaround to contraction.

3 Cyclic Cosmology

To make this talk self-contained, we include the calculation of $t_T \simeq 1.3Ty$ and, what is the same thing, $E_T \simeq 94$. Let us also briefly review CBE cyclic cosmology introduced in 2007 and pursued in subsequent papers, because assumptions made in the original paper have since been weakened.

CBE (= Comes Back Empty) cosmology is motivated by resolution of the 1931 Tolman no-go theorem which pointed out that the second law of thermodynamics and monotonic increase of entropy appeared at first sight, and at that time even at second sight, contradictory to infinitely cyclic cosmology because each period would be bigger and longer then its predecessor.

To avoid this no-go theorem, entropy must be periodically jettisoned, as is possible only due to the existence of the dark energy unknown to Tolman.

In the CBE model, entropy is jettisoned only at the turnaround from expansion to contraction. At turnaround, after a very long period of superluminal expansion, the universe fragments into a very large number, measured in googols, of causal patches. Almost all of these patches are empty meaning they contain no matter including black holes. The remaining tiny fraction of causal patches, almost none, do contain matter.

Our contracting universe is not one of the causal patches containing matter. Under contraction, black holes would merge and grow. Matter would clump and create structure. Contraction through phase transitions in reverse would violate the second law of thermodynamics. For all these reasons there will inevitably be a premature bounce leading to a failed universe.

By contrast, a patch among the vast majority of patches which is empty can contract successfully. It contracts adiabatically with a time-reverse of the radiation era of expansion and with close to zero entropy, thus explaining why the present expansion began with extremely low entropy.

The new much smaller scale factor $\hat{a}(t_T)$ for the contracting universe at turnaround is related to the scale factor of the previous expansion $a(t_T)$ by

$$\hat{a}(t_T) = f a(t_T), \tag{18}$$

where the coefficient $f << 1$ plays a role in the calculation of t_T, ut infra.

Before doing that calculation, let us mention two assumptions made in the original CBE paper which subsequent work has shown were not necessary.

Firstly, it was assumed that the dark energy equation of state satisfied $\omega < -1$*, so-called phantom dark energy, because the inspiration came from the Big Rip scenario in which time ends at a finite future time. However, this assumption is unnecessary and the CBE idea about cyclic entropy works equally well for* $\omega \geq -1$ *so long as there is lengthy superluminal expansion which will lead to the creation of a very large number of causal patches before the turnaround.*

Secondly, it was assumed that there is inflation near to the beginning of the expansion era. Such inflation is, however, not necessary because the successful predictions of inflation can be reproduced by cyclic contraction to a bounce. For example, flatness is a natural final state in a contracting FLRW metric and other issues like the horizon problem and the scalar index may be accommodated.

It is worth pointing out a subtlety concerning the infinite past of an infinitely cyclic cosmology. There is an ambiguity in the $t \rightarrow -\infty$ limit in that the number of universes either remains infinite or, perhaps surprisingly, can be finite e.g. one. To derive this result requires the use of set theory and transfinite numbers and can be of more interest to mathematicians than to physicists.

Let us begin by studying the present expansion era where important cosmic times are when radiation domination is replaced by matter domination (t_m), when matter domination is replaced by dark energy domination (t_{DE}), the present age (t_0) and the future turnaround time (t_T). For these we use the values

$$\begin{aligned} t_m &= 47ky, \\ t_{DE} &= 9.8Gy, \\ t_0 &= 13.8Gy, \\ t_T &= \text{to be determined, } ut\ infra. \end{aligned}$$

We must distinguish the radii of the introverse (R_{IV}) and extroverse (R_{EV}) which, while they coincide at $t = t_{DE}$,

$$R_{IV}(t_{DE}) = R_{EV}(t_{DE}) = 39Gly, \qquad (19)$$

for all later times satisfy $R_{EV}(t) > R_{IV}(t)$.

For example at $t = t_0$

$$R_{EV}(t_0) = 52Gly; \qquad R_{IV}(t_0) = 44Gly. \tag{20}$$

Taking cubes in Eq.(20), the ratio of EV *to* IV *volumes at present is*

$$\left(\frac{V_{EV}(t_0)}{V_{IV}(t_0)}\right) = \left(\frac{R_{EV}(t_0)^3}{R_{IV}(t_0)^3}\right) = 1.65. \tag{21}$$

Eq.(21) implies that approximately 40% *of the galaxies which were inside the visible universe at* $t = t_m = 9.8Gy$ *have exited and the present visible universe is surrounded by an extroverse containing hundreds of billions of galaxies rendered forever invisible. This is an early precursor of the causal patch separation which will take place at the turnaround time,* $t = t_T$.

Introverse means the same as visible universe, or particle horizon, and its radius $R_{IV}(t)$ *given by*

$$R_{IV}(t) = c\int_0^t \frac{dt}{a(t)}, \tag{22}$$

where $a(t)$ *characterises the expansion history of the universe and is the scale factor in a flat FLRW metric*

$$ds^2 = dt^2 - a(t)^2\left[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]. \tag{23}$$

As we have seen, the present value is $R_{IV}(t_0) = 44Gly$ *but, because of the finite speed of light, it increases relatively slowly to its asymptotic value which is nearly reached already when* $t \sim 50Gy$

$$R_{IV}(t > 50Gy) \simeq 58Gly. \tag{24}$$

The extroverse radius $R_{EV}(t)$ expands exponentially until the turnaround at $t = t_T$ when the number of causal patches can be estimated as

$$N_T = \left(\frac{R_{EV}(t_T)^3}{R_{IV}(t_T)^3} \right) = \frac{1}{f^3}, \qquad (25)$$

where f was defined in Eq.(18). We require N_T to be a very large number which means many googols because there are $\sim 10^{80}$ particles in the present extroverse and we need an overwhelming majority of empty causal patches. This requirement of very large N_T will be verified a posteriori.

To calculate $a(t_T)$, we need to find t_T from matching of the contraction and expansion scale factors, and use the value of $a(t_m = 47ky)$

$$\hat{a}(t_m) = a(t_m) = 2.1 \times 10^{-4}, \qquad (26)$$

at the time $t = t_m$ because the radiation-dominated behaviour for the expansion when $t < t_m$ matches the same behaviour of the entire contraction. The matching in Eq.(26) is necessary for a consistent infinite cyclicity. Provided $R_{IV}(t)$ is asymptotic, to be justified a posteriori, we know that

$$R_{EV}(t_T) = 58Gly, \qquad (27)$$

and therefore, since we use the normalisation $a(t_0) = 1$,

$$\hat{a}(t_T) = f a(t_T) = \left(\frac{R_{IV}(t_T)}{R_{EV}(t_T)}\right) a(t_T) = 1.11, \qquad (28)$$

independent of t_T, provided that $t_T > 50Gy$.

Now we use the time dependence appropriate to contraction of the empty introverse

$$\hat{a}(t) = \hat{a}(t_T) \left(\frac{t}{t_T} \right)^{\frac{1}{2}}, \qquad (29)$$

and the matching condition, Eq.(26), to calculate the turnaround time

$$t_T = \left(\frac{1.11}{2.2 \times 10^{-4}} \right) 47 Gy = 1.3 Ty. \qquad (30)$$

From the result Eq.(30), we can find the number, defined in Eq.(25), of causal patches at turnaround

$$N_T \simeq 2 \times 10^{122}, \qquad (31)$$

which is a very large number, as required. From the result Eq.(30), we can also compute the number E_T of necessary e-foldings between the present time and the turnaround time

$$E_T = H^{-1}(t_T - t_0) \simeq 94. \qquad (32)$$

Based on the swampland conjectures, according to Eq.(17), the number of allowed e-foldings before the SCs become violated is $E_T = O(1)$*, which is in tension with Eq.(32). To understand better this apparent disagreement, we can repeat our calculations for two O(1) values of* E_T*:* $E_T = 2$ *and* $E_T = 4$*. These values lead to smaller values for the turnaround time* $t_T \sim 41 Gy$ *and* $t_T \sim 69 Gy$ *and very much smaller values for the number of causal patches at turnaround* $N_T \sim 6,200$ *and* $N_T \sim 2.3 \times 10^6$*, respectively, numbers which are far too small, see Eq.(31), for cyclic cosmology to be possible. We are employing general relativity at all times except incrementally close to the turnaround and bounce, and do not expect short times with unknown mathematics to change our conclusions.*

4 Discussion

To confirm directly that string theory is the correct theory of quantum gravity is not yet possible because of the absence of relevant experimental or observational data. This situation seems unlikely to change soon, although even the best can underestimate future advances in technology. Early last century Einstein thought experiments to measure classical gravitational microlensing or gravitational waves were forever impracticable. Nevertheless, in 2000 and 2016 respectively, both have been accurately measured.

The swampland conjectures about string theory are extremely interesting because they are so extraordinarily strong in limiting the allowed low-energy effective field theories. Our understanding is that they were conjectured on the basis of studying many string theory models, but are not proven to be necessary. While they may play a role in confirming string theory, they also offer the possibility that it might be refuted. We have discussed the issue of the future of the universe and uncovered an apparent contradiction.

To resolve this contradiction, we may entertain three explanations. One possibility is that, from Eq.(16), to weaken $\Delta \sim O(1)$ in SC1 to $\Delta \sim O(100)$. A second possibility is that cyclic cosmology overestimates the number of e-foldings. A third possibility is that string theory is not the correct theory of quantum gravity. Time will tell which of these possibilities is the truth.

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Thank you for your attention.