Helical magnetic fields from Riemann coupling

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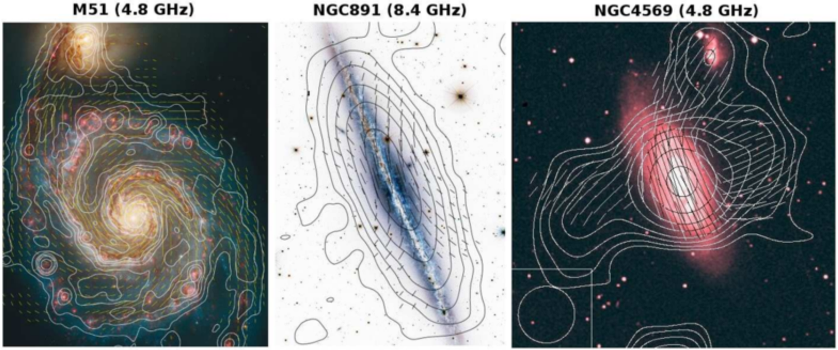
Observations of magnetic fields in universe



Picture source: Max Planck Institute for Radio Astronomy

Micro-Gauss strength magnetic field over $10kpc$ coherence length scale is present in galaxies.

$1pc = 2.1 \times 10^5\text{AU} = 3.1 \times 10^{16}\text{m}$

Two kinds of fields

- Electromagnetic field has two transverse degrees of freedom which can be associated with Left circular and right circular polarization.

- For massless particle helicity is the projection of the direction of spin (clockwise or anti-clockwise) along the direction of propagation. Hence giving $+1, -1$ **for right handed and left handed helicity modes.**

- Same propagation (**speed or dispersion relation**) of both polarization modes lead to non-helical, and differently propagating modes lead to helical fields.

- If both the polarization modes propagate differently $\rightarrow$ **Helicity Imbalance**

How to create helicity imbalance?

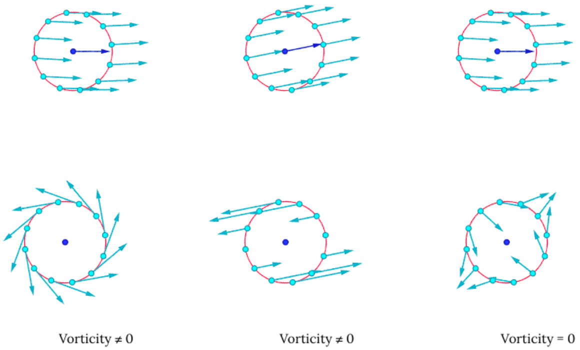
Helical magnetic fields

- Lorentz force, $\overrightarrow{F} = m\frac{d\overrightarrow{v}}{dt} = \overrightarrow{E} + \overrightarrow{v} \times \overrightarrow{B}$ implies that under parity transformation (changing the sign of coordinate system): $\overrightarrow{E} \longrightarrow -\overrightarrow{E}, \overrightarrow{B} \longrightarrow \overrightarrow{B}$.
- Because standard EM action, $F_{\mu\nu}F^{\mu\nu} \propto B^2 - E^2$, is quadratic in $\overrightarrow{E}$ and $\overrightarrow{B}$, it is invariant under parity symmetry.
- $F_{\mu\nu}\tilde{F}^{\mu\nu} = -4\overrightarrow{E} \cdot \overrightarrow{B}$ is parity non-invariant, where $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$.

Hence $F_{\mu\nu}\tilde{F}^{\mu\nu}$ can create the **Helicity imbalance**.

Vorticity

Vorticity is defined as $\overrightarrow{\Omega} = \overrightarrow{\nabla} \times \overrightarrow{v}$, where $\overrightarrow{v}$ is velocity field.

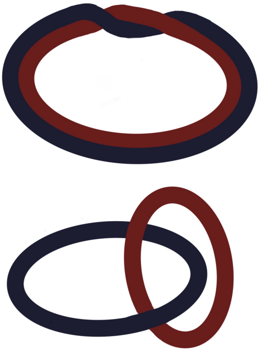


Source: Wikipedia

Magnetic helicity

- Magnetic helicity ($\mathcal{H}_M$) is defined as: $\int d^3x \vec{A} \cdot \vec{B}$ and $\vec{B} \cdot \vec{\nabla} \times \vec{B}$.
- It is a measure of twist and linkage of magnetic field lines.

Grasso and Rubinstein (2001), Blackman (2014)



$$\mathcal{H}_M = \int d^3x \vec{A} \cdot \vec{B} = 2 V_1 \cdot V_2$$

Why helical magnetic fields are interesting?

- Helical magnetic fields leave a very distinct signature as they **violate parity symmetry** which leads to observable effects, e.g. correlations between the anisotropies in the temperature and B-polarisation or in the E- and the B-polarisations in the CMB. Kahniashvili (2006)

- One of the interests in primordial magnetic helicity is that it can be a direct indication of parity violation (CP violation) in the early Universe. Vachaspati (2001)

How to generate magnetic fields ?

Problem with magnetic field generation during inflation

- EM action for an arbitrary 4-D metric

$$S_{em} = -\frac{1}{4}\int d^4x\sqrt{-g}g^{\alpha\mu}g^{\beta\nu}F_{\alpha\beta}F_{\mu\nu}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and A^μ is electromagnetic four vector.

- Under conformal transformation $\tilde{g}_{\mu\nu} = \omega^2(x)g_{\mu\nu}$

$$\tilde{S}_{em} = -\frac{1}{4}\int d^4x\sqrt{-g}\tilde{g}^{\alpha\mu}\tilde{g}^{\beta\nu}F_{\alpha\beta}F_{\mu\nu} = S_{em}$$

- EM action is conformally invariant.
- Because Flat FRW metric is conformally equivalent to Minikowski spacetime, $B \sim \frac{1}{a^2}$.

We need to break the conformal invariance of EM action !

(Helical fields) Models in the literature

Scalar field coupled models: $f(\phi)F_{\mu\nu}\tilde{F}^{\mu\nu}$

Durrer et al.(2011), Sharma et al.(2018)

where $f(\phi)$ is time-dependent coupling function.

- **Problems with these models :**
  - **Strong coupling** - Coupling between charged particles and the EM field is so strong that theory can not be treated perturbatively.
  - **Back-reaction** - Overproduction of gauge fields affect the background inflationary dynamics
  - Because magnetic fields are produced near the end of inflation, strength of the **fields generated depends on the reheating scale.**

To resolve strong coupling and back-reaction problem $f(\phi)$ is assumed to increase during inflation and decrease back to its initial value post inflation.
Sharma et al.(2018)

Helical magnetic fields from Riemann coupling

Motivation

- Non-minimal coupling to the Riemann tensor generates sufficient primordial helical magnetic fields at **all observable scales.**
- One of the helical states decay while the other helical mode increases, leading to a net non-zero helicity $\longrightarrow$ helicity imbalance
- **Necessary condition :** Conformal invariance breaking + parity violation

$$S = \overbrace{-\frac{M_{\mathrm{P}}^2}{2}\int d^4x\sqrt{-g}\,R}^{\text{Einstein-Hilbert term}} + \overbrace{\int d^4x\sqrt{-g}\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]}^{\text{Scalar field}}$$
$$-\frac{1}{4}\int d^4x\,\sqrt{-g}\,F_{\mu\nu}F^{\mu\nu} - \underbrace{\frac{\sigma}{M^2}\int d^4x\,\sqrt{-g}\,\tilde{R}^{\mu\nu\alpha\beta}F_{\alpha\beta}\,F_{\mu\nu}}_{\text{Conformal breaking}} \qquad (1)$$

where $\tilde{R}^{\mu\nu\alpha\beta} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}R_{\rho\sigma}{}^{\alpha\beta}$ and M is the energy scale, which sets the scale for the breaking of conformal invariance.

details

Electromagnetic energy density

To identify whether these modes lead to back-reaction on the metric, we define R, which is the ratio of the total energy density of the fluctuations and background energy density during inflation: Talebian et al.(2020)

$$R = \frac{(\rho_B + \rho_E)|_{k_* \sim \mathcal{H}}}{6 M_P^2 H^2} \tag{2}$$

α	ρ (in GeV^4)	R
$-\frac{1}{2} - \epsilon$	$\sim 10^{64}$	$\sim 10^{-4}$
$-\frac{3}{4}$	$\sim 10^{62}$	$\sim 10^{-6}$
-1	$\sim 10^{61}$	$\sim 10^{-7}$
-3	$\sim 10^{59}$	$\sim 10^{-9}$

No back-reaction on the background metric.

Estimating the strength of helical magnetic fields

- Assuming instantaneous reheating, and the Universe becomes radiation dominated after inflation. Due to flux conservation, the magnetic energy density will decay as $1/a^4$: Subramanian (2016)

- Using the fact that the relevant modes exited Hubble radius around 30 e-foldings of inflation, with energy density $\rho_B \approx 10^{64}\mathrm{GeV}^4$, the primordial helical fields at GPc scales is:

$$B_0 \approx 10^{-20}\mathrm{G} \tag{3}$$

- Helical magnetic fields that re-entered the horizon at two different epochs:

$$B|_{50\ \mathrm{MPc}} \sim 10^{-18}\ G\ (z \sim 20)\ ;\ \ B|_{1\ \mathrm{MPc}} \sim 10^{-14}\ G\ (z \sim 1000)$$

Conclusion and Future work...

- Our model does not require the coupling of the electromagnetic field with the scalar field. Hence, there are no extra degrees of freedom and will not lead to a strong-coupling problem.
- Since the curvature is large in the early Universe, the coupling term will introduce non-trivial corrections to the electromagnetic action.
- Power spectrum of the helical fields generated has a slight red-tilt for slow-roll inflation which is different compared to the scalar field coupled models where the power-spectrum has a blue-tilt.
- Currently we are looking at the effect of this helical field on baryon asymmetry during the early universe.

Thank you

Backup slides

Conformal transformation

$$\tilde{g}_{\mu\nu} = \omega^2(x) g_{\mu\nu} \quad \Longrightarrow \quad \tilde{\Gamma}^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu} + C^{\lambda}_{\mu\nu} \tag{4}$$

where $C^{\lambda}_{\mu\nu} = \omega^{-1} \left(\delta^{\lambda}_{\mu} \nabla_{\nu}\omega + \delta^{\lambda}_{\nu} \nabla_{\mu}\omega - g_{\mu\nu} g^{\rho\lambda} \nabla_{\rho}\omega \right)$

$$F_{\mu\nu} = \nabla_{\mu} A_{\nu} - \nabla_{\nu} A_{\mu} = \partial_{\mu} A_{\nu} - \Gamma^{\lambda}_{\mu\nu} A_{\lambda} - \partial_{\nu} A_{\mu} + \Gamma^{\lambda}_{\nu\mu} A_{\lambda} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} \tag{5}$$

$$\tilde{R}^{\lambda}_{\sigma\mu\nu} = R^{\lambda}_{\sigma\mu\nu} + \nabla_{\mu} C^{\lambda}_{\nu\sigma} - \nabla_{\nu} C^{\lambda}_{\mu\sigma} + C^{\lambda}_{\mu\rho} C^{\rho}_{\nu\sigma} - C^{\lambda}_{\nu\rho} C^{\rho}_{\mu\sigma} \tag{6}$$

$$\begin{aligned} \tilde{R}_{\mu\nu} &= R_{\mu\nu} - [2\delta^{\alpha}_{\mu}\delta^{\beta}_{\nu} + g_{\mu\nu} g^{\alpha\beta}] \omega^{-1} (\nabla_{\alpha}\nabla_{\beta}\omega) \\ &\quad + [4\delta^{\alpha}_{\mu}\delta^{\beta}_{\nu} - g_{\mu\nu} g^{\alpha\beta}] \omega^{-2} (\nabla_{\alpha}\omega)(\nabla_{\beta}\omega) \end{aligned} \tag{7}$$

$$\tilde{R} = \omega^{-2} R - -6 g^{\alpha\beta} \omega^{-3} (\nabla_{\alpha}\nabla_{\beta}\omega) \tag{8}$$

$$\tilde{\nabla}_{\mu}\tilde{\nabla}_{\nu}\phi = \nabla_{\mu}\nabla_{\nu}\phi - (\ \delta^{\alpha}_{\mu}\ \delta^{\beta}_{\nu}\ +\ \delta^{\beta}_{\mu}\ \delta^{\alpha}_{\nu}\ )\ \omega^{-1}\ (\nabla_{\alpha}\omega)(\nabla_{\beta}\omega) \tag{9}$$

Back

Energy densities

Gauge field decomposition:

$$A^i(\vec{x},\eta) = \int \frac{d^3k}{(2\pi)^3} \sum_{\lambda=1,2} \varepsilon^i_\lambda \left[A_\lambda(k,\eta) b_\lambda(\vec{k}) e^{ik\cdot x} + A^*_\lambda(k,\eta) b^\dagger_\lambda(\vec{k}) e^{-ik\cdot x}\right] \tag{10}$$

The EM energy densities with respect to the comoving observer are:

$$\rho_B(\eta,k) \equiv -\frac{1}{2}\langle 0|B_\mu B^\mu|0\rangle = \int \frac{dk}{k} \frac{1}{(2\pi)^2} \frac{k^5}{a^4} \left(|A_+(\eta,k)|^2 + |A_-(\eta,k)\ |^2 \right) \tag{11}$$

$$\rho_E(\eta,k) \equiv -\frac{1}{2}\langle 0|E_\mu E^\mu|0\rangle = \int \frac{dk}{k} \frac{1}{(2\pi)^2} \frac{k^3}{a^4} \left(|A'_+(\eta,k)|^2 + |A'_-(\eta,k)|^2 \right) \tag{12}$$

$$\rho_h(\eta,k) \equiv -\langle 0|A_\mu B^\nu|0\rangle = \int \frac{dk}{k} \frac{1}{2\pi^2} \frac{k^4}{a^3} \left(|A_+(\eta,k)|^2 - |A_-(\eta,k)|^2 \right). \tag{13}$$

where spectral energy density is given by $\frac{d\rho_\Upsilon}{d\ln k}$ for $\Upsilon \in (B, E, h)$

Evolution equation

In Flat FRW universe : $ds^2 = a^2(\eta)\left(d\eta^2 - \delta_{ij}dx^i dx^j\right)$. In the Coulomb gauge ($A^0 = 0, \partial_i A^i = 0$), equation of motion is

$$A_i'' + \frac{4\,\epsilon_{ijl}}{M^2}\left(\frac{a'''}{a^3} - 3\frac{a''a'}{a^4}\right)\partial_j A_l - \partial_j\partial_j A_i = 0 \tag{14}$$

Which in helicity basis can be written as:

$$A_h'' + \left[k^2 - \frac{4kh}{M^2}\,\Gamma(\eta)\right]A_h = 0 \tag{15}$$

where,

$$\Gamma(\eta) = \frac{a'''}{a^3} - 3\frac{a''a'}{a^4} = \frac{1}{a^2}\left(\mathcal{H}'' - 2\mathcal{H}^3\right) \tag{16}$$

which vanishes for de-sitter case. Back

Helical magnetic field generation

- For power law inflation: $a(\eta) = \left(-\frac{\eta}{\eta_0}\right)^{\beta+1}$, de-sitter $\beta = -2$, we have

$$A_h'' + \left[k^2 - \frac{8kh}{M^2}\frac{\beta(\beta+1)(\beta+2)}{\eta_0^3}\left(\frac{-\eta_0}{\eta}\right)^{(2\beta+5)}\right]A_h = 0 \qquad (17)$$

- Sub-horizon mode $|-k\eta| >> 1$ solution is: $A_h = \frac{1}{\sqrt{k}}e^{-ik\eta}$
- For super-horizon mode $|-k\eta| << 1$, with dimensionless variable, $\tau = \left(-\frac{\eta_0}{\eta}\right)^{\alpha}$ and $\alpha = \beta + \frac{3}{2}$

$$A_+(\tau,k) = \tau^{-\frac{1}{2\alpha}}\, J_{\frac{1}{2\alpha}}\left(\frac{\varsigma\sqrt{k}}{\alpha}\tau\right)C_1 + \tau^{-\frac{1}{2\alpha}}\, Y_{\frac{1}{2\alpha}}\left(\frac{\varsigma\sqrt{k}}{\alpha}\tau\right)C_2 \quad (18a)$$

$$A_-(\tau,k) = \tau^{-\frac{1}{2\alpha}}\, J_{\frac{1}{2\alpha}}\left(-i\frac{\varsigma\sqrt{k}}{\alpha}\tau\right)C_3 + \tau^{-\frac{1}{2\alpha}}\, Y_{\frac{1}{2\alpha}}\left(-i\frac{\varsigma\sqrt{k}}{\alpha}\tau\right)C_4 \quad (18b)$$

Taking $\mathcal{H} \sim {\eta_0}^{-1} \sim 10^{14}\mathrm{GeV}$, and $M \sim 10^{17}\mathrm{GeV}$ gives

$$|C_1| \approx |C_3| \approx 10^{-17/2}\mathrm{GeV}^{-\frac{1}{2}}, \quad \text{and} \quad |\mathrm{C}_2| \approx |\mathrm{C}_4| \approx 10^{-11/2}\mathrm{GeV}^{-\frac{1}{2}}. \tag{19}$$

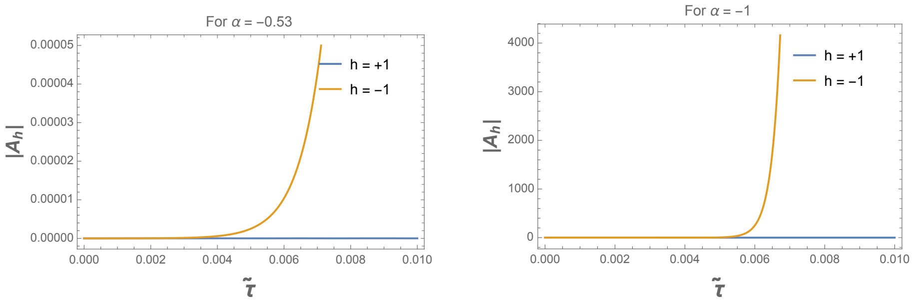


Figure: Figure showing the behaviour of positive and negative helicity mode for $\alpha = -0.53$ and $\alpha = -1$. $\tilde{\tau} = 10^{-\frac{63}{2}}\,\tau$ and the vertical axis is in $\mathrm{GeV}^{-1/2}$.

We can ignore the negative helicity mode.

Using the fact that we can approximate the super-horizon modes by power law, we have

$$A_+(\tau, k) = C\, k^{\frac{1}{4\alpha}} - C_2 \frac{\mathcal{F}^{-1}}{\pi} \Gamma\left(\frac{1}{2\alpha}\right) k^{-\frac{1}{4\alpha}} \tau^{-\frac{1}{\alpha}} \tag{20}$$

where

$$\mathcal{F}(\tau) = F(\tau) \left(\frac{\varsigma}{2\alpha}\right)^{\frac{1}{2\alpha}}, \tag{21}$$

$$C(\tau) = F(\tau) \left(\frac{\varsigma}{2\alpha}\right)^{\frac{1}{2\alpha}} \left[\frac{C_1}{\Gamma\left(1+\frac{1}{2\alpha}\right)} - \frac{C_2}{\pi} \Gamma\left(-\frac{1}{2\alpha}\right) \cos\left(\frac{\pi}{2\alpha}\right)\right], \tag{22}$$

and the approximate values are
$|\mathcal{F}| \sim 10^{-\frac{5}{\alpha}}\ \mathrm{GeV}^{-1/4\alpha}, |\mathrm{C}| \sim 10^{-\frac{5}{\alpha}-\frac{11}{2}} \mathrm{GeV}^{-\frac{1}{4\alpha}-\frac{1}{2}}$.