Bouncing cosmology in a curved braneworld

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Beyond Standard Model:
From Theory to Experiment (BSM-2021)

Reference: IB, Tanmoy Paul & Soumitra SenGupta,
JCAP, 02,041 (2021)

Motivation

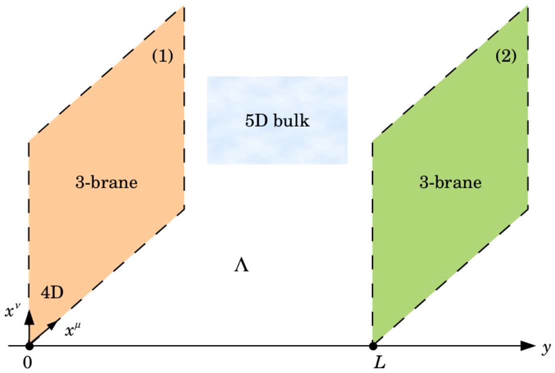
- Bouncing scenario is interesting because:
  - **It can generate an almost scale invariant power spectrum and hence confront the observational constraints.**
  - **Can give rise to a singularity free evolution of the early universe.**
- The bouncing scenario consists of two eras — an era of contraction and an era of expansion of the scale factor, both the eras being connected by a non-singular bounce
- In this talk, we will see the role of radion in inducing a bouncing universe.
- The scalar field radion arises in higher dimensional models—the warped geometry model proposed by Randall & Sundrum (RS)
- The RS model was proposed to resolve the gauge-hierarchy problem arising due to large radiative corrections to the Higgs mass

Plan of the talk

- Description of the non-flat warped braneworld scenario: Emergence of the radion field
- Radion Cosmology: Background evolution
- Radion Cosmology: Evolution of perturbations

EXTRA-DIMENSIONAL MODEL: THE NON-FLAT WARPED BRANEWORLD SCENARIO

RANDALL SUNDRUM MODEL



- Has two 3-branes extending in the x^{μ} directions, located at the fixed points y = 0 (hidden brane), π (visible brane) (y represents the extra space coordinate)
- RS model consists of a spacetime with a single S_1/Z_2 orbifold extra dimension $\Longrightarrow$ The points (x, y) and $(x, -y)$ are identified.

Randall & Sundrum (1999)

Randall-Sundrum model: The Original Setup

- The classical action is:

$$S = S_{gravity} + S_{vis} + S_{hid} \tag{1}$$

$$S_{gravity} = \int d^4x \int_{-\pi}^{\pi} dy \sqrt{-G}\{2M^3\mathbf{R} - \Lambda\} \tag{2}$$

$$S_{vis} = \int d^4x \sqrt{-g_{vis}}\{\mathcal{L}_{vis} - V_{vis}\}; \quad g^{vis}_{\mu\nu}(x^\mu) \equiv G_{\mu\nu}(x^\mu, y = \pi) \tag{3}$$

$$S_{hid} = \int d^4x \sqrt{-g_{hid}}\{\mathcal{L}_{hid} - V_{hid}\}; \quad g^{hid}_{\mu\nu}(x^\mu) \equiv G_{\mu\nu}(x^\mu, y = 0) \tag{4}$$

- 5-d Einstein's equations for the above action,

$$\sqrt{-G}(R_{MN} - \frac{1}{2}RG_{MN}) = -\frac{1}{4M^3}\Big[\Lambda\sqrt{-G}G_{MN} + V_{vis}\sqrt{-g_{vis}}g^{vis}_{\mu\nu}\delta^\mu_M\delta^\nu_N\delta(y-\pi) + V_{hid}\sqrt{-g_{hid}}g^{hid}_{\mu\nu}\delta^\mu_M\delta^\nu_N\delta(y)\Big] \tag{5}$$

- The metric ansatz

$$\mathbf{ds^2} = \mathbf{e}^{-\mathbf{2A}}\eta_{\mu\nu}\mathbf{dx}^\mu\mathbf{dx}^\nu + \mathbf{r_c^2dy^2} = \mathbf{e}^{-\mathbf{2}\sigma(\mathbf{y})}\eta_{\mu\nu}\mathbf{dx}^\mu\mathbf{dx}^\nu + \mathbf{r_c^2dy^2} \tag{6}$$

Warped geometry and Randall-Sundrum model

- Solving the Einstein's equations with the metric ansatz gives,

$$\sigma(y) = -r_c|y|\sqrt{\frac{-\Lambda}{24M^3}} = -r_c|y|k_0 \tag{1}$$

- The brane tensions are given by,

$$V_{vis} = -V_{hid} = -24M^3 k_0 \tag{2}$$

- The solution of the bulk metric is thus,

$$\mathbf{ds^2 = e^{-2kr_c|y|}}\eta_{\mu\nu}\mathbf{dx^{\mu}dx^{\nu} + r_c^2 dy^2} \tag{3}$$

- Any mass parameter v_0 in the fundamental higher-dimensional theory will correspond to a physical mass v on the visible 3-brane

$$v = v_0 e^{-k_0 r_c \pi} \tag{4}$$

- If $kr_c \sim 12$, the weak scale is generated on the "visible" brane from the Planck scale.

Randall & Sundrum (1999)

Success and deficiencies of the Randall Sundrum model

Success

- The RS two-brane model is particularly successful in resolving the fine tuning problem without bringing in any arbitrary intermediate scale between the Planck and the TeV scale.
- Masses of the graviton KK excitations $m_n = kx_n e^{-kr_c\pi}$ of TeV scale and couplings to Standard Model particles TeV suppressed $\rightarrow$ detectable in present day collider experiments.

Deficiencies

- In the RS model the 3-brane was taken to be flat $\Rightarrow$ cosmological constant induced on the visible brane is zero.
- But we know our universe has a very small cosmological constant that explains the acceleration of the universe.

Can we improve upon this situation?

Warped geometry models with non-flat branes

- We consider generalized RS set-up, (a) curved 3-branes and (b) treat the distance between the two branes as a 4-d field, the so called radion field or the modular field, $T(x)$.
- The action is given by:

$$S = S_{gravity} + S_{vis} + S_{hid} \tag{1}$$

where,

$$S_{gravity} = \int_{-\infty}^{\infty} d^4x \int_{-\pi}^{\pi} dy \sqrt{-G}(2M^3\mathbf{R} - \Lambda) \tag{2}$$

$$S_{vis} = \int_{-\infty}^{\infty} d^4x \sqrt{-g_{vis}}(L_{vis} - V_{vis}) \tag{3}$$

$$S_{hid} = \int_{-\infty}^{\infty} d^4x \sqrt{-g_{hid}}(L_{hid} - V_{hid}) \tag{4}$$

- The metric ansatz we consider is the following:

$$\mathbf{ds^2 = e^{-A(x_\mu, y)} g_{\mu\nu} dx^\mu dx^\nu + T(x)^2 dy^2} \tag{5}$$

$$e^{-A} = \omega \sinh\left(\ln \frac{c_2}{\omega} - kT(x)|y|\right) \qquad \omega = \frac{\Omega}{3k^2} \tag{6}$$

Effective action in de-Sitter 3-branes

- The 4-d effective action can be separated into 3 parts,

$$^{(4)}\mathcal{A}_{\rm tot} = \ ^{(4)}\mathcal{A}_{\rm curv} + \ ^{(4)}\mathcal{A}_{\rm kinetic} + \ ^{(4)}\mathcal{A}_{\rm pot} \tag{1}$$

where,

$$^{(4)}\mathcal{A}_{\rm curv} = 2M^3 \int d^4x \ \sqrt{-g} \ \hat{R} \ \left\{ \frac{c_2^2}{4k} + \frac{\omega^2}{k} \ln\left(\frac{\Phi}{f}\right) + \frac{\omega^4}{4kc_2^2}\left(\frac{f^2}{\Phi^2}\right) - \frac{\omega^4}{4kc_2^2} - \frac{c_2^2}{4k}\left(\frac{\Phi^2}{f^2}\right)\right\} \tag{2}$$

$$^{(4)}\mathcal{A}_{\rm kinetic} = \int d^4x \ \sqrt{-g} \ \left(-\frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi\right)\left\{1 + 8\frac{M^3}{k}\omega^2\left(\frac{1}{\Phi^2}\ln\frac{\Phi}{f}\right) - \frac{6M^3}{k}\frac{\omega^4}{c_2^2}\left(\frac{f^2}{\Phi^4}\right)\right\} \tag{3}$$

$$^{(4)}\mathcal{A}_{\rm pot} = \int d^4x \ \sqrt{-g}\hat{V}(\Phi/f) \tag{4}$$

$$\hat{V}\left(\frac{\Phi}{f}\right) = 6\omega^4 \ln\left(\frac{\Phi}{f}\right) - \frac{3}{2}\omega^2 c_2^2\left(\frac{\Phi^2}{f^2}\right) + \frac{3}{2}\omega^2 c_2^2 + \frac{3}{2}\frac{\omega^6}{c_2^2}\left(\frac{f^2}{\Phi^2}\right) - \frac{3}{2}\frac{\omega^6}{c_2^2} \tag{5}$$

where, $\Phi = fe^{-kT(x)\pi}$; $f = \sqrt{\frac{6M^3c_2^2}{k}}$; $c_2 = 1 + \sqrt{1+\omega^2}$

IB & S. SenGupta, EPJC, 77, 277 (2017);

IB, S. Chakraborty & S. SenGupta, PRD, 02, 023515 (2019)

Effective action in the Einstein frame

- The radion potential in the Einstein frame is given by,

$$V(\xi) = \frac{\hat{V}(\xi)}{h(\xi)^2} = \frac{6\omega^2}{h(\xi)} \qquad \xi \equiv \frac{\Phi}{f} \tag{1}$$

where the non-minimal coupling term is,

$$h(\xi) = \left\{ \frac{c_2^2}{4} + \omega^2 \ln(\xi) + \frac{\omega^4}{4c_2^2}\left(\frac{1}{\xi^2}\right) - \frac{\omega^4}{4c_2^2} - \frac{c_2^2}{4}\xi^2 \right\} = \frac{1}{6\omega^2}\hat{V}(\xi) \tag{2}$$

- The complete 4-d effective action in the Einstein frame is given by,

$$^{(4)}\mathcal{A}_{\text{tot}}^{\text{E}} = \int d^4x \sqrt{-\hat{g}} \left[\underbrace{\frac{2M^3}{k_0} R}_{L_{\text{curv}}} \underbrace{- \frac{1}{2} G(\Phi/f) \partial^\mu \Phi \partial_\mu \Phi}_{L_{\text{kinetic}}} \underbrace{- 2M^3 k_0 V(\Phi/f)}_{L_{\text{pot}}} \right] \tag{3}$$

where,

$$G(\xi) = \frac{\hat{G}(\xi)}{h(\xi)} + \frac{1}{c_2^2}\left[\frac{h'(\xi)}{h(\xi)}\right]^2 \tag{4}$$

$$\hat{G}(\xi) = 1 + \frac{4}{3}\frac{\omega^2}{c_2^2}\left(\frac{1}{\xi^2}\right)\ln(\xi) - \frac{\omega^4}{c_2^4}\left(\frac{1}{\xi^4}\right) \tag{5}$$

Interesting consequences related to the form of the warp factor

- Recall, $\Phi \equiv f \exp\{-kT(x)\pi\}$ where $f = \sqrt{6M^3 c_2^2/k}$ (order of Planck mass).
- Since $T(x)$ represents the distance between the two branes, it cannot be negative and hence $0 \lesssim (\Phi/f) \lesssim 1$.
- The warp factor can be written as,

$$\begin{aligned} e^{-A} &= \frac{\omega}{2}\left\{\exp\left[\left(\ln\frac{c_2}{\omega} - kT(x)|\phi|\right)\right] - \exp\left[-\left(\ln\frac{c_2}{\omega} - kT(x)|\phi|\right)\right]\right\} \\ &= \frac{c_2}{2}\exp(-kT(x)|\phi|) - \frac{\omega^2}{2c_2}\exp(kT(x)|\phi|) \end{aligned} \tag{1}$$

where, $c_2 = 1 + \sqrt{1+\omega^2}$.

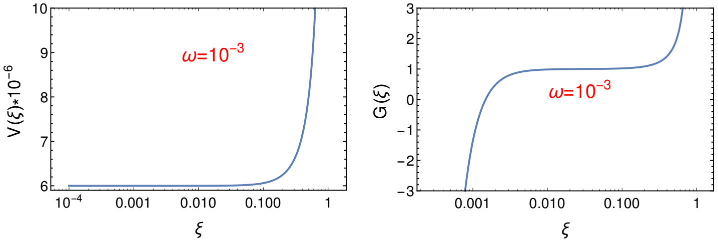
- The warp factor has to be positive on the visible brane, i.e., $\phi = \pi$,

$$\begin{aligned} & e^{-A} > 0 \\ & \Longrightarrow (\Phi/f) = \exp\{-kT(x)\pi\} > (\omega/c_2) \end{aligned} \tag{2}$$

- This implies $\frac{\omega}{c_2} \lesssim (\Phi/f) \lesssim 1$ is physically allowed region.

General properties of V and G

- The potential V has an inflection point at $\xi_i = \Phi_i/f = \omega/c_2$, where $c_2 = 1 + \sqrt{1+\omega^2}$, implies $\xi_i = \Phi_i/f < 1$.
- G becomes negative to positive at $\xi_f = \Phi_f/f > \xi_i$ where $\xi_f = \Phi_f/f \sim \omega$.



(a) (b)

Figure: The above figure depicts the variation of (a) the radion potential V and (b) the non-canonical coupling to the kinetic term G in the Eintein frame, within the allowed range of the radion field ξ for $\omega = 10^{-3}$.

- For any ω, $\xi_i < \xi_f$, e.g., for $\omega = 10^{-3}$ the value of $\xi_i = 5 \times 10^{-4}$ whereas $\xi_f = 1.483 \times 10^{-3}$, ξ exhibits phantom – like behavior when $\xi_i \leq \xi \leq \xi_f$

EARLY UNIVERSE COSMOLOGY WITH RADION: BACKGROUND EVOLUTION

Background evolution with radion

- We study early universe cosmology with the radion field.
- We consider the Einstein frame metric to be described by the FRW spacetime,

$$ds^2 = dt^2 - a(t)^2 \left[dx^2 + dy^2 + dz^2\right] \tag{1}$$

- The Friedmann equations are given by,

$$H^2 = \frac{\kappa^2}{3}\rho(t) = \frac{c_2^2}{4}G(\xi)\dot{\xi}^2 + \frac{k_0^2}{6}V(\xi); \qquad 2\kappa^2 = 16\pi G_N = \frac{k_0}{2M^3} \tag{2}$$

$$\dot{H} = -\frac{\kappa^2}{2}(\rho + p) = -\frac{3}{4}c_2^2 G(\xi)\dot{\xi}^2 \tag{3}$$

- The equation of motion for the radion field is given by,

$$\ddot{\xi} + 3H\dot{\xi} + \frac{G'(\xi)}{2G(\xi)}\dot{\xi}^2 + \frac{k_0^2}{3c_2^2}\frac{V'(\xi)}{G(\xi)} = 0 \tag{4}$$

- We solve these equations to get background evolution of $H(t)$ and $\xi(t)$.

Evolution equations for radion and Hubble parameter near bounce

- The model can show a bounce phenomena when $G(\xi) < 0$.
- Note: $\xi_i, \ \xi_f \sim \omega$, we analytically solve the background equations with $\xi \sim \omega$ near the bounce $(t = 0)$.
- We consider,

$$\xi(t) = \frac{\omega}{c_2}[1 + \delta(t)] \qquad \delta(t) \ll 1 \tag{5}$$

- Using 5 the evolution equations for the Hubble parameter $H(t)$ and the radion field $\xi(t)$ turn out to be,

$$\dot{H} + 3H^2 - \frac{12k_0^2\omega^2}{c_2^2} = 0 \tag{6}$$

and

$$\dot{\delta}^2 = \frac{c_2^2}{\omega^2}\frac{\dot{H}}{4ln\left(\frac{c_2}{\omega}\right)}\left[1 + \delta\left\{\frac{4 + 2ln\left(\frac{c_2}{\omega}\right)}{ln\left(\frac{c_2}{\omega}\right)}\right\}\right] \tag{7}$$

Analytic solution of $H(t)$ and $\xi(t)$ near bounce

- The background solution for $H(t)$ and $\delta(t)$ in the regime $\xi(t) = \frac{\omega}{c_2}(1+\delta(t))$ with $\delta(t) \ll 1$,

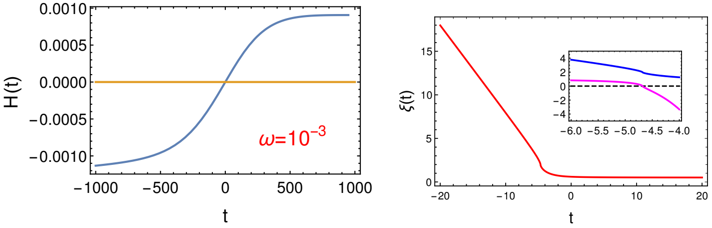
$$\xi(t) = \frac{\omega}{c_2}(1+\delta(t)), \begin{cases} H(t) = 2k_0 \frac{\omega}{c_2} tanh\left[6\frac{\omega}{c_2}k_0 t\right] \\ \delta(t) = \frac{2}{A}\left[exp\left\{-\frac{A}{6}\frac{\omega}{c_2}\sqrt{\frac{3}{ln\left(\frac{c_2}{\omega}\right)}}\left(tan^{-1}tanh\left(\frac{3\omega}{c_2}k_0 t\right)-\frac{\pi}{4}\right)\right\}-1\right], \end{cases} \quad (8)$$

where, $A = \frac{4+2ln\left(\frac{c_2}{\omega}\right)}{ln\left(\frac{c_2}{\omega}\right)}$. We use $\lim_{t\to\infty} \delta(t) \to 0$ to arrive at above result.

- 8 clearly indicates $H(0) = 0$ and $\dot{H} > 0$ at $t = 0$ (corresponding to the bounce time)
- The present model generically predits a bouncing universe in the visible brane when the radion field lies within the phantom regime i.e., $\xi \sim \omega$.
- In the phantom regime, the null energy condition (NEC) is violated, which makes the bounce possible at a certain finite time, in particular at $t = 0$.
- The violation of NEC occurs irrespective of any value of $\frac{\omega}{c_2}$ and k_0 (i.e the model parameters).

Numerical solution of $H(t)$ and $\xi(t)$ near bounce

- We check whether the radion field, starting from a value in the normal regime, will reach to the phantom regime by its dynamical evolution.
- We solve the Friedmann equations for a wide range of cosmic time.
- Boundary conditions used: $H(0) = 0$ and $\xi(0) = 6.0041 \times 10^{-4}$, where we consider $\omega = 10^{-3}$



(a) (b)

Figure: **The above figure depicts the time evolution of (a) the Hubble parameter $H(t)$ and (b) the non-canonical kinetic term $G(\xi)$ (magenta curve) and the background radion field magnified 1000 times, i.e. $\xi(t) * 1000$ (blue curve). 2b is illustrated near the zero crossing of $G(\xi)$, i.e. when $t \sim -4.7$. Note that bounce occurs after this at $t = 0$ when the kinetic term of the radion is in the phantom regime. Both the above figures are illustrated for $\omega = 10^{-3}$.**

EARLY UNIVERSE COSMOLOGY WITH RADION: EVOLUTION OF PERTURBATIONS

Generation era of perturbations: Behavior of comoving Hubble radius

- In our model the comoving Hubble radius $1/(aH)$ monotonically decreases with time and goes to zero asymptotically on both sides of the bounce.
- Hence, the perturbation modes generate near the bouncing regime where the Hubble radius has an infinite size such that all the perturbation modes are contained inside the horizon.
- Therefore we solve the perturbation equations near the bouncing point, i.e., $t = 0$.

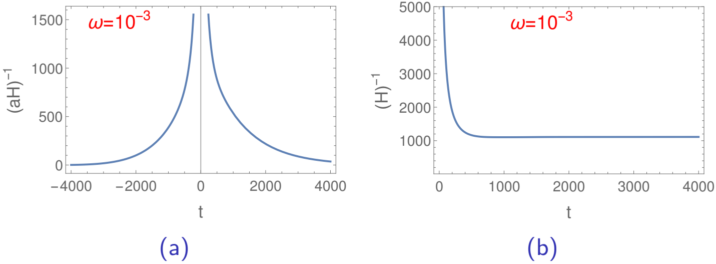


Figure: The above figure depicts the time evolution of (a) the comoving Hubble radius $\frac{1}{aH}$ and (b) the inverse Hubble parameter H^{-1}. Both the above figures are illustrated for $\omega = 10^{-3}$.

Scalar metric perturbations

- The scalar metric perturbation over FRW metric can be written in the longitudinal gauge as,

$$ds^2 = a^2(\eta)\Big[\Big(1+2\Psi\Big)d\eta^2 - \Big(1-2\Psi\Big)\delta_{ij}dx^i dx^j\Big] \tag{9}$$

where $d\eta = \frac{dt}{a(t)}$ and $\Psi(\eta,\vec{x})$ symbolizes the scalar metric fluctuation

- The spacelike and the timelike components of scalar perturbation are considered to be same as the background evolution has no anisotropic stress.

- We expand the radion field as,

$$\Phi(\eta,\vec{x}) = \Phi_0(\eta) + \delta\Phi(\eta,\vec{x}) \tag{10}$$

Perturbed Einstein's equations

- The perturbed Einstein's equations are given by,

$$\nabla^2\Psi - 3\mathcal{H}\Psi' - 3\mathcal{H}\Psi = \frac{\kappa^2}{2}\left[G(\Phi_0)\Phi_0'\delta\Phi' + \frac{1}{2}G'(\Phi_0)(\Phi_0')^2\delta\Phi + 2a^2M^3k_0V'(\Phi_0)\delta\Phi\right]$$

$$\Psi' + \mathcal{H}\Psi = \frac{\kappa^2}{2}\Phi_0'\delta\Phi$$

$$\Psi'' + 3\mathcal{H}\Psi' + \left(2\mathcal{H}' + \mathcal{H}^2\right)\Psi = \frac{\kappa^2}{2}\left[G(\Phi_0)\Phi_0'\delta\Phi' + \frac{1}{2}G'(\Phi_0)(\Phi_0')^2\delta\Phi - 2a^2M^3k_0V'(\Phi_0)\delta\Phi\right] \quad (11)$$

- Here the primes in $V(\Phi_0)$ and $G(\Phi_0)$ are with respect to Φ_0 while the primes in $\mathcal{H}$ and Φ_0 are with respect to η.

- From above we obtain evolution of $\Psi(t,\vec{x})$,

$$\ddot{\Psi} - \frac{1}{a^2}\nabla^2\Psi + \left[7H + \frac{2k_0^2\ V'(\xi_0)}{3c_2^2G(\xi_0)\dot{\xi}_0}\right]\dot{\Psi} + \left[2\dot{H} + 6H^2 + \frac{2k_0^2H\ V'(\xi_0)}{3c_2^2G(\xi_0)\dot{\xi}_0}\right]\Psi = 0 \quad (12)$$

  ξ_0 is the dimensionless unperturbed radion field.

- We solve the perturbation equations near $t = 0$ using background evolution of $H(t)$ and $\xi(t)$ near bounce.

Perturbed Einstein's equations

- Using background evolution of $H(t)$ and $\xi(t)$ in 12, the E.O.M. for $\Psi(t)$ becomes,

$$\ddot{\Psi} - \nabla^2\Psi + \Big[-\sqrt{\alpha}p + (q+14)\alpha t\Big]\dot{\Psi} + \Big[4\alpha - 2\alpha\sqrt{\alpha}p\ t\Big]\Psi(\vec{x},t) = 0 \qquad (13)$$

$$p = 16\sqrt{\frac{2}{3}}\left(\frac{B\sinh^2(B\pi/8)}{\left(3-2e^{B\pi/4}\right)\left(2-\ln\frac{\omega}{c_2}\right)}\right) \quad \text{and} \quad q = \frac{8\left(2-2\cosh(B\pi/4)+\sinh(B\pi/4)\right)}{\left(3-2e^{B\pi/4}\right)^2\left(2-\ln\frac{\omega}{c_2}\right)}$$

where $B = \frac{A}{6}\frac{\omega}{c_2}\sqrt{\frac{3}{\ln\left(\frac{c_2}{\omega}\right)}}$, $\alpha = \frac{6k_0^2\omega^2}{c_2^2}$, $A = \frac{4+2ln\frac{c_2}{\omega}}{ln\frac{c_2}{\omega}}$.

- In terms of the Fourier transformed scalar perturbation variable $\Psi_k(t) = \int d\vec{x}e^{-i\vec{k}.\vec{x}}\Psi(\vec{x},t)$, 13 can be written as,

$$\ddot{\Psi}_k + \Big[-\sqrt{\alpha}p + (q+14)\alpha t\Big]\dot{\Psi}_k + \Big[k^2 + 4\alpha - 2\alpha\sqrt{\alpha}p\ t\Big]\Psi_k(t) = 0 \qquad (14)$$

Perturbed Einstein's equations

- Solving 14 for $\Psi_k(t)$, we get

$$\Psi_k(t) = b_1(k)\ exp\left[\sqrt{\alpha}pt - 7\alpha t^2 - \frac{q}{2}\alpha t^2\right]\ H\left[-1 + \frac{k^2 + 4\alpha}{\alpha(q+14)}, \frac{-p + (q+14)\sqrt{\alpha}\ t}{\sqrt{2(q+14)}}\right] \quad (15)$$

with $H[n, x]$ is the n-th order Hermite polynomial.

- $b_1(k)$ is determined from the initial Bunch-Davies vacuum condition:

$$\lim_{t\to 0} \Psi_k(t) = \frac{\kappa^2 f}{2k^2} \lim_{t\to 0}\left[\sqrt{G(\xi)}\ \dot{\xi} v'_k(\eta)\right] = \frac{i\kappa^2 f}{2\sqrt{2}k^{3/2}} \lim_{t\to 0}\left[\sqrt{G(\xi)}\ \dot{\xi}\right] \quad (16)$$

where, $\lim_{\eta\to 0} v_k(\eta) = \frac{1}{\sqrt{2k}} e^{-ik\eta}$.

- Using the background evolution of $\xi(t)$ and $G(\xi)$ and comparing 15 and 16,

$$b_1(k) = \frac{\sqrt{3}}{2k^{3/2}}\left(\frac{\omega}{c_2}\right)\left(\frac{k_0}{M}\right)^{3/2}\left\{\frac{e^{B\pi/4}\left(3 - 2e^{B\pi/4}\right)^{1/2}}{H\left[-1 + \frac{k^2+4\alpha}{\alpha(q+14)}, \frac{-p}{\sqrt{2(q+14)}}\right]}\right\}$$

Scalar power spectrum

- The solution for the scalar perturbation variable,

$$\Psi_k(t) = \frac{\sqrt{3}}{2k^{3/2}}\left(\frac{\omega}{c_2}\right)\left(\frac{k_0}{M}\right)^{3/2} e^{B\pi/4}\left(3-2e^{B\pi/4}\right)^{1/2} e^{[p\sqrt{\alpha}\ t\ -7\alpha t^2-\frac{q}{2}\alpha t^2]} \times \left\{\frac{H\left[-1+\frac{k^2+4\alpha}{\alpha(q+14)}, \frac{-p+(q+14)\sqrt{\alpha}\ t}{\sqrt{2(q+14)}}\right]}{H\left[-1+\frac{k^2+4\alpha}{\alpha(q+14)}, \frac{-p}{\sqrt{2(q+14)}}\right]}\right\} \tag{17}$$

- The scalar power spectrum for k^{th} mode,

$$P_\Psi(k,t) = \frac{k^3}{2\pi^2}\left|\Psi_k(t)\right|^2$$
$$= \frac{3}{8\pi^2}\left(\frac{\omega}{c_2}\right)^2\left(\frac{k_0}{M}\right)^3 e^{B\pi/2}\left(3-2e^{B\pi/4}\right) e^{[2p\sqrt{\alpha}\ t\ -14\alpha t^2-q\alpha t^2]} \times \left\{\frac{H\left[-1+\frac{k^2+4\alpha}{\alpha(q+14)}, \frac{-p+(q+14)\sqrt{\alpha}\ t}{\sqrt{2(q+14)}}\right]}{H\left[-1+\frac{k^2+4\alpha}{\alpha(q+14)}, \frac{-p}{\sqrt{2(q+14)}}\right]}\right\}^2 \tag{18}$$

- To match with Planck 2018 observations we need to calculate $P_\Psi(k,t)$ at CMB scale $k_{CMB} \approx 0.02\text{Mpc}^{-1} \approx 10^{-40}\text{GeV}$.

Scalar power spectrum at horizon crossing

- With the background solution of Hubble parameter, we determine the time when k_{CMB} crosses the horizon, i.e., $k = aH$, and is given by,

$$t_h = \frac{k_{CMB}}{12k_0^2}\left(\frac{c_2^2}{\omega^2}\right) \quad , \tag{19}$$

- Correspondingly, the scalar power spectrum at horizon crossing can be expressed as,

$$P_\Psi(k,t)\Big|_{h.c} = \frac{3}{8\pi^2}\left(\frac{\omega}{c_2}\right)^2\left(\frac{k_0}{M}\right)^3 e^{B\pi/2}\left(3 - 2e^{B\pi/4}\right)e^{[2p\sqrt{\alpha}\;t_h - 14\alpha t_h^2 - q\alpha t_h^2]} \tag{20}$$
$$\times\left\{\frac{H\left[-1+\frac{k^2+4\alpha}{\alpha(q+14)},\frac{-p+(q+14)\sqrt{\alpha}\;t_h}{\sqrt{2(q+14)}}\right]}{H\left[-1+\frac{k^2+4\alpha}{\alpha(q+14)},\frac{-p}{\sqrt{2(q+14)}}\right]}\right\}^2$$

Tensor perturbation

- We consider the tensor perturbation on the FRW metric background,

$$ds^2 = -dt^2 + a(t)^2 \left(\delta_{ij} + h_{ij}\right) dx^i dx^j \ , \tag{21}$$

- The tensor perturbed action up to quadratic order is given by,

$$\delta S_h = \int dt d^3\vec{x} a(t) z_T(t)^2 \left[\dot{h}_{ij}\dot{h}^{ij} - \frac{1}{a^2}\left(\partial_l h_{ij}\right)^2\right] \ , \tag{22}$$

where, $z_T(t) = \frac{a(t)}{\kappa}$.

- Equation for the tensor perturbed variable h_{ij},

$$\frac{1}{a(t)z_T^2(t)}\frac{d}{dt}\left[a(t)z_T^2(t)\dot{h}_{ij}\right] - \frac{1}{a^2}\partial_l\partial^l h_{ij} = 0 \tag{23}$$

- In terms of the Fourier transformed tensor variable $h_k(t)$, 23 can be expressed as,

$$\frac{1}{a(t)z_T^2(t)}\frac{d}{dt}\left[a(t)z_T^2(t)\dot{h}_k\right] + \frac{k^2}{a^2}h_k(t) = 0 \tag{24}$$

- $h_{ij}(t,\vec{x}) = \int d\vec{k} \ \sum_\gamma \epsilon_{ij}^{(\gamma)} \ h_{(\gamma)}(\vec{k},t)e^{i\vec{k}.\vec{x}}$, where $\gamma =' +'$ and $\gamma =' \times'$ represent two polarization modes.
- $\epsilon_{ij}^{(\gamma)}$ are the polarization tensors satisfying $\epsilon_{ii}^{(\gamma)} = k^i\epsilon_{ij}^{(\gamma)} = 0$.

Near bounce equation for tensor perturbation

- Equation for the Fourier transformed tensor peturbation variable at leading order in t (near bounce where perturbation modes are generated)

$$\ddot{h}_k + 6\alpha \dot{h}_k \ t + k^2 h_k(t) = 0 \tag{25}$$

- Solving 25 for $h_k(t)$, we get,

$$h_k(t) = b_2(k) \ e^{-3\alpha t^2} \ H\left[-1 + \frac{k^2}{6\alpha}, \sqrt{3\alpha} \ t \right] \tag{26}$$

- We determine $b_2(k)$ assuming tensor perturbation field starts from the adiabatic vacuum: $\lim_{t\to 0} \left[z_T(t) h_k(t) \right] = \frac{1}{\sqrt{2k}}$ where at $t \to 0$, $a(t) \simeq 1 + \frac{6\omega^2}{c_2^2} k_0^2 t^2$ and $z_T(t \to 0) = a(t)/\kappa = 1/\kappa$
- Therefore the integration constant $b_2(k)$ is given by,

$$b_2(k) = \frac{1}{z_T(t \to 0)} \left[\frac{2\Gamma\left(1 - \frac{k^2}{12\alpha}\right)}{\sqrt{2\pi k} \ 2^{\frac{k^2}{6\alpha}}} \right] = \kappa \left[\frac{2\Gamma\left(1 - \frac{k^2}{12\alpha}\right)}{\sqrt{2\pi k} \ 2^{\frac{k^2}{6\alpha}}} \right] \quad . \tag{27}$$

Tensor power spectrum

- The solution of $h_k(t)$ is given by,

$$h_k(t) = \left(\frac{2\kappa\ \Gamma\left(1-\frac{k^2}{12\alpha}\right)}{\sqrt{2\pi k}\ 2^{\frac{k^2}{6\alpha}}}\right) e^{-3\alpha t^2}\ H\left[-1+\frac{k^2}{6\alpha}, \sqrt{3\alpha}\ t\right] \tag{28}$$

28 represents the solution of the tensor perturbation for both the polarization modes.

- The tensor power spectrum is ,

$$\begin{aligned} P_h(k,t) &= \frac{k^3}{2\pi^2}\sum_{\gamma}\left|h_k^{(\gamma)}(t)\right|^2 \\ &= \frac{2k^2}{\pi^3}\ \frac{\left(\kappa\ \Gamma\left(1-\frac{k^2}{12\alpha}\right)\right)^2}{2^{\frac{k^2}{3\alpha}}} e^{-6\alpha t^2}\ \left\{H\left[-1+\frac{k^2}{6\alpha}, \sqrt{3\alpha}\ t\right]\right\}^2 \end{aligned} \tag{29}$$

- Tensor power spectrum at horizon crossing $k = aH \simeq 2\alpha t_h$ $(\alpha = 6k_0^2\frac{\omega^2}{c_2^2})$

$$P_h(k,t)\Big|_{h.c} = \frac{12k_0^3\omega^2}{\pi^3 M^3 c_2^2}\ \alpha t_h^2\ \frac{\left(\kappa\ \Gamma\left(1-\frac{\alpha t_h^2}{3}\right)\right)^2}{2^{\frac{4\alpha t_h^2}{3}}} e^{-6\alpha t_h^2}\ \left\{H\left[-1+\frac{2}{3}\alpha t_h^2, \sqrt{3\alpha}\ t_h\right]\right\}^2 \tag{30}$$

Contact with observations

- Ee calculate the scalar spectral index of the primordial curvature perturbations n_s and the tensor-to-scalar ratio r.

$$n_s - 1 = \frac{\partial \ln P_\Psi}{\partial \ln k}\Big|_{H.C} \qquad , \qquad r = \frac{P_h(k,t)}{P_\Psi(k,t)}\Big|_{H.C} \tag{31}$$

- The perturbation modes are generated and also cross the horizon near the bounce $\Longrightarrow$ we can use the near-bounce scale factor in the horizon crossing condition to determine $k = aH = 2\alpha t_h$ (where t_h is the horizon crossing time).

$$n_s = 1 - \frac{16\alpha t_h^2}{(q+14)}\Bigg\{\frac{H^{(1,0)}\left[-1+\frac{4(\alpha t_h^2+1)}{(q+14)}, \frac{-p}{\sqrt{2(q+14)}}\right]}{H\left[-1+\frac{4(\alpha t_h^2+1)}{(q+14)}, \frac{-p}{\sqrt{2(q+14)}}\right]} - \frac{H^{(1,0)}\left[-1+\frac{4(\alpha t_h^2+1)}{(q+14)}, \frac{-p+(q+14)\sqrt{\alpha}\ t_h}{\sqrt{2(q+14)}}\right]}{H\left[-1+\frac{4(\alpha t_h^2+1)}{(q+14)}, \frac{-p+(q+14)\sqrt{\alpha}\ t_h}{\sqrt{2(q+14)}}\right]}\Bigg\}_{h.c} \tag{32}$$

where $q = \frac{8\left(2-2\cosh(B\pi/4)+\sinh(B\pi/4)\right)}{\left(3-2e^{B\pi/4}\right)^2\left(2-\ln\frac{\omega}{c_2}\right)}$ and $B = \frac{A}{6}\frac{\omega}{c_2}\sqrt{\frac{3}{\ln\left(\frac{c_2}{\omega}\right)}}$, $\alpha = \frac{6k_0^2\omega^2}{c_2^2}$, $A = \frac{4+2ln\frac{c_2}{\omega}}{ln\frac{c_2}{\omega}}$.

- n_s depend on the dimensionless parameters ω and αt_h^2

Contact with Planck 2018 data

- The tensor-to-scalar ratio

$$r = \frac{P_h(k,t)}{P_\Psi(k,t)}\bigg|_{H.C} \tag{33}$$

$$P_h(k,t)\bigg|_{h.c} = \frac{12k_0^3\omega^2}{\pi^3 M^3 c_2^2}\ \alpha t_h^2\ \frac{\left(\kappa\ \Gamma\left(1-\frac{\alpha t_h^2}{3}\right)\right)^2}{2^{\frac{4\alpha t_h^2}{3}}} e^{-6\alpha t_h^2}\ \left\{H\left[-1+\frac{2}{3}\alpha t_h^2, \sqrt{3\alpha}\ t_h\right]\right\}^2 \tag{34}$$

$$P_\Psi(k,t)\bigg|_{h.c} = \frac{3}{8\pi^2}\left(\frac{\omega}{c_2}\right)^2\left(\frac{k_0}{M}\right)^3 e^{B\pi/2}\left(3-2e^{B\pi/4}\right)e^{[2p\sqrt{\alpha}\ t_h - 14\alpha t_h^2 - q\alpha t_h^2]} \tag{35}$$
$$\times\left\{\frac{H\left[-1+\frac{k^2+4\alpha}{\alpha(q+14)}, \frac{-p+(q+14)\sqrt{\alpha}\ t_h}{\sqrt{2(q+14)}}\right]}{H\left[-1+\frac{k^2+4\alpha}{\alpha(q+14)}, \frac{-p}{\sqrt{2(q+14)}}\right]}\right\}^2$$

- n_s depend on the dimensionless parameters ω and $\alpha t_h^2 = \frac{R_h}{12\alpha} - 1$ (R_h is Ricci scalar at horizon crossing)
- The observable quantities n_s and r depend on ω and R_h/α.

Constraints from Planck 2018 data

- We estimate the allowed values of $\frac{R_h}{\alpha}$ and ω which in turn can give rise to n_s and r in agreement with the Planck data.

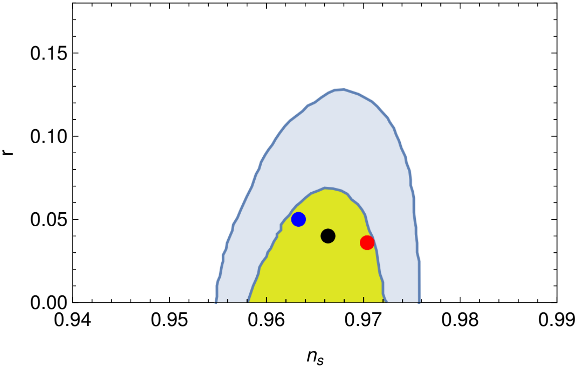


Figure: 1σ (yellow) and 2σ (light blue) contours for Planck 2018 results, on $n_s - r$ plane. Additionally, we present the predictions of the present bounce scenario with $\frac{R_h}{\alpha} = 14$ (blue point), $\frac{R_h}{\alpha} = 16$ (black point) and $\frac{R_h}{\alpha} = 19$ (red point). Here $\omega = 10^{-3}$.

Constraints from Planck 2018 data

- The scalar perturbation amplitude (A_s) is constrained to $\ln\left[10^{10}A_s\right] = 3.044 \pm 0.014$ from the Planck results.
- The amplitude of scalar perturbations A_s not only depends on ω and $\frac{R_h}{\alpha}$ but also on the ratio of the 5D bulk curvature (k_0) and the 5D Planck mass (M) i.e $\frac{k_0}{M}$.
- With $\omega = 10^{-3}$ and $\frac{R_h}{\alpha} = 16$, $A_s = 9.5 \times 10^{-9}\left(\frac{k_0}{M}\right)^3$.
- If $\frac{k_0}{M} = [0.601, 0.607]$ it is consistent with Planck data.
- Allowed range of $\frac{k_0}{M}$ is sensitive to the choice of ω, e.g. $\omega = 10^{-4}$ leads to the scalar perturbation amplitude as $A_s = 9.5 \times 10^{-11}\left(\frac{k_0}{M}\right)^3$ which becomes consistent with the Planck results for $\frac{k_0}{M} > 1$
- With $\frac{k_0}{M} > 1$, the assumption of the background classical solution ceases to hold true.
- The observable quantities n_s, r and A_s are simultaneously compatible with the Planck constraints for the parameter ranges : $\omega = 10^{-3}$, $14 \leq \frac{R_h}{\alpha} \leq 19$, $\frac{k_0}{M} = [0.601, 0.607]$ respectively.

Summary and main results

- We explore bouncing cosmology with radion which naturally arises in a non-flat warped braneworld scenario from compactification.
- In the effective 4-d theory it generates its own potential due to the presence of the brane cosmological constant and unlike most of the scalar-tensor bounce models where the scalar potentials are constructed by hand to explain the observations and often their origin remains unexplained.
- The radion exhbits a phantom era leading to violation of null energy condition and a non-singular bounce.
- Analysis of the background cosmological evolution of the Hubble parameter and the radion field reveals that the radion field starts its journey from the normal regime (i.e $G(\xi) > 0$ regime) and decreases monotonically in magnitude with cosmic time until it transits to the phantom era where the bounce occurs.
- The radion asymptotically stabilizes to $\frac{\omega}{c_2}$, the inflection point of the modulus potential. Such an asymptotic magnitude of the radion field can stabilize the modulus to the appropriate value where the gauge-hierarchy issue can also be adequately addressed.
- We then investigate the cosmological evolution of the scalar and tensor perturbations to the FRW metric from the present model. We compute n_s, r and A_s from the present model which turns out to be pleasantly in agreement with the latest Planck 2018 observations well within the 1-σ regime.