Gravitational Lensing by a black hole in Non–Riemannian space–times

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Beyond Standard Model: From Theory to Experiment (BSM- 2021)

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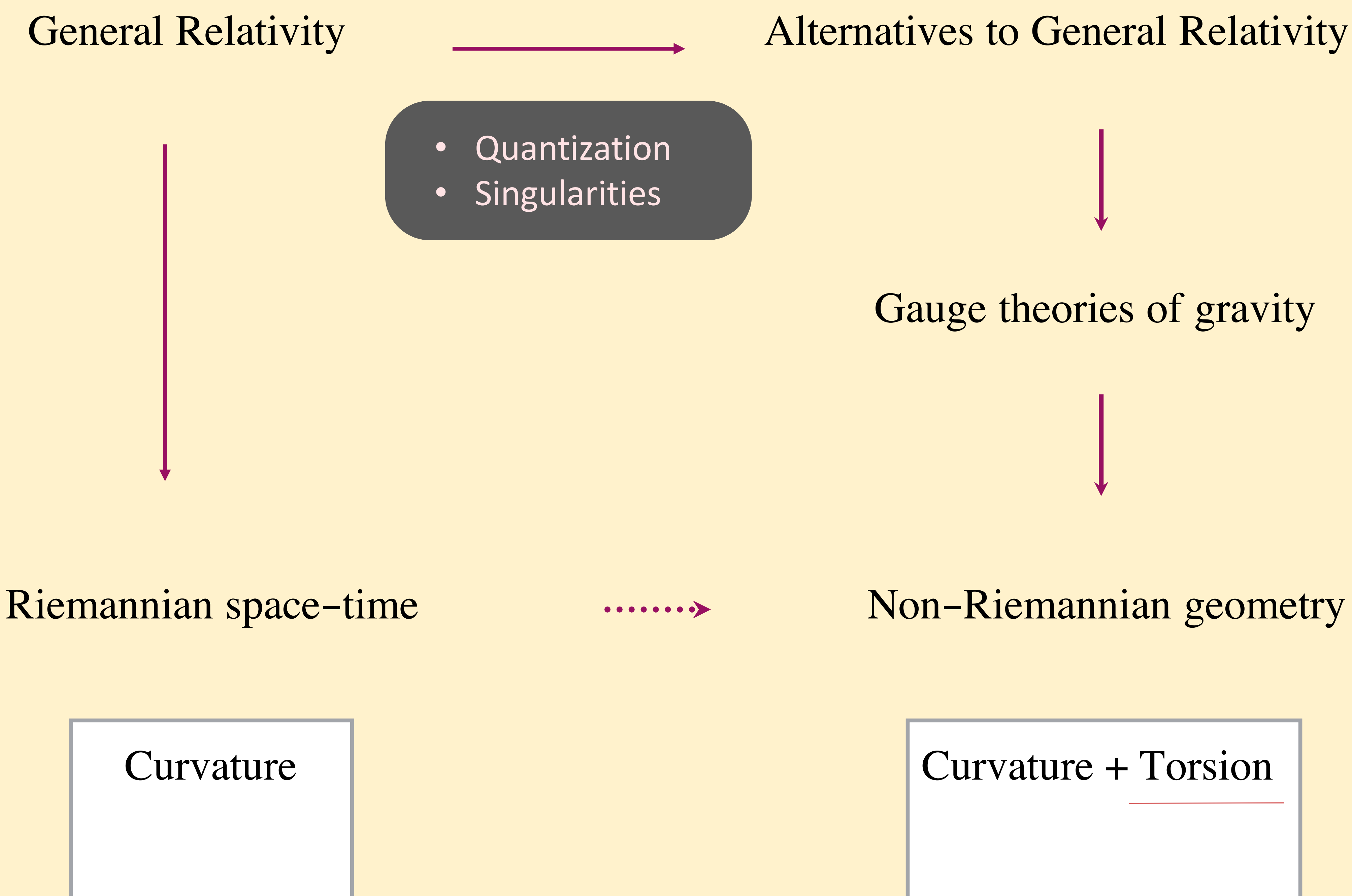

General Relativity
Alternatives to General Relativity
Quantization
Singularities
Gauge theories of gravity
Riemannian space–time
Non–Riemannian geometry
Curvature
Curvature + Torsion

Poincaré Gauge Theory (PGT)

The invariance under a local phase transformation requires the introduction of additional fields:

"Compensating Field"

Applying weyl's gauge principle

Standard Model of particle physics

Local symmetries

fundamental interactions

(Strong, Weak and Electromagnetic interactions)

Compensating fields cancel all the unwanted effects of local transformations and enable the existence of local symmetries.

The coframe $\vartheta^{\mu} = e_i{}^{\mu}\, dx^i$, which is essentially the frame field

Translational invariance.

- **Four** parameters
- Field strength: **torsion**
- Source for torsion: **spin–angular momentum**

The Lorentz connection $\Gamma_{\mu\nu}$, an additional gauge potential, Rotational invariance.

- **Six** parameters
- Field strength: **curvature**
- Source for curvature: **energy–momentum**

Gravitational Lensing by a black hole in PGT

Action

$$S = \frac{1}{16\pi}\int d^4x\,\sqrt{-g}\left[\mathcal{L}_m - R - \frac{1}{2}(2c_1 + c_2)\,\tilde{R}_{\lambda\rho\mu\nu}\,\tilde{R}^{\mu\nu\lambda\rho} + c_1\,\tilde{R}_{\lambda\rho\mu\nu}\,\tilde{R}^{\lambda\rho\mu\nu}\right.$$

$$\left. + c_2\,\tilde{R}_{\lambda\rho\mu\nu}\,\tilde{R}^{\lambda\mu\rho\nu} + d_1\,\tilde{R}_{\mu\nu}\left(\tilde{R}^{\mu\nu} - \tilde{R}^{\nu\mu}\right)\right]$$

Spherically symmetric line element

$$ds^2 = -e^{\nu(r)}\,dt^2 + e^{-\nu(r)}\,dr^2 + r^2\,(d\theta^2 + \sin^2\theta\,d\phi^2),$$

$$e^{\nu(r)} = \left(1 - \frac{2m}{r} + \frac{s}{r^2}\right)$$

Parameter related to the torsion of the space time

$$\frac{d^2\phi}{dp^2} - \left(\frac{2}{r}\right)\frac{dr}{dp}\frac{d\phi}{dp} = 0 \longrightarrow \frac{d}{dp}\left\{\ln\frac{d\phi}{dp} + \ln r^2\right\} = 0$$

$$\downarrow$$

$$\frac{d^2r}{dp^2} - \frac{3}{2}r\left(1 - \frac{2m}{r} + \frac{s}{r^2}\right)\left(\frac{d\phi}{dp}\right)^2 = 0$$

$$r^2\frac{d\phi}{dp} = j$$

$$\frac{d^2t}{dp^2} + 2\left(\frac{m}{r^2} - \frac{s}{r^3}\right)\left(\frac{dt}{dp}\right)^2 - \frac{r}{2}\left(\frac{d\phi}{dp}\right)^2 = 0$$

(Const, Angular momentum per unit mass)

Deflection Angle

From geodesic equation

$$d\tau^2 = E\,dp^2$$

$$E = 0\ (photon)$$

From assumptions

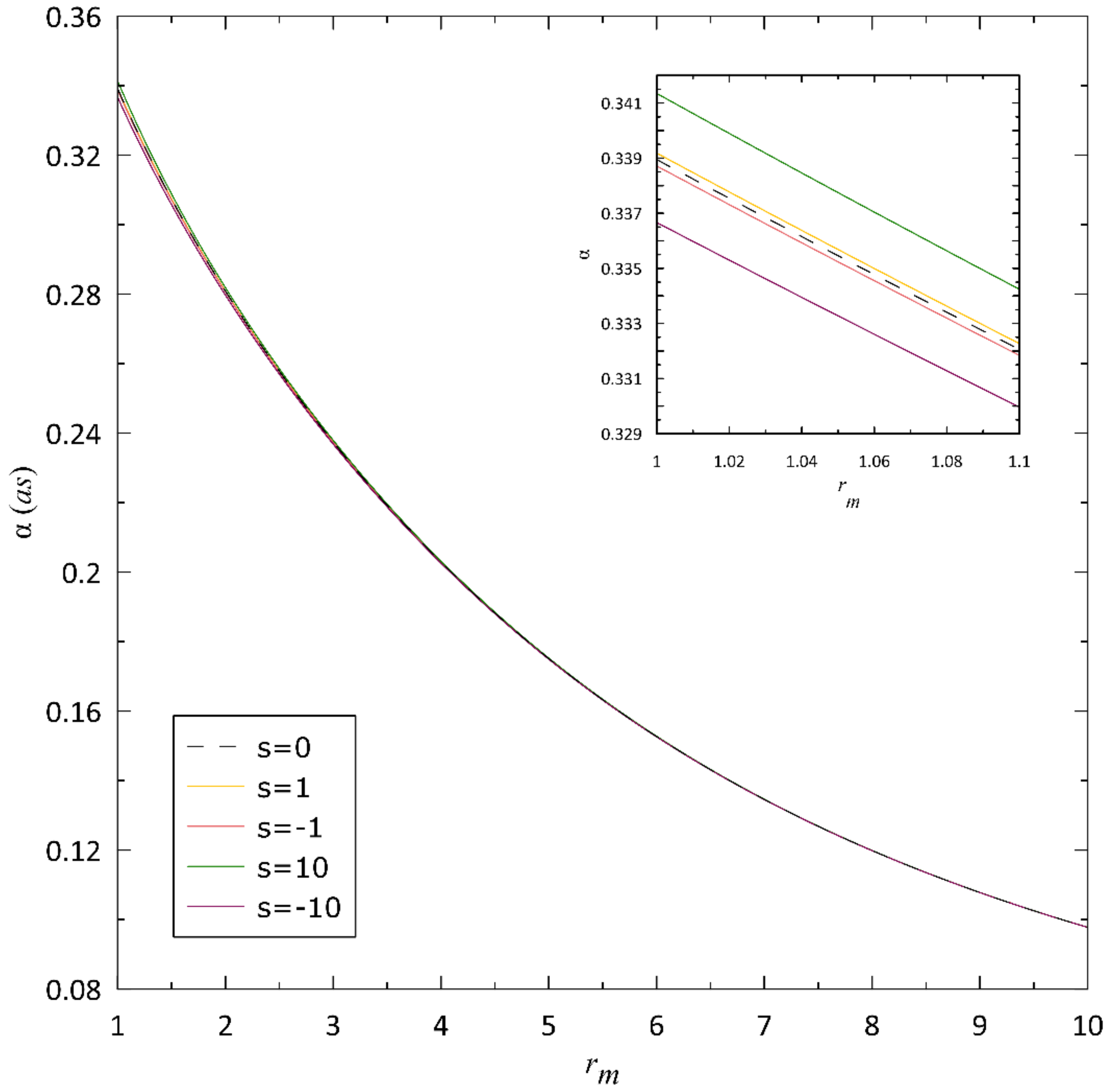
$$j^2 = r^4 (d\phi/dp)^2$$

$\theta = \pi/2$ (Equatorial Plane)

$$\alpha$$

$$\Phi = \pm\int_{r_m}^{r_s} \frac{2}{\sqrt{4r^4 C_1 - 2r^2 + 8mr - 5s}}\,dr - C_2$$

$$C_2 = \pi$$



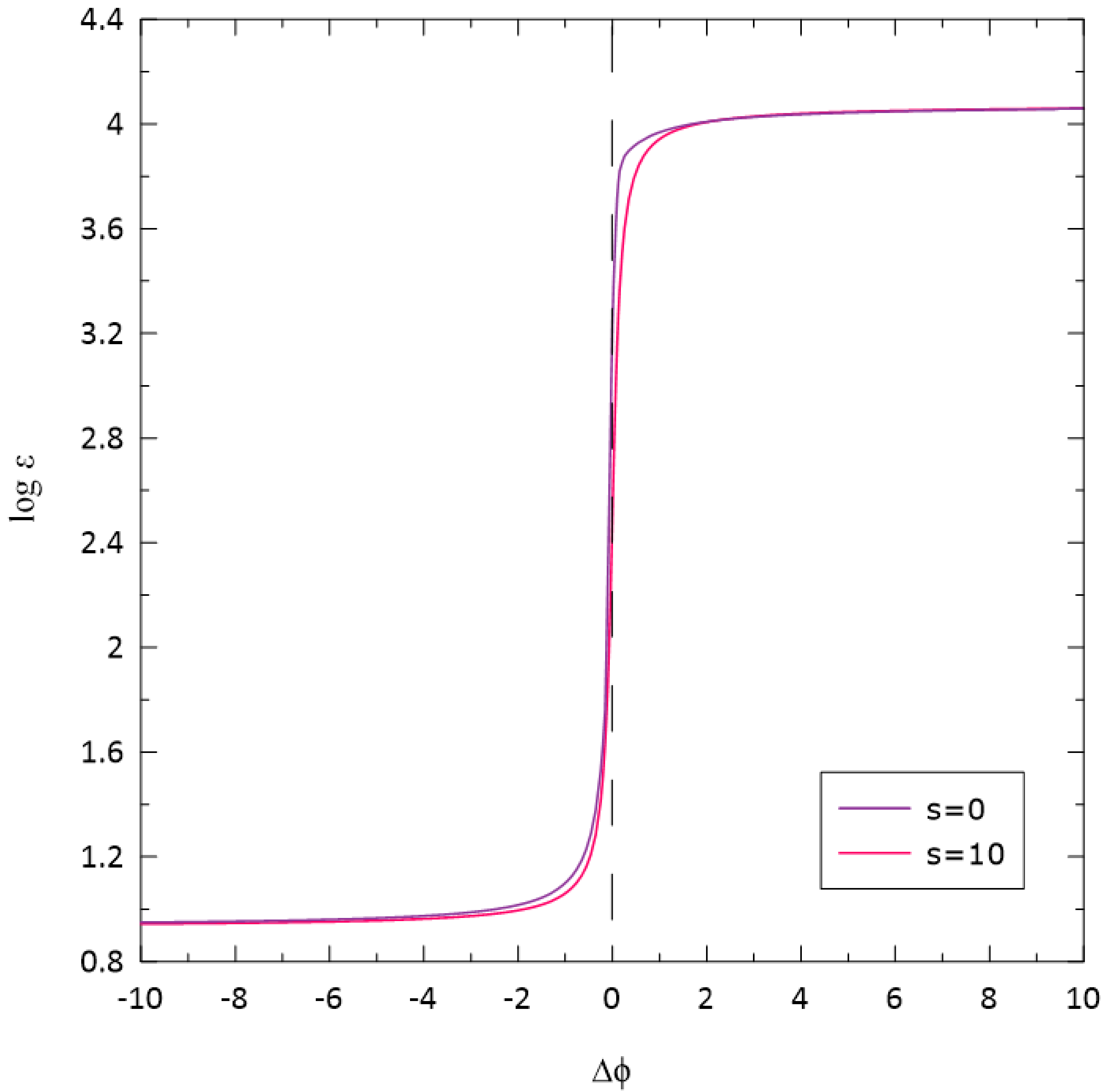
$$\pm \int_{r_m}^{r_s} \frac{2}{\sqrt{4r^4 C_1 - 2r^2 + 8mr - 5s}} \, dr$$

$C_1 = 1$

$m = 100$

$r_s = \infty$

$r_m: 1 - 10$



Angular distance (Lens & Source)

$$\Phi = \Delta\phi = \phi_o - \phi_s + 2n\pi$$

$$\alpha - \theta - \theta_s = \Delta\phi - \pi$$

Image position

Source position

$$n = 0$$

$$\epsilon \equiv \frac{\theta}{\bar{\theta}} - 1$$

Photon sphere position

Thanks!

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