GUT-GRAVITY INTERSECTION AND NEUTRINO MASS-GENERATING MECHANISM

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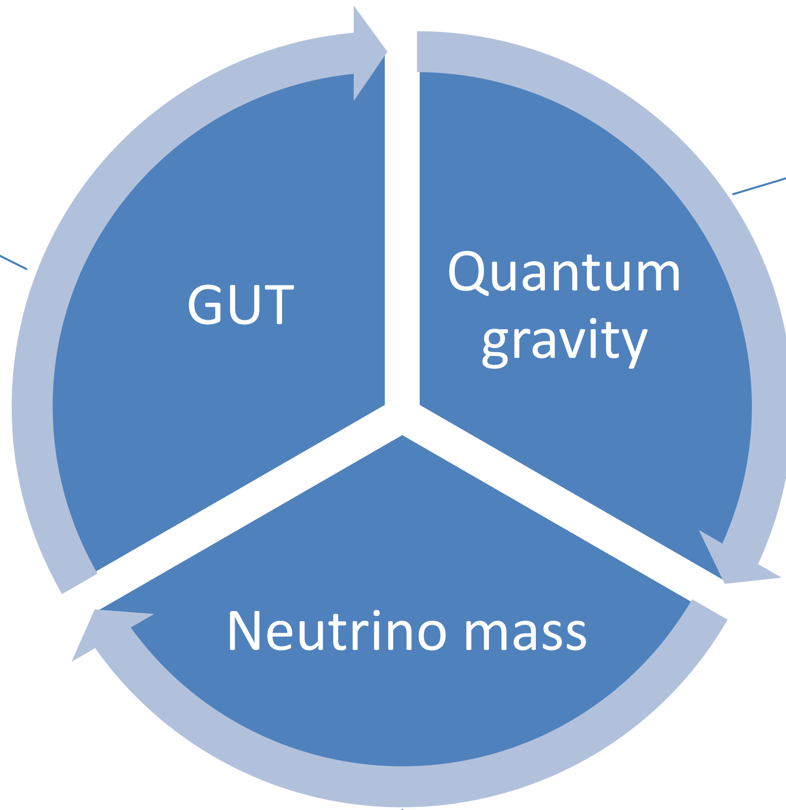
Abstract

✓ We propose that the faithful model of neutrino mass generating mechanism is lurked at the intersection of Grand Unified Theory (GUT) and the quantum theory of gravity. We show that in gravity background, equipped with two-step spontaneous symmetry breaking of GUT, the gravitational couplings can generate effective dimension-5 Weinberg mass term that is quite general for all kinds of neutrino mass models (see-saw or radiative). This robust theoretical framework allows us to calculate, for the first time, the individual neutrino masses m_1, m_2, m_3 and m_4. Our results agree excellently well with both experiments and cosmological observations.

Basic Problem

...many models, no testable predictions



...many models, no testable predictions

Neutrino's interaction-types $\Longrightarrow$ Weak interaction (governed by EW theory) & gravitational interaction (governed by $g_{\mu\nu}$).

EWT predicts massless neutrino: Wrong!!!

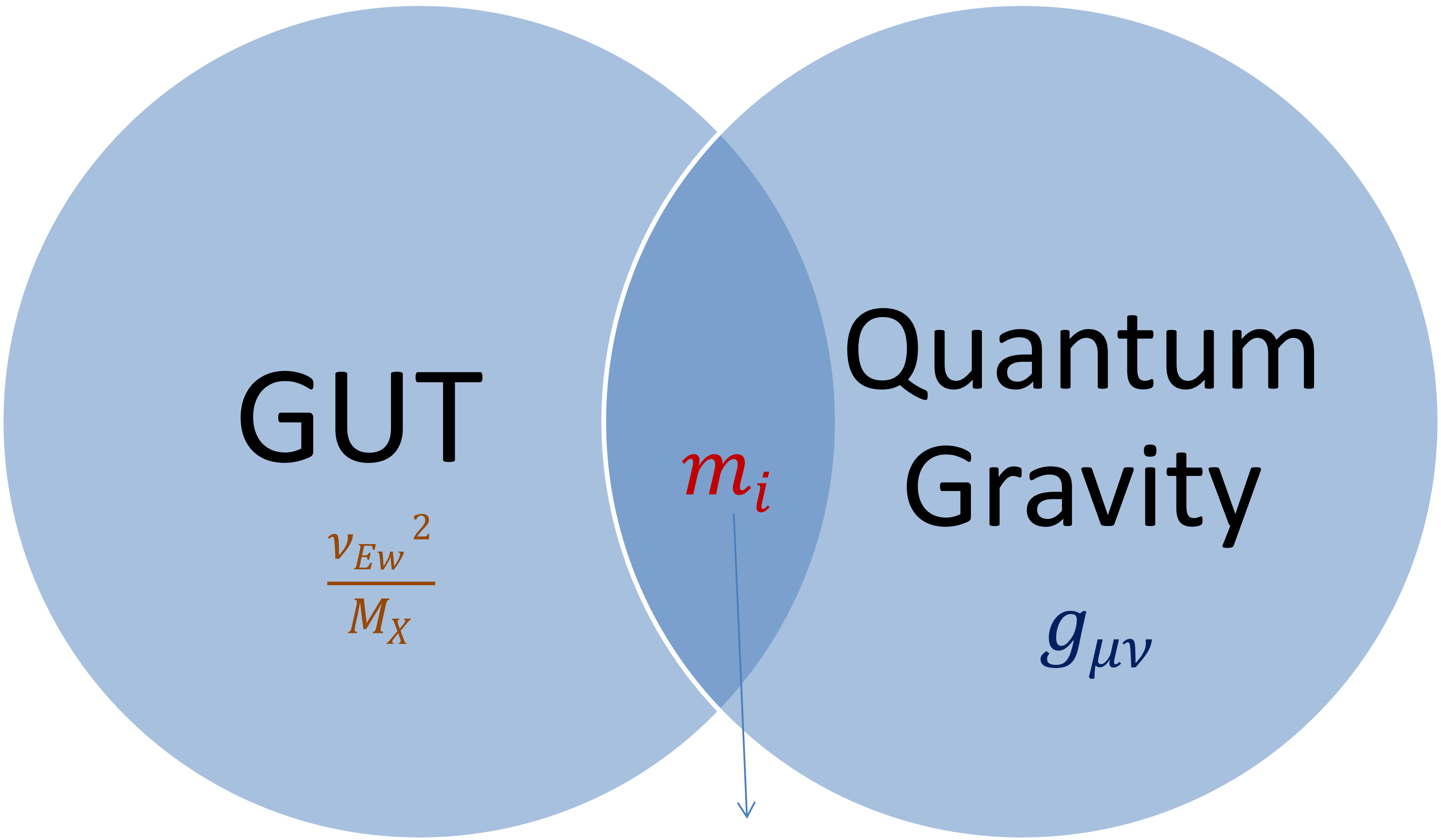
Extended EWT (GUT) needed!

... compound of problems altogether!

...too many models, no testable predictions

[O. F. Akinto & F. Tahir, 2021]

Basic Idea: Solution to the Basic Problem
(T.O.E. $\Longrightarrow$ GUT + Quantum Gravity)



⋆⋆⋆ Intersection $\Longrightarrow$ neutrino masses $m_i \equiv \frac{{\nu_{Ew}}^2}{M_X} g_{\mu\nu}$ ⋆⋆⋆

[O. F. Akinto & F. Tahir, 2021]

Outline

- Research questions
- Beyond Standard Model (BSM): from theory to experiments
- Formulation of covariant Weinberg operator (CWO) in curved space-time/gravity background.
- Majorana mass term/ Weinberg mass term and GUT.
- Our solution to quantum gravity
- Calculation of individual neutrino masses m_i and mass-squared differences Δm_{ji}^2.
- Comparison of our results with neutrino experimental results and cosmological observations.
- Conclusion.
- References.

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Research Questions

- ✓ What is the physics that leads to nonzero neutrino masses?
- ✓ Are the neutrinos Majorana or Dirac particles?
- ✓ Can we use neutrinos to test for the theory of quantum gravity?

BSM: from theory to experiments

- ✓ The existence of neutrino masses is the first and, so far, the only well established <u>new physics</u> beyond Standard Model (BSM) of Particle physics.
- ✓ It is expected that the new physics should modify SM predictions.
- ✓ The modifications would be described by non-renormalizable interactions organized by dimensions in an effective Weinberg-operator Lagrangian:
- ✓ $\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_{\mathrm{SM}} + \mathrm{M_X}^{-1}\mathcal{L}_5 + \mathrm{M_X}^{-2}\mathcal{L}_6 + \ldots$ [1]
- ✓ Dimension-5 operator is sufficiently adequate to describe the physics of Majorana neutrino.

[O. F. Akinto & F. Tahir, 2021]

FORMULATION OF CWO IN GRAVITY BACKGROUND

- ✓ By using the SM fields and the dimension-5 operator from Eq. (1), one can construct a simple covariant Weinberg-operator Lagrangian in gravity background [O. F. Akinto & F. Tahir, 2021]:

$$\mathcal{L}_{eff}{}^{(5)} = \frac{1}{M_X}\sum_{\mu,\nu} g_{\mu\nu}\left[\left(\psi^T{}_{\mu L}\tau_2\phi\right)C^\dagger(\phi^T\tau_2\psi_{\nu L})\right] + H.c \qquad [2]$$

SSB mass-scale of GUT   metric tensor   second Pauli Spin matrix   Higgs field   left-handed fermionic field   charge-conjugate operator

$g_{\mu\nu}$ is equipped with covariant derivatives and must enter Eq.(2) as a purely diagonal matrix-valued gravity (c-number) and not as operator-valued metric [S. L. Adler, 2013].

MAJORANA MASS TERM/WEINBERG MASS TERM AND GUT

✓ Once the Higgs field picks up its vacuum expectation value, Eq. (2) turns into a Majorana mass term:

$$\mathcal{L}_{mass} = \frac{{\nu_{Ew}}^2}{2M_X} \sum_{\mu,\nu} g_{\mu\nu} ({\nu_{\mu L}}^T) C^\dagger (\nu_{\nu L}) + H.c \qquad [3]$$

VEV of Higgs field (= 246GeV)

four linearly independent neutrino fields

From Eq. (3), the Majorana mass matrix elements are given as:

$$m_i \equiv {M_{\mu\nu}}^L = {M_{\nu\mu}}^L = \frac{{\nu_{Ew}}^2}{M_X} g_{\mu\nu} \; ; i = 1,2,3,4 \; \& \; \mu, \nu = 0,1,2,3 \qquad [4]$$

Equation 4 is called modified Weinberg mass term (where $g_{\mu\nu}$ replaces the dimensionless coefficients $\lambda_{\alpha\beta}$) [A. de Gouvea, 2016]; N. Fornengo, C. Giunti, C.W. Kim and J. Song obtained similar result from their quantum-mechanical neutrino-phase in gravity background (where $m_k \equiv {v_{Ew}}^2/M_X$)[N. Fornengo, et al., 1997].

And SSB of GUT is given as:

$$G_{GUT} \xrightarrow{M_X} SU(3)_C \times SU(2)_L \times U(1)_Y \xrightarrow{\nu_{Ew}} SU(2)_L \times U(1)_Q \quad [5]$$

Gauge group of GUT

Gauge group of minimal SM

Electroweak SB

[O. F. Akinto & F. Tahir, 2021]

GUT

✓ GUT

Non-supersymmetric G(331): $M_X = 1.63 \times 10^{16}$ GeV [R. A. Diaz, et al., 2007]

Minimal supersymmetric SU(5): $M_X \backsim 2.0 \times 10^{16}$ GeV [V.N. Senoguz & Q. Shafi, 2003]

Minimal supersymmetric SO(10): $M_X \backsim 2.0 \times 10^{16}$ GeV [J.M. Gipson & K.E. Marshak, 1985]

All the above listed GUTs have one-step symmetry breaking down to the SM gauge group.

✓ Types A and B Non-supersymmetric G(331), without supersymmetric sophistications, is the faithful description of nature. Hence we have $M_X = 1.63 \times 10^{16}$ GeV. The validity of this choice will be shown as we progress in this presentation.

From Eqs 4 & 5 we have:

[★★Gravity couples universally to all energy and momentum★★]

$$g_{\mu\nu}$$

$$G_{GUT} \xrightarrow{M_X} G_{SM} \xrightarrow{{v_{Ew}}^2} G(1) \qquad [6]$$

Where $G(1) \equiv SU(2)_L \times U(1)_Q$ & $G_{SM} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y$

$\frac{1}{d}$

Quantum gravity is an attempt to explain how gravity works on the universe's smallest massive particles (i.e., neutrinos).

➢Our solution to quantum theory of gravity rests on three premises (3 P's)

P1⟹ Principle of Equivalence (Equivalence of gravitation and inertia) states that at any infinitesimally small region of space-time (i.e., quantum-world), we may erect a locally inertial coordinate system in which matter/energy obeys the laws of special relativity[S. Weinberg, 1972].

P2⟹ Gravity couples universally to all energy and momentum [GR].

P3⟹ Weinberg's quantum statistical interpretation of gravitational radiation: Imagine a black-body cavity in a body of temperature T that is so large and dense that it is opaque to gravitational radiation. The cavity will be filled with both electromagnetic and gravitational radiation / field in equilibrium with the container. By using the same quantum statistical arguments that give the Planck distribution law for electromagnetic radiation, one can derive simple expression for graviton number density [S. Weinberg, 1972]. That is:

Graviton number density ($\mathcal{N}_g$) $= 2\times$ photon (or any spin-1 particle) number density ($\mathcal{N}_\gamma$)

$$\mathcal{N}_g = 2\mathcal{N}_\gamma \qquad [7]$$

Theory of gravitational radiation provides a crucial link between gravity and the elementary-particle frontier of physics[P3].

@microscopic scale GR$\Longrightarrow$SR [P1]

Any gravitational radiation is itself a distribution of energy and momentum ($T_{\mu\nu}$)that contributes to the gravitational field $g_{\mu\nu}$[P2&P3].

- From p2 & p3, we have:
- $\langle 0| \, \overline{T}_{\mu\nu} \, |0\rangle = T_{\mu\nu}{}^{vac} = g_{\mu\nu}E^4 = 2\mathcal{N}_{\gamma}E$ [8]

For proper vacuum energy density (ρ_{vac}), we have

- $T_{00}{}^{vac} = \rho_{vac} = g_{00}E^4 = 2\mathcal{N}_{\gamma}E$ [9]

(Due to the spherically symmetric/isotropic condition in a comoving Lorentz frame [S. Weinberg, 1972]).

Hence

- $g_{00} = 2\mathcal{N}_{\gamma} E^{-3}$ [10]

But $\mathcal{N}_{\gamma} = 8\mathcal{g}_f E^3/27k_B{}^3$, $\mathcal{g}_f = 2.5305 \times 10^7 m^{-3}k^{-3}$ and $k_B = 1.3807 \times 10^{-23} Jk^{-1}$.

Thus,

- $g_{00} = 0.1797.$ [11]

[O. F. Akinto & F. Tahir, 2021]

… apparent deviation from previous predictions of gravity:

$$0 \leq g_{00} \leq 1 \qquad [12]$$

$g_{00} = 0.1797$ modifies the horizon structure(R_{sch}) of Schwarzschild black-hole as

$$R_{sch} = \frac{GM}{c^2}(2 + \alpha)$$

Where $\alpha = 0.439$.

…new predictions from Our theory [13]

[O. F. Akinto & F. Tahir, 2021]

The plateau-limit of g_{00} at subatomic scale is $\Longrightarrow 0.1 < g_{00}(x) < 0.2$. See figure 1 [S.L. Adler & F.M. Ramazanoglu, 2015]. New gravity-like fundamental interaction?

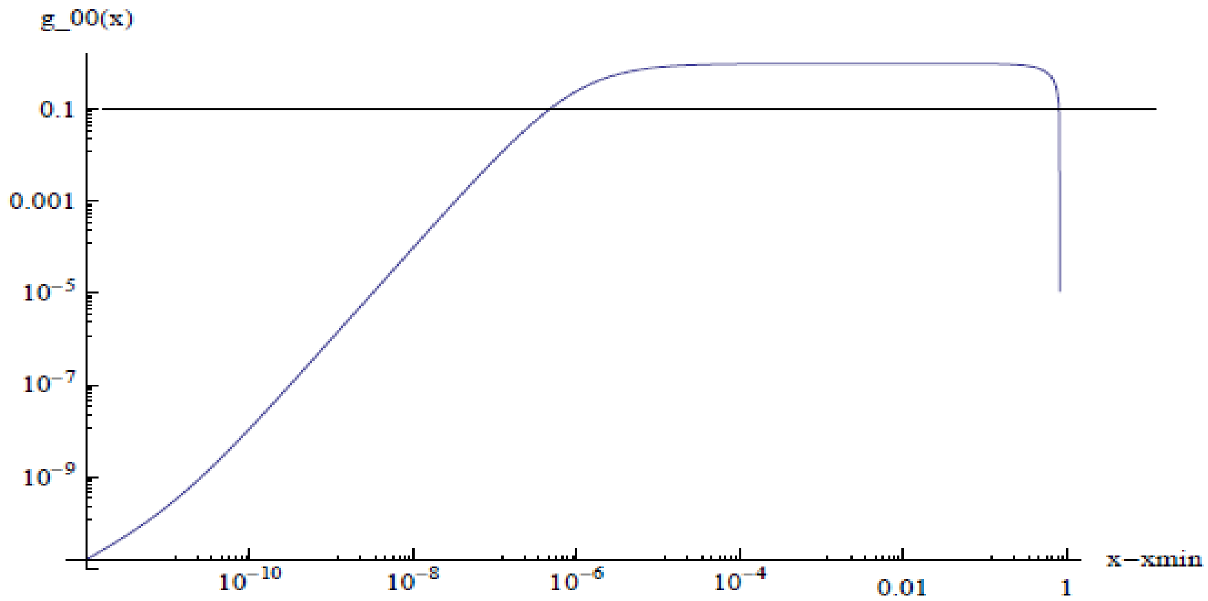


Fig. 1: Numerical result for $g_{00}(x)$ in isotropic coordinates versus $x - x_{min}$.

[O. F. Akinto & F. Tahir, 2021]

Classically looking quantum gravity: Start with Lorentz-invariant quantum theory of gravity and end with classical gravity theory (Weinberg's condition)

$g_{\mu\nu} = \underset{i}{\lambda_{\mu}} \underset{i}{\lambda_{\nu}}$ [14] $\Longrightarrow$ no pre-existence of space(-time) background

Quantum theory of space-time $\Leftrightarrow$ Quantum gravity

Euclidean space-time (++++) is the mathematical device used for constructing quantum theory of gravitation [S. Hawking, 2002].

Effective Riemannian metric (Eq.(14)) $\Longrightarrow$ 'self-interaction' of two quanta of absolute parallelism (AP)-space $\Longrightarrow$ formation of Euclidean spacetime [M.I. Wanas, 2007].

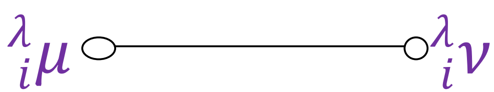
[O. F. Akinto & F. Tahir, 2021]

AP-space is an affinely connected space

…affine connection $\Longrightarrow$ linear connection of zero curvature & the associated parallel field is a tensor field having constant coordinates wrt the field frames e (Lorentz condition)

Hence from Eq.(14), we have

$g_{\mu\nu}$ $\Longrightarrow$ [can only exist for minimum of two points]



For any vanishingly small region of space $X(= x - x_0)$ in an arbitrarily strong gravitational field $g_{\mu\nu}$, one can define a locally flat inertial coordinates system such that $g_{\mu\nu}(X) = \eta_{\mu\nu}$ and $\left(\frac{\partial g_{\mu\nu}(x)}{\partial x^{\gamma}}\right)_{x=X} = 0$, provided that x_0 (of ${}^{\lambda}_{i}\mu$) is situated at the origin (i.e., $x_0 = 0$) $\Longrightarrow$ Radial/Euclidean distance ($r \equiv X = x - x_0$) [S. Weinberg, 1972].

The conditions imposed on any would-be quantum theory of gravitation by P1 are:

AP-space conditions   Weinberg conditions

no singular point (or singularity) at subatomic scale

no curvature at subatomic scale	existence of Lorentz invariance at subatomic scale	corresponding classical gravity theory must obey P1

Is it possible to construct one gravitational rulebook that would satisfy all the conditions listed above?

M. I. Wanas obtained Reissner-Nordström metric (which reduces to the familiar isotropic Schwarzschild vacuum solution to GR when the electric-charge parameter, in the theory, is set to zero) from Eq.(14). [To facilitate comparison with GR, the author used $ds^2 =\star\, g_{\mu\nu}\, dx^\mu dx^\nu$, $\star\, g_{\mu\nu} \equiv \sum_{i=1}^{4} e_i \underset{i}{\lambda_\mu} \underset{i}{\lambda_\nu}$ & $e_i = (1, -1, -1, -1)$].

And thus Eq.(14) satisfies all the four conditions listed above.

From this the following components of gravitational (Yukawa-like) potential are given as:⋆⋆⋆quantum gravitational couplings⋆⋆⋆

$$g_{11} = \frac{1}{g_{00}}\left[1 - \frac{\left(1-g_{00}{}^{1/2}\right)^2}{\left(1+g_{00}{}^{1/2}\right)^2}\right]^2 = 3.8921 \qquad [15]$$

$$g_{22}\left(r = \sqrt{3}\right) = g_{11} r^2 = 11.6763 \qquad [16]$$

Where $r^2(x, y, z) = 2(x^2 + y^2 + z^2) + xy + yz + zx = 3.$

$$g_{33} = 0 \qquad [17]$$

...new predictions from our theory

NB: $g_{00} = 0.1797$

[O. F. Akinto & F. Tahir, 2021]

Calculation of individual neutrino masses m_i and mass-squared differences Δm_{ji}^2

By putting Eqs (11), (15), (16) & (17) (with the values of M_X and v_{Ew}) into Eq. (4), we have the following: ★★★ Breakthrough!!!★★★

$$m_1 = 0.6672 meV, \qquad [18]$$

$$m_2 = 14.4498 meV, \qquad [19]$$

$$m_3 = 43.3494 meV, \qquad [20]$$

$$m_4 = 0 meV. \qquad [21]$$

...new predictions from our theory

Hence $m_3 > m_2 > m_1 \Longrightarrow Normal\ Mass\ Hierarchy\ (NMH)$

The mass-sum of the three active neutrinos is given as:

$$\sum_{i=1}^{3} m_i = m_1 + m_2 + m_3 = 0.0585 eV \approx 0.06 eV \quad [22]$$

For $\Delta m_{ji}{}^2 = m_j{}^2 - m_i{}^2$, we have (from Eqs 18-20)

$$\Delta m_{21}{}^2 = \Delta m_{\odot}{}^2 = m_2{}^2 - m_1{}^2 = 2.08 \times 10^{-4} eV^2 \quad [23]$$

&

$$\Delta m_{31}{}^2 = \Delta m_A{}^2 = m_3{}^2 - m_1{}^2 = 1.88 \times 10^{-3} eV^2 \quad [24]$$

...major predictions from our theory

[O. F. Akinto & F. Tahir, 2021]

Hence Eq. (22) can be rewritten (by using Eqs (18), (23) & (24)) as:

$$\sum_{i=1}^{3} m_i = m_1 + \sqrt{{m_1}^2 + {\Delta m_{21}}^2} + \sqrt{{m_1}^2 + {\Delta m_{31}}^2} = 0.0585eV \approx 0.06eV \quad [25]$$

Comparison of our results with neutrino-oscillation experimental results and cosmological observations

⇒ From solar and reactor neutrino experiments (⋆⋆⋆ with $m_1 = 0$⋆⋆⋆), we have [A. de Gouvea, 2016]:

$\Delta m_{21}{}^2 \simeq 7.5 \times 10^{-5} eV^2$ [26]

Bad approxi mation!

⇒ From atmospheric and accelerator beam experiments (⋆⋆⋆ with $m_1 = 0$ ⋆⋆⋆),

we have [A. de Gouvea, 2016]:

$\Delta m_{31}{}^2 \simeq \pm 2.5 \times 10^{-3} eV^2$ [27]

Bad approxi mation!

⋆⋆⋆Note that $m_1 \neq 0$ in Eqs (23 & 24)⋆⋆⋆ Excellent!

By putting Eqs (26 & 27) into Eq.(25), with $m_1 = 0$, we have

$$\sum m^{NMH} = \sqrt{\Delta m_{21}{}^2} + \sqrt{\Delta m_{31}{}^2} = 0.0587 eV \quad [28]$$

֍֍֍Compare Eq.(28) with Eq.(22)֍֍֍ ⟹ comparison of experiments with theory!

⇒ From Solar and kamLAND neutrino experiments, Δm_{21}^{2}

gives a tiny region above $\Delta m_{21}^{2} = 2.0 \text{ X } 10^{-4} eV^{2}$ at 99.73% C.L. [H. Nunokawa, et al., 2003]. This value is to be compared with our Eq. (23).

⇒ The measurement of dominant atmospheric neutrino and antineutrino oscillations in the MINOS far detector gives $\Delta m_{31}^{2} = 1.9 \text{ X } 10^{-3} eV^{2}$ at 90% C.L. [A. Adamson, et al., 2012]. This is to be compared with our Eq. (24).

⋆⋆⋆Comparison of experiments with theory⋆⋆⋆

$\Rightarrow$ The Planck Base-ΛCDM Model (Standard Model of Cosmology) fixed the mass-sum of neutrinos – as one of its six fiducial parameters – at $\sum m^{\mathrm{fid}} \simeq 0.06\mathrm{eV}$ in order to give an acceptable fit to current cosmological observations [N. Aghanim, et al., 2019]. This value is to be compared with Eq. (22).

⋆⋆⋆Comparison of observations with theory⋆⋆⋆

Concluding Remarks

We have shown that the description of a simple, particle-in-a-box-like universe including gravity is mathematically equivalent to a picture of a flat universe with only quantum physics (and no gravity) at subatomic scale. This ability to jump back and forth between two distinct descriptions (occasioned by the Principle of Equivalence) suggests that space may not be a fundamental ingredient of the universe but rather a side effect that emerges from particle interactions. In this regard, GUT$\left(with\ \frac{{\nu_{Ew}}^2}{M_X}\right)$ and quantum gravity $\left(with\ g_{\mu\nu} = \frac{\lambda_\mu}{i}\frac{\lambda_\nu}{i}\right)$ are seen as two sides of a coin and the neutrino mass m_i $\left(with\ {\nu_{\mu L}}^T C^\dagger \nu_{\nu L}\right)$ is the coin!

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THANK YOU!