Trimaximal mixing with one texture zero from Type-II seesaw and Δ(54) flavor symmetry

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and Faculty of Engineering and Natural Sciences at Sabancı University

Plan of the talk

I- Introduction and Motivations

II- Neutrino Model based on Type-II seesaw and $\Delta(54)$ flavor symmetry

III- Phenomenological implications

IV- Conclusion and perspectives

I- Introduction and Motivations

- **Before 2012, the oscillation data were consistent the well-known Tri-bimaximal (TBM) mixing pattern.**

$$U_{TBM} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \qquad \begin{aligned} \theta_{12} &= 35.3^\circ \\ \theta_{23} &= 45^\circ \\ \theta_{13} &= 0 \end{aligned}$$

Harrison, Perkins, Scott (2002)

$$m^\nu = \begin{pmatrix} a & b & b \\ b & a+c & b-c \\ b & b-c & a+c \end{pmatrix}$$

$\mu - \tau$ symmetry : $\nu_\mu \leftrightarrow \nu_\tau$ + Magic symmetry

NuFIT 5.0 (2020), www.nu-fit.org, JHEP 09 (2020) 178 [arXiv:2007.14792]

	$\theta_{12}/^\circ$	$\theta_{23}/^\circ$	$\theta_{13}/^\circ$
NH (best-fit$^{+3\sigma}_{-3\sigma}$)	$33.44^{+2.42}_{-2.17}$	$49.2^{+2.5}_{-9.1}$	$8.57^{+0.36}_{-0.37}$
IH (best-fit$^{+3\sigma}_{-3\sigma}$)	$33.45^{+2.42}_{-2.18}$	$49.3^{+2.5}_{-9.0}$	$8.60^{+0.36}_{-0.36}$

$$\left.\begin{array}{l} \textbf{The observation of nonzero } \theta_{13} \\ \textbf{The octant of } \theta_{13} \end{array}\right\} \textbf{exclude the TBM pattern}$$

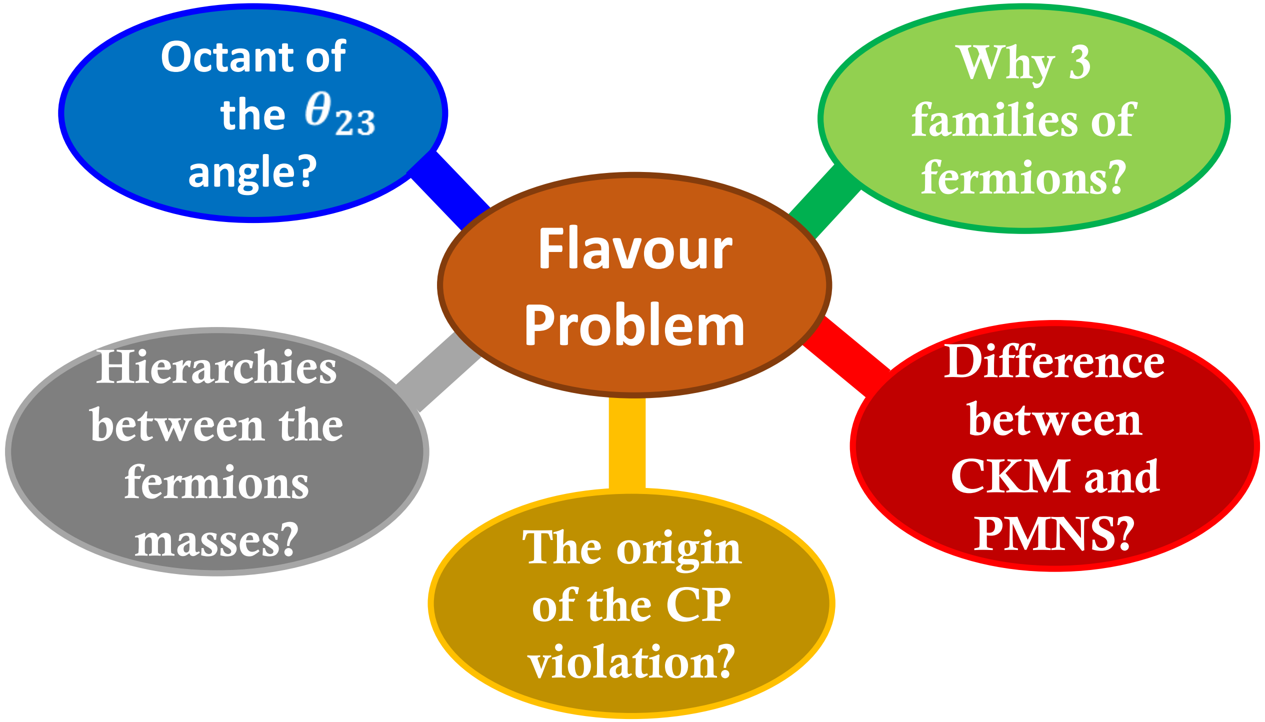
TBM can be used as a good first approximation for the observed neutrino mixing angles

I- Introduction and Motivations

- Corrections from m^{ν} : $\boxed{m^{\nu} \to m^{\nu} + \delta m^{\nu} \longrightarrow U_{\nu} \to U_{TBM} + \delta U_{\nu}}$

- Trimaximal mixing is a good candidate for the neutrino mixing, and it is consistent with current observations.

- Discrete non-Abelian symmetries such as S_3, S_4, A_4, D_4 … are nice candidates to realize these mixings.

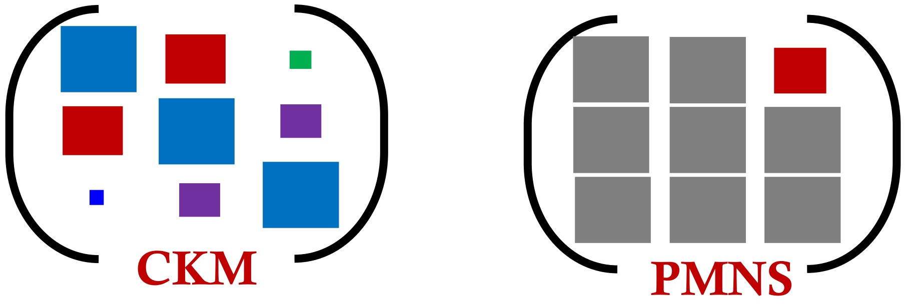


Octant: $\theta_{23} > \pi/4$ or $\theta_{23} < \pi/4$

$$m_e < m_{\mu} < m_{\tau}$$

Normal or inverted neutrino mass hierarchy

CP violation $\longrightarrow$ $\delta \neq \{0, \pi\}$



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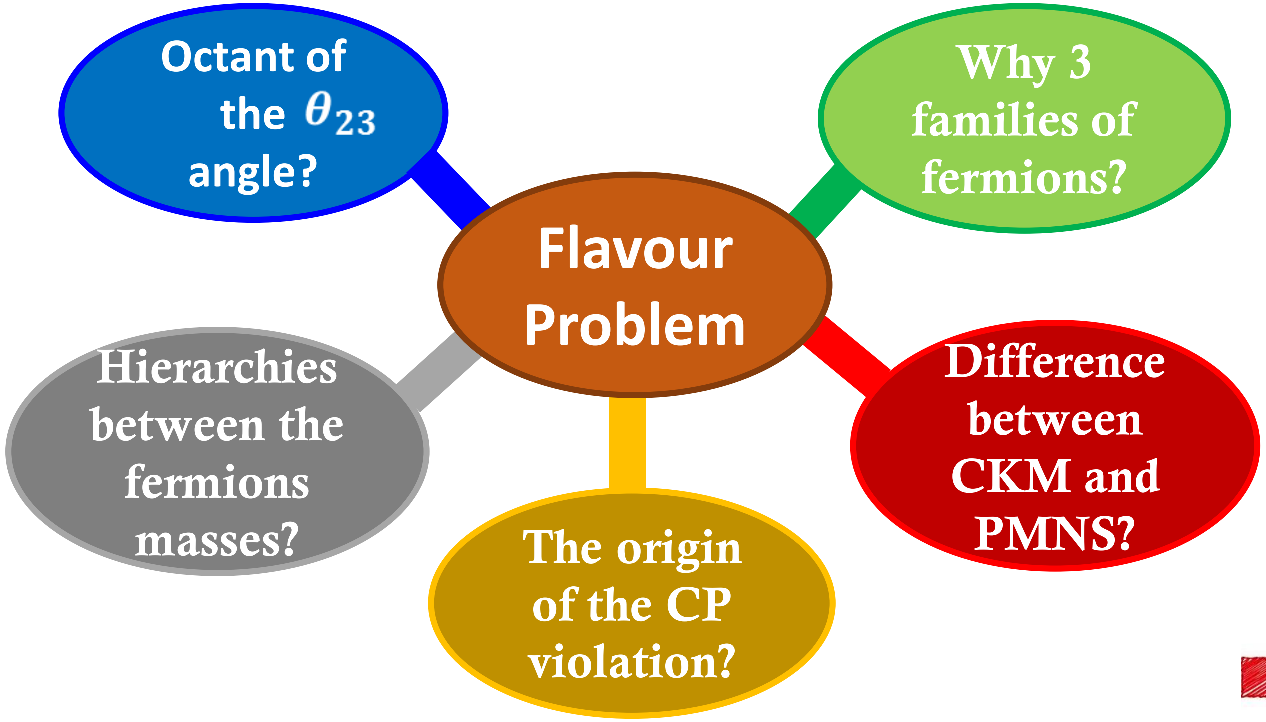

These structures could be explained by flavor (family) symmetries,

- Assign charges under a flavor symmetry.
- To make the Lagrangian invariant, we need to add Higgs-like fields ‘flavons’.
- The structure of the flavon VEVs leads to the desired fermion mass matrices.

⇒ The flavor problem can be addressed by extending the SM to include a family symmetry.

II- Neutrino Model based on Type-II seesaw and Δ(54) flavor symmetry

Some basics on $\Delta(54)$ family symmetry

- $\Delta(54)$ is the smallest non trivial group of $\Delta(6n^2)$ series:

$$n = 1 \rightarrow \Delta(6) = \mathbb{S}_3; \quad n = 2 \rightarrow \Delta(24) = \mathbb{S}_4; \quad n = 3 \rightarrow \Delta(54)$$

$$\Delta(54) = (\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes \mathbb{S}_3$$

- $\Delta(54)$ has four generators that satisfy : $S^3 = T^2 = (ST)^3 = U^3 = V^3 = \mathbb{1}_{id}$

$\Delta(54)$ has ten irreducible representations

- ❖ Two singlets $\mathbf{1}_+$ (trivial) and $\mathbf{1}_-$
- ❖ Four doublets $\mathbf{2}_{1,2,3,4}$
- ❖ Four triplets: $\mathbf{3}_1$; $\mathbf{3}_2$; $\bar{\mathbf{3}}_1$ and $\bar{\mathbf{3}}_2$

$$\mathbf{3}_1 \otimes \bar{\mathbf{3}}_1 = \mathbf{1}_+ + \mathbf{2}_1 + \mathbf{2}_2 + \mathbf{2}_3 + \mathbf{2}_4$$

$$\mathbf{3}_2 \otimes \bar{\mathbf{3}}_2 = \mathbf{1}_+ + \mathbf{2}_1 + \mathbf{2}_2 + \mathbf{2}_3 + \mathbf{2}_4$$

$\Delta(54)$ can serve as a flavor symmetry in particle physics, but remains almost unexplored.

For more details on $\Delta(54)$, see *H. Ishimori et al, arXiv:1003.3552 and J. A. Escobar, C. Luhn. arXiv: 0809.0639*

II- Neutrino Model based on Type-II seesaw and Δ(54) flavor symmetry

The Δ(54) charge assignments of lepton and scalar fields

Fields	$L_i = (L_1, L_2, L_3)$	T	χ	ϕ	ρ
$SU(2)_L$	**2**	**3**	**1**	**1**	**1**
$\Delta(54)$	$\mathbf{3_2}$	$\mathbf{1_-}$	$\mathbf{3_2}$	$\mathbf{3_1}$	$\mathbf{1_-}$

Type-II Seesaw (T)

TBM mixing (χ, ϕ)

TM2 mixing (ρ)

The Δ(54)-invariant Lagrangian for neutrinos is given by

$$\mathscr{L}_{typeII} = \boxed{\frac{Y_\chi^{ij}}{\Lambda}\left(\overline{L}_i^c i\sigma_2 T L_j\right)\chi + \frac{Y_\phi^{ij}}{\Lambda}\left(\overline{L}_i^c i\sigma_2 T L\right)_i \phi} + \boxed{\frac{Y_{\phi\rho}^{ij}}{\Lambda^2}\left(\overline{L}_i^c i\sigma_2 T L_j\right)\phi\rho} + h.c.$$

After symmetry breaking, the neutral component of the triplet T acquires a VEV υ_T while the flavon fields develop VEVs along the directions

$$\langle\chi\rangle = \begin{pmatrix} \upsilon_\chi \\ 0 \\ 0 \end{pmatrix}, \quad \langle\phi\rangle = \begin{pmatrix} 0 \\ \upsilon_\phi \\ 0 \end{pmatrix}, \quad \langle\rho\rangle = \upsilon_\rho$$

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The resulting mass matrix for light neutrino masses is expressed as

$$M_\nu^{II} = \underbrace{\begin{pmatrix} a+b & 0 & \varepsilon \\ 0 & \varepsilon+b & a \\ \varepsilon & a & b \end{pmatrix}}_{\text{Magic symmetry}} \quad \text{with} \quad a = Y_\chi \frac{\upsilon_T \upsilon_\chi}{\Lambda}, \quad b = Y_\phi \frac{\upsilon_T \upsilon_\phi}{\Lambda}, \quad \varepsilon = Y_{\phi\rho} \frac{\upsilon_T \upsilon_\rho \upsilon_\phi}{\Lambda^2}$$

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$\theta \to 0$ corresponds to TBM mixing

$$U_{TM_2} = \begin{pmatrix} \sqrt{\frac{2}{3}}\cos\theta & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\sin\theta\, e^{-i\sigma} \\ -\frac{\cos\theta}{\sqrt{6}} - \frac{\sin\theta}{\sqrt{2}} e^{i\sigma} & \frac{1}{\sqrt{3}} & \frac{\cos\theta}{\sqrt{2}} - \frac{\sin\theta}{\sqrt{6}} e^{-i\sigma} \\ -\frac{\cos\theta}{\sqrt{6}} + \frac{\sin\theta}{\sqrt{2}} e^{i\sigma} & \frac{1}{\sqrt{3}} & -\frac{\cos\theta}{\sqrt{2}} - \frac{\sin\theta}{\sqrt{6}} e^{-i\sigma} \end{pmatrix} .U_P$$

σ is related to the Dirac CP phase δ

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Magic symmetry

Zeros in the mass matrix is the simplest way of inducing relations among the physical quantities and thereby reducing the number of free parameters

There are six possible patterns of one texture zero in the neutrino mass matrix which are compatible with the present data. Radha Raman Gautam, arXiv: 1802.00425,

II- Neutrino Model based on Type-II seesaw and $\Delta(54)$ flavor symmetry

- Assuming that the parameters a and b are real while ε is complex; $\varepsilon = |\varepsilon| e^{i\phi_\varepsilon}$.

The diagonalization of M_ν^{II} using the U_{TM_2} leads to the following eigenvalues

$$|m_1| = \sqrt{(a+b)^2 - |\varepsilon|(\cos\phi_\varepsilon)(a-b)} \qquad |m_2| = \sqrt{(a+b)^2 + 2|\varepsilon|\cos\phi_\varepsilon(a+b)}$$

$$|m_3| = \sqrt{(a-b)^2 - |\varepsilon|(\cos\phi_\varepsilon)(a-b)}$$

Under the conditions

$$\tan 2\theta = \sqrt{3}\frac{|\varepsilon|\sqrt{a^2\sin^2\phi_\varepsilon + b^2\cos^2\phi_\varepsilon}}{|\varepsilon|b\cos\phi_\varepsilon - 2ab}$$

$$\tan\sigma = \frac{a}{b}\tan\phi_\varepsilon$$

Regarding the mixing, they may be expressed in the case of Trimaximal mixing as a function of θ and σ

$\sin^2\theta_{13} = \frac{2}{3}\sin^2\theta$	$\sin^2\theta_{12} = \frac{1}{3 - 2\sin^2\theta}$	$\sin^2\theta_{23} = \frac{1}{2} - \frac{\sqrt{3}\sin 2\theta}{2(3-\sin^2\theta)}\cos\sigma$

- A relationship between the arbitrary phase σ and the Dirac phase δ_{CP} can be obtained by means of the Jarlskog invariant parameter defined as $J_{CP} = Im(U_{e1}U_{\mu 1}^{*}U_{\mu 2}U_{e2}^{*})$

$$\left.\begin{aligned} J_{CP} &= \frac{1}{8}\sin 2\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23}\cos\theta_{13}\sin\delta_{CP} \\ J_{CP}|_{TM_2} &= (1/6\sqrt{3})\sin 2\theta\sin\sigma \end{aligned}\right\} \longrightarrow \sin 2\theta_{23} = (\sin\sigma / \sin\delta_{CP})$$

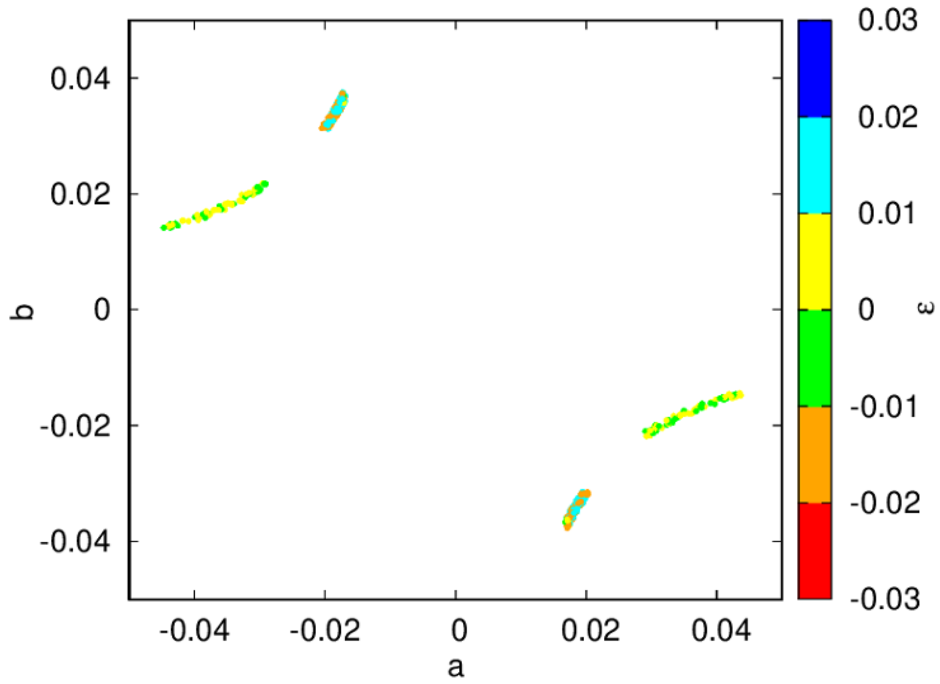
S. T. Petcov, arXiv: 1711.10806

III- Phenomenological implications

- Using the obtained expressions of neutrino masses, the solar and atmospheric mass-squared differences are expressed as

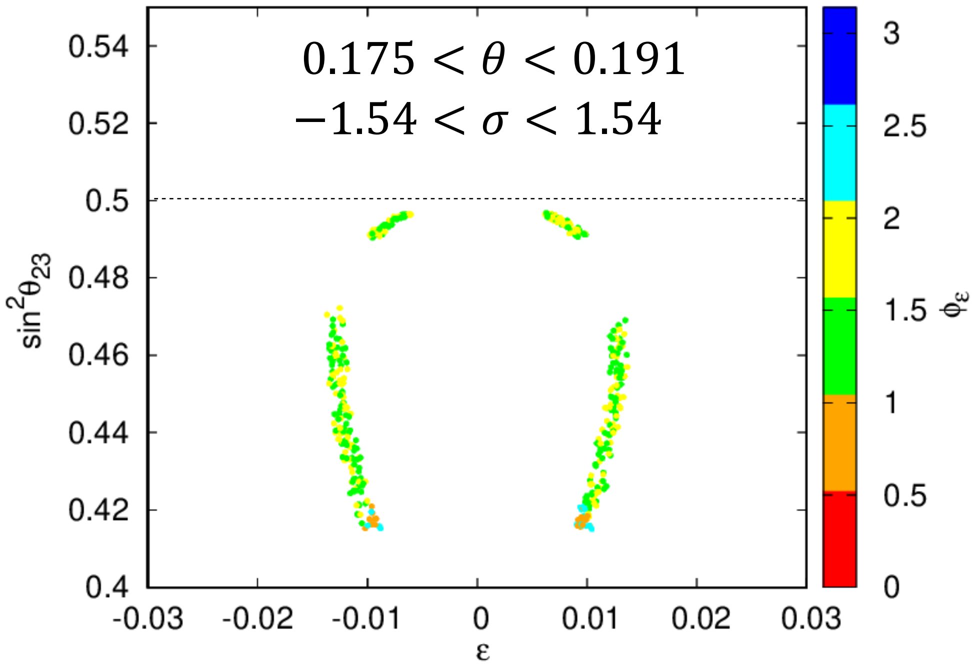
$$\Delta m_{31}^2 = -4ab \qquad \Delta m_{21}^2 = |\varepsilon| \cos\phi_\varepsilon \,(3a+b)$$

- We plot the correlation among the parameters a, b and ε



$\Delta m_{31}^2 > 0$ **The model predicts NH**

- We plot θ_{23} as a function of ε



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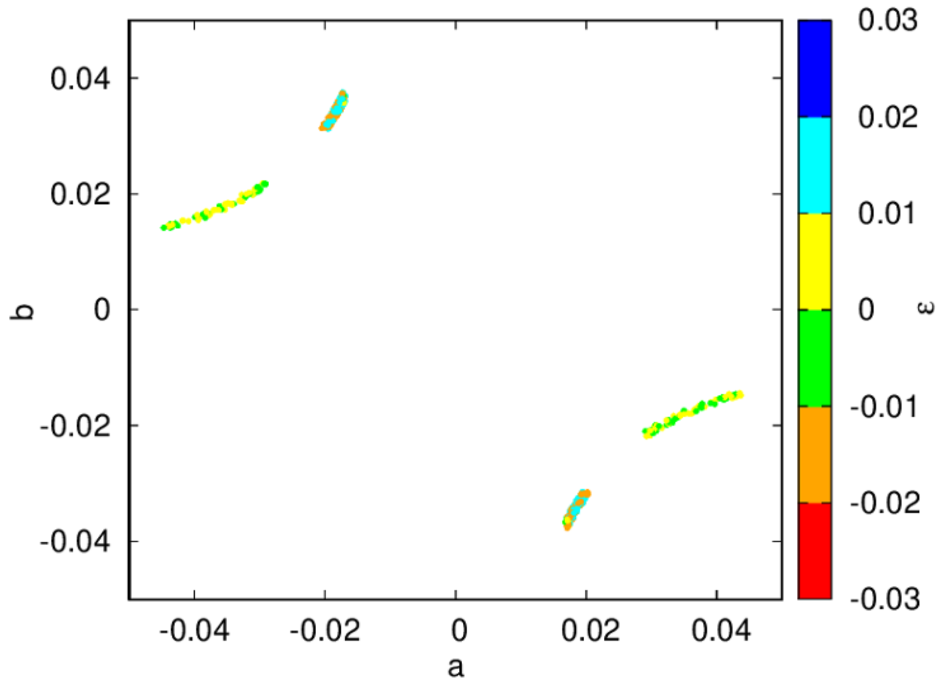

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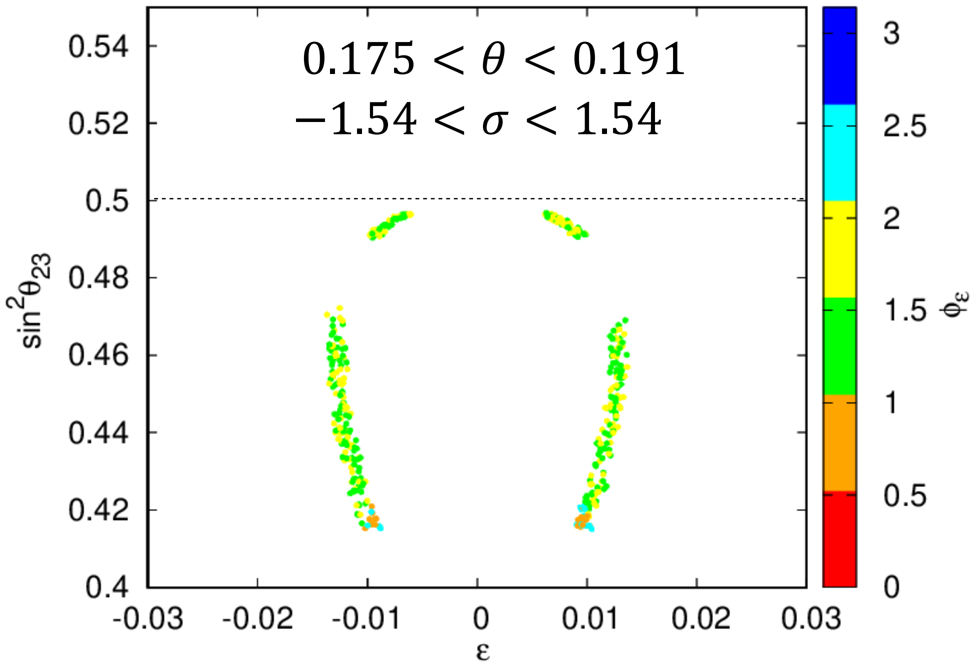

$$\sin^2\theta_{23} = \frac{1}{2} - \frac{\sqrt{3}\sin 2\theta}{2\left(3-\sin^2\theta\right)}\cos\sigma \;\Rightarrow\; \frac{\sqrt{3}\sin 2\theta}{2\left(3-\sin^2\theta\right)}\cos\sigma < \frac{1}{2}$$

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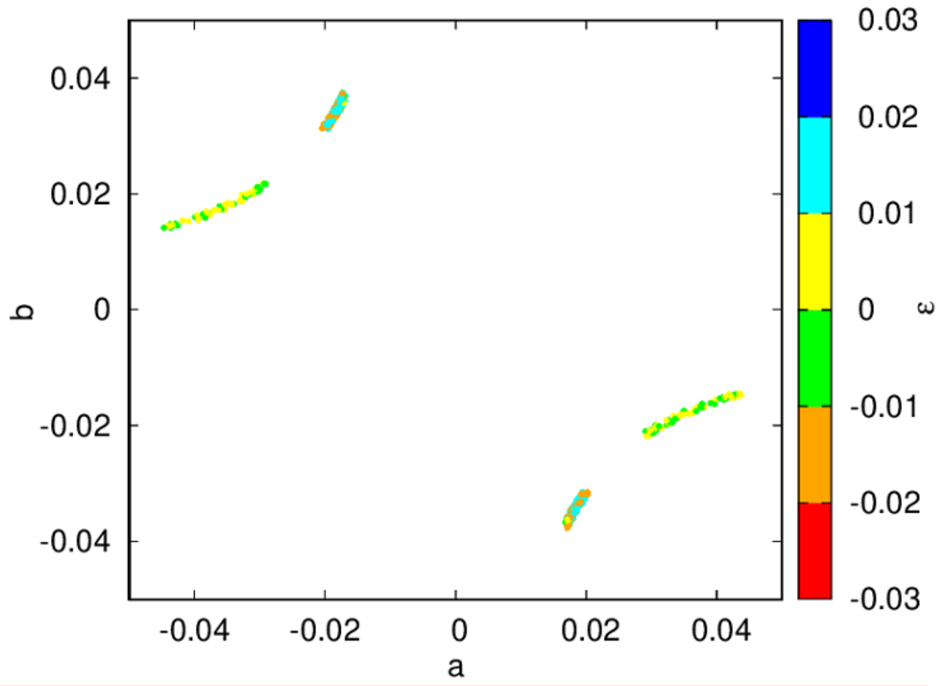

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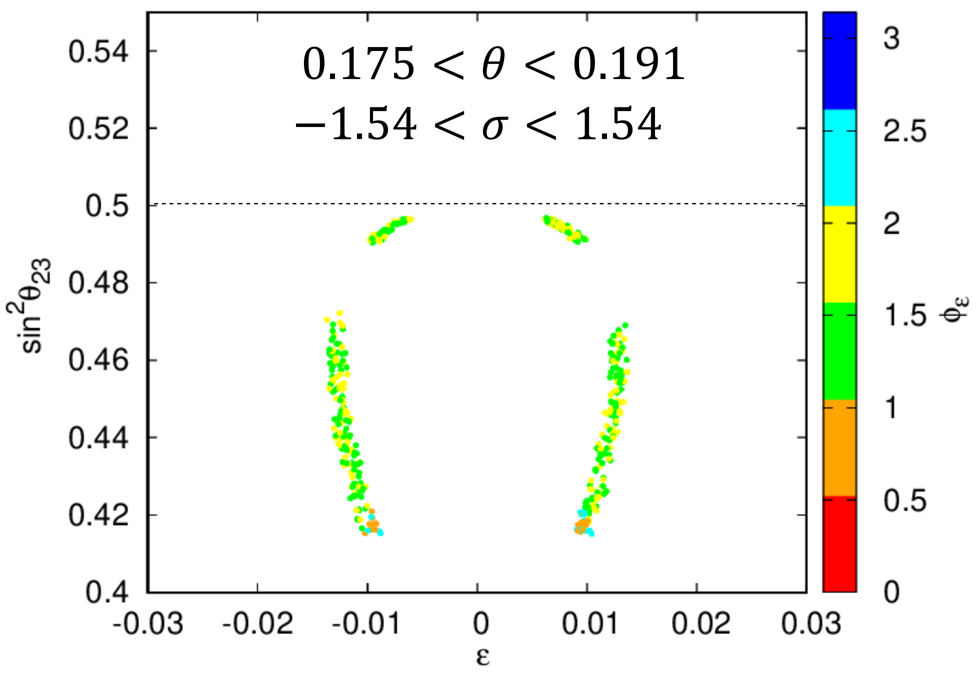


The model predicts lower octant

III- Phenomenological implications

What about the absolute scale of neutrino masses?

- Direct determination of the neutrino mass by measuring the energy spectrum of electrons produced in the β-decay of nuclei which allows to get information on the effective electron antineutrino mass defined by

$$m_{\nu_e} = \left(\sum_{i=1,2,3} m_i^2 \left|U_{ei}\right|^2 \right)^{1/2}$$

The current limit from tritium beta decay is given by the KATRIN project, which aims at a detection of m_{ν_e} with a sensitivity of 0.2eV.

- Search for 0νββ decay processes having a decay amplitude proportional to the effective Majorana neutrino mass defined as

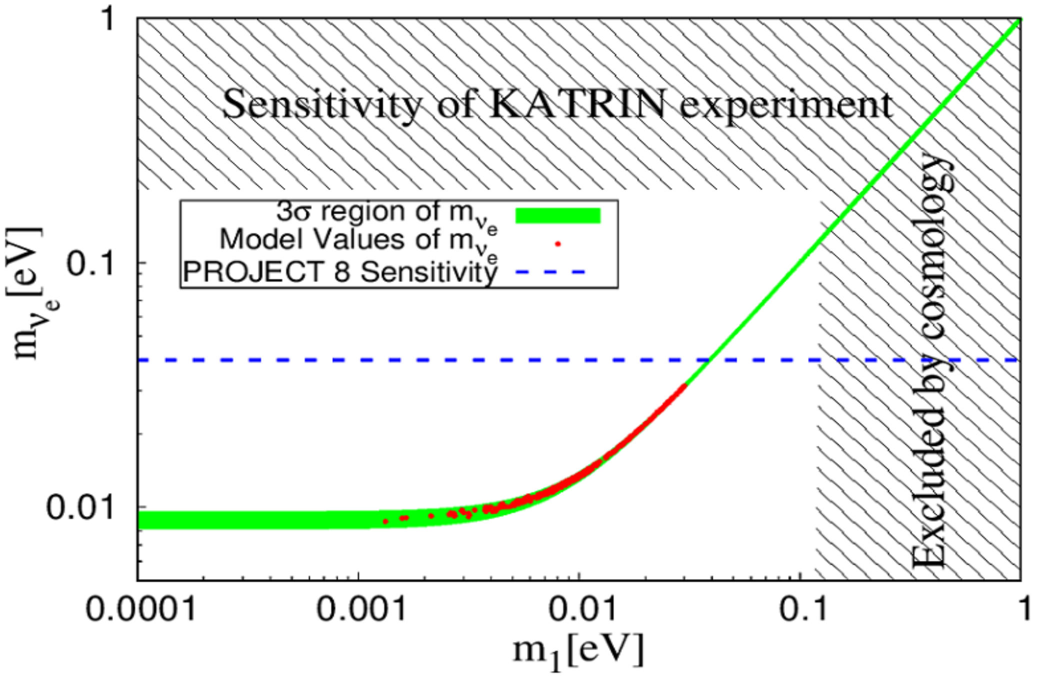
$$|m_{ee}| = \left| \sum_{i=1,2,3} U_{ei}^2 m_i \right|$$

This is also considered as the unique probe for the Majorana nature of neutrinos.

- Constraints from cosmological observations providing an upper bound on the sum of the three active neutrino masses; $\Sigma m_i = m_1 + m_2 + m_3$. The present upper bound on Σm_i from the Planck collaboration is given by Σm_i<0.12eV.

III- Phenomenological implications

Constraints on the sum of neutrino masses and the effective anti-neutrino mass



The red points correspond to the range:

$$0.00873 < m_{\nu_e} < 0.03135$$

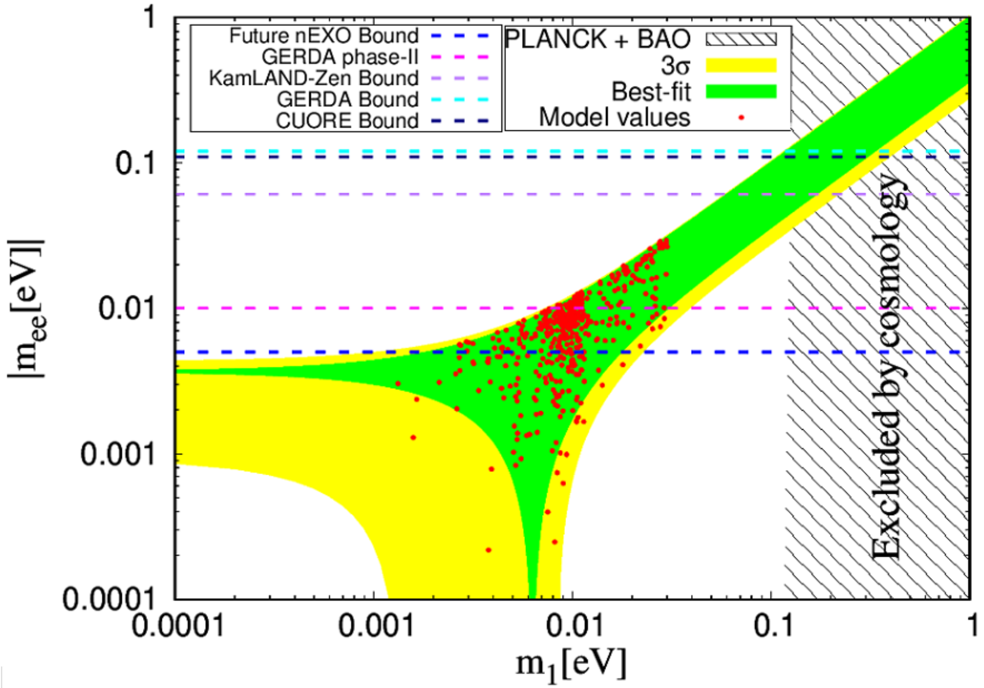
The values in this interval are very small when compared with the forthcoming experiment sensitivities such as KATRIN, HOLMES, and Project 8 which will investigate m_{ν_e} at 0.2eV, 0.1eV and 0.04eV respectively.

III- Phenomenological implications

- There are many ongoing and upcoming experiments around the world setting as their purpose the detection of $0\nu\beta\beta$ process

- The present bounds on $|m_{ee}|$ come from the KamLAND-Zen, CUORE and GERDA experiments corresponding to $|m_{ee}|<(0.061-0.165)$eV, $|m_{ee}|<(0.11-0.5)$eV and $|m_{ee}|<(0.15-0.33)$eV respectively.

$$|m_{ee}| = \left|\frac{2}{3}m_1\cos^2\theta + \frac{1}{3}\sqrt{m_1^2+\Delta m_{21}^2}e^{i\alpha_{21}} + \frac{2}{3}\sqrt{m_1^2+\Delta m_{31}^2}\sin^2\theta e^{-i(2\sigma-\alpha_{31})}\right|$$



- Using the upper limit $\Sigma m_i<0.12$eV and the neutrino oscillation parameters within their currently allowed 3σ ranges

$$\boxed{0.00021 < m_{ee} < 0.02923}$$

- The anticipated sensitivities of the next-generation experiments such as GERDA Phase II and nEXO will cover the model predictions on $|m_{ee}|$.

IV - Conclusion and perspectives

- The problem of fermions masses and mixing has become more interesting in recent years with the discovery of the oscillation phenomenon.
- Non-abelian discrete groups are an interesting tool to explain masses and mixing of fermions.
- One-texture zero + $\Delta(54)$ flavor symmetry with Trimaximal mixing has led to interesting predictions.
- However, many important questions are still unanswered.

Work in progress

- Detailed Study of the scalar sector
- CP-violation and Leptogenesis

Thanks for your

ATTENTION