

# $S_3$ symmetric scotogenic model for realistic neutrino mixing

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# Three-flavour oscillation

- Neutrinos are massive and they mix !!

Three flavor oscillation probability

$$P_{\nu_\alpha \nu_\beta} = \delta_{\alpha\beta} - 4 \sum_{j>i} U_{\alpha i} U_{\beta i} U_{\alpha j}^* U_{\beta j}^* \sin^2 \left( \frac{\pi L}{\lambda_{ij}} \right)$$

2 independent  $\Delta m^2$ , 3 mixing angles, 1 phase

$$U^T M_\nu U = \text{diag}(m_1, m_2, m_3)$$

- The Pontecorvo, Maki, Nakagawa, Sakata – PMNS – matrix:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

A measure of CP-violation is given by the basis-independent leptonic Jarlskog(J) parameter:

$$J = \text{Im}[U_{e1}U_{\mu 2}U_{e2}^*U_{\mu 1}^*]$$

# Popular Lepton Mixing and oscillation data

The current  $3\sigma$  global fits of the three mixing angles:

$$\theta_{12} = (31.42 - 36.05)^\circ,$$

$$\theta_{23} = (40.3 - 51.5)^\circ,$$

$$\theta_{13} = (8.09 - 8.98)^\circ.$$

- $\theta_{13} \neq 0$  but small.
- Best fit  $\theta_{23} \neq \pi/4$ . Octant not known.

**Popular lepton mixings (By construction:  $\theta_{13} = 0$ ,  $\theta_{23} = \pi/4$ . Differ only in  $\theta_{12}$ ):**

$$U^0 = \begin{pmatrix} \cos \theta_{12}^0 & \sin \theta_{12}^0 & 0 \\ -\frac{\sin \theta_{12}^0}{\sqrt{2}} & \frac{\cos \theta_{12}^0}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{\sin \theta_{12}^0}{\sqrt{2}} & \frac{\cos \theta_{12}^0}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix},$$

Tribimaximal (TBM) [ $\theta_{12} = 35.3^\circ$ ]; Bimaximal (BM) [ $\theta_{12} = 45^\circ$ ]; Golden Ratio (GR) [ $\theta_{12} = 31.7^\circ$ ]

- Corresponding structure of left-handed Majorana neutrino mass matrix:

$$M_{\nu L}^{flavour} = U^0 M_{\nu L}^{mass} U^{0T} = U^0 \text{diag}(m_1, m_2, m_3) U^{0T} = \begin{pmatrix} a & c & c \\ c & b & d \\ c & d & b \end{pmatrix}.$$

Here,

$$a = m_1 \cos^2 \theta_{12}^0 + m_2 \sin^2 \theta_{12}^0$$

$$b = \frac{1}{2} (m_1 \sin^2 \theta_{12}^0 + m_2 \cos^2 \theta_{12}^0 + m_3)$$

$$c = \frac{1}{2\sqrt{2}} \sin 2\theta_{12}^0 (m_2 - m_1)$$

$$d = \frac{1}{2} (m_1 \sin^2 \theta_{12}^0 + m_2 \cos^2 \theta_{12}^0 - m_3)$$

$$\tan 2\theta_{12}^0 = \frac{2\sqrt{2}c}{b+d-a}$$

# Objective

- Note  $a, b, c$  and  $d$  have to be non-zero for neutrino masses to be non-degenerate and realistic.

## Objective

Construct a scotogenic model using  $S3 \times Z_2$  with two right-handed neutrinos that at one-loop level can generate:

- The structure of the left-handed Majorana neutrino mass matrix with  $\theta_{13} = 0$ ,  $\theta_{23} = \pi/4$  when the two right-handed neutrinos are maximally mixed.
- Non-zero  $\theta_{13}$ , deviations of  $\theta_{23}$  from maximality and small corrections to solar mixing by a small shift from maximal mixing in the right-handed neutrino sector.

## Discrete Flavour symmetry $S3$

- Permutation group of three objects. Has two generators  $A, B$  and three irreducible representations viz. 2 (dimension 2) and 1, 1' (dimension 1).

- Product rule:**  $1' \times 1' = 1$ ,  $2 \times 2 = 1 + 1' + 2$ .

- Combining two doublets of  $S3$  viz.  $\Phi \equiv \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$  and  $\Psi \equiv \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$ , we get:  
$$\phi_1\psi_2 + \phi_2\psi_1 \equiv 1 \quad , \quad \phi_1\psi_2 - \phi_2\psi_1 \equiv 1' \quad \text{and} \quad \begin{pmatrix} \phi_2\psi_2 \\ \phi_1\psi_1 \end{pmatrix} \equiv 2 \quad .$$

- In case one of the two  $S3$  doublet fields is a hermitian conjugate we get:

$$\phi_2^\dagger\psi_2 + \phi_1^\dagger\psi_1 \equiv 1 \quad , \quad \phi_2^\dagger\psi_2 - \phi_1^\dagger\psi_1 \equiv 1' \quad \text{and} \quad \begin{pmatrix} \phi_1^\dagger\psi_2 \\ \phi_2^\dagger\psi_1 \end{pmatrix} \equiv 2 \quad .$$

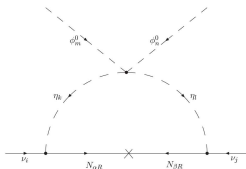
# Fields in the model

Leptons	$SU(2)_L$	$S3$	$Z_2$
$L_{eL} \equiv (\nu_e \quad e^-)_L$	2	1	1
$L_{\zeta L} \equiv \begin{pmatrix} \nu_\mu & \mu^- \\ \nu_\tau & \tau^- \end{pmatrix}_L$	2	2	1
$N_{\alpha R} \equiv \begin{pmatrix} N_{1R} \\ N_{2R} \end{pmatrix}$	1	2	-1
Scalars	$SU(2)_L$	$S3$	$Z_2$
$\Phi \equiv \begin{pmatrix} \phi_1^+ & \phi_1^0 \\ \phi_2^+ & \phi_2^0 \end{pmatrix}$	2	2	1
$\eta \equiv \begin{pmatrix} \eta_1^+ & \eta_1^0 \\ \eta_2^+ & \eta_2^0 \end{pmatrix}$	2	2	-1

- Here,  $S3$  is acting vertically,  $SU(2)_L$  is acting horizontally.
- Inert  $SU(2)_L$  doublet scalars  $\eta_j \equiv (\eta_j^+, \eta_j^0)^T$ , ( $j = 1, 2$ ) and right-handed neutrinos  $N_{\alpha R}$ , ( $\alpha = 1, 2$ ) are odd under  $Z_2$ . Thus after spontaneous symmetry breaking (SSB),  $\eta_j$  does not acquire vev. Lightest among  $\eta_j$  can be a potential dark matter candidate.
- After SSB,  $\Phi_i$  gets vev  $v_i$ .

# The Model

At one-loop level neutrino mass can be radiatively generated by the following diagram:



Relevant part of the  $S3$  conserving potential at the scalar four-point vertex:

$$V_{relevant} \supset \lambda_1 \left[ \left\{ (\eta_2^\dagger \phi_2 + \eta_1^\dagger \phi_1)^2 \right\} + h.c. \right] + \lambda_2 \left[ \left\{ (\eta_2^\dagger \phi_2 - \eta_1^\dagger \phi_1)^2 \right\} + h.c. \right] \\ + \lambda_3 \left[ \left\{ (\eta_1^\dagger \phi_2)(\eta_2^\dagger \phi_1) + (\eta_2^\dagger \phi_1)(\eta_1^\dagger \phi_2) \right\} + h.c. \right].$$

From  $S3$  conservation we get:

- The Yukawa vertices conserving  $S3 \times Z2$  is given by:

$$\mathcal{L}_{Yukawa} = y_1 \left[ (\overline{N}_{2R} \eta_2^0 + \overline{N}_{1R} \eta_1^0) \nu_e \right] + y_2 \left[ (\overline{N}_{1R} \eta_2^0 + \overline{N}_{2R} \eta_1^0) \nu_\mu \right] + h.c.$$

- The direct mass term for the right-handed neutrinos:

$$\mathcal{L}_{right-handed\ neutrinos} = \frac{1}{2} m_{R12} \left[ N_{1R}^T C^{-1} N_{2R} + N_{2R}^T C^{-1} N_{1R} \right].$$

Introduce soft  $S3$  breaking terms:

$$\mathcal{L}_{soft} = \frac{1}{2} \left[ m_{R11} N_{1R}^T C^{-1} N_{1R} + m_{R22} N_{2R}^T C^{-1} N_{2R} \right]$$

# The right-handed neutrino mass

This leads to right-handed neutrino Majorana mass matrix:

$$M_{\nu_R} = \frac{1}{2} \begin{pmatrix} m_{R11} & m_{R12} \\ m_{R12} & m_{R22} \end{pmatrix}.$$

- $m_{R11} = m_{R22} \Rightarrow$  structure of the left-handed neutrino mass matrix with  $\theta_{13} = 0$  and  $\theta_{23} = \pi/4$ .
- $m_{R11} \neq m_{R22}$  i.e., small shift from  $m_{R11} = m_{R22}$  can produce realistic mixings i.e., non-zero  $\theta_{13}$ , deviation of  $\theta_{23}$  from maximality and small corrections to the solar mixing.

Let the average mass of the right-handed neutrinos be given by  $m_R$  and  $m_0$  is the common mass of the  $\eta_i$  fields.

If  $m_R^2 \gg m_0^2$ , the one-loop diagram gives the following contribution to the left-handed Majorana neutrino mass matrix:

$$(M_{\nu_L}^{flavour})_{22} = \lambda \frac{v_m v_n}{8\pi^2} \frac{y_2^2}{m_{R22}} [\ln z - 1] \text{ and } (M_{\nu_L}^{flavour})_{23} = \lambda \frac{v_m v_n}{8\pi^2} \frac{y_2^2 m_{R12}}{m_{R11} m_{R22}} [\ln z - 1].$$

where  $z \equiv \frac{m_R^2}{m_0^2}$  and  $y_i$  are the Yukawa couplings. Similarly, one can write the expressions for (1,1), (1,2) and (1,3) entries.

Neglecting  $M_\alpha$  dependence of  $z_\alpha$  one can absorb everything else in the loop expression other than the vevs and the quartic couplings in right-handed loop contributing factors  $r_{\alpha\beta}$  given by:

$$r_{11} \equiv \frac{1}{8\pi^2 m_{R11}} [\ln z - 1], \quad r_{22} \equiv \frac{1}{8\pi^2 m_{R22}} [\ln z - 1] \text{ and } r_{12} \equiv \frac{m_{R12}}{8\pi^2 m_{R11} m_{R22}} [\ln z - 1].$$

# The left-handed neutrino mass matrix

Thus,  $m_{R11} = m_{R22} \Rightarrow r_{11} = r_{22} = r$  and  $m_{R11} \neq m_{R22} \Rightarrow r_{11} \neq r_{22}$  i.e.,  $r_{22} = r_{11} + \epsilon$ .

Let us consider  $r_{11} \neq r_{22}$  and  $v_1 \neq v_2$  first and obtain the most general left-handed Majorana neutrino mass matrix as:

$$M_{\nu L}^{flavour} = \begin{pmatrix} \chi_1 & \chi_4 & \chi_5 \\ \chi_4 & \chi_2 & \chi_6 \\ \chi_5 & \chi_6 & \chi_3 \end{pmatrix}$$

with,

$$\begin{aligned} \chi_1 &\equiv y_1^2 [4r_{12}v_1v_2(\lambda_3 + \lambda_1 - \lambda_2) + (r_{11}v_1^2 + r_{22}v_2^2)(\lambda_1 + \lambda_2)] \\ \chi_2 &\equiv y_2^2 [r_{22}(\lambda_1 + \lambda_2)v_1^2] \\ \chi_3 &\equiv y_2^2 [r_{11}(\lambda_1 + \lambda_2)v_2^2] \\ \chi_4 &\equiv y_1y_2 [r_{12}(\lambda_1 + \lambda_2)v_1^2 + 2r_{22}(\lambda_3 + \lambda_1 - \lambda_2)v_1v_2] \\ \chi_5 &\equiv y_1y_2 [r_{12}(\lambda_1 + \lambda_2)v_2^2 + 2r_{11}(\lambda_3 + \lambda_1 - \lambda_2)v_1v_2] \\ \chi_6 &\equiv y_2^2 [2r_{12}(\lambda_3 + \lambda_1 - \lambda_2)v_1v_2]. \end{aligned}$$

**For  $v_1 = v_2 = v$  and  $r_{11} = r_{22} = r$  i.e.,  $m_{R11} = m_{R22}$  we get:**

$$M_{\nu L}^{flavour} = v^2 \begin{pmatrix} y_1^2 [4r_{12}\lambda_{123} + 2r\lambda_{12}] & y_1y_2 [r_{12}\lambda_{12} + 2r\lambda_{123}] & y_1y_2 [r_{12}\lambda_{12} + 2r\lambda_{123}] \\ y_1y_2 [r_{12}\lambda_{12} + 2r\lambda_{123}] & y_2^2 r\lambda_{12} & y_2^2 (2r_{12}\lambda_{123}) \\ y_1y_2 [r_{12}\lambda_{12} + 2r\lambda_{123}] & y_2^2 (2r_{12}\lambda_{123}) & y_2^2 r\lambda_{12} \end{pmatrix}$$

This corresponds to  $\theta_{13} = 0$  and  $\theta_{23} = \pi/4$

Here,  $\lambda_{12} = \lambda_1 + \lambda_2$  and  $\lambda_{123} = \lambda_3 + \lambda_1 - \lambda_2$ .

Recall, for  $\theta_{13} = 0$  and  $\theta_{23} = \pi/4$ :

$$M_{\nu L}^{flavour} = \begin{pmatrix} a & c & c \\ c & b & d \\ c & d & b \end{pmatrix}.$$



# Results

For  $v_1 = v_2 = v$  and  $r_{22} = r_{11} + \epsilon$  we get:  $M_{\nu L}^{flavour} = M^0 + M'$  where,

$$M^0 = \begin{pmatrix} y_1^2[4r_{12}\lambda_{123} + 2r_{11}\lambda_{12}] & y_1y_2[r_{12}\lambda_{12} + 2r_{11}\lambda_{123}] & y_1y_2[r_{12}\lambda_{12} + 2r_{11}\lambda_{123}] \\ y_1y_2[r_{12}\lambda_{12} + 2r_{11}\lambda_{123}] & y_2^2r_{11}\lambda_{12} & y_2^2(2r_{12}\lambda_{123}) \\ y_1y_2[r_{12}\lambda_{12} + 2r_{11}\lambda_{123}] & y_2^2(2r_{12}\lambda_{123}) & y_2^2r_{11}\lambda_{12} \end{pmatrix} \text{ and}$$

$$M' = \epsilon \begin{pmatrix} x & y & 0 \\ y & x' & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Here,  $x = y_1^2v^2$ ,  $x' = y_2^2v^2$  and  $y = y_1y_2v^2\lambda_{123}$ . Note,  $M^0$  and  $M'$  are symmetric owing to the Majorana nature and we define  $M_{11}^0 \equiv a'$ ,  $M_{22}^0 = M_{33}^0 \equiv b'$ ,  $M_{12}^0 = M_{13}^0 \equiv c'$  and  $M_{23}^0 \equiv d'$ .

Thus  $M^0$  corresponds to  $\theta_{13} = 0$  and  $\theta_{23} = \pi/4$ .  $M'$  gives  $\theta_{13} \neq 0$  and  $\theta_{23} \neq \pi/4$  and small corrections to solar mixing angle.

The third first-order corrected ket: 
$$|\psi_3\rangle = \begin{pmatrix} \frac{\epsilon}{\gamma^2 - \rho^2} [\rho(\sqrt{2}y \cos 2\theta_{12}^0 - x' \sin 2\theta_{12}^0) - \gamma\sqrt{2}y] \\ -\frac{1}{\sqrt{2}}[1 + \xi\epsilon] \\ \frac{1}{\sqrt{2}}[1 - \xi\epsilon] \end{pmatrix}.$$

Thus, 
$$\sin \theta_{13} = \frac{\epsilon}{\gamma^2 - \rho^2} [\rho(\sqrt{2}y \cos 2\theta_{12}^0 - x' \sin 2\theta_{12}^0) - \gamma\sqrt{2}y],$$

$$\tan \varphi \equiv \tan(\theta_{23} - \pi/4) = \xi\epsilon,$$

$$\tan \theta_{12} = \frac{\sin \theta_{12}^0 + \epsilon\zeta \cos \theta_{12}^0}{\cos \theta_{12}^0 - \epsilon\zeta \sin \theta_{12}^0}.$$

with, 
$$\gamma \equiv (b' - 3d' - a') \text{ and } \rho \equiv \sqrt{a'^2 + b'^2 + 8c'^2 + d'^2 - 2a'b' - 2a'd' + 2b'd'},$$

$$\xi \equiv [\gamma x' + \rho(x' \cos 2\theta_{12}^0 + \sqrt{2}y \sin 2\theta_{12}^0)]/(\gamma^2 - \rho^2),$$

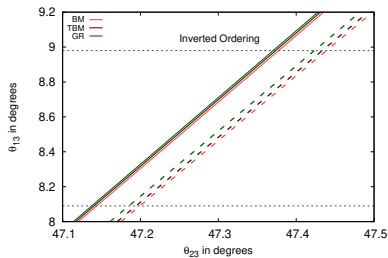
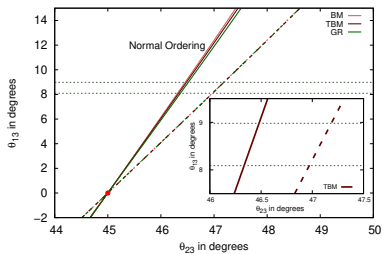
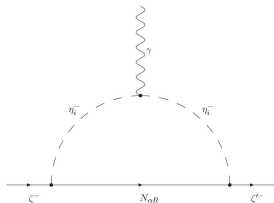
$$\zeta \equiv \frac{[\frac{y}{\sqrt{2}} \cos 2\theta_{12}^0 + \frac{1}{2}(x - \frac{x'}{2}) \sin 2\theta_{12}^0]}{\rho}.$$

# Conclusions

- Neutrino masses and mixings in general.
- A short discussion on discrete flavour symmetry:  $S_3$ .
- Realistic neutrino mixing radiatively at one-loop level using  $S_3 \times Z_2$  symmetry.
- Two right-handed neutrinos, maximally mixed to produce the structure of the left-handed Majorana neutrino mass matrix characterized by  $\theta_{13} = 0$ ,  $\theta_{23} = \pi/4$  and any value of  $\theta_{12}^0$  particular to the Tribimaximal (TBM), Bimaximal (BM) and Golden Ratio (GR) or other mixings.
- Small deviation from this maximal mixing between the two right-handed neutrinos could generate non-zero  $\theta_{13}$ , shifts of the atmospheric mixing angle  $\theta_{23}$  from  $\pi/4$  and correct the solar mixing angle  $\theta_{12}$  by a small amount.
- Two  $Z_2$  odd inert scalar  $SU(2)_L$  doublets were used, the lightest of which can serve as a dark matter candidate.
- Flavour violating decays of the charged leptons are forbidden by the  $S_3$  symmetry.

*Thank you*

# *Backup Slides*



# Scalar Potential

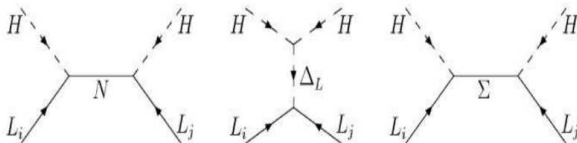
The scalar sector of the model as can be seen from Table. 2, comprises of two inert  $SU(2)_L$  doublets,  $\eta_i \equiv (\eta_i^+ \eta_i^0)^T$ , ( $i = 1, 2$ ), forming a doublet under  $S3$  denoted by  $\eta$  and two other  $SU(2)_L$  doublet scalar fields  $\Phi_j \equiv (\phi_j^+ \phi_j^0)^T$ , ( $j = 1, 2$ ), represented by  $\Phi$ , transforming as a doublet under  $S3$ . Under the unbroken  $Z_2$ ,  $\eta$  is odd whereas  $\Phi$  is even. Thus after SSB,  $\phi_j^0$  can acquire vevs  $v_j$ , ( $j = 1, 2$ ), but the  $\eta_i^0$  cannot. The complete scalar potential consisting of all the terms allowed by the SM gauge symmetry and  $S3 \times Z_2$  is given by:

$$\begin{aligned}
 V_{total} = & m_\eta^2 \left( \eta_2^\dagger \eta_2 + \eta_1^\dagger \eta_1 \right) + m_\phi^2 \left( \phi_2^\dagger \phi_2 + \phi_1^\dagger \phi_1 \right) \\
 & + \tilde{\lambda}_1 \left( \eta_2^\dagger \eta_2 + \eta_1^\dagger \eta_1 \right)^2 + \tilde{\lambda}_2 \left( \eta_2^\dagger \eta_2 - \eta_1^\dagger \eta_1 \right)^2 + \tilde{\lambda}_3 \left( \phi_2^\dagger \phi_2 + \phi_1^\dagger \phi_1 \right)^2 + \tilde{\lambda}_4 \left( \phi_2^\dagger \phi_2 - \phi_1^\dagger \phi_1 \right)^2 \\
 & + \tilde{\lambda}_5 \left[ \left( \eta_2^\dagger \eta_2 + \eta_1^\dagger \eta_1 \right) \left( \phi_2^\dagger \phi_2 + \phi_1^\dagger \phi_1 \right) \right] + \tilde{\lambda}_6 \left[ \left( \eta_2^\dagger \eta_2 - \eta_1^\dagger \eta_1 \right) \left( \phi_2^\dagger \phi_2 - \phi_1^\dagger \phi_1 \right) \right] \\
 & + \tilde{\lambda}_7 \left[ \left( \phi_1^\dagger \phi_2 \right) \left( \phi_2^\dagger \phi_1 \right) \right] + \tilde{\lambda}_8 \left[ \left( \eta_1^\dagger \eta_2 \right) \left( \eta_2^\dagger \eta_1 \right) \right] \\
 & + \tilde{\lambda}_9 \left[ \left\{ \left( \phi_1^\dagger \phi_2 \right) \left( \eta_2^\dagger \eta_1 \right) \right\} + \left\{ \left( \phi_2^\dagger \phi_1 \right) \left( \eta_1^\dagger \eta_2 \right) \right\} \right] + V_{relevant}
 \end{aligned} \tag{B.1}$$

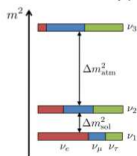
where,

$$\begin{aligned}
 V_{relevant} = & \lambda_1 \left[ \left\{ \left( \eta_2^\dagger \phi_2 + \eta_1^\dagger \phi_1 \right)^2 \right\} + h.c. \right] + \lambda_2 \left[ \left\{ \left( \eta_2^\dagger \phi_2 - \eta_1^\dagger \phi_1 \right)^2 \right\} + h.c. \right] \\
 & + \lambda_3 \left[ \left\{ \left( \eta_1^\dagger \phi_2 \right) \left( \eta_2^\dagger \phi_1 \right) + \left( \eta_2^\dagger \phi_1 \right) \left( \eta_1^\dagger \phi_2 \right) \right\} + h.c. \right].
 \end{aligned} \tag{B.2}$$

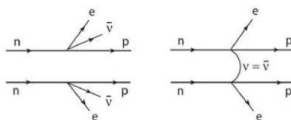
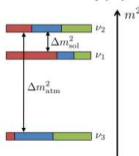
Since at the four-point scalar vertex in Fig. 1, two  $\phi$  are destroyed and two  $\eta$  are created, the terms only of  $(\eta^\dagger \phi)(\eta^\dagger \phi)$  type play a crucial role in determining the neutrino mass matrix. Thus we call these terms as the relevant part of the scalar potential, represented by  $V_{relevant}$  in Eq. (B.2). The quartic couplings  $\lambda_j$  ( $j = 1, 2, 3$ ) appearing in Eq. (B.2) were taken to be real for the analysis.



normal hierarchy (NH)



inverted hierarchy (IH)



**Golden Ratio:**

$$\frac{a+b}{a} = \frac{a}{b} \stackrel{\text{def}}{=} \varphi, \quad \varphi = \frac{1+\sqrt{5}}{2} = 1.6180339887$$

$$\theta_{12} = 31.7^\circ \text{ for GR mixing} \Rightarrow \frac{\cos 31.7^\circ}{\sin 31.7^\circ} = 1.618..$$

# Seesaw in brief

Extend the *SM* by a singlet *RH* neutrino  $N_R$  per family.

$$\text{Neutrino Majorana mass term: } m\psi_{L(R)}^T C^{-1}\psi_{L(R)}$$

↓

$$\mathcal{L}_{mass} = \frac{1}{2}\alpha_L^T C^{-1} \mathcal{M}_{D+M} \alpha_L + h.c.$$

where,  $\mathcal{M}_{D+M} = \begin{pmatrix} 0 & M_D^T \\ M_D & M_R \end{pmatrix} \Rightarrow 6 \times 6$  matrix and  $\alpha_L = \begin{pmatrix} \nu_L \\ C(\bar{N}_R)^T \end{pmatrix}$

Diagonalize:  $W^T \mathcal{M}_{D+M} W = \begin{pmatrix} M_{light} & 0 \\ 0 & M_{heavy} \end{pmatrix}$

$$M_{light} = M_D^T M_R^{-1} M_D \quad \text{and} \quad M_{heavy} = M_R$$



⇒



Can be done in three ways:

**Type I**

Fermion Singlet

**Type II**

Scalar Triplet

**Type III**

Fermion Triplet



# Popular lepton mixings

**Recall:**

$$U_{PMNS} \equiv V_l^\dagger U_\nu = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}.$$

$$\downarrow$$

$\theta_{13}=0, \theta_{23}=\pi/4$

$$\downarrow$$

$$U^0 = \begin{pmatrix} \cos \theta_{12}^0 & \sin \theta_{12}^0 & 0 \\ -\frac{\sin \theta_{12}^0}{\sqrt{2}} & \frac{\cos \theta_{12}^0}{\sqrt{2}} & \sqrt{\frac{1}{2}} \\ \frac{\sin \theta_{12}^0}{\sqrt{2}} & -\frac{\cos \theta_{12}^0}{\sqrt{2}} & \sqrt{\frac{1}{2}} \end{pmatrix} \rightarrow \theta_{12}^0 = 0^\circ (\text{NSM}) \rightarrow U^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

$\downarrow$

Studied in A4 case

$\theta_{12}^0$  :  $35.26^\circ$  [TriBimaximal(TBM)],  $45^\circ$  [Bimaximal (BM)],  $31.7^\circ$  [Golden Ratio(GR)].

$$\begin{aligned} \Delta m_{21}^2 &= (7.02 - 8.08) \times 10^{-5} \text{ eV}^2, \quad \theta_{12} = (31.52 - 36.18)^\circ, \\ |\Delta m_{31}^2| &= (2.351 - 2.618) \times 10^{-3} \text{ eV}^2, \quad \theta_{23} = (38.6 - 53.1)^\circ, \\ \theta_{13} &= (7.86 - 9.11)^\circ, \quad \delta = (0 - 360)^\circ. \end{aligned}$$

**Amendment Required!!**

# Neutrino Oscillations

A Purely Quantum Mechanical Phenomenon.

Oscillation conserves probability hence *Hamiltonian* is *Hermitian*.  $\Rightarrow$  Diagonalized by Unitary transformation ( $U$ ).

**Consider two flavors oscillation first:**

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \xRightarrow{U_{2 \times 2}} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

Flavor Eigenstates

Stationary States

**Flavor Eigenstates:** Participate in weak interactions

**Stationary States:** Mass eigenstates and are admixtures of the flavor states.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$P_{\nu_e \rightarrow \nu_e}(t)$ : **Probability of  $\nu_e$  emitted from the source to remain an  $\nu_e$  after time  $t$ .**

Calculate Time Evolution :

$$|\nu_i(t)\rangle = e^{-iE_i t} |\nu_i(0)\rangle \quad (i = 1, 2) \text{ in Natural units.}$$

# Oscillation continued ...

$$P_{\nu_e \rightarrow \nu_e}(t) \equiv |\langle \nu_e(t) | \nu_e(0) \rangle|^2 = 1 - \sin^2 2\theta \sin\left(\frac{\Delta m^2 L}{4E}\right)$$

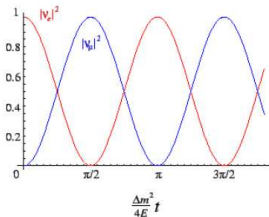
$L \rightarrow$  distance between source and detector.

In ultra relativistic limit:  $E_i - E_j = (m_i^2 - m_j^2)/2E = \frac{\Delta m^2}{2E}$

$$P_{\nu_e \rightarrow \nu_\mu}(t) = 1 - P_{\nu_e \rightarrow \nu_e}(t) = \sin^2 2\theta \sin\left(\frac{\Delta m^2 L}{4E}\right)$$

**Necessary Requirements:**  $\Delta m^2 \neq 0$  and  $\sin^2 2\theta \neq 0$ .

**Maximal mixing when  $\theta = \frac{\pi}{4}$ .**



# Three flavour Oscillation

Oscillation with three flavors:  $\nu_e, \nu_\mu, \nu_\tau$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

↓

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

The Pontecorvo, Maki, Nakagawa, Sakata – PMNS – matrix

$$U^T M_\nu U = \text{diag}(m_1, m_2, m_3)$$

Oscillation probability

$$P_{\nu_\alpha \nu_\beta} = \delta_{\alpha\beta} - 4 \sum_{j>i} U_{\alpha i} U_{\beta i} U_{\alpha j}^* U_{\beta j}^* \sin^2 \left( \frac{\pi L}{\lambda_{ij}} \right)$$

2 independent  $\Delta m^2$ , 3 mixing angles, 1 phase