Neutrino oscillations in extended theories of gravity

Luciano Petruzziello

Università degli Studi di Salerno, Dipartimento di Fisica "E. R. Caianiello"
INFN Sezione di Napoli, Gruppo collegato di Salerno

Beyond Standard Model: From Theory to Experiment 2021

Outline

- Quadratic theories of gravity
- Neutrino oscillations in curved spacetime
- Strong equivalence principle violation
- Future perspectives

Quadratic theories of gravity

The most general gravitational Lagrangian quadratic in the curvature invariants*

$$\mathcal{L} = \frac{\sqrt{-g}}{2\kappa^2}\left\{\mathcal{R} + \frac{1}{2}\left[\mathcal{R}\mathcal{F}_1(\Box)\mathcal{R} + \mathcal{R}_{\mu\nu}\mathcal{F}_2(\Box)\mathcal{R}^{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{F}_3(\Box)\mathcal{R}^{\mu\nu\rho\sigma}\right]\right\}$$

For a suitable choice of the form factors we have:

- Non-locality
- UV completion of General Relativity valid at all energy scales
- Freedom from ghost fields (i.e. avoids unitarity problem)

---

*T. Biswas, E. Gerwick, T. Koivisto and A. Mazumdar, Phys. Rev. Lett. (2012)

Linearized regime

In the limit

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

The Lagrangian then becomes

$$\mathcal{L} = \frac{1}{4}\Big\{\frac{1}{2}h_{\mu\nu}f(\Box)\Box h^{\mu\nu} - h^{\sigma}_{\mu}f(\Box)\partial_{\sigma}\partial_{\nu}h^{\mu\nu} + h\,g(\Box)\partial_{\mu}\partial_{\nu}h^{\mu\nu}$$
$$-\frac{1}{2}h\,g(\Box)\Box h + \frac{1}{2}h^{\lambda\sigma}\frac{f(\Box)-g(\Box)}{\Box}\partial_{\lambda}\partial_{\sigma}\partial_{\mu}\partial_{\nu}h^{\mu\nu}\Big\}$$

$$h = \eta_{\mu\nu}h^{\mu\nu} \quad f(\Box) = 1 + \frac{1}{2}\mathcal{F}_2(\Box)\Box \quad g(\Box) = 1 - 2\mathcal{F}_1(\Box)\Box - \frac{1}{2}\mathcal{F}_2(\Box)\Box$$

Solutions

For a static point-like source of gravity

$$T_{\mu\nu} = m\delta_\mu^0\delta_\nu^0\delta^{(3)}(\mathbf{r})$$

$$ds^2 = (1+2\phi)dt^2 - (1-2\psi)(dr^2 + r^2 d\Omega^2)$$

$$\phi(r) = -\frac{4Gm}{\pi r}\int_0^\infty \frac{f-2g}{f(f-3g)}\frac{\sin(kr)}{k}\,dk$$

$$\psi(r) = \frac{4Gm}{\pi r}\int_0^\infty \frac{g}{f(f-3g)}\frac{\sin(kr)}{k}\,dk$$

$$f = g = 1 \Rightarrow \phi = \psi = -\frac{Gm}{r} \Rightarrow \mathrm{GR}$$

Neutrino oscillations

According to the standard picture*

$$|\nu_\alpha\rangle = \sum_{k=1,2} U_{\alpha k}(\theta)\,|\nu_k\rangle$$

In a semiclassical fashion

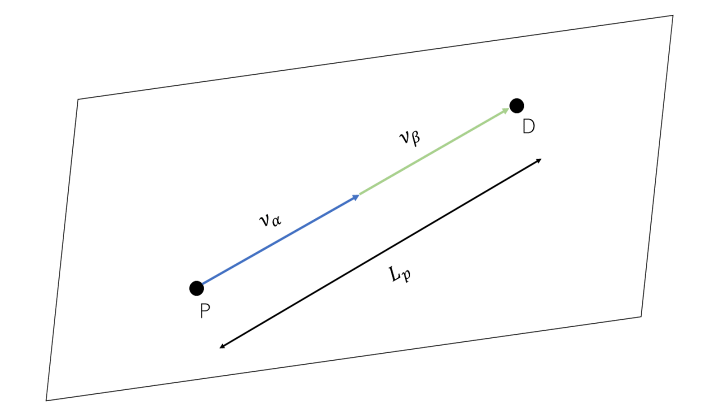
$$|\nu_k(x)\rangle = \exp\left[-i\varphi_k(x)\right]|\nu_k\rangle, \qquad \varphi_k = \int_{\lambda_P}^{\lambda_D} P_{\mu,k}\frac{dx^\mu_{null}}{d\lambda}d\lambda$$

The flavor transition probability

$$\mathcal{P}_{\alpha\to\beta} = \left|\langle\nu_\beta(t_D,\mathbf{x}_D)|\nu_\alpha(t_P,\mathbf{x}_P)\rangle\right|^2 = \sin^2(2\theta)\,\sin^2\left(\frac{\varphi_{12}}{2}\right)$$

*S. M. Bilenky and B. Pontecorvo, Phys. Rep. (1978)

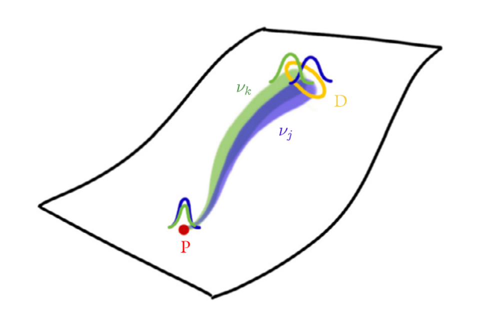
Setup (flat case)



$$U(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \qquad \varphi_{12} = \frac{\Delta m^2}{2E} L_p \qquad \Delta m^2 = m_2^2 - m_1^2$$

Setup (Schwarzschild case)

In curved spacetime the situation is different*



$$\varphi_{12} = \varphi_0 + \varphi_{GR}$$

$$\varphi_0 = \frac{\Delta m^2}{2E_\ell} L_p$$

$$\varphi_{GR} = \frac{\Delta m^2 L_p}{2E_\ell}\left[\frac{Gm}{r_D} - \frac{Gm}{L_p}\ln\left(\frac{r_D}{r_P}\right)\right]$$

*C. Y. Cardall and G. M. Fuller, Phys. Rev. D (1997)

Extended model scenario

If the spacetime is not simply described by GR*

$$\varphi_{12} = \varphi_0 + \varphi_{GR} + \varphi_Q$$

Assuming $\phi = \phi_{GR} + \phi_Q$ and $\psi = \psi_{GR} + \psi_Q = \phi_{GR} + \psi_Q$

$$\varphi_Q \;=\; \frac{\Delta m^2 L_p}{2E_\ell}\left[\frac{1}{L_p}\int_{r_P}^{r_D}\phi_Q(r)\,dr - \phi_Q(r_D)\right]$$

Equivalently, for an observer at infinity $E_\ell = \; e^0{}_{\hat{0}}E$

$$\varphi_{12} = \frac{\Delta m^2}{2E}\int_{r_P}^{r_D}\left[1 + \phi(r) - \psi(r)\right]dr$$

---

*L. Buoninfante, G. G. Luciano, L. Petruzziello and L. Smaldone, Phys. Rev. D (2020)

Some examples

- $\mathcal{R}^2$ gravity ($\mathcal{F}_1 = \alpha,\ \mathcal{F}_2 = 0$)

$$\varphi_Q = \frac{\Delta m^2 L_p}{2E_\ell} \left\{ \frac{Gm\, e^{-m_0 r_D}}{3r_D} - \frac{Gm}{3L_p} \left[\mathrm{Ei}\left(-m_0 r\right) \right]_{r_P}^{r_D} \right\}$$

- Fourth-order gravity ($\mathcal{F}_1 = \alpha,\ \mathcal{F}_2 = \beta$)

$$\begin{aligned} \varphi_Q &= \frac{\Delta m^2 L_p}{2E_\ell} \left\{ \frac{Gm\, e^{-m_0 r_D}}{3r_D} - \frac{4Gm\, e^{-m_2 r_D}}{3r_D} \right. \\ &- \left. \frac{Gm}{3L_p} \left[\mathrm{Ei}\left(-m_0 r\right) \right]_{r_P}^{r_D} + \frac{4Gm}{3L_p} \left[\mathrm{Ei}\left(-m_2 r\right) \right]_{r_P}^{r_D} \right\} \end{aligned}$$

Some examples (2)

- Infinite derivative gravity ($\mathcal{F}_1 = -\frac{1}{2}\mathcal{F}_2 = \frac{1-e^{\Box/M_s^2}}{2\,\Box}$)

$$\varphi_Q = \frac{\Delta m^2 L_p}{2E_\ell}\left\{-\frac{Gm}{r_B}\mathrm{Erfc}\left[\frac{M_s r_D}{2}\right] + \frac{Gm}{L_p}\ln\left(\frac{r_D}{r_P}\right) - \frac{Gm}{L_p}\left[\frac{M_s r}{\sqrt{\pi}}\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{M_s^2 r^2}{4}\right)\right]_{r_P}^{r_D}\right\}$$

- IR modification of GR ($\mathcal{F}_1 = \frac{\alpha}{\Box},\ \mathcal{F}_2 = 0$)

$$\varphi_Q = \frac{\alpha}{3\alpha - 1}\varphi_{GR}.$$

How to constrain extended models

Due to the linearized approximation

$$|\varphi_0| > |\varphi_{GR}|$$

Due to gravitational phenomenology

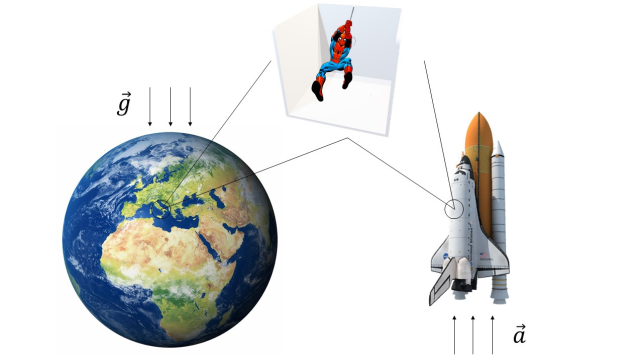
$$|\varphi_{GR}| \gtrsim |\varphi_Q|$$

Therefore

$$\left|\frac{\varphi_Q}{\varphi_0}\right| < 1$$

Physics entails a restriction on the free parameters of the extended theories

Weak Equivalence Principle (WEP)



$$|\mathbf{a}| = |\mathbf{g}| \quad \Longrightarrow \quad m_i = m_g$$

$$\lambda \equiv \frac{Gm}{rc^2} \ll 1$$

Strong Equivalence Principle (SEP)

Simultaneous requirements*

$$\text{SEP} = \text{GWEP} + \text{LLI} + \text{LPI}$$

In order to quantify SEP, it is possible to resort to the PPN formalism†

Parameter	Meaning	Value in GR
γ	Space curvature produced by unit rest mass	1
β	Nonlinearity effects for gravity	1
ξ	Preferred-location effects	0
α_1		0
α_2	Preferred-frame effects	0
α_3		0
α_3		0
ζ_1		0
ζ_2	Violation of total momentum conservation	0
ζ_3		0
ζ_4		0

Nordtvedt parameter

$$\eta = 4(\beta - 1) - (\gamma - 1)$$

$$\eta \neq 0 \Longleftrightarrow \text{No SEP}$$

*C. M. Will, Living Rev. Rel. (2006)
†S. Weinberg, *Gravitation and Cosmology* (1972)

Neutrino oscillations as a witness of SEP

In the linearized regime

$$\beta = 1 \qquad \Longrightarrow \qquad \eta = \frac{\phi_Q - \psi_Q}{\phi}$$

$$\varphi_{12} = \frac{\Delta m^2}{2E} \int_{r_P}^{r_D} \left[1 + \phi(r)\,\eta(r)\right] dr$$

The flavor transition rate *does* depend on the hypothetical SEP violation

$$\eta \to 0 \Longrightarrow \mathcal{P}^{\text{curved}}_{\alpha\to\beta} \to \mathcal{P}^{\text{flat}}_{\alpha\to\beta}$$

Future perspective

- To go beyond the linearized approximation
- To go beyond the semiclassical approximation
- To further explore the interplay between mixed particles and gravity*
- To further explore the interplay between mixed particles and the equivalence principle†

*M. Blasone, G. Lambiase, G. G. Luciano and L. Petruzziello, Phys. Rev. D (2018); Phys. Lett. B (2020); Eur. Phys. J. C (2020); G. G. Luciano and L. Petruzziello, Int. J. Mod. Phys. D (2020); M. Blasone, G. Lambiase, G. G. Luciano, L. Petruzziello and L. Smaldone, Class. Quant. Grav. (2020)

†M. Blasone, P. Jizba, G. Lambiase and L. Petruzziello, Phys. Lett. B (2020)

BÏYAN
SHUKRIA
DANKSCHEEN
TINGKI
YAQHANYELAY
CHALTU
NUHUN
SPASSIBO
SNACHALHUYA
TASHAKKUR ATU
YUSPAGARÀTAM
THANK
GRACIAS
WABEEJA
MAITEKA
HUI
SUKSAMA
DHANYAVAAD
ANIHA
EKHMET
UNALCHÉESH
GUI
HATUR
MERSI
SPASIBO
DENKAUJA
ATTO
NENACHALHYA
EKOJU
SIKOMO
ARIGATO
SANCO
KOMAPSUMNIDA
MAAKE
GRAZIE
MEHRBANI
YOU
MAKETAI
SHUKURIA
MERASTAWHY
GAEJTHO
LAH
PALDIES
TAVTAPUCH
MEDAWAGSE
GOZAIMASHITA
JUSPAXAR
EFCHARISTO
AGUYJE
BAIKA
FAKAAUE
MINMONCHAR
BOLZÏN
MERCI