Neutrino masses and mixing in D_4 model

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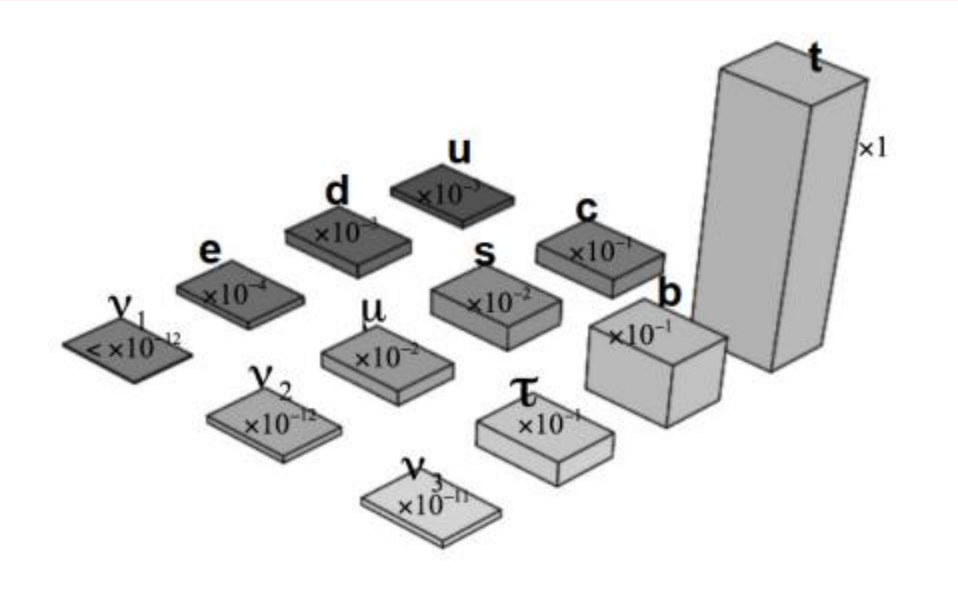
Beyond Standard Model: From Theory to Experiment-2021
CPE & FENS 31-03-2021

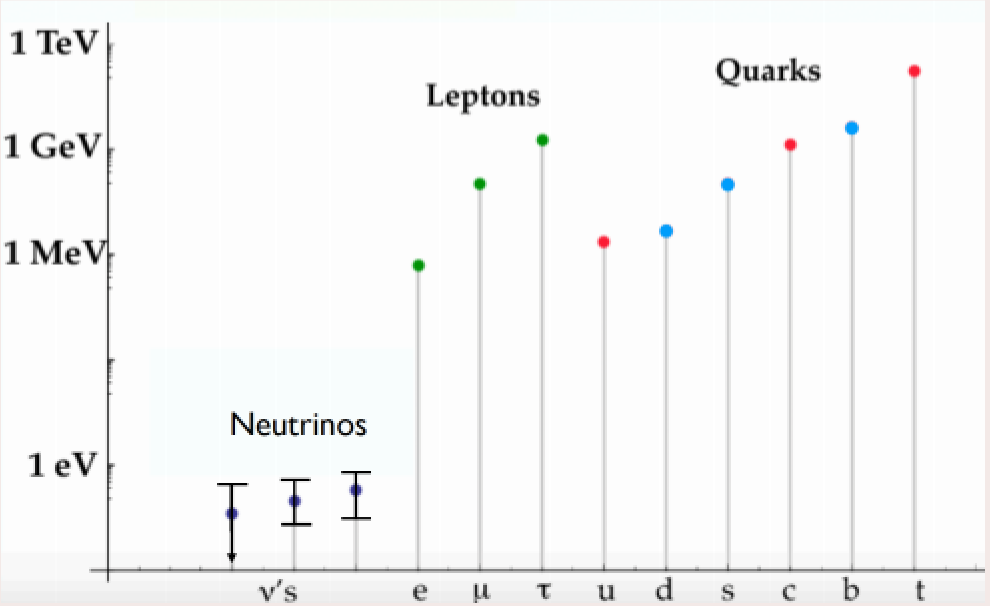
Beyond Standard Model

- Despite the standard model being the most successful theory of particle physics to date, going Beyond it is required
- Many of the unresolved problems combine with the so-called Flavor Problem.

Flavor Problem

Why such large hierarchy among fermion masses ?

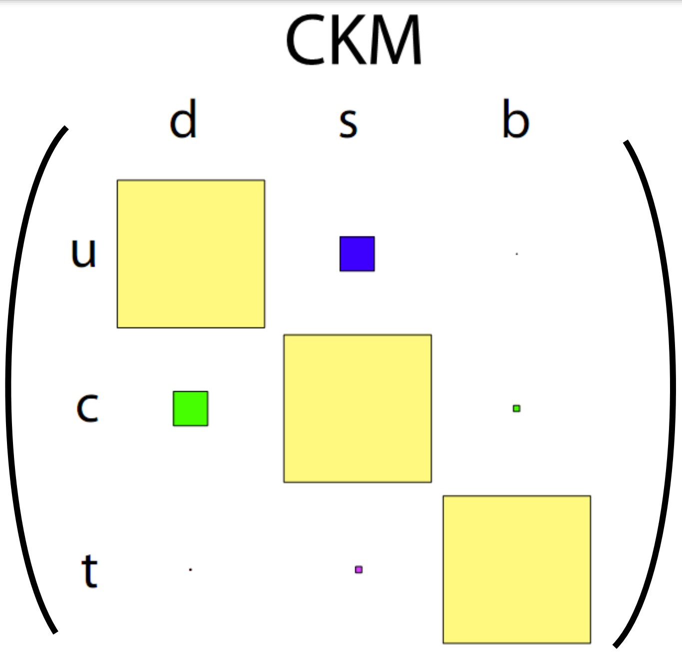


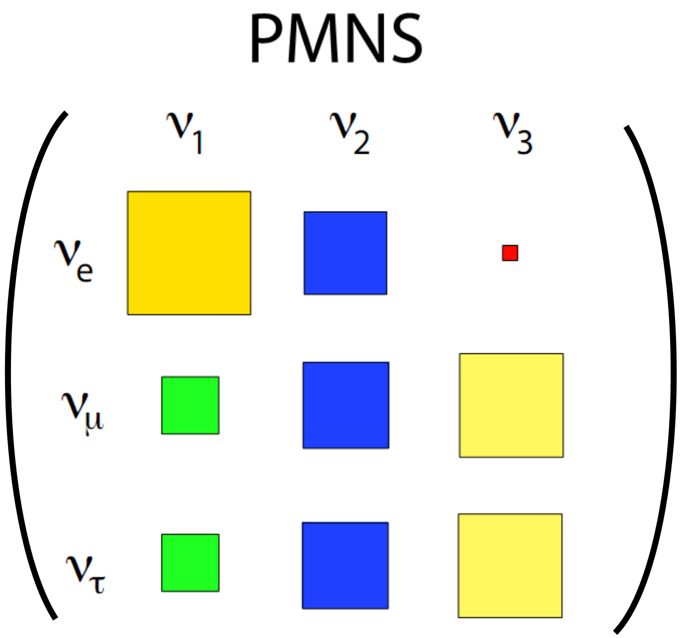


[S, King '17]

Flavor Problem

Why is flavor mixing in the quark sector small compared to the lepton sector?





[S, King '17]

They are so much different!

Flavor Problem

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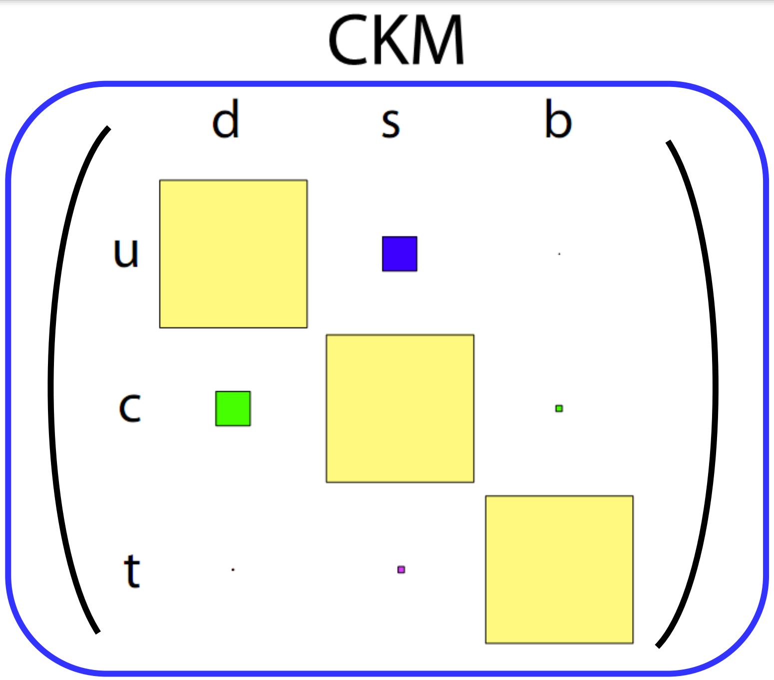
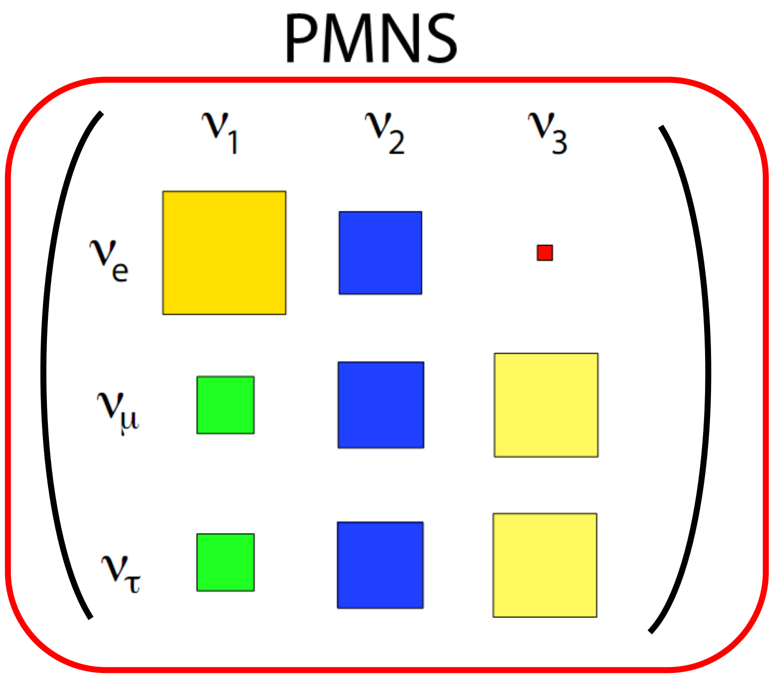




[S, King '17]

U_{CKM} is Close to the identity

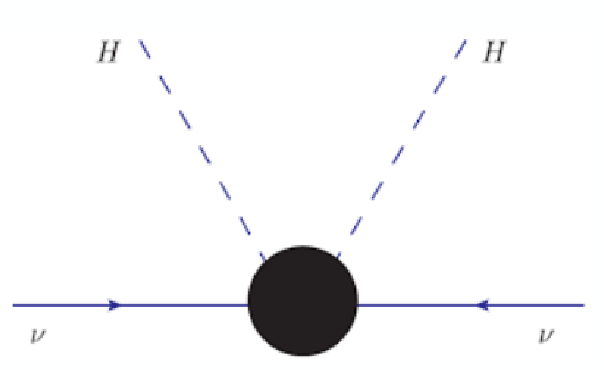
Large mixing

What the origine of neutrino masses?

Seesaw mechanism:

Seesaw mechanisms from Weinberg dimension 5 operator

- Type I seesaw → (Fermion Singlet N_R)
- Type II seesaw → (Scalar Triplet Δ)
- Type III seesaw → (Fermion Triplet Σ)



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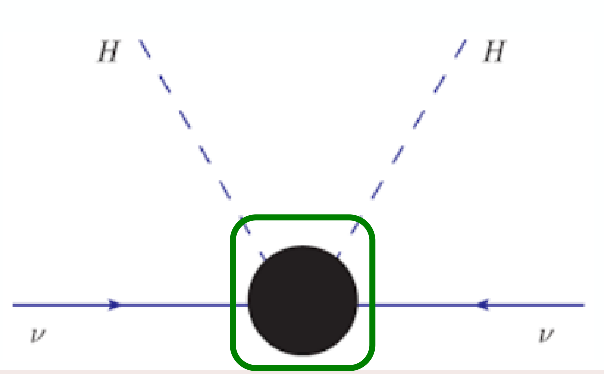
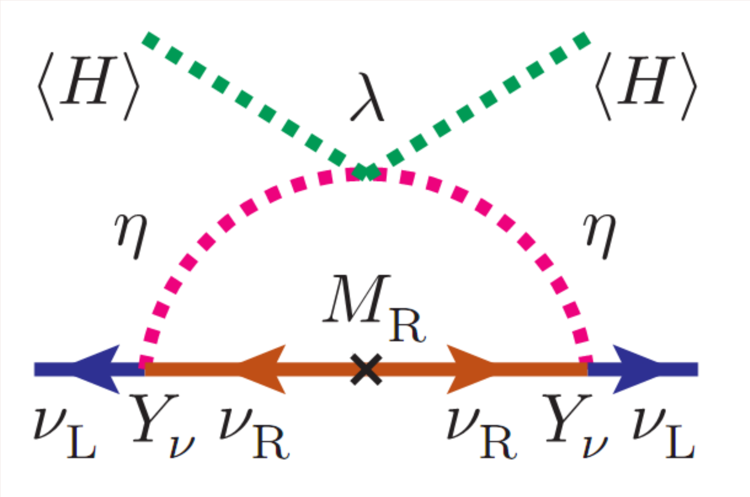
Superheavy Particles



What the origine of neutrino masses?

Radiative neutrino mass generation:

complete Weinberg operator via loops



[T, Ohlsson and S, Zhou 2013]

What the origine of neutrino masses?

Radiative neutrino mass generation:

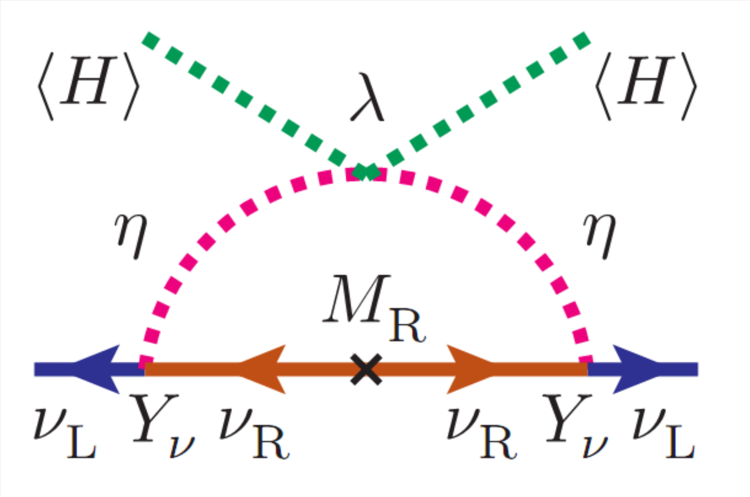
complete Weinberg operator via loops

- A canonical example: "scotogenic model"

[E, Ma 2006]

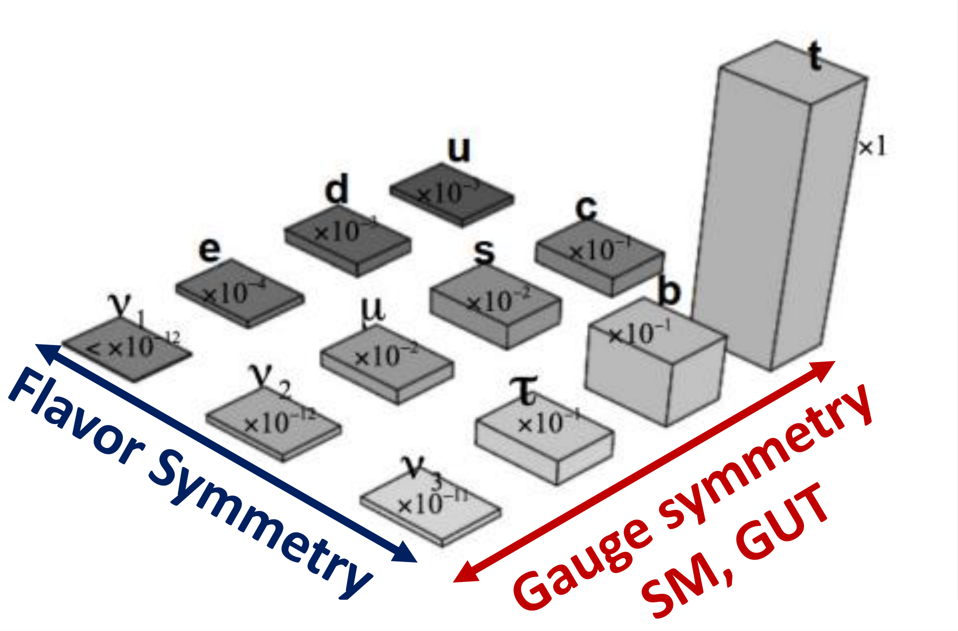
  - Introduce new electroweak doublet(s) and right-handed neutrinos

(new states can be DM candidates)



[T, Ohlsson and S, Zhou 2013]

Flavor Symmetries approach

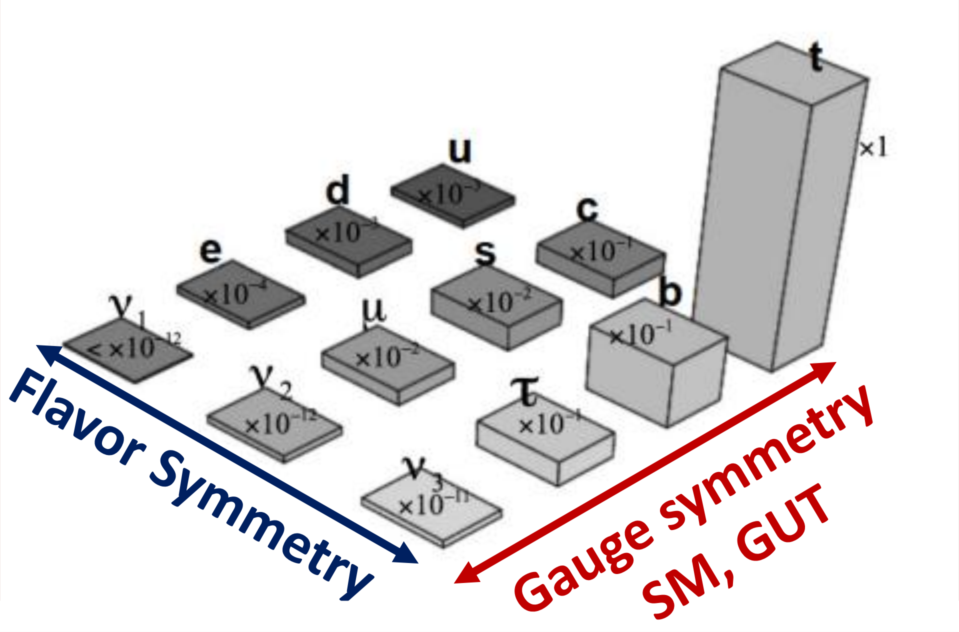


Beyond Standard model

↓ Needed

New Symmetries
+
New Particles

Flavor Symmetries approach



Beyond Standard model

Needed

New Symmetries
+
New Particles

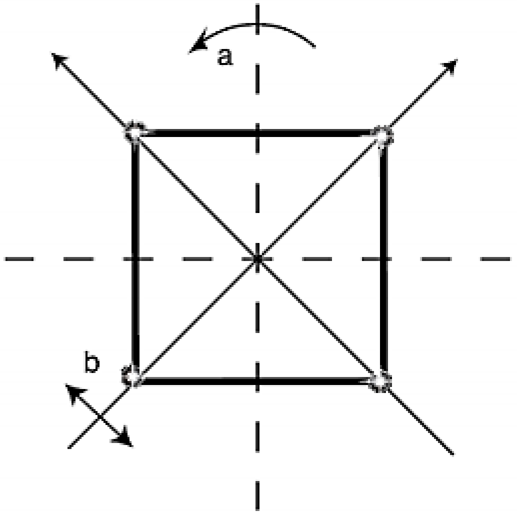
Flavor Symmetry

- <u>Non-abelian discrete groups as FS</u>

Finite groups with triplet representations (S_4, A_4, A_5) are often used in Flavor Symmetry based models.

D_4 is a group of square

$$b^2 = a^4 = Id$$



D_4 Irreducible representations

Four singlets $\mathbf{1}_{++}$, $\mathbf{1}_{+-}$, $\mathbf{1}_{-+}$ *and* $\mathbf{1}_{--}$
One doublet $\mathbf{2}_{00}$

$$\mathbf{2}_{00} \otimes \mathbf{2}_{00} = \mathbf{1}_{++} \oplus \mathbf{1}_{+-} \oplus \mathbf{1}_{-+} \oplus \mathbf{1}_{--}$$

Models based on D_4 flavor symmetry

D_4 models		Predictions on neutrino sector
$SU(3)_C x SU(2)_L x U(1)_Y x D_4 x Z_2$ [W, Grimus and L, Lavoura 2003]	Non-SUSY →	• Predicts $\mu-\tau$ symmetry in a natural way • Neutrino masses are generated from Type I seesaw mechanism

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Lepton mixing is described by the Tribimaximal mixing (TBM) pattern →

- The atmospheric angle is maximal $\theta_{23} = \frac{\pi}{4}$
- Vanishing reactor angle $\boldsymbol{\theta_{13}}=0$

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In **2012** Reactor angle $\boldsymbol{\theta_{13}}$ was measured by:
Double Chooz, Daya bay, RENO, MINOS
→
- Non-zero value of $\boldsymbol{\theta_{13}} \neq 0$
- Suggest that CP might be violated in lepton sector

Our D4xU(1) model

LH lepton RH lepton RH neutrino

Fields	L_e	(L_μ, L_τ)	l_e^c	l_μ^c	l_τ^c	N_1^c	$N_{2,3}^c$	H_u	H_d
D_4	$\mathbf{1}_{+,+}$	$\mathbf{2}_{0,0}$	$\mathbf{1}_{+,+}$	$\mathbf{1}_{+,+}$	$\mathbf{1}_{+,+}$	$\mathbf{1}_{+,+}$	$\mathbf{2}_{0,0}$	$\mathbf{1}_{+,+}$	$\mathbf{1}_{+,+}$
$U(1)$	2	2	1	-1	-2	-1	-1	-1	1

Matter fields Higgses

Our D4xU(1) model

LH lepton | RH lepton | RH neutrino

Fields	L_e	(L_μ, L_τ)	l^c_e	l^c_μ	l^c_τ	N^c_1	$N^c_{2,3}$	H_u	H_d
D_4	$\mathbf{1}_{+,+}$	$\mathbf{2}_{0,0}$	$\mathbf{1}_{+,+}$	$\mathbf{1}_{+,+}$	$\mathbf{1}_{+,+}$	$\mathbf{1}_{+,+}$	$\mathbf{2}_{0,0}$	$\mathbf{1}_{+,+}$	$\mathbf{1}_{+,+}$
$U(1)$	2	2	1	-1	-2	-1	-1	-1	1

Matter fields | Higgses

Flavons	ρ_1	ρ_2	ρ_3	η	σ
D_4	$\mathbf{1}_{+,+}$	$\mathbf{1}_{+,-}$	$\mathbf{1}_{-,-}$	$\mathbf{2}_{0,0}$	$\mathbf{2}_{0,0}$
$U(1)$	2	2	2	2	2

Flavon fields for neutrino sector

Our D4xU(1) model

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D_4	$\mathbf{1}_{+,+}$	$\mathbf{2}_{0,0}$	$\mathbf{1}_{+,+}$	$\mathbf{1}_{+,+}$	$\mathbf{1}_{+,+}$	$\mathbf{1}_{+,+}$	$\mathbf{2}_{0,0}$	$\mathbf{1}_{+,+}$	$\mathbf{1}_{+,+}$
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Matter fields — Higgses

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$U(1)$ ✓	2	2	2	2	2

Extra U(1) symmetry

- To separate the charged lepton and neutrino sector
- Accommodate Diagonal charged lepton mass matrix

Neutrino superpotential

The neutrino superpotential invariant under $D_4 \times U(1)$ reads as

$$\begin{aligned}\mathcal{W}_\nu = & \lambda_1 N_1^c L_e H_5 + \lambda_2 N_{2,3}^c L_{\mu,\tau} H_5 + \lambda_3 N_1^c N_1^c \rho_1 + \lambda_4 N_{2,3}^c N_{2,3}^c \rho_1 \\ & + \lambda_5 N_1^c N_{2,3}^c \eta + \lambda_7 N_{2,3}^c N_{2,3}^c \rho_2 + \lambda_6 N_1^c N_{2,3}^c \sigma + \lambda_8 N_{2,3}^c N_{2,3}^c \rho_3\end{aligned}$$

Vacuum alignment

Vacuum alignment required for the symmetry breaking pattern

$$\begin{aligned}\langle \rho_1 \rangle = v_{\rho_1} \quad , \quad \langle \rho_2 \rangle = v_{\rho_2} \quad , \quad \langle \rho_3 \rangle = v_{\rho_3} \\ \langle H_u \rangle = v_u \quad , \quad \langle \eta \rangle = (v_\eta, v_\eta)^T \quad , \quad \langle \sigma \rangle = (v_\sigma, 0)^T\end{aligned}$$

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$$\mathcal{W}_\nu = \boxed{\lambda_1 N_1^c L_e H_5 + \lambda_2 N_{2,3}^c L_{\mu,\tau} H_5} + \lambda_3 N_1^c N_1^c \rho_1 + \lambda_4 N_{2,3}^c N_{2,3}^c \rho_1 + \lambda_5 N_1^c N_{2,3}^c \eta + \lambda_7 N_{2,3}^c N_{2,3}^c \rho_2 + \lambda_6 N_1^c N_{2,3}^c \sigma + \lambda_8 N_{2,3}^c N_{2,3}^c \rho_3$$

- The Dirac mass matrix is obtained from the first two terms in $\boldsymbol{W_\nu}$

Dirac mass matrix

$$m_D = \upsilon_u \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_2 \end{pmatrix}$$

Neutrino superpotential

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$$+ \lambda_5 N_1^c N_{2,3}^c \eta + \lambda_7 N_{2,3}^c N_{2,3}^c \rho_2 + \lambda_6 N_1^c N_{2,3}^c \sigma + \lambda_8 N_{2,3}^c N_{2,3}^c \rho_3$$

The other terms lead to the Majorana mass matrix

Majorana mass matrix

$$m_M = \Lambda \begin{pmatrix} \mathbf{a} & \mathbf{b} & \mathbf{b}+\mathbf{k} \\ \mathbf{b} & \mathbf{k} & c \\ \mathbf{b}+\mathbf{k} & c & 0 \end{pmatrix}$$

Where

$$a = \frac{\lambda_3 v_{\rho_1}}{\Lambda}, \quad b = \frac{\lambda_5 v_\gamma}{\Lambda}$$

$$c = \frac{2\lambda_4 v_{\rho_1}}{\Lambda}, \quad \mathrm{k} = \frac{\lambda_6 v_\sigma}{\Lambda}$$

Neutrino superpotential

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- TB mixing is violated by small term $k \neq 0$ (k is taken to be complex $k = |k| e^{i\phi_k}$)

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→ Dirac CP violating phase

Neutrino sector

Neutrino masses

Assuming that $\lambda_1 = \lambda_2$ and $c = a + b$ The light neutrino mass matrix is then given by the see-saw formula

$$m_\nu = m_D m_M^{-1} \, m_D^T$$ **(Type-I see-saw mechanism)**

Where we obtain the three light neutrino masses

$$|m_1| = \frac{m_0}{\sqrt{(a-b)^2 - |\mathrm{k}|\,(a-b)\cos\phi_k + (|\mathrm{k}|^2/4)}}$$

$$|m_2| = \frac{m_0}{\sqrt{(a+2b)^2 + 2\,|\mathrm{k}|\,(a+2b)\cos\phi_k + |\mathrm{k}|^2}}$$

$$|m_3| = \frac{m_0}{\sqrt{(a+b)^2 - |\mathrm{k}|\,(a+b)\cos\phi_k + (|\mathrm{k}|^2/4)}}$$

Where $m_0 = \frac{(\lambda_1 v_u)^2}{\Lambda}$

Trimaximal mixing

Therefore, Neutrino matrix m_ν is diagonalized by Trimaximal mixing matrix

$$\mathcal{U}_{TM_2} = \begin{pmatrix} \sqrt{\frac{2}{3}}\cos\theta & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\sin\theta e^{-i\gamma} \\ -\frac{\cos\theta}{\sqrt{6}} - \frac{\sin\theta}{\sqrt{2}}e^{i\gamma} & \frac{1}{\sqrt{3}} & \frac{\cos\theta}{\sqrt{2}} - \frac{\sin\theta}{\sqrt{6}}e^{-i\gamma} \\ -\frac{\cos\theta}{\sqrt{6}} + \frac{\sin\theta}{\sqrt{2}}e^{i\gamma} & \frac{1}{\sqrt{3}} & -\frac{\cos\theta}{\sqrt{2}} - \frac{\sin\theta}{\sqrt{6}}e^{-i\gamma} \end{pmatrix} .\mathcal{U}_P$$

θ and γ : arbitrary angle and phase

Mixing angles

Comparing U_{PMNS} matrix with trimaximal mixing matrix U_{TM2} we obtain

$$\sin^2\theta_{13} = \frac{2}{3}\sin^2\theta \qquad \sin^2\theta_{12} = \frac{1}{3-2\sin^2\theta}$$

$$\sin^2\theta_{23} = \frac{1}{2} - \frac{3\sin 2\theta}{2\sqrt{3}(3-2\sin^2\theta)}\cos\gamma$$

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$\boldsymbol{\theta}$ and $\boldsymbol{\gamma}$: arbitrary angle and phase

Model free parameters

Five independent parameters (m_0 ,a, b, k and ϕ_k) in the neutrino sector

→

Observables

- Squared-mass differences Δm_{ij}
- Three mixing angles $\boldsymbol{\theta_{ij}}$
- Dirac CP violating phase $\boldsymbol{\delta_{CP}}$

Constraining parameters from neutrino mixing

Observable	Best fit	3σ range
$\sin^2\theta_{13}$	0.02219	$0.02032 \to 0.02410$
$\sin^2\theta_{12}$	0.304	$0.269 \to 0.343$
$\sin^2\theta_{23}$	0.573	$0.415 \to 0.616$
$\Delta m^2_{21}/10^{-5}$	7.42	$6.82 \to 8.04$
$\Delta m^2_{3l}/10^{-3}$	2.517	$2.435 \to 2.598$
δ°_{CP}	197	$120 \to 369$

Updated compared to the published version, to use nu-fit v5]
2020

Both normal and inverted mass hierarchies are possible in this scenario.

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$|k| \lesssim 0{,}5$

$|a| \lesssim 1$

$|b| \lesssim 1$

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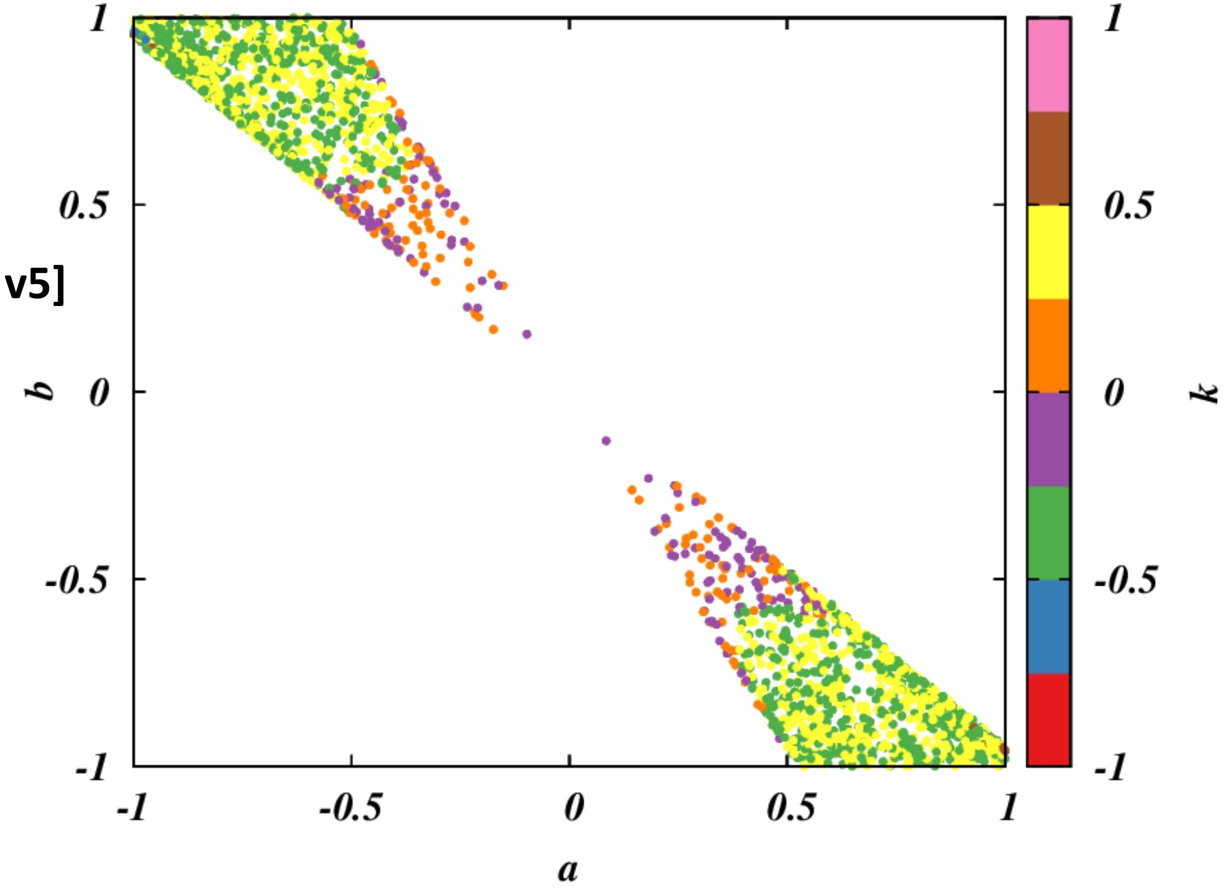

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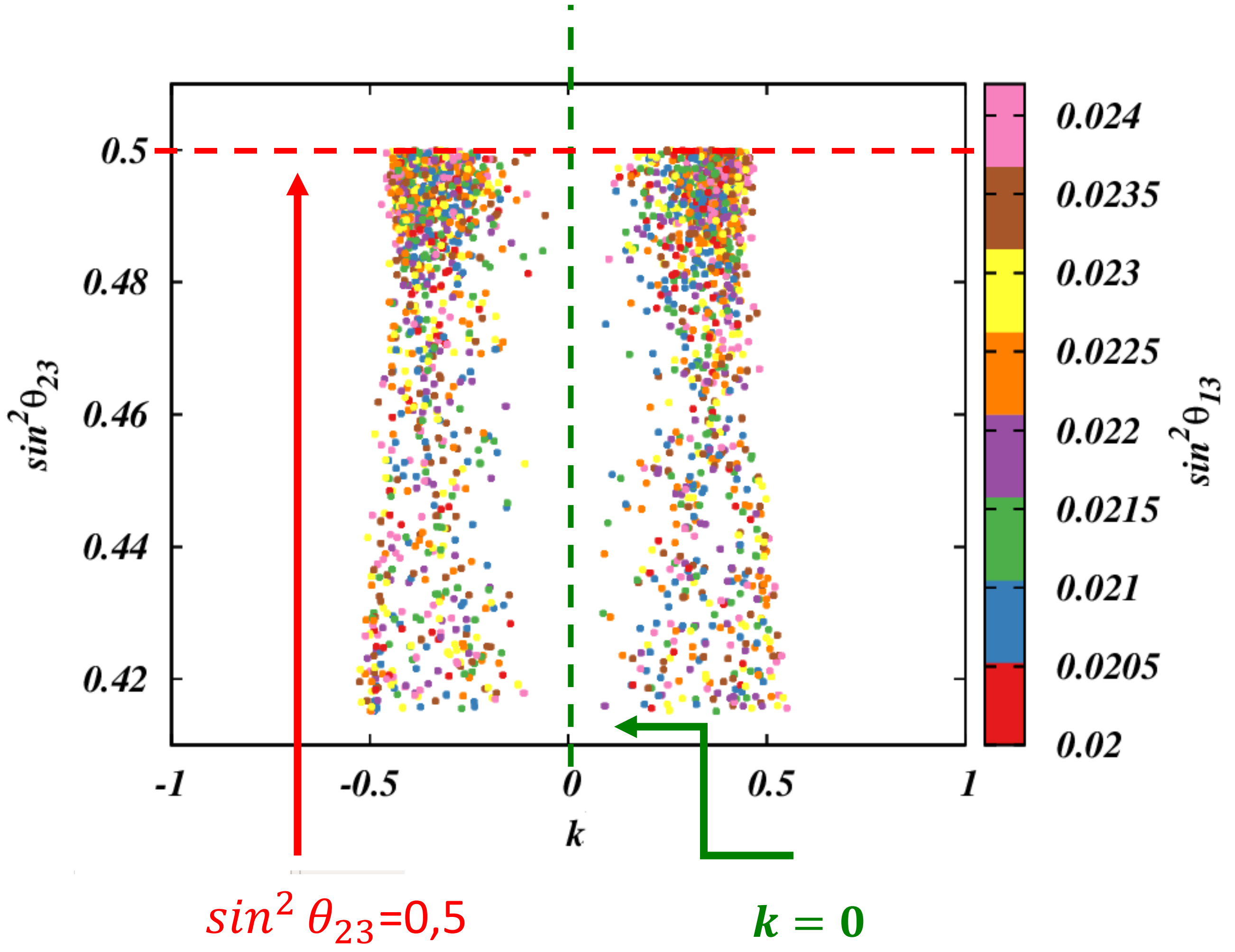
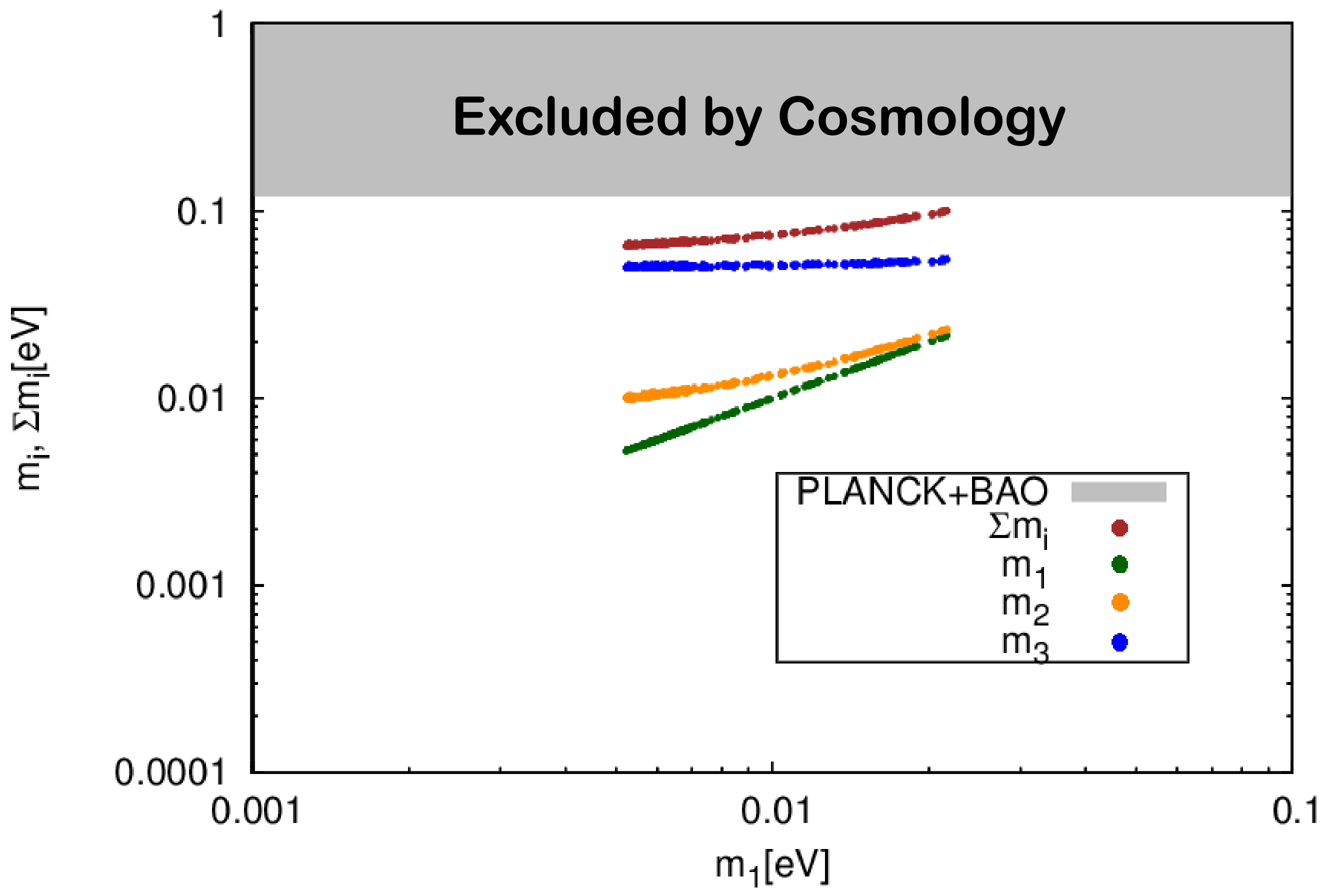


Model predictions

$k \neq 0$

- **Non-zero** value of the reactor angle $\boldsymbol{\theta_{13}} \neq 0$
- **Lower octant** on atmospheric angle $\theta_{23} < \frac{\pi}{4}$

Search for absolute mass scale (NH)



The sum of neutrino masses

Cosmological limite ($\sum_i m_i$ <0,12 eV)

[Planck Collaboration 2018]

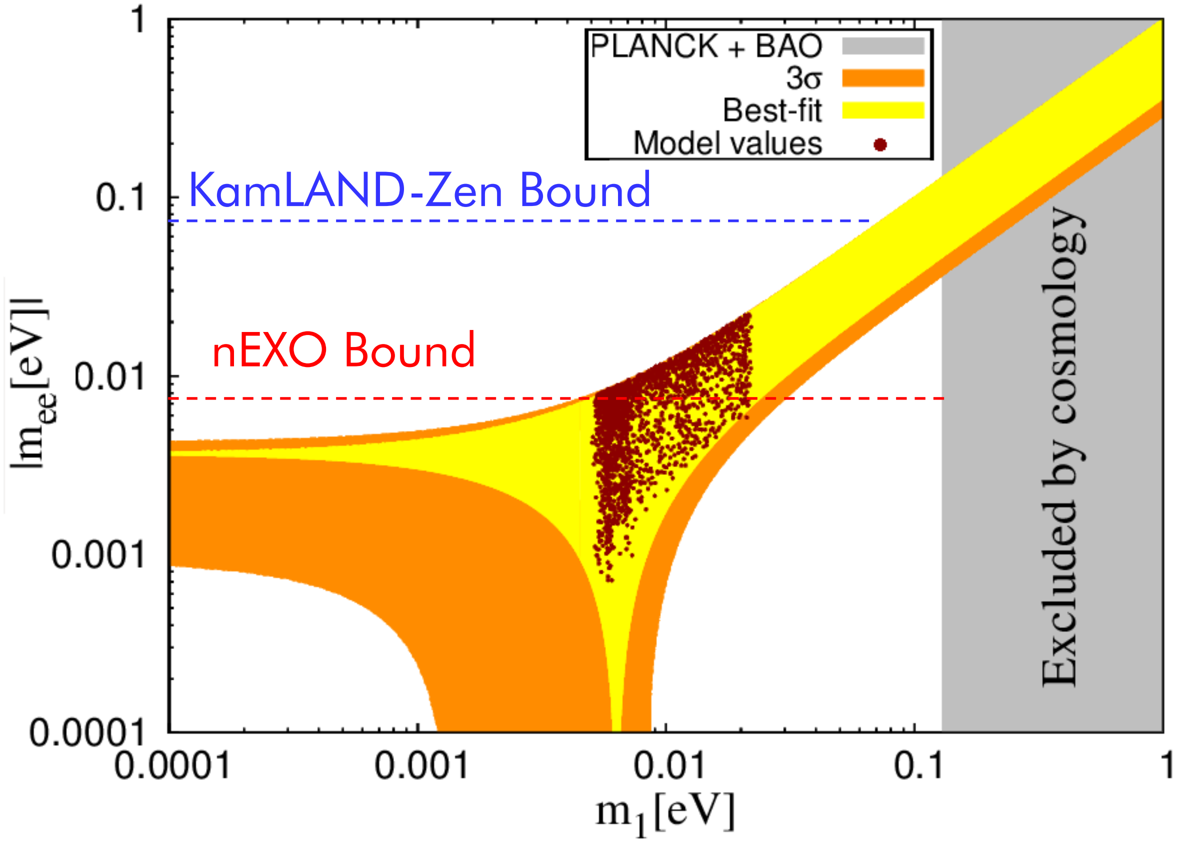
- The sum of neutrinos

$$0{,}0613 \lesssim \sum_i m_i \,[\mathrm{eV}] \lesssim 0.117$$

- The lightest neutrino mass (NO)

$$0{,}005 \lesssim m_1 \,[\mathrm{eV}] \lesssim 0{,}021$$

Search for absolute mass scale (NH)



$$0{,}000715 \lesssim m_{\beta\beta}\,[\mathrm{eV}] \lesssim 0{,}022$$

Neutrinoless double beta decay $m_{\beta\beta}$

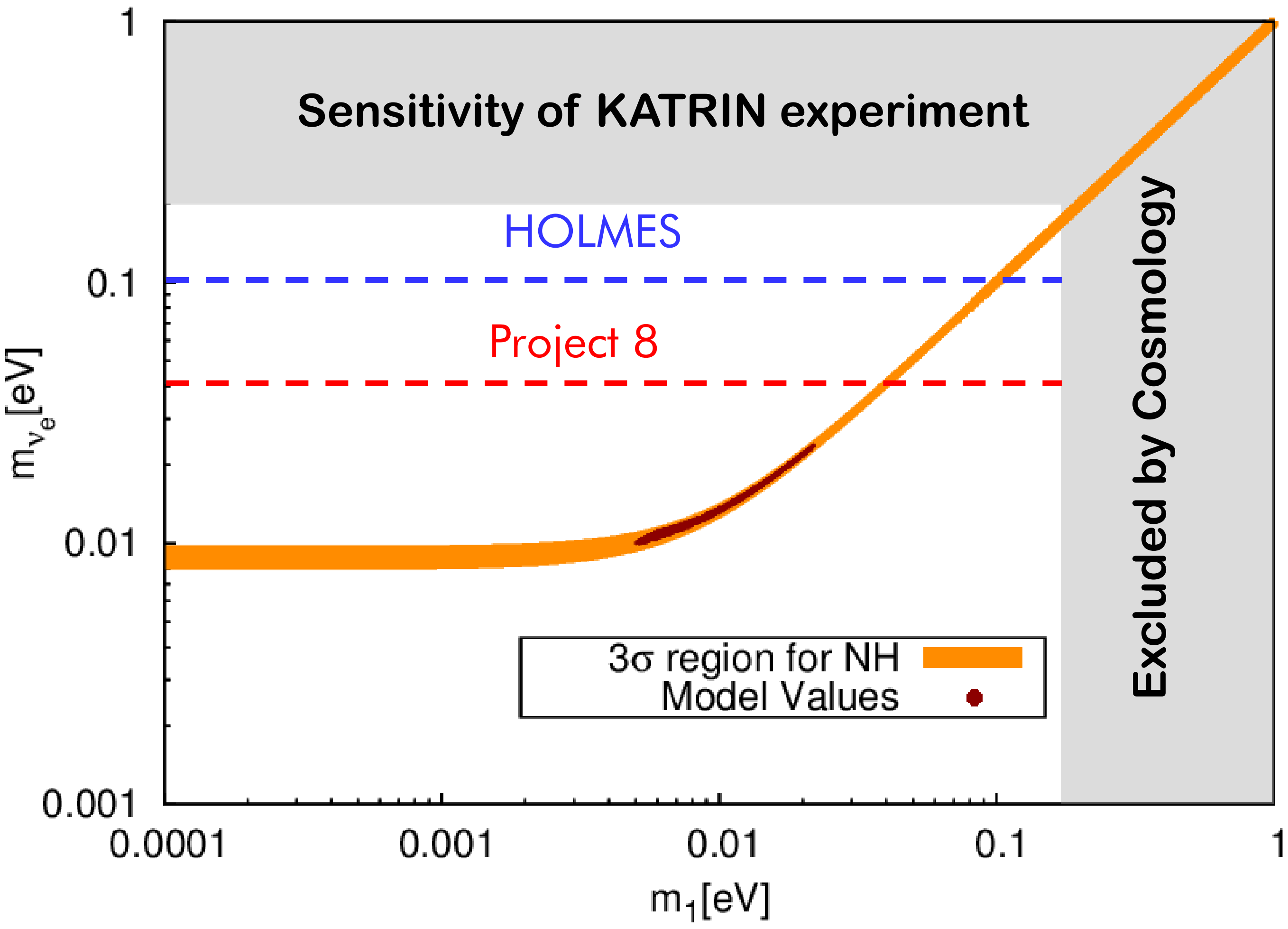
$$|m_{\beta\beta}| = \left|\sum_i U_{ei}^2 m_i\right|$$

KamLAND–Zen Bound $m_{\beta\beta}$ **[eV]** < 0.061 – 0.165
[KamLAND-Zen Collaboration 2016]

GERDA $m_{\beta\beta}$ **[eV]** < 0,104 – 0,228
[GERDA 2018]

Expected nEXO Bound $m_{\beta\beta}$ **[eV]** < 0.005
[nEXO Collaboration 2018]

Search for absolute mass scale (NH)



Tritium Beta decay

$$m_\beta = \left(\sum_i |U_{ei}|^2 m_i^2\right)^{1/2}$$

The sensitivity of KATRIN is to $m_\beta \lesssim 0{,}2$ eV

HOLMES $m_\beta \lesssim 0{,}1$ eV

Project 8 $m_\beta \lesssim 0{,}04$ eV

$$0{,}01 \lesssim \boldsymbol{m_\beta}[\text{eV}] \lesssim 0{,}023$$

THANK YOU!