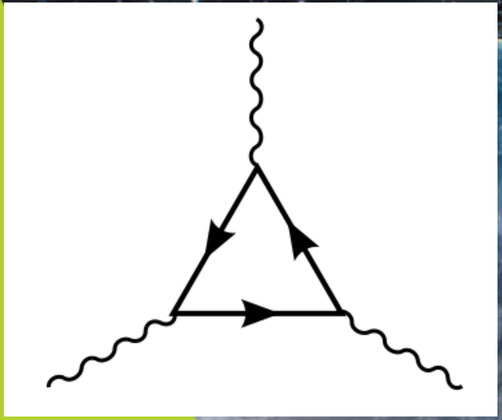
Gauge Anomaly Cancellation in Effective Field Theories



Beyond Standard Model: From Theory to Experiment (BSM-2021)

BSM

March 29-
April 2, 2021

Ferruccio Feruglio
University of Padova

April 1st, 2021

based on F.F. 2012.13989 to appear in JHEP

See also: Q. Bonnefoy, L. Di Luzio, C. Grojean, A. Paul and A. N. Rossia 2012.07740

EFTs

$\mathcal{L}_{EFT}$ $\mathcal{L}_{EFT}$

New Physics ?

$10^{18}\ GeV$

$10^{-22}\ eV$

New Physics ?

$$\mathcal{L}_{EFT} = \mathcal{L}_4 + \frac{1}{\Lambda}\sum c_k^{(5)}\, O_k^{(5)} + \frac{1}{\Lambda^2}\sum c_k^{(6)}\, O_k^{(6)} + \cdots$$

constraints on c_k from

- data
- consistency requirements, such as causality, unitarity, Lorentz invariance

constraints from anomalies

PHYSICAL REVIEW D   VOLUME 6, NUMBER 2   15 JULY 1972

Gauge Theories Without Anomalies*

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(Received 27 March 1972)

standard criterion
about $\mathcal{L}_4$:

$$tr\left(t^a\{t^b,t^c\}\right) = 0$$

t^a fermion
generators
of gauge group
in a Weyl basis

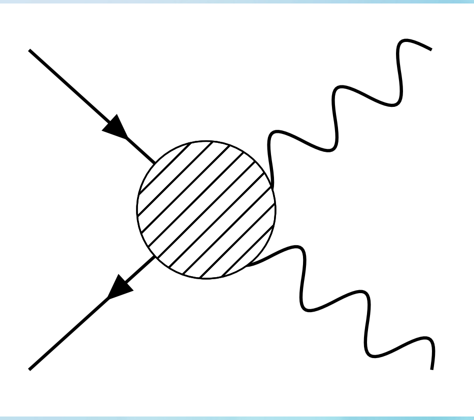
any constraint on c_k from gauge anomaly cancellation ?

given an EFT satisfying the standard criterion,
the question is not IF anomalies cancel, but

HOW they cancel ?

a concrete problem

- you have been asked to compute $e^+e^- \to W^+W^-$ in the SMEFT at 1-loop and $1/\Lambda^2$ accuracy



- requirements:
  - generic gauge
  - dimensional regularization

- is $\overline{MS}$ prescription sufficient to fully define the renormalization scheme?

Tools

- classical, gauge invariant, action S

$$S = \int d^4x \; \mathcal{L}(\psi, \varphi, A)$$

$$\delta_\alpha S = 0$$

⇒ $\partial_\mu j^\mu = 0$

$$j^\mu = -\frac{1}{g}\frac{\delta S}{\delta A_\mu}$$

- gauge invariance of effective, action W

$$e^{iW[\varphi,A]} = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \; e^{i\,S}$$

$$\delta_\alpha W = 0$$

$\delta_\alpha W \neq 0$ ⇔ Anomaly

$\delta_\alpha W$ efficiently studied through differential operators $L(x)$

$$\delta_\alpha W = \int d^4x\, \alpha(x)\, L(x) W[\varphi, A]$$

$$\delta_\alpha W = 0 \leftrightarrow L(x)W = 0$$

$$L(x) = \left[-\frac{1}{g}\partial_\mu \frac{\delta}{\delta A_\mu(x)} + i\varphi(x)\frac{\delta}{\delta\varphi(x)} - i\varphi^\dagger(x)\frac{\delta}{\delta\varphi^\dagger(x)} \right]$$

for instance:
abelian theory,
one charged scalar

regularization $W \to W_r$

$L(x)W_r \neq 0$ ➡ is the theory anomalous ?

inspect the entire class $\{W\} = W_r + \int d^4y\, P(y)$

$P(y)$ local polynomial in the bosonic fields.

if $P_c(y)$ exists such that

$$L(x)\,[W_r + \int d^4y\, P_c(y)] = 0$$

define $W = W_r + \int d^4y\, P_c(y)$

$$L(x)W = 0$$

$P_c(y)$ defined up to a gauge invariant contribution

an anomaly is a non trivial equivalence class $\{L(x)W\}$

Physics Letters B
Volume 37, Issue 1, 1 November 1971, Pages 95-97

Consequences of anomalous ward identities

J. Wess [a, b], B. Zumino

The anomalies of Ward identities are shown to satisfy consistency or integrability relations, which restrict their possible form. For the case of SU(3) × SU(3) we verify that

$$L_a(x)L_b(y)W_{\mathtt{r}} - L_b(y)L_a(x)W_{\mathtt{r}} = \delta^4(x-y)f_{ab}^{c}L_c(x)W_{\mathtt{r}}$$

- we can regard the class $\{L(x)W\}$ as the unknown in this equation
  the general solution is known from cohomology

Physics Reports
Volume 338, Issue 5, November 2000, Pages 439-569

Local BRST cohomology in gauge theories

Glenn Barnich [a, b], Friedemann Brandt [c], Marc Henneaux [a, d]

We shall also consider "effective Yang–Mills theories" for which the Lagrangian contains all possible terms compatible with gauge invariance [118], [223] and thus involves derivatives of arbitrarily high order.

semisimple gauge group:
the anomaly is a polynomial of dimension 4 in the fields and derivatives

$$\{L_a(x)W\} = \frac{ie^2}{24\pi^2}\varepsilon^{\mu\nu\lambda\rho}\mathrm{tr}\, T^a \partial_\mu \left(A_\nu \partial_\lambda A_\rho + \frac{1}{2} A_\nu A_\lambda A_\rho \right)$$

The anomaly does not depend on c_k in semisimple gauge theories

abelian theory, one charged scalar
non-trivial candidate anomalies $L(x)W$

$$\varphi^\dagger \varphi\, \varepsilon^{\mu\nu\rho\sigma} \partial_\mu A_\nu\, \partial_\rho A_\sigma \quad , \qquad i\, \varphi^\dagger \varphi\, \varepsilon^{\mu\nu\rho\sigma} \partial_\mu (\varphi^\dagger D_\nu \varphi - D_\nu \varphi^\dagger \varphi)\, \partial_\rho A_\sigma$$

these are solutions of

$$L_a(x) L_b(y) W_{\mathrm{r}} - L_b(y) L_a(x) W_{\mathrm{r}} = \delta^4(x-y) f^c_{ab} L_c(x) W_{\mathrm{r}}$$

dependence on c_k not forbidden by BRST cohomology if G_{gauge} is not semisimple

PHYSICAL REVIEW D VOLUME 35, NUMBER 6 15 MARCH 1987

Spinor loop anomalies with very general local fermion Lagrangians

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(Received 27 October 1986)

fundamental level.] An effective local Lagrangian will typically include renormalizable couplings[33] which play a dominant dynamical role, and nonrenormalizable couplings[34] (involving operators of dimension larger than four in the case of four spacetime dimensions) which may be less important dynamically but still crucial for some processes. There is a question which arises naturally in an effective low-energy (chiral or nonchiral) gauge theory with spinor fields. Will there not be certain restrictions to the structure of allowed higher dimensional local spinor couplings (besides naive gauge invariance of the forms) because of possible gauge anomaly problems? We can now give a definite answer to that—as long as the spinor field contents are such that the usual gauge anomaly cancellation condition[2] is satisfied, the effective gauge theory Lagrangian may include any gauge-invariant, renormalizable or nonrenormalizable, local spinor couplings without encountering gauge inconsistency by spinor loop effects.

no dependence on c_k for general gauge group G

diagrammatic proof uses two regularizations
- gauge invariant for diagrams containing d > 4 operators
- non gauge-invariant for diagrams containing renormalizable operators

What happens if we use a unique regulator?

a miniature SM

F.F. 2012.13989

$$\mathcal{L} = \mathcal{L}_4 + \mathcal{L}_6 + \ldots$$

$$G_{gauge} = U(1)$$

	Q
φ	-1
l_L	-1
l_R	0
q_L	$+1$
q_R	0

$$\mathrm{tr}\, Q^3\Big|_F = 0$$

no anomaly according to usual criterion

$$\mathcal{L}_6 = \sum_k c_k O_k + \ldots$$

$$\mathcal{L}_4 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{l_L}i\gamma^\mu D_\mu l_L + \overline{l_R}i\gamma^\mu \partial_\mu l_R + \overline{q_L}i\gamma^\mu D_\mu q_L + \overline{q_R}i\gamma^\mu \partial_\mu q_R + D_\mu\varphi^\dagger D^\mu\varphi - V(\varphi^\dagger\varphi) - (y_l\, \varphi\overline{l_L}l_R + y_q\, \varphi^\dagger\overline{q_L}q_R + h.c.) \quad ,$$

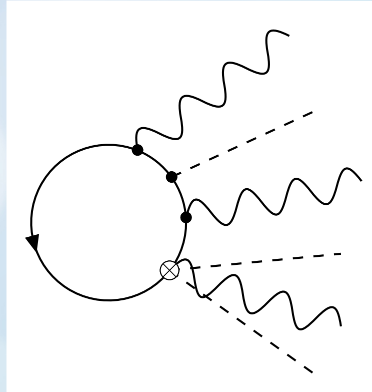
O_φ	$(\varphi^\dagger\varphi)^3$
$O_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$
$O_{\varphi D}$	$(\varphi^\dagger D^\mu\varphi)^*(\varphi^\dagger D_\mu\varphi)$
$O_{\varphi A}$	$g^2(\varphi^\dagger\varphi)F_{\mu\nu}F^{\mu\nu}$
$O_{\varphi\widetilde{A}}$	$g^2(\varphi^\dagger\varphi)F_{\mu\nu}\widetilde{F}^{\mu\nu}$

$O_{l\varphi}$	$(\varphi^\dagger\varphi)\varphi\overline{l_L}l_R$
$O_{q\varphi}$	$(\varphi^\dagger\varphi)\varphi^\dagger\overline{q_L}q_R$
O_{lA}	$g\varphi\overline{l_L}\sigma^{\mu\nu}l_R F_{\mu\nu}$
O_{qA}	$g\varphi^\dagger\overline{q_L}\sigma^{\mu\nu}q_R F_{\mu\nu}$
$O_{\varphi l_L}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu\varphi)\overline{l_L}\gamma^\mu l_L$
$O_{\varphi l_R}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu\varphi)\overline{l_R}\gamma^\mu l_R$
$O_{\varphi q_L}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu\varphi)\overline{q_L}\gamma^\mu q_L$
$O_{\varphi q_R}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu\varphi)\overline{q_R}\gamma^\mu q_R$

Table 1: Dimension-six operators other than the four-fermion ones.

compute $W[\varphi, A]$

$$W[\varphi, A] = \sum$$



+ 4-fermion interactions

$$\mathcal{L}_{\bar{f}f} = \bar{f}\left(\mathcal{S} + \mathcal{P}\gamma_5 + \mathcal{V}_\mu\gamma^\mu + \mathcal{A}_\mu\gamma^\mu\gamma_5 + \mathcal{T}_{\mu\nu}\sigma^{\mu\nu}\right) f$$

$$f = (l, q)^T$$

$$\mathcal{R}_\mu = \mathcal{V}_\mu + \mathcal{A}_\mu = i\, C_{\phi R}(\varphi^+ \overleftrightarrow{D_\mu}\varphi)$$

$$\mathcal{L}_\mu = \mathcal{V}_\mu - \mathcal{A}_\mu = -g\, Q A_\mu + i\, C_{\phi L}(\varphi^+ \overleftrightarrow{D_\mu}\varphi)$$

$$Q = \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix} \qquad C_{\phi R(L)} = \begin{pmatrix} c_{\varphi l_{L(R)}} & 0 \\ 0 & c_{\varphi q_{L(R)}} \end{pmatrix}$$

$O_{\varphi l_L}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)\overline{l_L}\gamma^\mu l_L$
$O_{\varphi l_R}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)\overline{l_R}\gamma^\mu l_R$
$O_{\varphi q_L}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)\overline{q_L}\gamma^\mu q_L$
$O_{\varphi q_R}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)\overline{q_R}\gamma^\mu q_R$

result in DR t'Hooft-Veltman γ_5

$$L(x)W_{\mathtt{r}} = -\frac{1}{24\pi^2}\varepsilon^{\mu\nu\rho\sigma} \Big\{\xi_{AA}\,\partial_\mu A_\nu(x)\cdot\partial_\rho A_\sigma(x) + i\,\xi_{\varphi A}\,\partial_\mu(\varphi^\dagger \overleftrightarrow{D}_\nu\varphi)(x)\cdot\partial_\rho A_\sigma(x) - \xi_{\varphi\varphi}\,\partial_\mu(\varphi^\dagger \overleftrightarrow{D}_\nu\varphi)(x)\cdot\partial_\rho(\varphi^\dagger \overleftrightarrow{D}_\sigma\varphi)(x)\Big\}$$

$$\begin{aligned}\xi_{AA} &= g^2\,\mathtt{tr}\,Q^3\\ \xi_{\varphi A} &= 2g\,\mathtt{tr}\,Q^2(C_{\phi R}-C_{\phi L})\\ \xi_{\varphi\varphi} &= \mathtt{tr}\,Q(C_{\phi R}-C_{\phi L})^2 \quad .\end{aligned}$$

$\xi_{\varphi A}$ $\xi_{\varphi\varphi}$ terms are trivial

$$W_{\mathtt{c}} = \frac{1}{24\pi^2}\varepsilon^{\mu\nu\rho\sigma}\int d^4y \Big\{+i\,\xi_{\varphi A}\,gA_\mu(y)(\varphi^\dagger \overleftrightarrow{D}_\nu\varphi)(y)\cdot\partial_\rho A_\sigma(y) - \xi_{\varphi\varphi}\,gA_\mu(y)(\varphi^\dagger \overleftrightarrow{D}_\nu\varphi)(y)\cdot\partial_\rho(\varphi^\dagger \overleftrightarrow{D}_\sigma\varphi)(y)\Big\}$$

local polynomial in bosonic fields

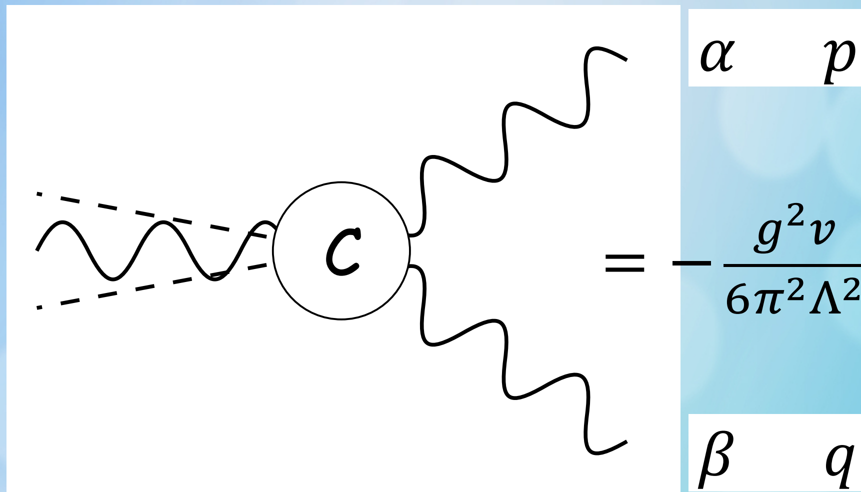
$$W = W_{\mathtt{r}} + W_{\mathtt{c}}$$

$$L(x)W = -\frac{g^2\mathtt{tr}Q^3}{24\pi^2}\varepsilon^{\mu\nu\rho\sigma}\,\partial_\mu A_\nu(x)\cdot\partial_\rho A_\sigma(x)$$

$\mathrm{tr}\,Q^3\big|_F = 0$ usual condition for anomaly cancellation

an $\mathcal{O}(\hbar)$ counterterm at $\mathcal{O}(1/\Lambda^2)$

$W_c =$



$$= -\frac{g^2 v}{6\pi^2\Lambda^2} tr\left(C_{\varphi R} - C_{\varphi L}\right)\epsilon^{\alpha\beta\mu\nu} p_\mu q_\nu + \dots$$

- applies to both broken and unbroken phase

- depends on c_k coefficients

- regularization-dependent: unphysical by itself
  only the combination [diagrams + counterterm] is physical

- W_c should be included when evaluating a physical amplitude at 1-loop order as in $e^+e^- \to W^+W^-$

G. Durieux, J. Gu, E. Vryonidou and C. Zhang, “Probing top-quark couplings indirectly at Higgs factories,” Chin. Phys. C **42** (2018) no.12, 123107 [arXiv:1809.03520 [hep-ph]].

$e^+e^- \to W^+W^-$ in SMEFT

G. Durieux, J. Gu, E. Vryonidou and C. Zhang, "Probing top-quark couplings indirectly at Higgs factories," Chin. Phys. C **42** (2018) no.12, 123107 [arXiv:1809.03520 [hep-ph]].

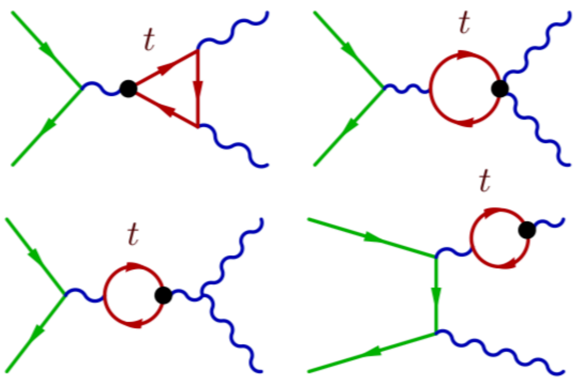


Fig. 1. Selected diagrams for dimension-six top-quark contributions to $e^+e^- \to W^+W^-$. Red lines represent the top quark. Blobs represent dimension-six operator insertions.

can be sensitive to 3rd generation d=6 operators as

$$O_{\varphi Q}^{(1)} = (\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar{Q}\gamma^\mu Q),$$

$$O_{\varphi Q}^{(3)} = (\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar{Q}\gamma^\mu \tau^I Q),$$

$$O_{\varphi Q}^{(+)} \equiv \frac{1}{2}\left(O_{\varphi Q}^{(1)} + O_{\varphi Q}^{(3)}\right)$$

Ward identities of γWW vertex require counterterms such as

$$\mathrm{R2}_{\mathrm{O}_{\varphi \mathrm{Q}}^{(+)}}(WW\gamma) = \frac{C_{\varphi Q}^{(+)} e^3 v^2}{96\pi^2 s_W^2 \Lambda^2}\epsilon^{\mu\nu\rho\sigma}(p_{2\sigma} - p_{3\sigma})$$

- $\overline{MS}$ prescription insufficient to fully define the renormalization scheme

conclusion

- condition for gauge anomaly cancellation in EFTs

  $$tr\left(t^a\{t^b,t^c\}\right) = 0$$

  no extra conditions on coefficients c_k

- consistent amplitudes evaluated at ≥ 1 loop, $1/\Lambda^p$ orders require (c_k ,regularization)-dependent counterterms

- the computation outlined in previous miniature SMEFT might allow to determine the whole set of such counterterms.

THANK YOU

four-fermion operators

$$\mathcal{L}_6^{4F} = \frac{1}{2}\,\bar{l}\Gamma_I l\; C_{IJ}^{ll}\;\bar{l}\Gamma_J l + \frac{1}{2}\,\bar{q}\Gamma_I q\; C_{IJ}^{qq}\;\bar{q}\Gamma_J q + \bar{l}\Gamma_I l\; C_{IJ}^{lq}\;\bar{q}\Gamma_J q$$

$\Gamma_I = (1, \gamma_5, \gamma^\mu, \gamma^\mu\gamma_5, \sigma^{\mu\nu})$

can be rewritten as fermion bilinears through gauge invariant auxiliary fields $\chi_I = (s, p, v^\mu, a^\mu, t^{\mu\nu})$

$$\mathcal{L}_4^{\text{aux}} = \chi_I\;\bar{f}\begin{pmatrix} X_{IJ}^l & 0 \\ 0 & X_{IJ}^q \end{pmatrix}\Gamma_J f - \frac{1}{2}\chi_I M_I^2 \chi_I$$

$$\mathcal{V}_\mu = \frac{1}{2}\left[-gQA_\mu + i(C_{\phi R} + C_{\phi L})(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi) + C_V v_\mu\right]$$
$$\mathcal{A}_\mu = \frac{1}{2}\left[+gQA_\mu + i(C_{\phi R} - C_{\phi L})(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi) + C_A a_\mu\right]$$

$$L(x)W_r[\varphi, A] = \varepsilon^{\mu\nu\rho\sigma}\partial_\mu B_\nu^a(x)\cdot\partial_\rho B_\sigma^b(x)$$

$$B_\mu^a = (A_\mu, i(\varphi^\dagger D_\nu\varphi - D_\mu\varphi^\dagger\varphi), v_\mu, a_\mu)$$

as before, the non-trivial part comes from $B_\mu^a = A_\mu$

PHYSICAL REVIEW D VOLUME 35, NUMBER 6 15 MARCH 1987

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$$\mathscr{L}(x)=\bar{\psi}(x)[i\gamma^\mu D_\mu+f(D_\mu,\Gamma(x))]\psi(x)+\mathscr{L}'(x)\ , \qquad (2.1)$$

where $(\psi,\bar{\psi})$ are dynamical freedoms of our theory and we shall take them to be $2^{D/2}$-component Dirac spinor fields. Here, $D_\mu\equiv\partial_\mu-iB_\mu(x)$ is the gauge-covariant derivative, $\Gamma(x)=\{\Gamma^i(x)\}$ denotes collectively various possible external Bose fields (no restriction on spins) other than B_μ, and $f(D_\mu,\Gamma(x))$ is a differential operator of the general form

$$\begin{aligned}f(D_\mu,\Gamma(x))=\ &a_0(\Gamma(x))+a_1(\Gamma(x))^\mu D_\mu\\&+a_2(\Gamma(x))^{\mu\nu}D_\mu D_\nu+\cdots\\&+a_r(\Gamma(x))^{\mu_1\cdots\mu_r}D_{\mu_1}D_{\mu_2}\cdots D_{\mu_r}\ .\end{aligned} \qquad (2.2)$$

$$\mathscr{L}_{\bar{R}}(x)=\bar{\psi}(x)\left\{i\gamma^\mu D_\mu\left[1+\left[-\frac{1}{M^2}D^2\right]^q\right]+f(D_\mu,\Gamma(x))\right\}\psi(x)+\mathscr{L}'(x)\ , \quad ($$

$$iS_F(p)_R=\frac{M^{2q}}{(p^2)^q+M^{2q}}\frac{i\gamma^\mu p_\mu}{p^2+i\epsilon}$$

The Lagrangian of the form (2.12) can be considered for a general, γ_{D+1}-dependent, gauge field matrix $B_\mu(x)$. Furthermore, $\mathscr{L}_{\bar{R}}(x)$ possesses manifestly the gauge invariance of the original theory, and the separation we have made in Eq. (2.16) for the effective action is clearly a gauge-invariant one.[22] We may thus immediately conclude that under arbitrary, chiral or nonchiral, gauge transformations (2.5), we have

$$\delta_\Lambda(W_{\bar{R}}[B_\mu,\Gamma]_{\rm II}+W_{\bar{R}}[B_\mu,\Gamma]_{\rm III})=0\ ; \qquad (2.17)$$

i.e., the functional $W_{\bar{R}}[B_\mu,\Gamma]_{\rm II}+W_{\bar{R}}[B_\mu,\Gamma]_{\rm III}$ is fully gauge invariant. Of course, no meaningful statement of

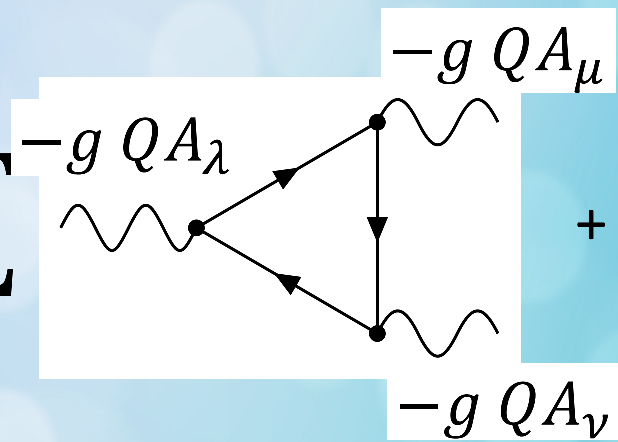
only triangle diagrams with $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$ insertions contribute to the anomaly

W. A. Bardeen, "Anomalous Ward identities in spinor field theories," Phys. Rev. **184** (1969), 1848-1857.

T. E. Clark and S. T. Love, "The Axial Anomaly and Antisymmetric Tensor Fields," Nucl. Phys. B **223** (1983), 135-143.

- expand $W[\varphi, A]$ powers of $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$

$W[\varphi, A] = \sum$



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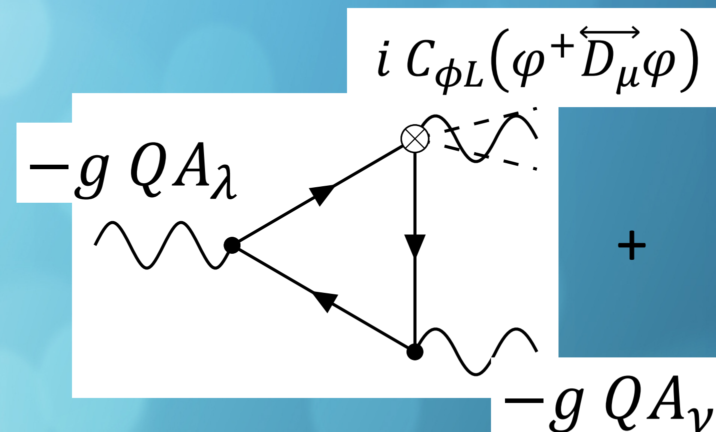

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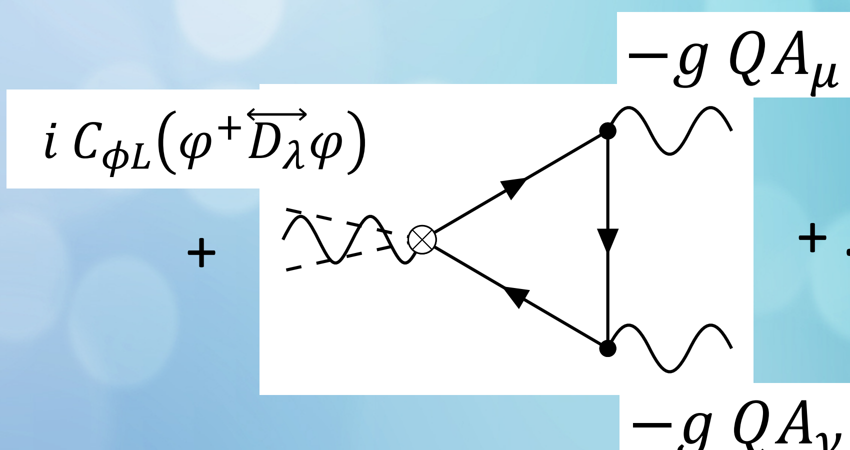


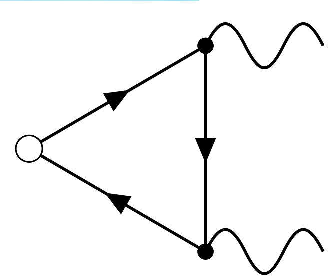
$+ \ldots.$

compute $L(x)W[\varphi, A]$

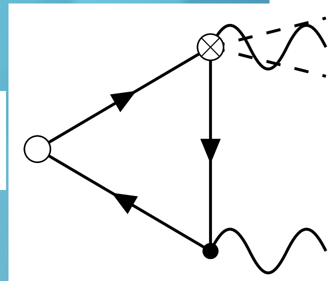
$$L(x) = \left[-\frac{1}{g}\partial_\mu \frac{\delta}{\delta A_\mu(x)} + i\varphi(x)\frac{\delta}{\delta\varphi(x)} - i\varphi^\dagger(x)\frac{\delta}{\delta\varphi^\dagger(x)} \right]$$

$$L(x)\mathcal{L}_\mu(y) = Q\ \partial_\mu \delta^4(x-y) \quad , \qquad L(x)\mathcal{R}_\mu(y) = 0$$

$L(x)W[\varphi, A] = \sum Q$



$+\ Q$



$+ \ldots.$

Physics Letters B
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$$L_a(x)L_b(y)W_{\mathrm{r}} - L_b(y)L_a(x)W_{\mathrm{r}} = \delta^4(x-y)f_{ab}^{c}L_c(x)W_{\mathrm{r}}$$

- we can regard the class $\{L(x)W\}$ as the unknown in this equation
  the general solution is known from cohomology

by quantizing (φ, A_μ) gauge invariance is replaced by BRST invariance

- $\delta_{BRST}\, W = 0 \qquad \delta_{BRST}^2 = 0$

WZ consistency condition reads

$$\delta_{BRST}\left(\delta_{BRST} W_r\right) = 0$$

solution of WZ equations

$$\omega^{1,n} \sim \sum_{I:\text{abelian}} C^I I_I^n + B^{1,n} + V^{1,n} + W^{1,n}$$

two abelian ghosts. Hence, solutions $V^{1,n}$ exist only if the gauge group contains at least two abelian factors. They are given by

$$V^{1,n} = \sum_{I,J:\text{abelian}} \lambda_{IJ}^{\Delta} \left[K_\Delta C^I C^J + j_\Delta (A^I C^J - A^J C^I) \right] . \qquad (12.5)$$

j_Δ^μ are the nontrivial gauge invariant Noether currents,

$$sK_\Delta + dj_\Delta = 0, \quad j_\Delta \in \mathcal{I}$$

$$n = 4, \quad L = \sum_{I=1}^{3} \left[-\tfrac{1}{4} F_{\mu\nu}^I F^{\mu\nu I} - \tfrac{1}{2} (\partial_\mu \phi^I) \partial^\mu \phi^I + \tfrac{1}{4} \phi^I \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^I F_{\rho\sigma}^3 \right]$$

where $F_{\mu\nu}^I = \partial_\mu A_\nu^I - \partial_\nu A_\mu^I$ are abelian field strengths, and ϕ^I are real scalar fields. A solution $W^{1,4}$ is

$$W^{1,4} = d^4x \sum_{I,J,K=1}^{3} \epsilon_{IJK} \left[\tfrac{1}{4} \epsilon^{\mu\nu\rho\sigma} C^I A_\mu^J A_\nu^K F_{\rho\sigma}^3 + C^I A_\mu^J \partial^\mu \phi^K - \tfrac{1}{2} C^I C^J \phi_K^* \right]. \quad (12.6)$$

We know turn to $H^{1,n}(s|d)$, i.e., to anomalies. The most general solution of the consistency condition with ghost number 1 and form-degree n is

$$\omega^{1,n} \sim \sum_{I:\text{abelian}} C^I I_I^n + B^{1,n} + \cdots\cdots$$

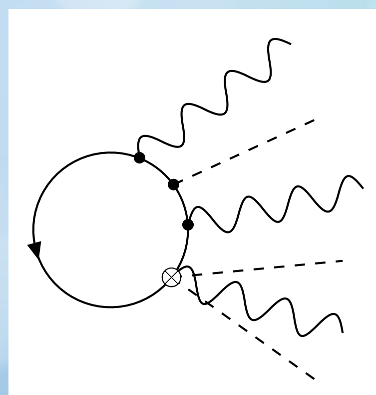
only if there are at least two abelian factors in G_{gauge}

In 4 dimensions, $B^{1,4}$ is the non abelian gauge anomaly [17]:

$$B^{1,4} = d_{IJK} C^I d[A^J dA^K + \tfrac{1}{4} e f_{LM}{}^J A^K A^L A^M], \qquad (12.7)$$

where d_{IJK} is the general symmetric $\mathcal{G}$-invariant tensor. Hence, the gauge group is

$L(x)$ $\sum$



semisimple gauge group: the anomaly is a polynomial of dimension 4 in the fields and derivatives

dependence on c_k only in d > 4 contributions

The anomaly does not depend on c_k in semisimple gauge theories

$I_I^n \in \mathcal{I}$, i.e., $\{I_I^n\}$ is a set of strictly gauge invariant n-forms.

no limits on the dimensionality of the anomaly for abelian factors

J. A. Dixon, "Anomalies, BRS cohomology and effective theories," Phys. Rev. Lett. **67** (1991), 797-800.

abelian theory, one charged scalar

non-trivial candidate anomalies $L(x)W$

$$\varphi^\dagger\varphi\, \varepsilon^{\mu\nu\rho\sigma}\partial_\mu A_\nu\, \partial_\rho A_\sigma \quad , \qquad i\, \varphi^\dagger\varphi\, \varepsilon^{\mu\nu\rho\sigma}\partial_\mu(\varphi^\dagger D_\nu\varphi - D_\nu\varphi^\dagger\varphi)\, \partial_\rho A_\sigma$$

these are solutions of

$$L_a(x)L_b(y)W_{\mathtt{r}} - L_b(y)L_a(x)W_{\mathtt{r}} = \delta^4(x-y)f_{ab}^c L_c(x)W_{\mathtt{r}}$$

dependence on c_k not forbidden by BRST cohomology if G_{gauge} is not semisimple