Hierarchy problem and dimension-6 effective operators

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Brief

- Without any mechanism to protect its mass, the self-energy of the Higgs boson diverges quadratically, leading to the hierarchy or fine-tuning problem. One bottom-up solution is to postulate some yet-to-be-discovered symmetry which forces the sum of the quadratic divergences to be zero, or almost negligible; this is known as the Veltman Condition.

- We study such divergences in an effective theory framework, and construct the Veltman Condition with dimension-6 operators. We show that there are two classes of diagrams, the one-loop and the two-loop ones, that contribute to quadratic divergences, but the contribution of the latter is suppressed by a loop factor of $\frac{1}{16\pi^2}$.

- There are only six dimension-6 operators that contribute to the one-loop category, and the Wilson coefficients of these operators play an important role towards softening the fine-tuning problem.

- We find the parameter space for the Wilson coefficients that satisfies the extended Veltman Condition, and also discuss why one need not bother about the $d > 6$ operators.

The parameter space is consistent with the theoretical and experimental bounds of the Wilson coefficients, and should act as a guide to the model builders.

Veltman Condition with dimension-4 operators

- We start from the SM Higgs potential with only $d \leq 4$ terms

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 \tag{1}$$

where Φ is the SM doublet, with $< \Phi >= \frac{v}{\sqrt{2}}$

- One may write the physical Higgs mass, m_h , in terms of a bare mass term $m_{h,0}$ and higher-order self-energy corrections

$$m_h^2 = m_{h,0}^2 + \delta m_h^2 \tag{2}$$

- The Higgs self-energy receives a quadratically divergent correction

$$\delta m_h^2 = \frac{\Lambda^2}{16\pi^2} \left(6\lambda + \frac{3}{4} g_1^2 + \frac{9}{4} g_2^2 - 6 g_t^2 \right) , \tag{3}$$

where Λ is the cutoff scale of the theory, g_1 and g_2 are the $U(1)_Y$ and $SU(2)_L$ gauge couplings, and $g_t = \frac{2m_t}{v}$ is the top quark Yukawa coupling. All other fermions are treated as massless.

- the VC fails badly for the SM.
- There are numerous attempts in the literature by introducing more particles, like extra scalars or fermions. While these attempts provided some important constraints on the parameter space, the VC could hardly be stabilised over the entire energy scale from v to Λ if one considers the RG evolution of the couplings.
- We will take the bottom-up approach to its extreme limit. For us, whatever New Physics (NP) exists there at the high energy scale can be effectively integrated out at the scale Λ to give us the SM, plus some effective operators involving only the SM fields, which is known as the SM Effective Field Theory (SMEFT).
- In SMEFT, the first interesting higher dimensional operators come at $d = 6$ (the $d = 5$ Weinberg operator is not relevant for scalar self-energies). We will use the Hagiwara, Ishihara, Szalapski, and Zeppenfeld (HISZ) basis. Only a handful among the 59 dimension-6 operators contribute to the quadratically divergent part of the scalar self-energy.

- We will use the SMEFT basis as in keeping in mind that only operators with two or more Higgs fields are relevant and the divergence should be quartic.

The relevant operators are as follows:

$$O_{WW} = \Phi^\dagger \hat{W}_{\mu\nu} \hat{W}^{\mu\nu} \Phi\,, \quad O_{BB} = \Phi^\dagger \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} \Phi\,, \quad O_{GG} = \Phi^\dagger \Phi \hat{G}_{\mu\nu} \hat{G}^{\mu\nu}\,,$$

$$O_W = (D_\mu\Phi)^\dagger \hat{W}^{\mu\nu} (D_\nu\Phi)\,, \quad O_B = (D_\mu\Phi)^\dagger \hat{B}^{\mu\nu} (D_\nu\Phi)\,, \quad O_{\phi,1} = (D_\mu\Phi)^\dagger \Phi\, \Phi^\dagger (D^\mu\Phi)\,,$$

$$O_{\phi,2} = \frac{1}{2}\,\partial^\mu(\Phi^\dagger\Phi)\,\partial_\mu(\Phi^\dagger\Phi)\,, \quad O_{\phi,3} = \frac{1}{3}\,(\Phi^\dagger\Phi)^3\,, \quad O_{\phi,4} = (D_\mu\Phi)^\dagger (D^\mu\Phi)\,\Phi^\dagger\Phi \tag{4}$$

where

$$\hat{B}_{\mu\nu} = \frac{ig_1}{2}\,B_{\mu\nu}\,, \quad \hat{W}_{\mu\nu} = \frac{ig_2}{2}\,\sigma^a W^a_{\mu\nu}\,, \hat{G}_{\mu\nu} = \frac{ig_s}{2}\,\lambda^A G^A_{\mu\nu}\,.$$

g_2, g_1 being the $SU(2)_L$ and $U(1)_Y$ gauge couplings respectively, and λ^A , σ^a are the Gell-Mann and the Pauli matrices. Note that the mixed gauge operator $O_{BW} = \phi^\dagger BW\phi$ cannot generate a self-energy amplitude, either at one- or at two-loop. There is no contribution from O_B either, due to the abelian nature of the field tensor, but we keep it for completeness. Only the O_{VV} $(V = W, B, G)$ and $O_{\phi,i}$ $(i = 1, 2, 4)$ operators are relevant for one-loop diagrams.

- With these set of nine operators, we can write the dimension-6 part of the Lagrangian as

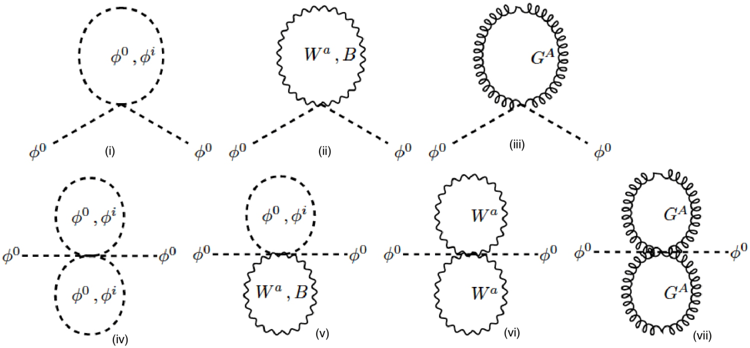
$$\mathcal{L}_6 = \frac{1}{\Lambda^2} \sum_{i=1}^{9} c_i O_i \, , \tag{5}$$

- Eqn (3) takes the form:

$$\delta m_h^2 = \frac{\Lambda^2}{16\pi^2} \left(6\lambda + \frac{3}{4} g_1^2 + \frac{9}{4} g_2^2 - 6 g_t^2 \right) + \frac{\Lambda^2}{16\pi^2} \sum_i f_i + \frac{\Lambda^2}{(16\pi^2)^2} \sum_i g_i \, , \tag{6}$$

where f_i and g_i terms come respectively from the one-loop and two-loop quartic divergences with the insertion of the operator O_i , any of the eight (except O_B) dimension-6 operators listed above.

The Feynman diagrams that contribute to the Λ^4 divergences



The first row shows the one-loop diagrams with momentum-dependent vertices; the second row shows the two-loop diagrams where the vertex factor does not contain the loop momentum. The latter set is suppressed by an extra $1/16\pi^2$ compared to the former set. ϕ^0, ϕ^i, W^a, B, and G^A stand for the Higgs boson, the Goldstone bosons, the $SU(2)_L$ and $U(1)_Y$ gauge bosons, and the gluons respectively. The indices i and a run from 1 to 3, while A runs from 1 to 8.

The contributions from the diagrams are given by

$$
\begin{aligned}
&f_{\phi,1} = -3\, c_{\phi,1}\,, \quad f_{\phi,2} = -6\, c_{\phi,2}\,, \quad g_{\phi,1} = -\frac{9}{2}\,\left(g_1{}^2 + 3g_2^2\right)\, c_{\phi,1}\,,\\
&g_{\phi,2} = g_{BB} = g_B = 0\,, \quad f_{\phi,3} = f_B = f_W = 0\,, \quad g_{\phi,3} = 18\, c_{\phi,3}\,,\\
&f_{\phi,4} = -3\, c_{\phi,4}\,, \quad g_{\phi,4} = -\frac{9}{2}\,\left(g_1{}^2 + 3g_2^2\right)\, c_{\phi,4}\,,\\
&f_{WW} = -\frac{9}{4}\, g_2^2\, c_{WW}\,, \quad g_{WW} = -27\, g_2^4\, c_{WW}\,,\\
&f_{BB} = -\frac{3}{4}\, g_1{}^2\, c_{BB}\,, \quad g_W = -\frac{27}{2}\, g_2^4\, c_W\,,\\
&f_{GG} = -6\, g_s^2\, c_{GG}\,, \quad g_{GG} = -72\, g_s^4\, c_{GG}\,.
\end{aligned}
\tag{7}
$$

Couple of important points:

- we can safely neglect the two-loop g_i terms, as they are suppressed by at least two orders of magnitude coming from $\frac{1}{16\pi^2}$
- At what scale should the VC be satisfied? Obviously, it should be at the matching scale Λ. The cancellation need not be exact, it should be of the order of $\frac{v^2}{\Lambda^2}$.
- Thus, it is meaningless to talk about the fine-tuning problem if $\Lambda = 1$ TeV, and anyway we already know that there is no new physics (at least strongly interacting) at that scale. $\Lambda = 100$ TeV makes the fine-tuning problem come back in a softened avatar, so this should be the correct ballpark to study the issue. Even higher values, like $\Lambda = 10^6$ TeV, makes the fine-tuning problem seriously uncomfortable.

- We will study the VC for two values of Λ, namely, 100 TeV and 10^6 TeV. To start with, let us assume that only one of the six SMEFT operators (neglecting $O_{\phi,3}$, O_W, and O_B which do not contribute to f_i) is present at the matching scale.
- One gets, for exact cancellation of the quadratic divergence,

$$100 \text{ TeV} \;\mid\; c_{\phi,1} = c_{\phi,4} = 2c_{\phi,2} = -1.15\,, \quad c_{BB} = -21.5\,,$$

$$c_{WW} = -4.13\,, \quad c_{GG} = -0.78\,,$$

$$10^6 \text{ TeV} \;\mid\; c_{\phi,1} = c_{\phi,4} = 2c_{\phi,2} = -1.03\,, \quad c_{BB} = -17.3\,,$$

$$c_{WW} = -4.20\,, \quad c_{GG} = -1.11\,.$$

$$2 \text{ TeV} \;\mid\; c_{\phi,1} = c_{\phi,4} = 2c_{\phi,2} = -1.34\,, \quad c_{BB} = -26.2\,,$$

$$c_{WW} = -4.53\,, \quad c_{GG} = -0.66\,.$$

- However, there is hardly any UVC theory that generates only one of these six operators at the matching scale. As the sign of the WCs can be either positive or negative, the six free parameters do not even give a closed hypersurface in the 6-dimensional plot, and therefore marginalisation is of very limited use. Let us consider two distinct cases where only a pair of WCs are nonzero at Λ:

  (1) Only $c_{\phi,2}, c_{\phi,4} \neq 0$: The approximate condition to satisfy the VC is

$$\begin{aligned} c_{\phi,4} + 2c_{\phi,2} + 1.150 &= 0 \quad (\Lambda = 100 \text{ TeV}), \\ c_{\phi,4} + 2c_{\phi,2} + 1.030 &= 0 \quad (\Lambda = 10^6 \text{ TeV}). \end{aligned} \tag{8}$$

  (2) Only $c_{WW}, c_{BB} \neq 0$:

$$\begin{aligned} c_{BB} + 5.212c_{WW} + 21.544 &= 0 \quad (\Lambda = 100 \text{ TeV}), \\ c_{BB} + 4.122c_{WW} + 17.306 &= 0 \quad (\Lambda = 10^6 \text{ TeV}). \end{aligned} \tag{9}$$

The exact conditions broaden out to finite-width bands if we allow a finite amount of fine-tuning, the bands getting narrower for higher values of Λ.

Why we can neglect $d = 8$ and higher operators

- For $d = 6$ operators, there are two types of diagrams that come with a Λ^4 divergence. First are the two-loop diagrams, like the one from $\left(\Phi^\dagger \Phi\right)^3$. The second class consists of one-loop diagrams but momentum-dependent vertices, like the one coming from $(D^\mu \Phi)^\dagger (D_\mu \Phi) \Phi^\dagger \Phi$. The final result is: only those dimension-6 operators contribute quartic divergences at one-loop for which both the derivatives act on the field in the loop.

- Thus, among the $d = 8$ operators, one should look only for those operators that come with four derivatives, D^4. There are only three such operators, and all of them have a generic structure of $(D\Phi)^\dagger (D\Phi)(D\Phi)^\dagger (D\Phi)$. As two of the derivatives act on the external leg fields and hence give the square of the external leg momentum, the vertex factor can only have a k^2 dependence, and the divergence remains only Λ^4 and not Λ^6.

- Similarly, operators of the form $D^2(\Phi^2 W^2)$, where W is the generic gauge tensor, do not generate any Λ^6 divergence.

- Here we have discussed the Veltman condition leading to the cancellation of the quadratic divergence of the Higgs self-energy in the context of an SMEFT framework. In other words, we assume the existence of a cut-off scale , below which we have the SM, while the theory above introduces higher-dimensional operators in the low-energy domain.
- We show that the higher dimensional operators lead to quadratic divergences too, but there are two distinct sources of them. For example, with $d = 6$ operators, such divergences can come from one-loop diagrams with momentum-dependent vertices, or two-loop diagrams with momentum-independent vertices. The latter, however, are suppressed by an extra loop factor of $\frac{1}{16\pi^2}$ and hence can be neglected as a first approximation.
- We find that there are only six operators that contribute to the Veltman condition at the one-loop level. It turns out that at least one of the WCs has to be negative, but they are all consistent with a high-scale perturbative theory. The parameter space that we find is compatible with other theoretical and experimental constraints. Thus, this study should set a benchmark for the model builders.

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