Spin-1 polarization observable as a probe of anomalous gauge-Higgs vertex

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April 2, 2021

Plan of the talk

- Motivation of the work
- Spin Density Matrix
- Helicity Amplitudes for the process
- Asymmetries and Sensitivities
- Summary

Motivations of the work

- Precise measurement of the couplings of the Higgs to electroweak gauge bosons is needed to uncover the exact mechanism of EWSB.
- Apart from the usual observables namely total cross section,angular distribution, observables like spin polarizations can provide deeper insight into underlying physics.
- Focus is to use the information contained in the polarization of EW gauge bosons to study its coupling to Higgs, using spin density matrix formalism.

Goal of The Work

- Study anomalous ZZH vertex in the associated ZH production at the e^+e^- and LHC using the Z polarization observables

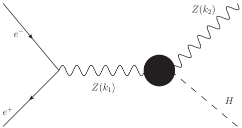


Figure: Feynman diagram for ZH production.

where the vertex $Z_\mu(k_1) \to Z_\nu(k_2)H$ takes the following Lorentz invariant structure

$$\Gamma^V_{\mu\nu} = \frac{g_w}{\cos\theta_w} m_z \left[a_z g_{\mu\nu} + \frac{b_z}{m_z^2}\left(k_{1\nu}k_{2\mu} - g_{\mu\nu}k_1.k_2\right) + \frac{\tilde{b}_z}{m_z^2}\epsilon_{\mu\nu\alpha\beta}k_1^\alpha k_2^\beta \right]$$

The form factors a_z, b_z and $\tilde{b}_z$ are in general complex. The first two couplings would correspond to CP-even terms in the interaction, while the third term is odd under CP.

Formalism

Polarization Parameters of Z boson

The 2×2 density matrix for spin-1/2 system-

$$\rho = \frac{1}{2} I + \frac{1}{2} \mathcal{P}.\sigma$$

where the Pauli matrices σ serve the basis for this expansion and $\mathcal{P}$ is called the **spin- polarization vector** for the ensemble

$$\mathcal{P} = \langle \sigma \rangle = Tr(\rho\sigma)$$

For spin-1,the elements of 3×3 spin density matrix written as

$$\rho = \frac{1}{3} I + \frac{1}{2} \sum_{M=-1}^{M=1} \langle S_M \rangle^* S_M + \sum_{M=-2}^{M=2} \langle T_M \rangle^* T_M$$

where $S_0 = S_3$,$S_{\pm 1} = \mp \frac{1}{\sqrt{2}}(S_1 + iS_2)$ are the spin operators in spherical basis and T_Ms are five rank 2 irreducible tensors built from S_M .

The production and decay density matrices

For a generic process $AB \to VX$, $V \to f\bar{f}'$. Total rate with V being on-shell is given as

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_f} = \frac{2s+1}{4\pi}\sum_{\lambda,\lambda'} \mathbf{P}(\lambda,\lambda')\Gamma(\lambda,\lambda') \quad (1)$$

$\sigma = \sigma_V BR(V \to f\bar{f}')$ is the total cross section for production of V.
$\mathbf{P}(\lambda,\lambda')(\lambda,\lambda' = \pm 1, 0)$ is the polarization density matrix for V and in terms of a hermitian 3×3 production density matrix given as

$$\mathbf{P}(\lambda,\lambda') = \frac{1}{\sigma_v}\int \rho(\lambda,\lambda')d\Omega_v = \frac{1}{\sigma_v}\rho_T(\lambda,\lambda') \quad (2)$$

with σ_v the production cross section of V without decay.

$$\rho(\lambda,\lambda') = \frac{Phase\ space}{Flux}\mathcal{M}(\lambda)\mathcal{M}^\dagger(\lambda')$$

P parametrized in terms of a vector $P = (P_x, P_y, P_z)$ and a rank 2 traceless,symmetric tensor T_{ij} (E.Leader," Spin in particle physics")

The production and decay density matrices

$$\mathbf{P}(\lambda,\lambda^{'}) = \begin{bmatrix} \frac{1}{3}+\frac{P_z}{2}+\frac{T_{zz}}{\sqrt{6}} & \frac{P_x-iP_y}{2\sqrt{2}}+\frac{T_{xz}-iT_{yz}}{\sqrt{3}} & \frac{T_{xx}-T_{yy}-2iT_{yz}}{\sqrt{6}} \\ \frac{P_x+iP_y}{2\sqrt{2}}+\frac{T_{xz}+iT_{yz}}{\sqrt{3}} & \frac{1}{3}-\frac{2T_{zz}}{\sqrt{6}} & \frac{P_x-iP_y}{2\sqrt{2}}-\frac{T_{xz}-iT_{yz}}{\sqrt{3}} \\ \frac{T_{xx}-T_{yy}-2iT_{xy}}{\sqrt{6}} & \frac{P_x+iP_y}{2\sqrt{2}}-\frac{T_{xz}+iT_{yz}}{\sqrt{3}} & \frac{1}{3}-\frac{P_z}{2}+\frac{T_{zz}}{\sqrt{6}} \end{bmatrix} \quad (3)$$

The decay density matrix with the interaction vertex $Vf\bar{f} : \gamma^{\mu}(c_L^f P_L + c_R^f P_R)$ in its rest frame is given by

$$\Gamma(\lambda,\lambda^{'}) = \begin{bmatrix} \frac{(1+\cos^2\theta+2\alpha\cos\theta}{4} & \frac{\sin\theta(\alpha+\cos\theta)e^{i\phi}}{2\sqrt{2}} & \frac{(1-\cos\theta^2)e^{2i\phi}}{4} \\ \frac{\sin\theta(\alpha+\cos\theta)e^{-i\phi}}{2\sqrt{2}} & \frac{\sin^2\theta}{2} & \frac{\sin\theta(\alpha-\cos\theta)e^{i\phi}}{2\sqrt{2}} \\ \frac{(1-\cos\theta^2)e^{-2i\phi}}{4} & \frac{\sin\theta(\alpha-\cos\theta)e^{-i\phi}}{2\sqrt{2}} & \frac{(1+\cos^2\theta-2\alpha\cos\theta}{4} \end{bmatrix} \quad (4)$$

$\alpha \rightarrow \frac{c_R^2-c_L^2}{c_R^2+c_L^2}$ for massless final state fermions

Therefore the angular distribution of the fermion in the rest frame of V

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_f} = \frac{3}{8\pi}[(\frac{2}{3} - \frac{T_{zz}}{\sqrt{6}}) - P_z \cos\theta] \tag{5}$$

$$+\sqrt{\frac{3}{2}}T_{zz}\cos^2\theta + (-P_x + 2\sqrt{\frac{2}{3}}T_{xz}\cos\theta)\sin\theta\cos\phi$$

$$+(-P_y + 2\sqrt{\frac{2}{3}}T_{yz}\cos\theta)\sin\theta\sin\phi$$

$$+(\frac{T_{xx} - T_{yy}}{\sqrt{6}})\sin^2\theta\cos 2\phi + \sqrt{\frac{2}{3}}T_{xy}\sin^2\theta\sin 2\phi$$

Extracting the various polarization parameters of Z -
At production level, by using the polarization matrix elements (R.Rahaman and R.K.Singh, Eur.Phys.J. C76 (2016) no.10, 539)

$$
\begin{aligned}
P_x &= \frac{\{\rho_T(+,0)+\rho_T(+,0)\}+\{\rho_T(0,-)+\rho_T(-,0)\}}{\sqrt{2}\sigma_v} \\
P_y &= \frac{-i\{[\rho_T(0,+)-\rho_T(+,0)]+[\rho_T(-,0)-\rho_T(0,-)]\}}{\sqrt{2}\sigma_v} \\
P_z &= \frac{[\rho_T(+,+)]-[\rho_T(-,-)]}{2\sigma_v}
\end{aligned}
$$

$$
\begin{aligned}
T_{xy} &= \frac{-i\sqrt{6}[\rho_T(-,+) - \rho_T(+,-)]}{4\sigma_v} \\
T_{xz} &= \frac{\sqrt{3}\{[\rho_T(+,0) + \rho_T(+,0)] - [\rho_T(0,-) + \rho_T(-,0)]\}}{\sqrt{2}\sigma_v} \\
T_{yz} &= \frac{-i\sqrt{3}\{[\rho_T(0,+) - \rho_T(+,0)] - [\rho_T(-,0) - \rho_T(0,-)]\}}{\sqrt{2}\sigma_v} \\
T_{xx} - T_{yy} &= \frac{\sqrt{6}[\rho_T(-,+) - \rho_T(+,-)]}{2\sigma_v} \\
T_{zz} &= \frac{\sqrt{6}}{2}\left\{\frac{[\rho_T(+,+)] - [\rho_T(-,-)]}{\sigma_v} - \frac{2}{3}\right\} = \frac{\sqrt{6}}{2}\left[\frac{1}{3} - \frac{\rho_T(0,0)}{\sigma_v}\right]
\end{aligned}
$$

Here T_{xx} and T_{yy} can be separately calculated by using the tracelessness property of T_{ij}.

At decay level, by using partial integration of the differential distribution (**equation**(5)) and then constructing various asymmetries.(R.Rahaman and R.K.Singh, Eur.Phys.J. C76 (2016) no.10, 539)

$$A_x = \frac{3\alpha P_x}{4} \equiv \frac{\sigma(\cos\phi > 0) - \sigma(\cos\phi < 0)}{\sigma(\cos\phi > 0) + \sigma(\cos\phi < 0)}$$

$$A_y = \frac{3\alpha P_y}{4} \equiv \frac{\sigma(\sin\phi > 0) - \sigma(\sin\phi < 0)}{\sigma(\sin\phi > 0) + \sigma(\sin\phi < 0)}$$

$$A_z = \frac{3\alpha P_z}{4} \equiv \frac{\sigma(\cos\theta > 0) - \sigma(\cos\theta < 0)}{\sigma(\cos\theta > 0) + \sigma(\cos\theta < 0)}$$

$$A_{xz} = \frac{-2}{\pi}\sqrt{\frac{2}{3}}T_{xz} \equiv \frac{\sigma(\cos\theta\cos\phi < 0) - \sigma(\cos\theta\cos\phi > 0)}{\sigma(\cos\theta\cos\phi > 0) + \sigma(\cos\theta\cos\phi < 0)}$$

$$A_{yz} = \frac{2}{\pi}\sqrt{\frac{2}{3}}T_{yz} \equiv \frac{\sigma(\cos\theta\sin\phi > 0) - \sigma(\cos\theta\sin\phi < 0)}{\sigma(\cos\theta\sin\phi > 0) + \sigma(\cos\theta\sin\phi < 0)}$$

$$A_{x^2-y^2} = \frac{1}{\pi}\sqrt{\frac{2}{3}}(T_{xx} - T_{yy}) \equiv \frac{\sigma(\cos 2\phi > 0) - \sigma(\cos 2\phi < 0)}{\sigma(\cos 2\phi > 0) + \sigma(\cos 2\phi < 0)}$$

$$A_{xy} = \frac{2}{\pi}\sqrt{\frac{2}{3}}T_{xy} \equiv \frac{\sigma(\sin 2\phi > 0) - \sigma(\sin 2\phi < 0)}{\sigma(\sin 2\phi > 0) + \sigma(\sin 2\phi < 0)}$$

$$A_{zz} = \frac{3}{8}\sqrt{\frac{3}{2}}T_{zz} \equiv \frac{\sigma(\sin 3\theta > 0) - \sigma(\sin 3\theta < 0)}{\sigma(\sin 3\theta > 0) + \sigma(\sin 3\theta < 0)}$$

Helicity Amplitudes for $e^- + e^+ \to Z + H$

$$e^-(p_1) + e^+(p_2) \to Z^\alpha(k_2) + H(k)$$

In the limit of massless initial states

$$\begin{aligned}
M(-,+,\pm) &= \frac{g_w^2 m_z \sqrt{s}}{\cos^2\theta_w((s-m_z^2)+i\Gamma_z m_z)} \frac{(c_v+c_a)}{2} \left[1 - \frac{\sqrt{s}}{m_z^2}(E_z b_z \pm i\tilde{b_z} P_z)\right] \\
&\quad \times \frac{(1\mp\cos\theta)}{\sqrt{2}} \\
M(\mp,\pm,0) &= \frac{g_w^2 \sqrt{s}}{\cos^2\theta_w((s-m_z^2)+i\Gamma_z m_z)} \frac{(c_v\pm c_a)}{2} \left[E_z - \sqrt{s} b_z\right] \sin\theta \\
M(+,-,\pm) &= \frac{g_w^2 m_z \sqrt{s}}{\cos^2\theta_w((s-m_z^2)+i\Gamma_z m_z)} \frac{(c_v-c_a)}{2} \left[-1 + \frac{\sqrt{s}}{m_z^2}(E_z b_z \pm i\tilde{b_z} P_z)\right] \\
&\quad \times \frac{(1\pm\cos\theta)}{\sqrt{2}}
\end{aligned}$$

where the first two entries in M denote the helicities $+1/2$ and $-1/2$ of the electron and positron respectively

$\sqrt{s}$ = total center of mass energy , $C_v = -0.5 + \sin^2\theta_w$, $C_a = -0.5$ where θ_w is the weak mixing angle.
we adopt the following representations for the polarization vectors of Z

$$\varepsilon_\mu(s = \pm 1) = \mp\frac{1}{\sqrt{2}}(0, -\cos\theta, \mp i, \sin\theta) \tag{6}$$

$$\varepsilon_\mu(s = 0) = \frac{1}{m_z}(\mid p_z \mid, -E_z\sin\theta, 0, -E_z\cos\theta) \tag{7}$$

where E_z,$\mid p_z \mid$ are the energy and momentum of the Z respectively,with θ being the polar angle made by Z with respect to the e^- coming along the positive z axis.

Sensitivities at $\sqrt{s} = 500$GeV, $\int \mathcal{L} dt = 500$ fb^{-1} for unpolarized and polarized beams

Observable	Coupling	Limit ($\times 10^{-3}$) for $P_L = 0$, $\bar{P}_L = 0$	Limit ($\times 10^{-3}$) for $P_L = -0.8$, $\bar{P}_L = 0.3$
σ	Re b_z	3.32	2.8
A_x	Re b_z	394	54.2
A_y	Re $\tilde{b}_z$	204	28.2
A_z	Im $\tilde{b}_z$	47.9	40.4
A_{xy}	Re $\tilde{b}_z$	33.7	28.5
A_{yz}	Im b_z	77.7	10.7
A_{xz}	Im $\tilde{b}_z$	72.0	9.93
$A_{x^2-y^2}$	Re b_z	46.7	39.4
A_{zz}	Re b_z	12.8	10.8

- σ sensitive to only Re b_z and A_x,$A_{x^2-y^2}$,A_{zz} being CP even observables depend on CP even Re b_z
- Remaining asymmetries being either CP even and T odd or CP odd, has dependence on the CP odd coupling (Im b_z, $\tilde{b}_z$)

(K.Rao, S.D. Rindani, P.Sarmah, Nucl.Phys.B 950 114840 (2020))

Sensitivities at $\sqrt{s} = 250\text{GeV}$, $\int \mathcal{L} dt = 2\ \text{ab}^{-1}$ for unpolarized and polarized beams

Observable	Coupling	Limit ($\times 10^{-3}$) for $P_L = 0$, $\bar{P}_L = 0$	Limit ($\times 10^{-3}$) for $P_L = -0.8$, $\bar{P}_L = 0.3$
σ	Re b_z	1.36	1.15
A_x	Re b_z	3480	478
A_y	Re $\tilde{b}_z$	303	41.7
A_z	Im $\tilde{b}_z$	32.3	27.2
A_{xy}	Re $\tilde{b}_z$	22.7	19.2
A_{yz}	Im b_z	189	26.1
A_{xz}	Im $\tilde{b}_z$	107	14.7
$A_{x^2-y^2}$	Re b_z	94.5	80.2
A_{zz}	Re b_z	26.8	22.8

- Better sensitivities for oppositely polarized beams
- Slight improvement on limits with increasing c.m. energy
- Minimal acceptance cut leads to 1% change in all the observables

(K.Rao, S.D. Rindani, P.Sarmah, Nucl.Phys.B 950 114840 (2020))

Asymmetries and Sensitivities at LHC

$$q(p_1) + \bar{q}(p_2) \to Z^{\alpha}(p) + H(k)$$

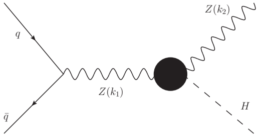


Figure: Feynman diagram for ZH production.

where the vertex $Z_\mu(k_1) \to Z_\nu(k_2)H$ takes the following Lorentz invariant structure

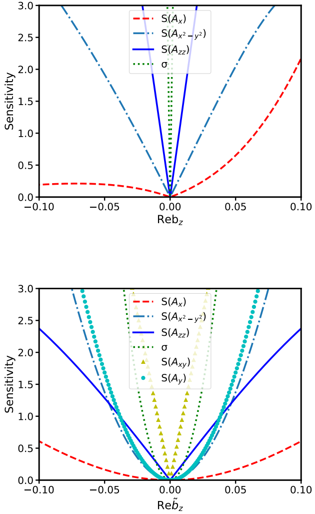
$$\Gamma^V_{\mu\nu} = \frac{g_w}{\cos\theta_w} m_z \left[a_z g_{\mu\nu} + \frac{b_z}{m_z^2}\left(k_{1\nu}k_{2\mu} - g_{\mu\nu}k_1.k_2\right) + \frac{\tilde{b}_z}{m_z^2}\epsilon_{\mu\nu\alpha\beta}k_1^\alpha k_2^\beta \right]$$

Sensitivities at $\sqrt{s} = 14$ TeV LHC

Observable	Coupling	Limit ($\times 10^{-3}$)
σ	Re b_Z	0.70
A_x	Re b_Z	136
A_y	Re $\tilde{b}_Z$	37.9
A_z	Im $\tilde{b}_Z$	13.5
A_{xy}	Re $\tilde{b}_Z$	9.53
A_{yz}	Im b_Z	16.5
A_{xz}	Im $\tilde{b}_Z$	13.3
$A_{x^2-y^2}$	Re b_Z	24.4
A_{zz}	Re b_Z	6.88

TABLE: 1σ limit obtained from cross section and various leptonic asymmetries calculated upto linear order in couplings at $\sqrt{s} = 14$ TeV with integrated luminosity $\int \mathcal{L}dt = 1000$ fb^{-1}.

Sensitivities at $\sqrt{s} = 14$ TeV and $\int \mathcal{L}dt = 1000$ fb^{-1}



Observable	Coupling	Limit ($\times 10^{-3}$)
σ	$\lvert\text{Re } b_Z\rvert$	0.70
σ	$\lvert\text{Im } b_Z\rvert$	15.9
A_{xy}	$\lvert\text{Re } \tilde{b}_Z\rvert$	9.54
A_{xz}, A_z	$\lvert\text{Im } \tilde{b}_Z\rvert$	13.3

TABLE: The best 1σ limit on couplings and the corresponding observables at $\sqrt{s} = 14$ TeV

(K.Rao, S.D.Rindani, P.Sarmah, Nucl.Phys.B 964 115317 (2021))

Summary

- Studied anomalous ZZH vertex by making use of the full density matrix of Z boson at the e^+e^- and LHC. The 8 angular asymmetries corresponding to different polarization states of Z, help probing all the anomalous couplings.
- We see that most of the 1σ limits are of the order of a few times 10^{-3} for 500 GeV e^+e^- colliders and find that beams with opposite polarization provides better limits on the couplings.
- $\sqrt{s} = 14$ TeV LHC with $\int \mathcal{L}dt = 1000$ fb^{-1} could provide a limit on the couplings Reb_z in the interval $[-0.7, 0.7] \times 10^{-3}$ and Imb_z in the interval $[-15.9, 15.9] \times 10^{-3}$.
- Couplings Re$\tilde{b}_z$ and Im$\tilde{b}_z$ get a best bound of $|\text{Re}\tilde{b}_z| \leq 9.54 \times 10^{-3}$ and $|\text{Im}\tilde{b}_z| \leq 13.3 \times 10^{-3}$respectively.

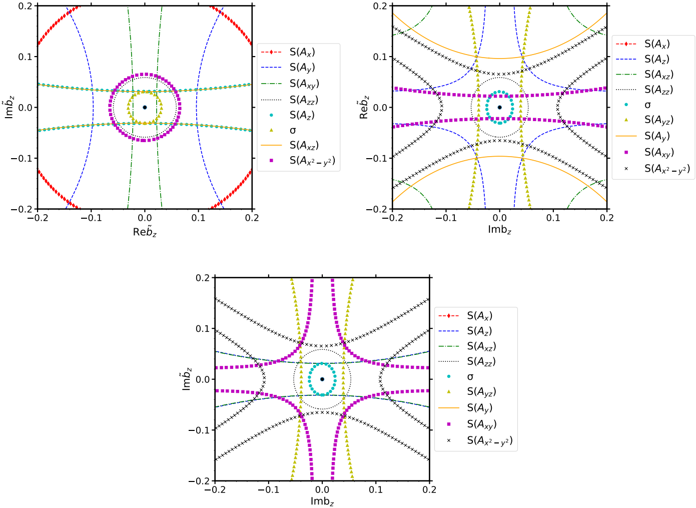
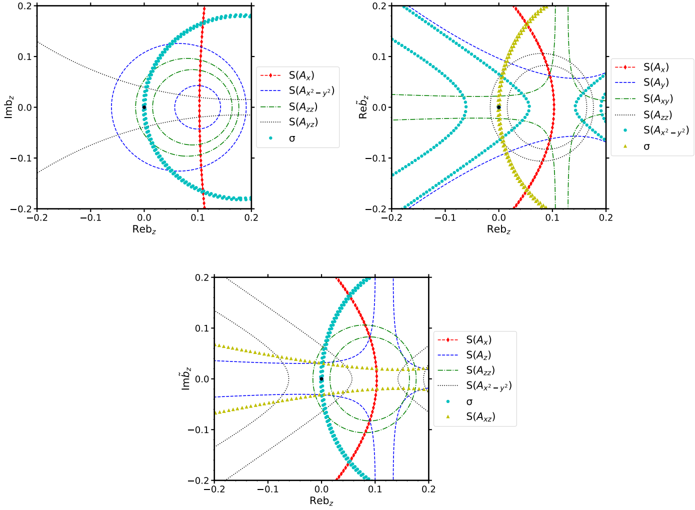


FIGURE: 1σ sensitivity contours for cross-section and asymmetries obtained by varying two parameters simultaneously.



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