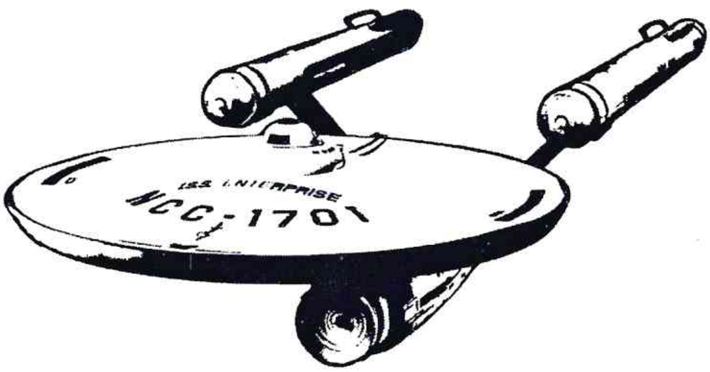
Effective Field Theories Beyond Colliders

NExT PhD Workshop

2021



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Effective Field Theories

Are valid quantum field theories with a limited range of validity and often constructed with a specific scope in mind.

→ Given the observational and theoretical shortcomings of the SM, we expect it to be an EFT as well.

→ EFTs allow us to disentangle the relevant from the less relevant effects in a given problem.

Quiz

Problems or Puzzles in Particle Physics

Effective Field Theories associated/helpful to current issues

-
-
-
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- ......

ν - Masses

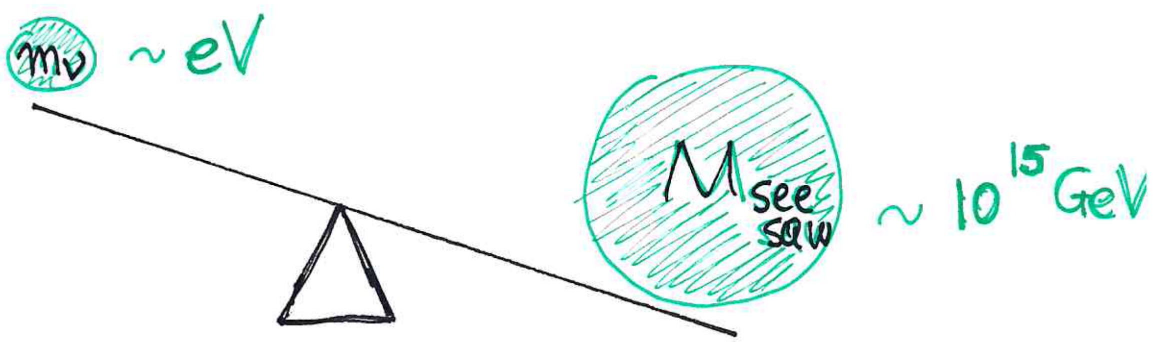
$m_\nu \sim 1 eV$

Dim-4 Standard Model

$m_\nu = 0$

UV Models for ν- massgeneration

Seesaw Models



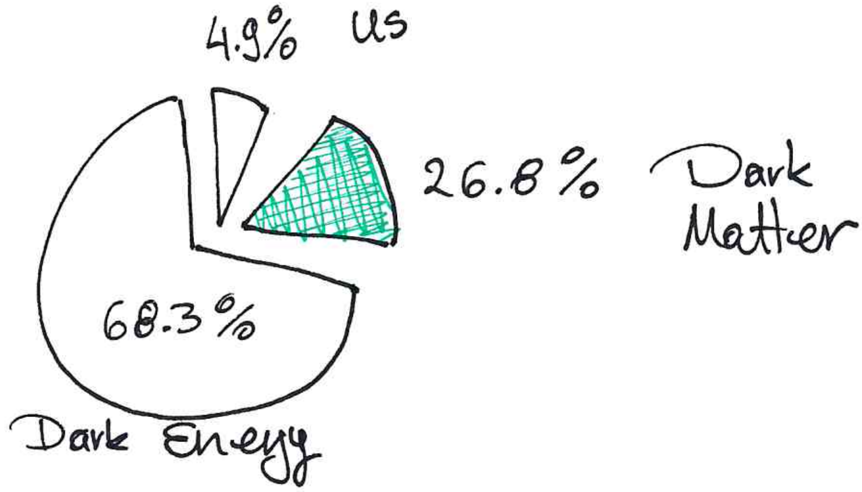
EFT : dim-5 Weinberg Operator

$$\mathcal{L} \supset \frac{C}{\Lambda_{seesaw}} \left(\overline{L^c}\,\tilde{H}^*\right)\left(\tilde{H}^\dagger L\right) + h.c$$

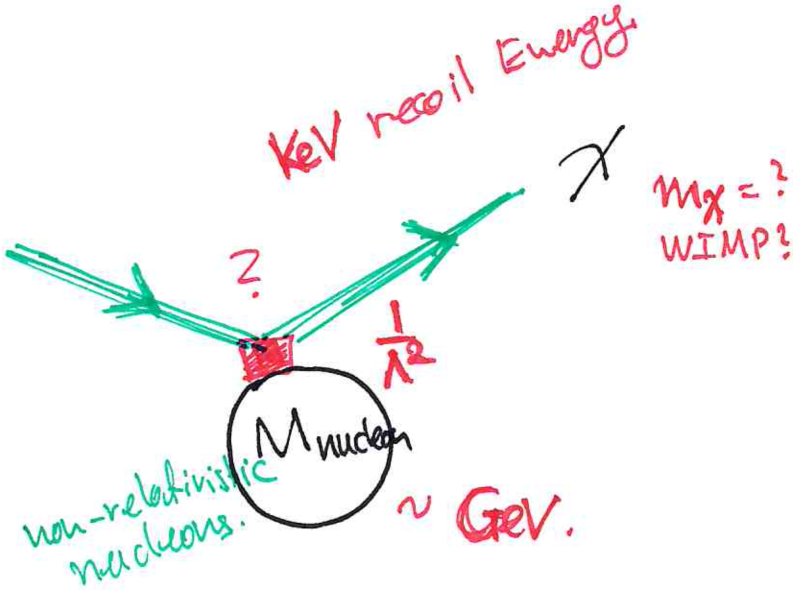
L - number violation

Dark Matter

Energy Budget of Universe



Direct Detection Experiments



<u>Dark-Matter-EFT</u> from quarks to non-rel. nucleons.

$$\mathcal{L}_\chi \overset{?}{\supset} \frac{C}{\Lambda^2}(\bar{\chi}\gamma_\mu\chi)(\bar{q}\gamma^\mu P_{L/R}\, q)$$

or $\frac{C}{\Lambda}(\bar{\chi}\sigma^{\mu\nu}\chi)\, F_{\mu\nu}$

or ...

Baryogenesis

Origin of Matter-Antimatter Asymmetry.

<u>One</u> possible solution EW Baryogenesis

→ need new sources of CP violation

→ modify EW Phase-Transition to 1^{st} order Phase-Tr.

$$\mathcal{L} \supset \frac{c}{\Lambda^2_{\cancel{CP}}} H^\dagger H \bar{Q} H t + h.c.$$

$$+ \frac{c'}{\Lambda_{CP}} (H^\dagger H)^3$$

[Huber, Pospelov, Ritz(?), Ramsey-Musolf 12]

$\Lambda \sim 1\,\text{TeV}$ for successful EW Baryogen.

Strong CP Problem

Why is the neutron EDM so small? $\Rightarrow \theta_{QCD} \simeq 0$

$$\mathcal{L} \supset \underset{< 10^{-10} \; ???}{\theta_{QCD}} \, G \tilde{G}$$

Axion - EFT

$$\mathcal{L} = \frac{1}{2} \partial_\mu a \, \partial^\mu a + \frac{a}{f_{PQ}} G \tilde{G}$$

Dynamically solves the Strong CP Problem

Invisible Axion Models $\quad f_{PQ} >> v_{ew} \simeq 246 \text{ GeV}$

Hierarchy Problem of Higgs Mass

→ Stability / Calculability of the Higgs mass in the presence of New-Dynamics

Only mass-parameter in SM.

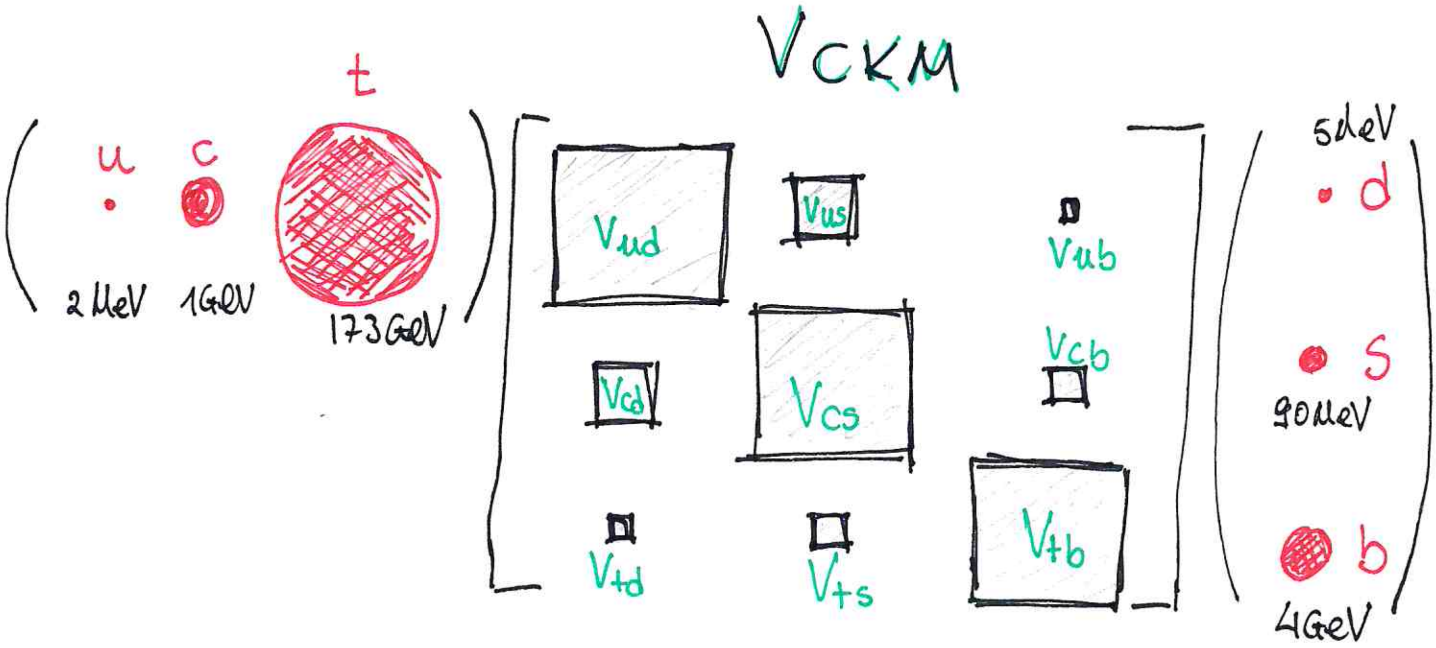
$$\mathcal{L} \supset -\left(\mu^2 + \Lambda_{NP}^2 \tfrac{1}{16\pi^2}\#\right) H^\dagger H + \dots$$

$$\overset{?}{2}$$

$$0$$

If $\Lambda_{NP} \sim g\, v_{ew}$ where are the new states.

Flavour Puzzle



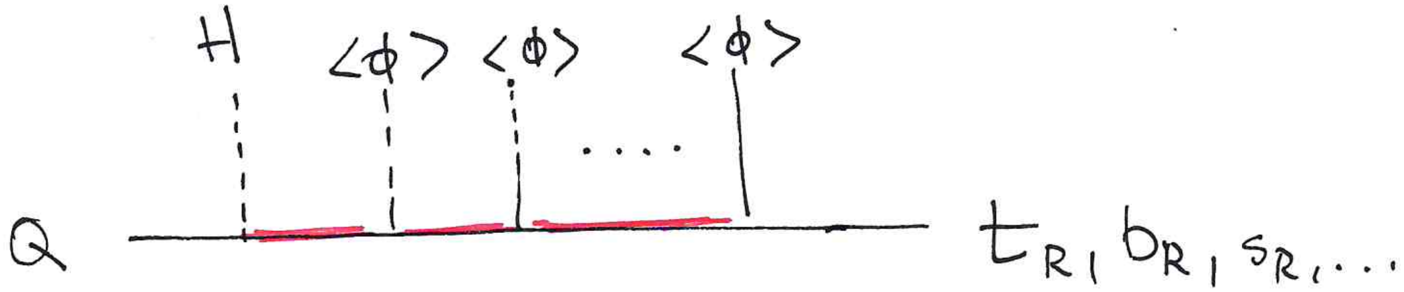
One Solution: Froggatt-Nielsen Models.

Flavour-VEVs.

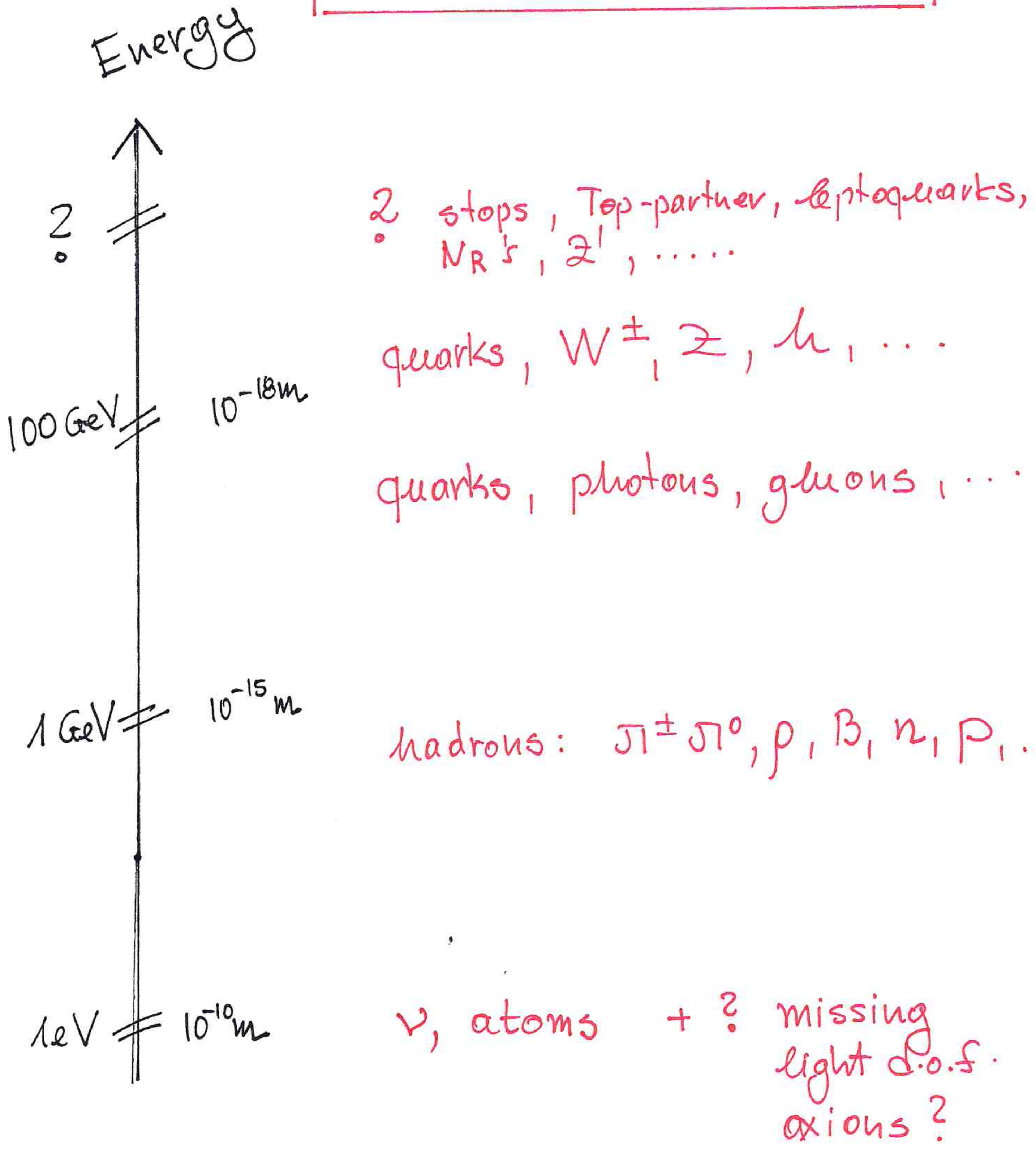
FN charges.

$$\mathcal{L}_{Yukawa} \supset \underbrace{\left(\frac{\langle \phi \rangle}{\Lambda_{\cancel{Flavour}}} \right)}_{\text{dimensionless.}}{}^{q_Q + q_t} \bar{Q} H t + h.c.$$

dimensionless. $\Lambda_{\cancel{Flavour}}$ not set!

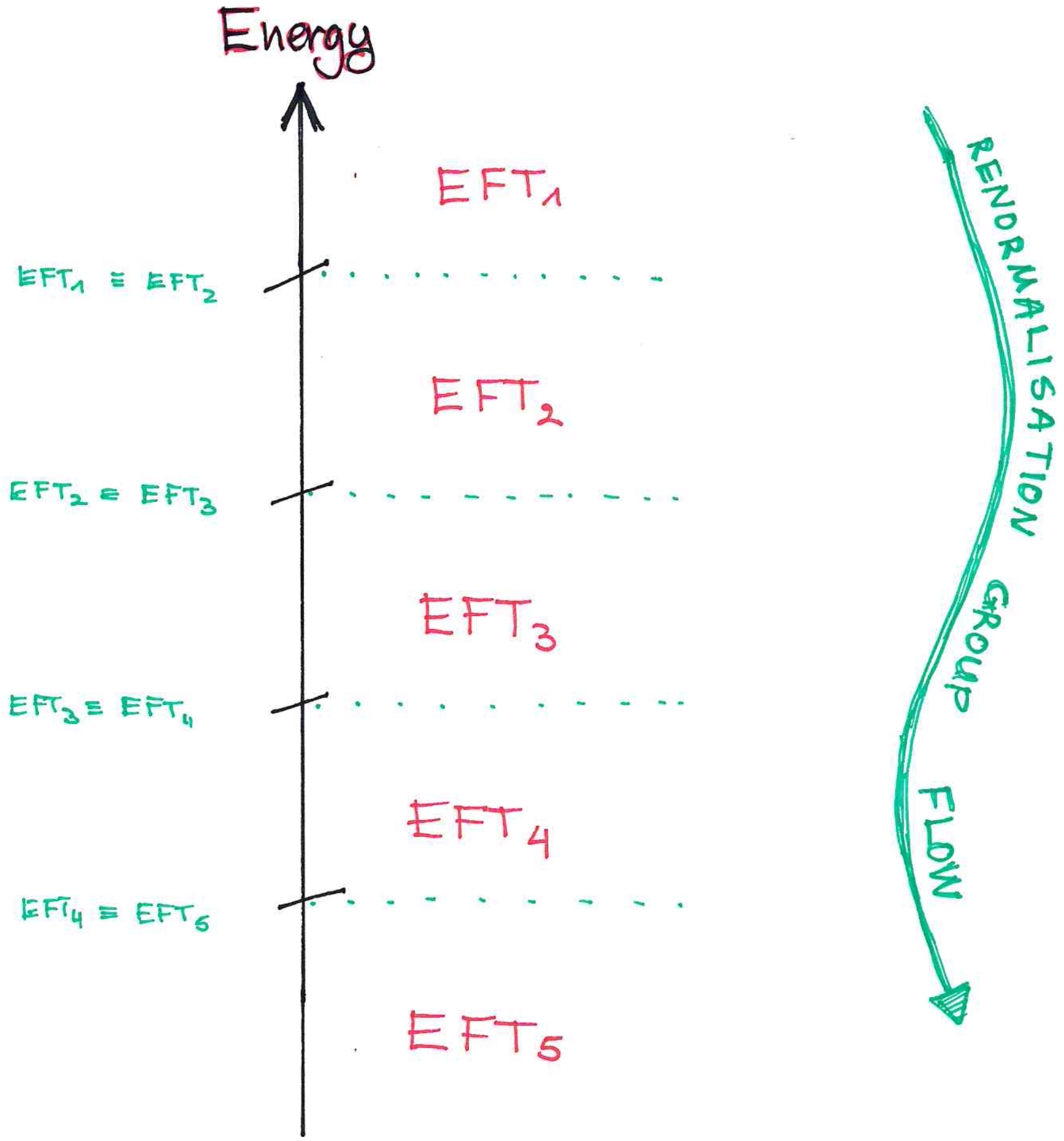


Tower of EFTs



→ Every energy scale has its own periodic table

→ The appropriate d.o.f depend on the Energy



The study of the Renormalisation Group enables us to combine EFTs and obtain both qualitative and quantitative control of dynamics at various scales

The power of EFTs at the era of LHC.

Precision

Correlation of Signals

Modelbuilding Guidance

Goal of these lectures

Highlight the usefulness and required methodology for EFTs in terms of phenomenologically relevant examples.

Precision:

most relevant for SM predictions with NP sensitivity

example: FCNC observable ε_K.

Correlation of signals:

Higgs $\cancel{CP}$ and EDMs within SMEFT

Modelbuilding: Guidance

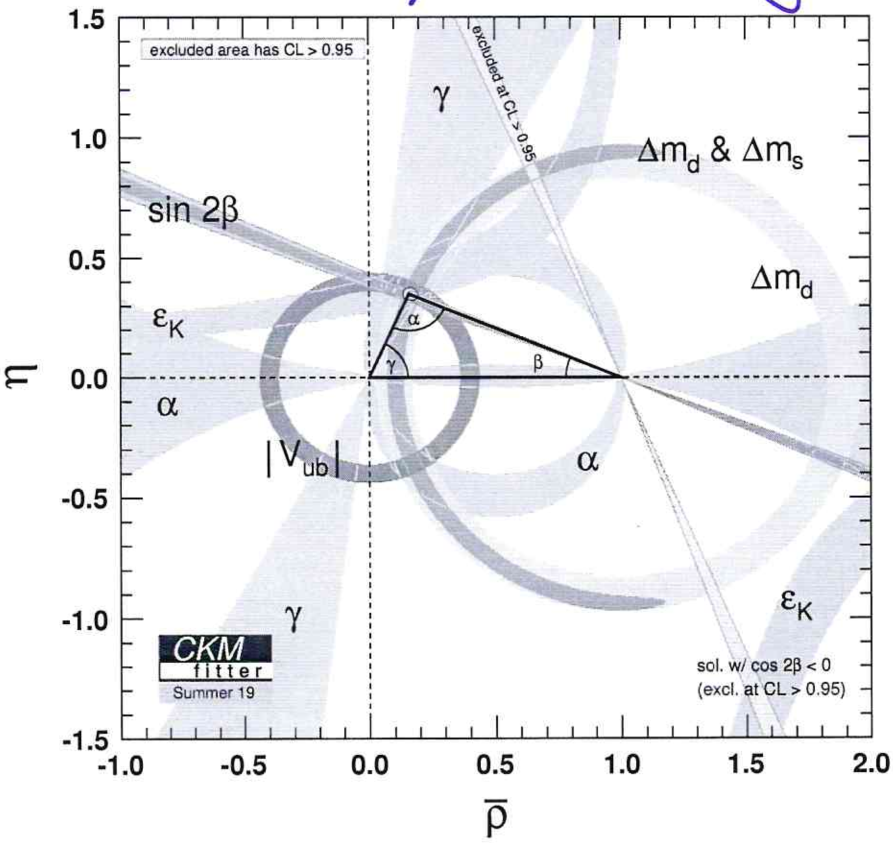
The Strong-CP Problem and Axion EFT

EFTs for Precision SM

Quark-flavour-violating observables strongly constrain NP models with $\Lambda_{NP} \sim \Lambda_{EW}$

NP Flavour Problem

CKM Unitarity Triangle



Question: Why are q-FCNCs quite suppressed in the SM?

$$\mathcal{L}^{SM}_{Fl} \supset y^u_{ij}\, \bar{Q}_i \tilde{H} u_j + y^d_{ij}\, \bar{Q}_i H d_j$$

Many interesting observables at LHC, Belle-II
+ anomalies at LHCb R_K, $B \to K^* \mu\mu$.

$\boxed{\text{Kaon Sector}}$ $\longrightarrow$ Most stringent constraint

ε_K, ε'/ε, $K \to \pi\nu\nu$, $K_L \to \mu\mu$

$$\boxed{\varepsilon_K : \not{CP} \text{ in } K - \bar{K}}$$

$$\mathcal{L}_{NP} = \frac{C}{\Lambda_{NP}^2} (\bar{s}\Gamma d)(\bar{s}\Gamma d) + h.c.$$

ε_K excluded composite-Higgs models with anarchic Yukawas!

Success of flavour observables as NP probes relies heavily on EFT computations.

ε_K : indirect $\cancel{CP}$ in $K-\bar{K}$

(if CP is conserved $K_L \not\to \pi\pi$, but mixing allows it)

$$\varepsilon_K \equiv \frac{\langle (\pi\pi)_{I=0} \mid K_L \rangle}{\langle (\pi\pi)_{I=0} \mid K_S \rangle}$$

= interesting details

= more interesting details

$$= e^{i\varphi_\varepsilon} \sin\varphi_\varepsilon \left[\frac{\mathrm{Im} \langle \bar{K}^0 | \mathcal{H} | K^0 \rangle}{\Delta M_K} + \xi \right.$$

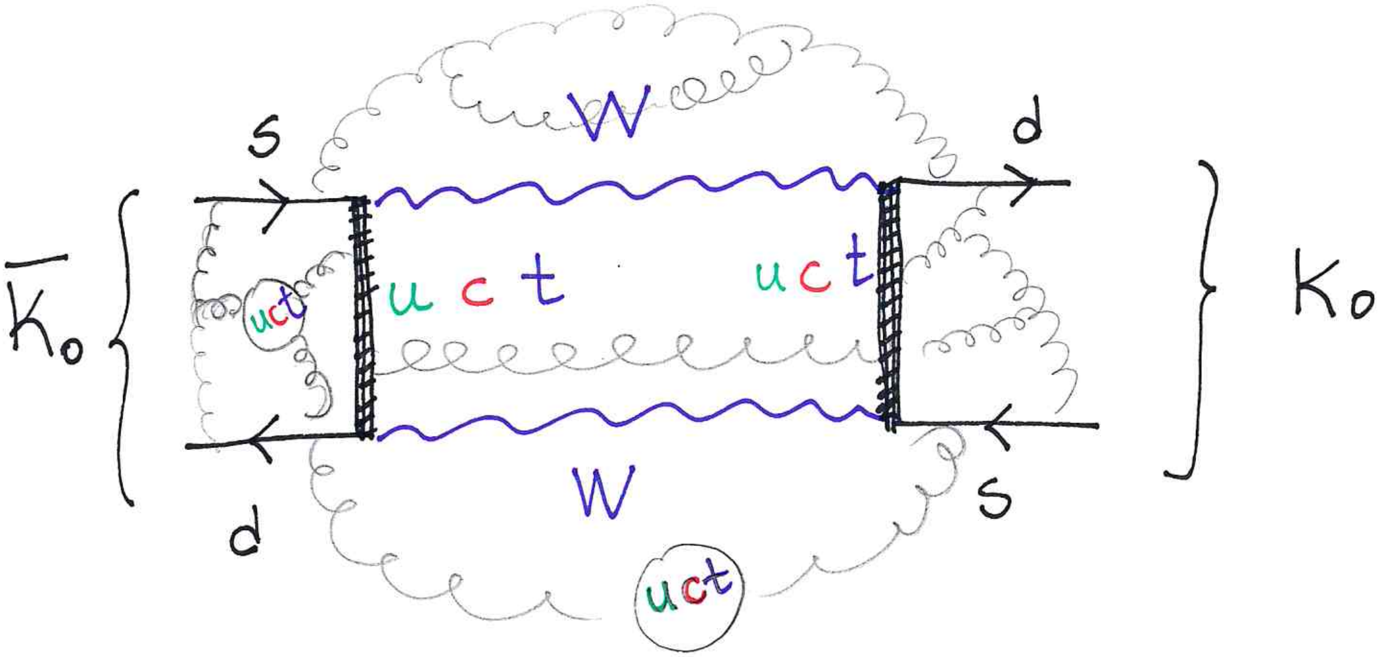
from experiment.

non pertur (ignore he

How would you compute

$$\mathrm{Im} \langle \bar{K}^0 | \mathcal{H} | K^0 \rangle \quad ?$$

$\mathrm{Im} \langle \bar{K}_0 | \mathcal{H} | K_0 \rangle$



Problem has many dynamical scales

TOO COMPLICATED

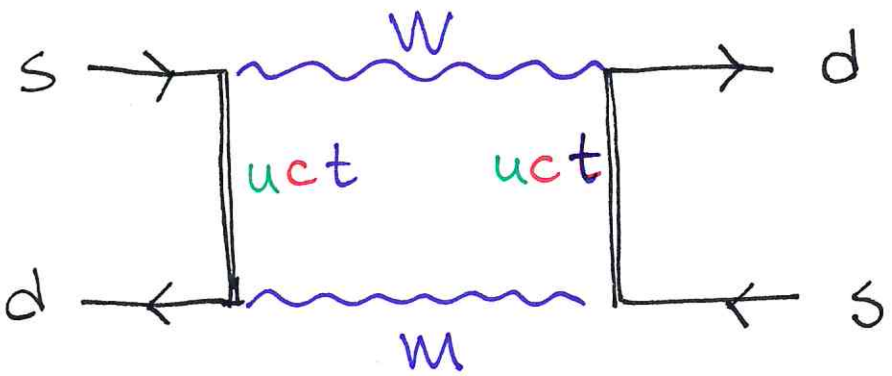
$\Lambda_{EW} \sim M_W, m_t \sim 100\,\mathrm{GeV}$

$\Lambda_{charm} \sim m_c \sim 1\,\mathrm{GeV}$

$\Lambda_{QCD} \sim m_K \sim 500\,\mathrm{MeV}$

- Are the gluons hard or soft?
- What value of α_s shall we use in logarithms (large) $\frac{\alpha_s}{4\pi} \log \frac{m_t}{m_c}$?

Dimensional and Wolfenstein parameter counting for ε_K



$$\lambda_q \equiv V_{qd} V_{qs}^*$$

CKM unitarity: $\lambda_u + \lambda_c + \lambda_t = 0$

$\varepsilon_K \longrightarrow$ Imaginary Part of Amplitude

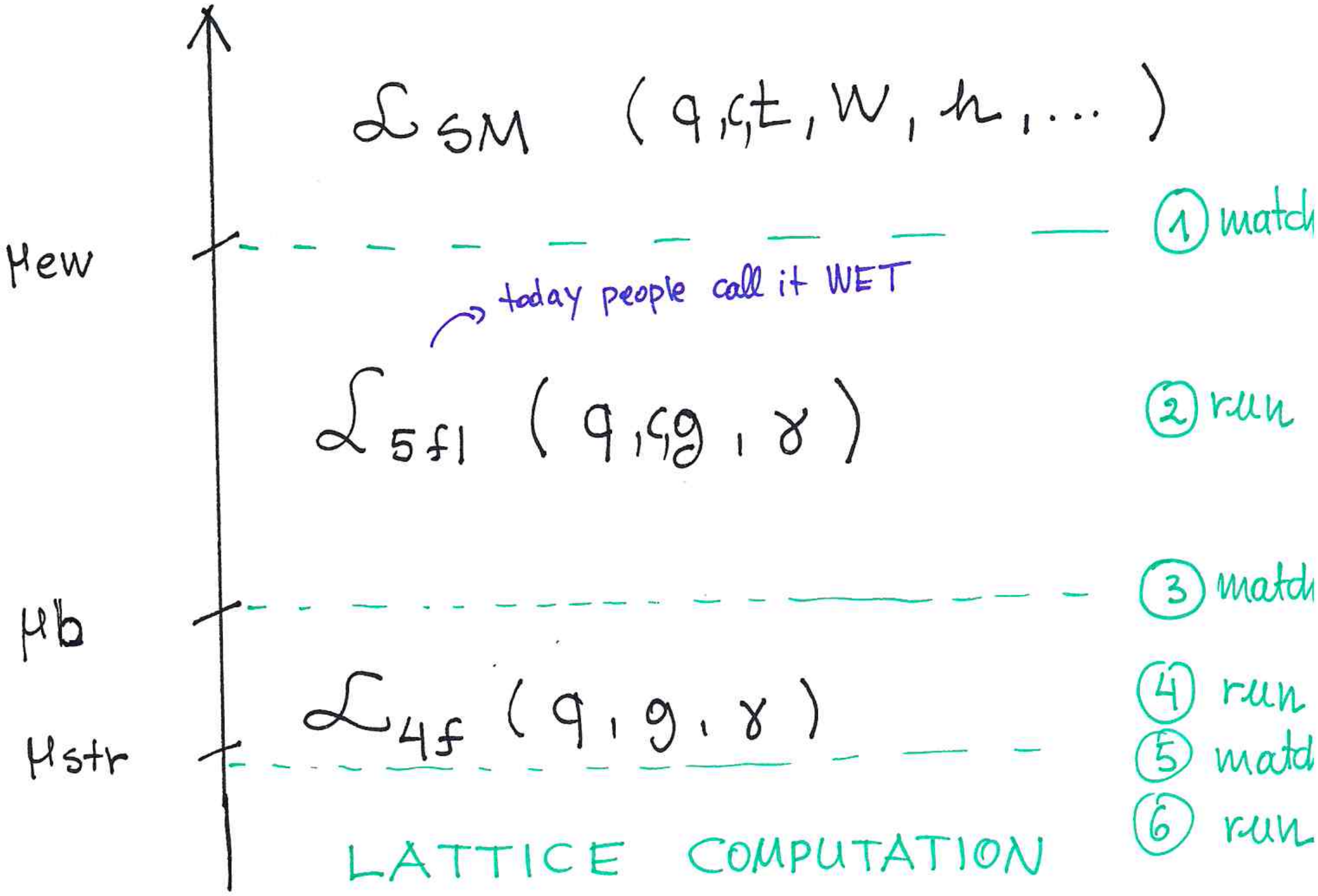
~~Three~~ Two contributions $\qquad \lambda \simeq V_{us} = 0.23$

$$\lambda_t^2 \frac{m_t^2}{M_W^2} \simeq \lambda^{10} \frac{m_t^2}{M_W^2} \sim 10^{-6}$$

$$\lambda_u \lambda_t \underbrace{\frac{m_c^2}{M_W^2}}_{\text{GIM}} \underbrace{\log \frac{m_c}{M_W}}_{\text{EFT result}} \simeq \lambda^6 \frac{m_c^2}{M_W^2} \log \frac{m_c}{M_W} \sim 10^{-7}$$

cannot neglect "c" !! compared

Setting up the EFT computation



Each step is an individual computation, which – in this case – can be performed in perturbation theory.

Step 1: Match SM with 5_{fl}-theory.

Power of EFT computation

- neglect all masses apart from those we integrate out (M_W, m_t, M_Z, M_h) (or expand when no LO effect)

- here no relevant kinematic scale (like p_T). Can expand in powers of external momenta (like in Fermi Theory)

- Multiple ways to perform the matching. One that extends well beyond LO is to match off shell 1LPI amplitudes between SM and 5_{fl}.

$$\boxed{\mathcal{A}^{1LPI}_{SM}(\mu_{EW}) \stackrel{!}{=} \mathcal{A}^{1LPI}_{5fl}(\mu_{EW})}$$

Other ways also possible (functional method, the Covariant Derivative Expansion, etc).

Step 1: The SM side

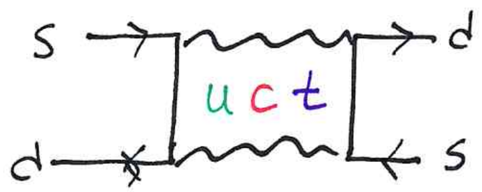
The top-quark contribution

$\mathcal{L}_{SM}$: massive t, W, Z, h

Compute the part of the Amplitude proportional to $\lambda_t^2 = (V_{td} V_{ts}^*)^2$ in perturbation theory and renormalise

LO

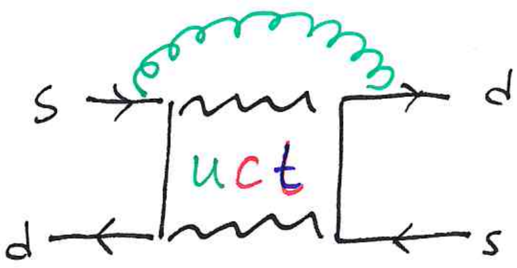
$\mathcal{A}^{ILPI,t}_{SM}(\mu_{ew}) =$



$\Big|_{\lambda_t^2}$ $\Big\}\ \alpha_s^0$

NLO

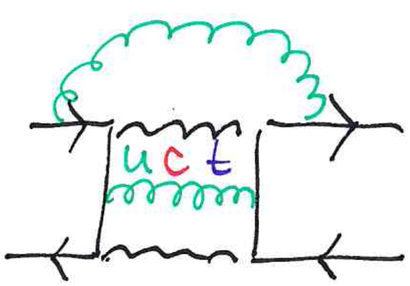
$+$



$\Big|_{\lambda_t^2} + \ldots \Big\}\ \dfrac{\alpha_s^{6fl}}{4\pi}$

NNLO

$+$



$\Big|_{\lambda_t^2} + \ldots \Big\}\ \left(\dfrac{\alpha_s^{6fl}}{4\pi}\right)^2$

$+ \ldots$

Step 1: The EFT side
The top-quark contribution

$$\mathcal{L}_{5fl}^{\Delta S=2} = \frac{G_F^2 M_W^2}{4\pi^2} Q_{S2} \left(\lambda_t^2 C^{tt} + \lambda_u^2 C^{uu} + \lambda_u \lambda_t C^{ut} \right) + h.c.$$

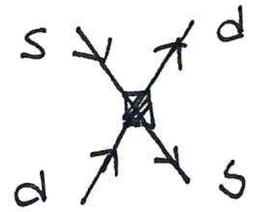
$$Q_{S2} = (\bar{s}_L \gamma^\mu d_L)(\bar{s}_L \gamma_\mu d_L)$$

Expansion of Wilson Coefficient

$$C^{tt}(\mu_{ew}) = C^{tt(0)}(\mu_{ew}) + \frac{\alpha_s^{5fl.}(\mu_{ew})}{4\pi} C^{tt(1)}(\mu_{ew}) + \ldots$$

$$\mathcal{A}_{5fl}^{1LPI,t}(\mu_{ew}) = i \frac{G_F^2 M_W^2}{4\pi^2} \Bigg($$

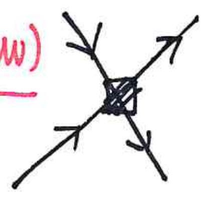
LO



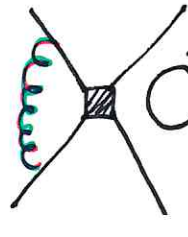
$$\cdot\, C^{tt(0)}(\mu_{ew})$$

NLO

$$+ \frac{\alpha_s(\mu_{ew})}{4\pi}$$



$$\cdot\, C^{tt(1)}(\mu_{ew}) +$$



$$C^{tt(0)}(\mu_{ew})$$

Step 1: Matching

$$\mathcal{A}^{SM,t}_{1LPI}(\mu_{ew}) \overset{!}{=} \mathcal{A}^{5fl,t}_{1LPI}(\mu_{ew})$$

By comparing the Amplitudes we obtain in perturbation theory

- $C^{tt\,(0)}(\mu_{ew})$ = function of m_t, M_W
- $C^{tt\,(1)}(\mu_{ew})$ = function of m_t, M_W
- $\vdots$

EFT "Magic": We don't need to about IR divergences because per definition the IR structure of full and EFT theory must be the same. IR poles must cancel in matchin

Step 2: RG Flow in EFT

$$\mathcal{L} \supset \frac{G_F^2 M_W^2}{4\pi^2} \lambda_t^2 \, C^{tt} \, Q_{S2}$$

Question: We computed $C^{tt}(\mu_{ew})$, but how do we evolve this to the next scale?

Hints:

- An EFT is still a QFT.
- How do you compute the running of α_s (and receive the Nobel price for asymtotic freedom)?

Step 2: RG Flow in EFT

Just like in renormalisable theories also here divergences must be absorbed in a redefinition/renormalisation of couplings.

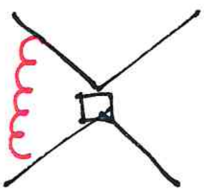
$$\mathcal{L} = \frac{G_F^2 M_W^2}{4\pi^2}\lambda_t^2 \, C^{tt,\,\mathrm{bare}} \left(\bar{s}_L^{\,\mathrm{bare}}\gamma^\mu d_L^{\mathrm{bare}}\right)^2$$

$$= \frac{G_F^2 M_W^2}{4\pi^2}\lambda_t^2 \, Z\, C^{tt}\, Z_f^2 \left(\bar{s}_L\gamma^\mu d_L\right)^2$$

$$Z_f = 1 + \frac{\alpha_s}{4\pi} Z_f^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 Z_f^{(2)} + \dots$$

from +  +  + ...

What about Z ? Renormalise $\bar{s}_L d_L \to \bar{d}_L s_L$ amplitude

 $+ \frac{\alpha_s}{4\pi}\left(Z_f^{(1)}\right)^2$  $+ \frac{\alpha_s}{4\pi} Z^{(1)}$ 

must be finite $\to Z^{(1)}$

Step 2: RG Flow in EFT

From Z - constants to RGE (in $\overline{MS}$

Bare quantities do not depend on μ.
$(\alpha_s^{bare} = Z_{\alpha_s} \alpha_s(\mu) \mu^{2\varepsilon})$

$$0 \stackrel{!}{=} \frac{d}{d\log\mu} C^{bare}$$

$$= \frac{d}{d\log\mu} Z C$$

$$= \frac{dZ}{d\log\mu} C + Z \frac{dC}{d\log\mu}$$

$$\Longleftrightarrow$$

$$\frac{dC}{d\log\mu} = \underbrace{-\frac{d\log Z}{d\log\mu}} C$$

$$\boxed{\frac{dC(\mu)}{d\log\mu} \equiv \gamma\, C(\mu)}$$

Anomalous Dimension γ and the RGE of C

Step 2: RG Flow in EFT

We can solve and resum large logarithms $\left(\alpha_s \log \frac{\mu_{low}}{\mu_{ew}} \right)^n$ by solving the RGE

$$\frac{dC}{d\log\mu} = \gamma C$$

Solution

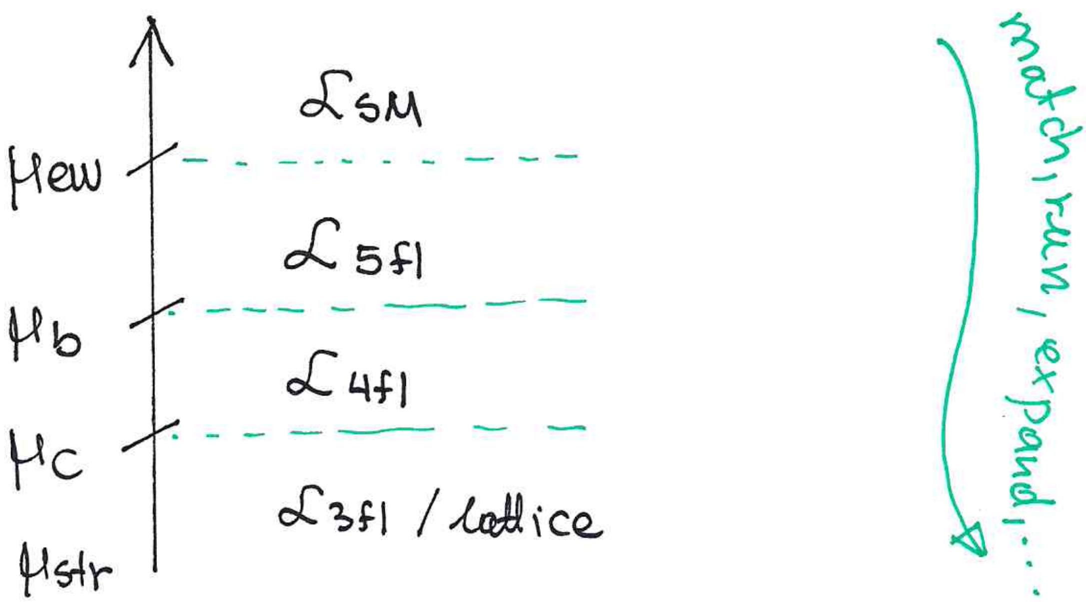
$$\boxed{C(\mu_{low}) = U(\mu_{low}, \mu_{EW})\, C(\mu_{EW})}$$

$$U(\mu_{low}, \mu_{EW}) = \exp\left[\int_{g_s(\mu_{ew})}^{g_s(\mu_{low})} dg' \, \frac{\gamma(g')}{\beta(g')} \right]$$

It can be shown that this resums all powers $(\alpha_s \log(\mu_{low}/\mu_{ew}))^n$!

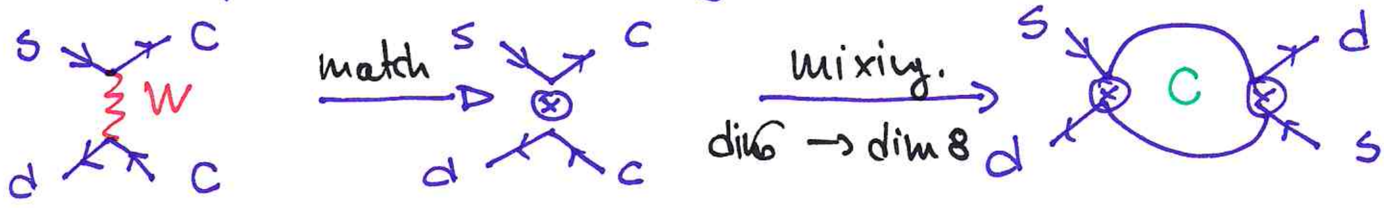
ε_K

The EFT tower



[The top-charm contribution is tougher but the principles are the same!

Complication: Mixing of operators



$\sim \frac{m_c^2}{M_W^2} \log \frac{m_c}{M_W}$

EFT $\longrightarrow$ FACTORISATION of Scales

$$\langle \bar{K}_0 | \mathcal{H} | K_0 \rangle = \underbrace{\langle Q \rangle (\mu_{had})}_{\text{LATTICE}}\, U(\mu_{had}, \mu_c) \times U(\mu_b, \mu_{ew}) \times C(\mu_{ew})$$

perturbative EFT

Precision EFT

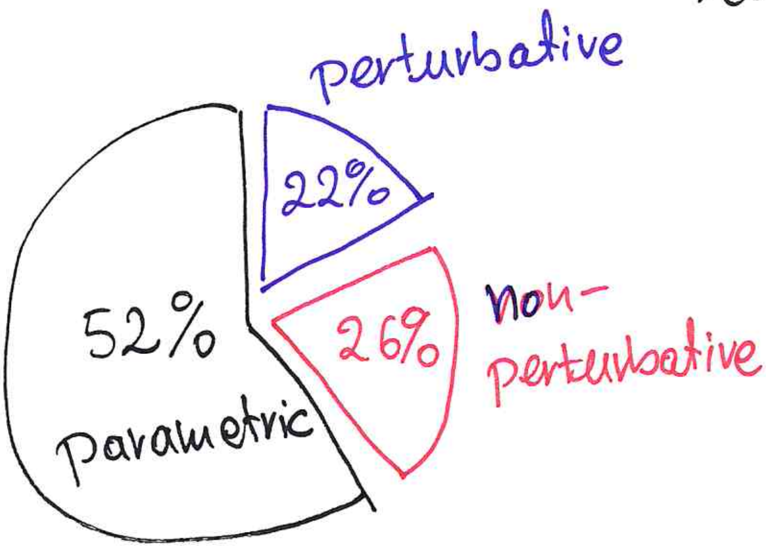
$$|\varepsilon_K| = \kappa_\varepsilon C_\varepsilon \hat{B}_K |V_{cb}|^2 \lambda^2 \bar{\eta}^2 \times$$

$$\times \left[|V_{cb}|^2 (1-\bar{\rho}) \underbrace{\eta_{tt}}_{\substack{\text{QCD} \\ \text{correction} \\ \text{for top-quark} \\ \text{contr.}}} S_t - \underbrace{\eta_{ut}}_{\substack{\text{QCD} \\ \text{correction} \\ \text{for charm-top} \\ \text{contr.}}} S_{ut} \right]$$

$$\eta_{tt} = 1 \text{ @ LO} \; , \; 0.55(2) \text{ @ NLL}$$

$$\eta_{ut} = 1 \text{ @ LO} \; , \; 0.402(5) \text{ @ N}^2\text{LL}$$

large QCD corrections.



$$|\varepsilon_K|^{SM} = 2.16(6)(8)(15) \times 10^{-3}$$

$$|\varepsilon_K|^{exp} = 2.228(11) \times 10^{-3}$$

The power of EFTs at the era of LHC

Precision

Take-Away Points

- Tower of EFTs $\equiv$ RG flow
- Separation / Factorisation of dynamical scales
- EFTs only way towards accurate prediction.
- Precision computations most impactful once a model (e.g SM) fixed. (not easy without)

The power of EFTs at the era of LHC

Correlation of NP signals

Careful: Correlations are only possible with a certain model-framework.

EFT are models, they come with their own assumptio

[e.g. SMEFT linear realisation of $SU(2) \times U(1)_Y \to U(1)_{em}$

$\vee$

HEFT/chiL$_{EW}$ non-linear realisatio of $SU(2) \times U(1)_Y \to U(1$

Nevertheless: As we saw, many UV models can match to the same EFT

$\Longrightarrow$ EFT analysis captures many mod

Correlation of NP Signals

Setup: - so far no clear signal of particles beyond the SM from <u>direct</u> production. at the LHC (Hierarchy of $m_{ew} \ll \Lambda_{NP}$)

- EW-precision tests compatible with linear EW-symmetry-breaking pattern (The Higgs particle is part of the Higgs Doublet field.)

SMEFT is the EFT framework that extends dim-4 SM with higher-dim operators with the power counting assumption $m_{ew} \ll \Lambda_{NP}$

SMEFT

E

scale separation

$\mathcal{L}_{UV}$ (your favourite NP with particles much heavier than Λ_{ew})

Λ - - - - - - - - - - - - -

$\mathcal{L}_{SMEFT}(Q, u, d, H, W^a, A, G, L \ldots$

Λ_{ew} - - - - - - - - - - - - -

$\mathcal{L}_{WET}$ (theory without $W^\pm, Z, h, t$)

Example: Integrate / Match out ν_R from Seesaw models $\longrightarrow$ dim-5 Weinberg operator for ν-Masses.

PROS : we can map data from multiple frontiers (intensity / high-p_T) to same theory

CONS :

#	$N_f = 1$	$N_f = 3$	$\cdots$
dim-5	2	12	
dim-6	84	3045	
$\vdots$			

[see Henning et al 15]

SMEFT vs SM

SM : 19 free real parameters

($g_1\, g_2\, g_3\ \theta_{QCD}\ \mu^2\ \lambda\ \ m_e\, m_\mu\, m_\tau\ m_u\, m_c\, m_t$
$m_d\, m_s\, m_b\ \lambda\ A\ \bar{\rho}\ \bar{\eta}$)

SMEFT: 3129 free real parameters
dim5 + 6

Need either : a concrete Moder

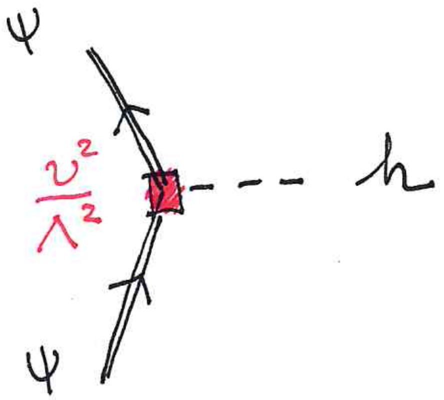
or : additional assumptions

(e.g. MFV couplings for flavour-violatio
restrict to operators with
tree-level contributions to
certain observables, etc...)

SMEFT
Higgs CP Violation
&
EDMs

Motivation: EW Baryogenesis

$\rightarrow$ new sources of $\cancel{CP}$



$$\frac{C}{\Lambda^2_{\cancel{CP}}} (H^\dagger H)(\bar{Q} H t) + h.c.$$

$\hookrightarrow$ 1 TeV for successful Baryogenesis.

Flavour Violation + CP Violation $\longrightarrow$ FCNCs.

Flavour Conservation + CP Violation $\longrightarrow$ EDMs.

LHC limited sensitivity to Higgs $\cancel{CP}$

EDM / LHC complementarity

SMEFT Model-Assumption

Restrict to operators that modify Higgs (flavour conserving) Yukawas including CP violation

CP conserving

$$\mathcal{L}_{SMEFT} = \sum_{ij} \frac{C_{ij}^{(+)}}{\Lambda^2} (H^\dagger H)(\bar{F}_L^i H f_R^j + \bar{f}_R^j H^\dagger F_L$$

CP violating.

$$+ \sum_{ij} \frac{C_{ij}^{(-)}}{\Lambda^2} (H^\dagger H) i (\bar{F}_L^i H f_R^j - \bar{f}_R^j H^\dagger F_L$$

BROKEN PHASE

ROTATION TO MASS BASIS

like in extended K-frame-work.

$$\mathcal{L} \supset \left[\frac{C^+ \cos\theta - C^- \sin\theta}{2\sqrt{2}} \frac{v^2}{\Lambda^2} - \frac{m_f}{v} \right] h\bar{f}f$$

$$+ \left[\frac{C^- \cos\vartheta + C^+ \sin\vartheta}{2\sqrt{2}} \frac{v^2}{\Lambda^2} \right] h\bar{f} i\gamma_5 f$$

$$\left\{ \sin\vartheta \equiv \frac{C^- v}{2\sqrt{2}\, m} \frac{v^2}{\Lambda^2} \right\}$$

SMEFT Restrictions
"model" assumptions

Restrict to operators that modify Higgs Yukawas including CP violation

CP conserving

$$\mathcal{L}_{SMEFT} = \frac{C_+}{\Lambda^2} (H^\dagger H)(\bar{Q}_L H q_R + h.c) + \frac{C_-}{\Lambda^2} i (H^\dagger H)(\bar{Q}_L H q_R - h.c)$$

CP violating.

Broken Phase

Rotation to mass eigenstates

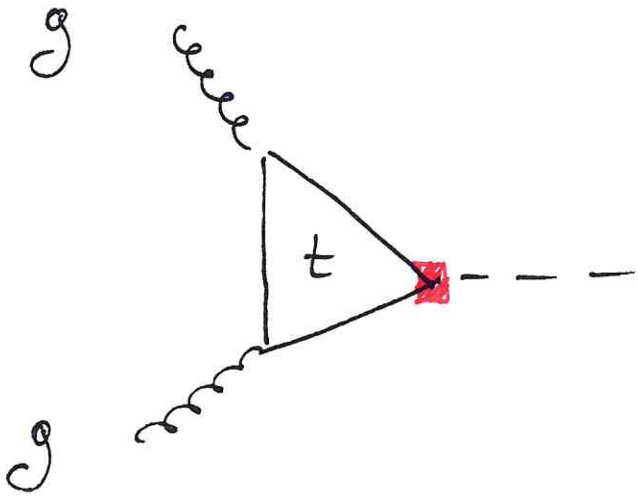
$$\mathcal{L}_\kappa \supset -\frac{y^{SM}}{\sqrt{2}} \kappa h \bar{q} (\cos\phi + i \sin\phi \gamma_5) q$$

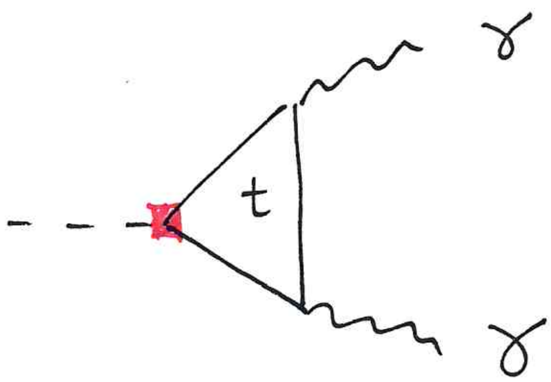
$$= \left[\frac{C_+ \cos\theta - C_- \sin\theta}{2\sqrt{2}} \frac{v^2}{\Lambda^2} - \frac{m_f}{v} \right] h \bar{f} f + \left[\frac{C_- \cos\theta + C_+ \sin\theta}{2\sqrt{2}} \frac{v^2}{\Lambda^2} \right] h \bar{f} i \gamma_5 f$$

with $\sin\theta \equiv \frac{C_- v}{2\sqrt{2}\, m} \frac{v^2}{\Lambda^2}$

$\not{C}\!\!P$ Higgs Yukawas @ LHC

Limited sensitivity at LHC in particular for light fermions

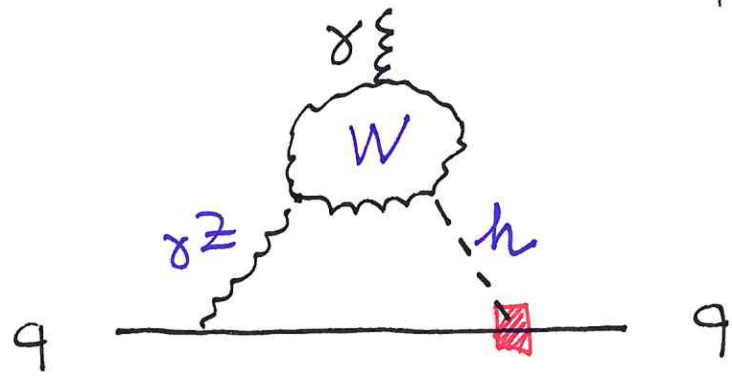




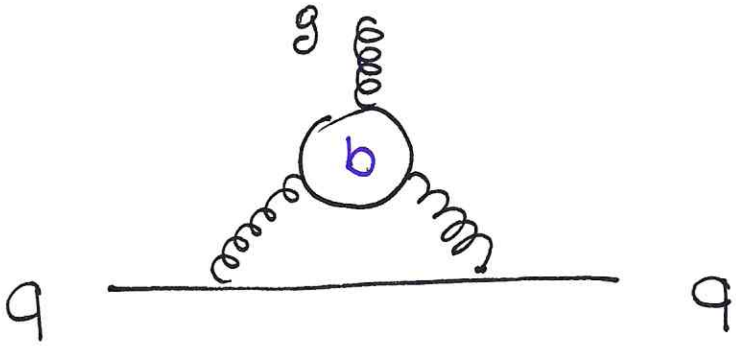
Difficult to disentangle CP violating from CP conserving contributions.

$\not{CP}$ Higgs Yukawas
&
EDMs

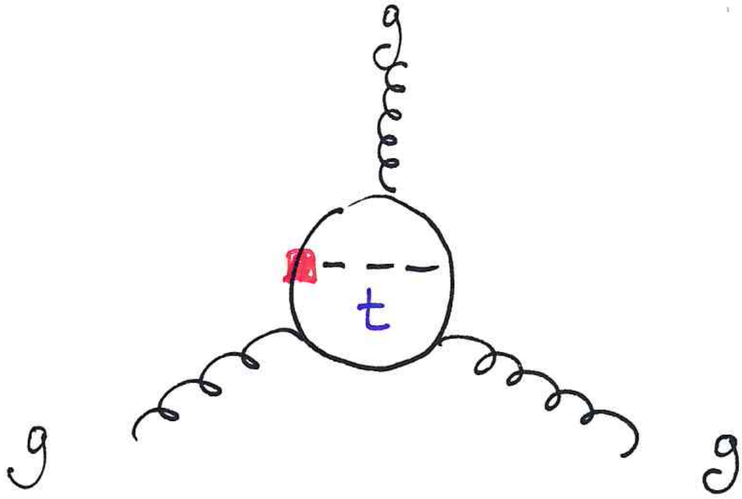
Contributions to electron, nuclear EDM



EDM



Chromo EDM



$\not{CP}$ Weinberg Operator

Many observables → correlations/cancellations

Excellent experimental prospects!

EDMs

Experimental Situation

[ecm]	d_e	d_n	$d_{p,D}$
current	1.1×10^{-29}	2.9×10^{-26}	—
expected	10^{-34}	1.0×10^{-28}	10^{-29}

	d_{Hg}	d_{Xe}	d_{Ra}
current	7.4×10^{-30}	$5.5\ 10^{-27}$	$4.2\ 10^{-22}$
expected	1.0×10^{-29}	$5.0\ 10^{-29}$	10^{-27}

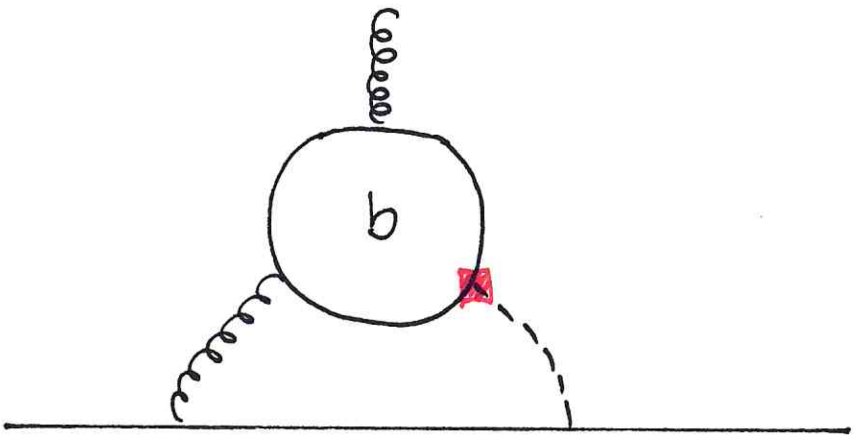
[Hewett et al 12,
Baker et al 06,
ACME 18,
Graner et al 16
EDM3 collaboration]

Impressive Prospects!

EDM Predictions

Example: bottom $\cancel{CP}$ Yukawa

Naive Barr-Zee Type Computation



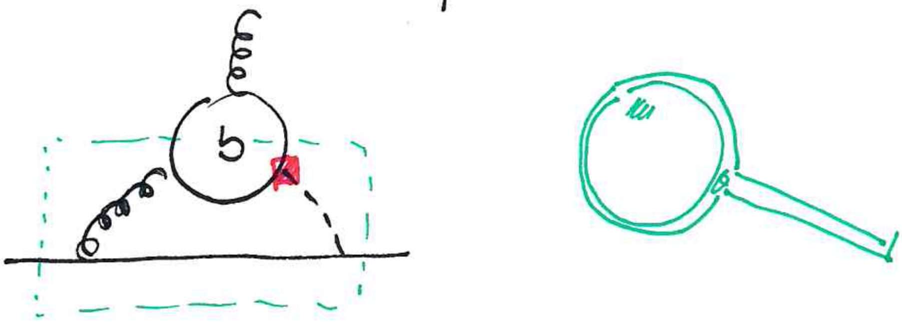
$$\tilde{d}_q = g_s^3 \left(\# + \# \log \frac{m_b}{m_h} \right) \kappa_b \sin\varphi$$

- Like for ε_K naive computation is a bad approximation
- Faktor 5 uncertainty in qCEDM
- Why? Multi-scale Problem $\alpha_s \log \frac{m_b}{m_h} \sim 1$

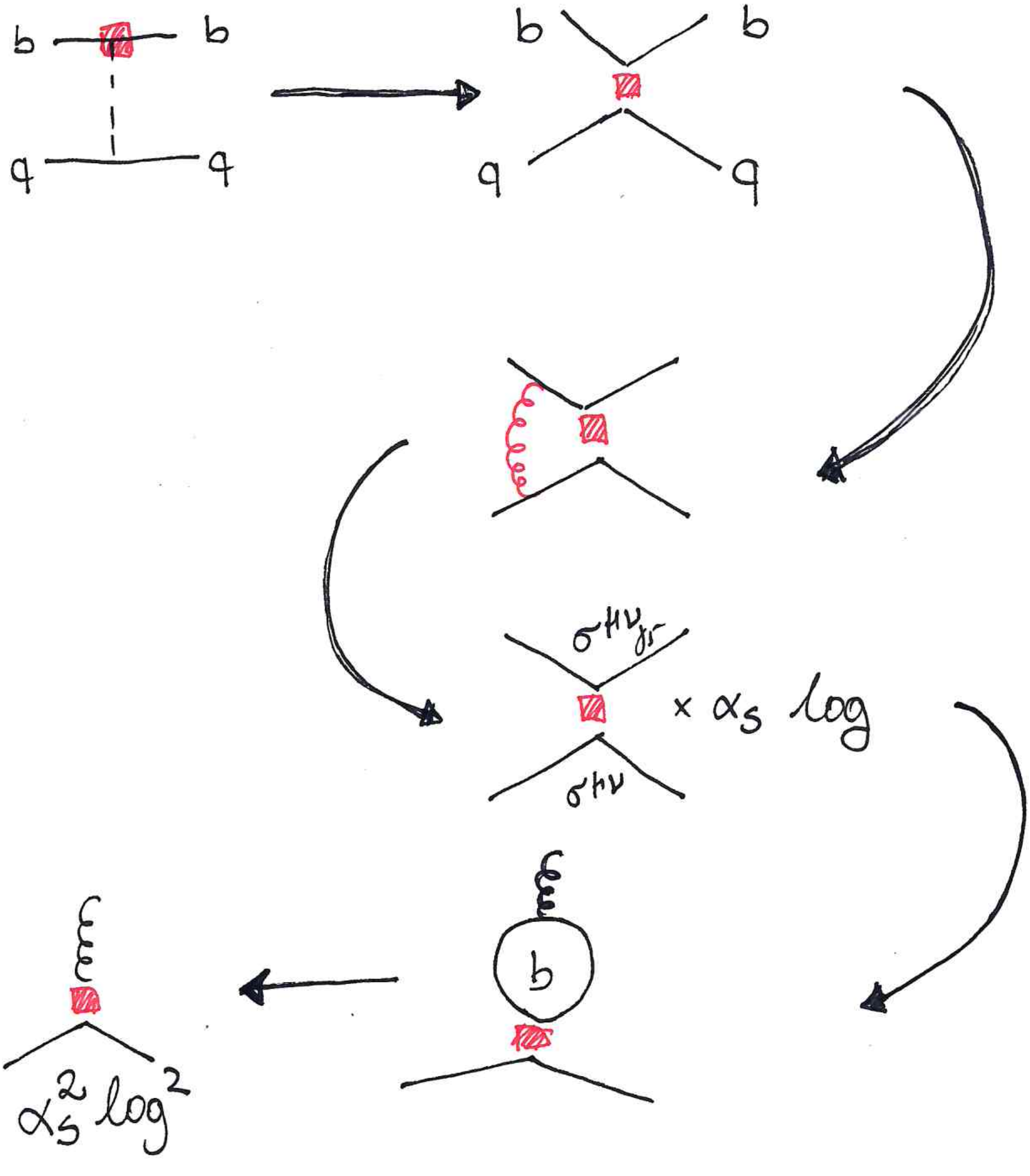
  $\longrightarrow$ EFT resums Logs.

What is the RGE doing for us?

Naive Fixed Order Computation

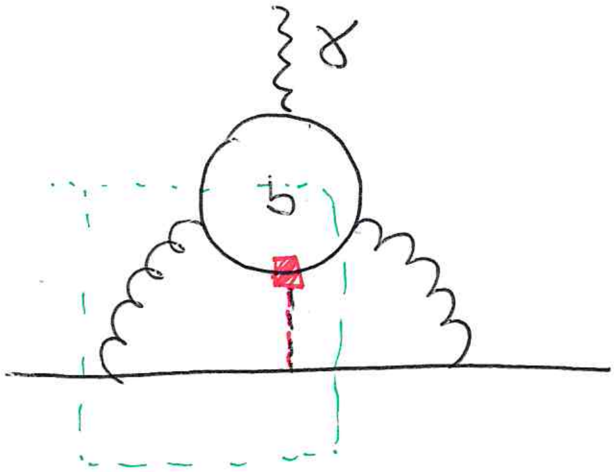


Matching + Running in EFT

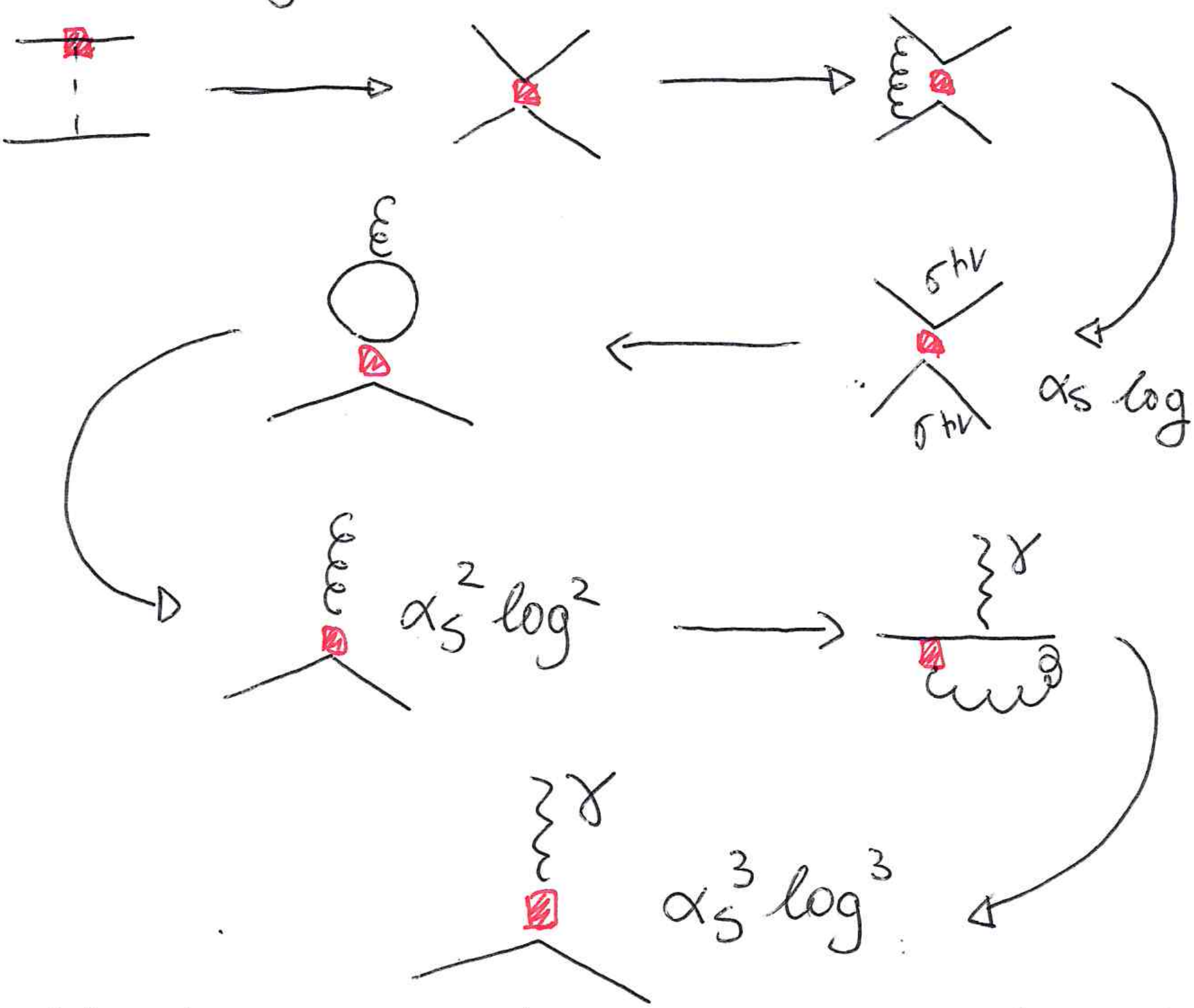


what is the RGE doing for us?

It captures the most relevant effects!

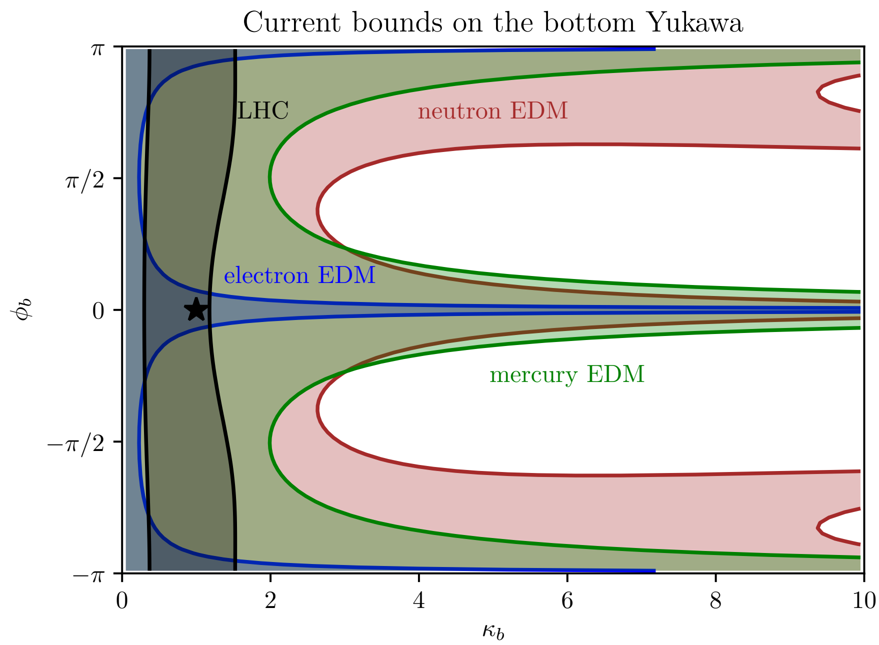


Zooming with the RGE

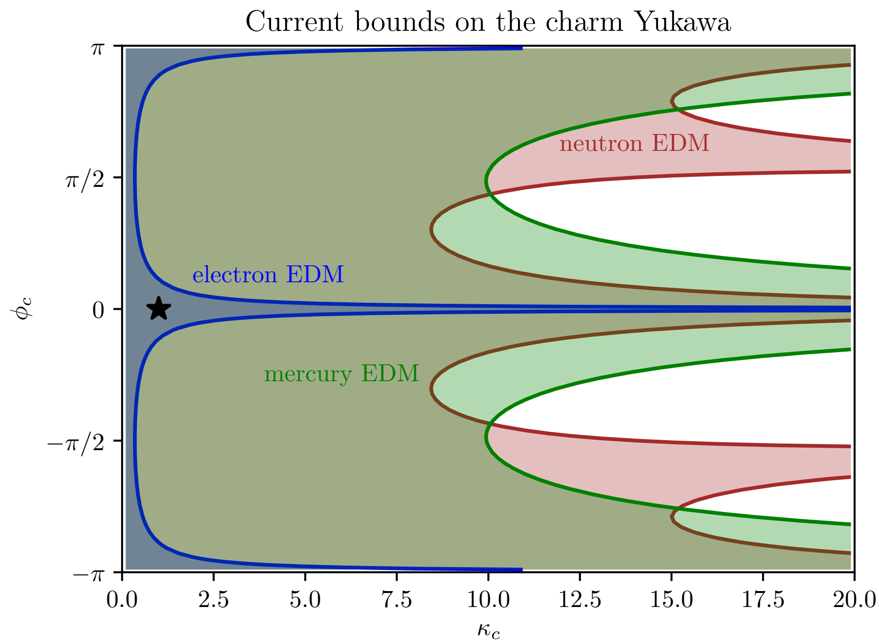


This "3-loop" contribution dominates the naive Barr-Zee by a factor of 10!

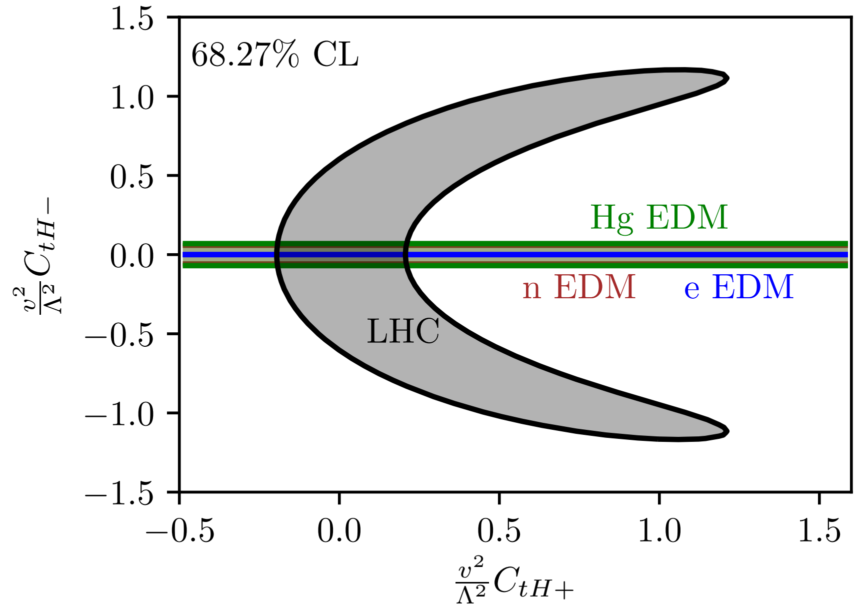
Bottom Yukawa



Charm Yukawa



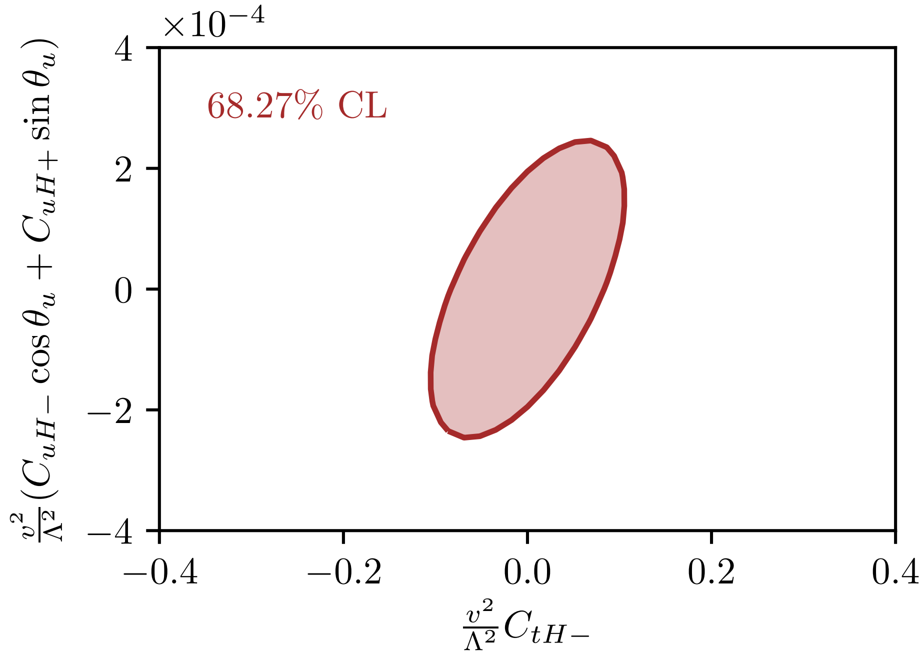
Top Yukawa



EDM constraints depend on electron and light-quark Yukawas

Cancellations possible, need simultaneous fit

Simultaneous top-/up-Yukawa fit on d_n and $d_{\mathbf{Hg}}$



[in progress with Brod, Skodras, Cornell]

The power of EFTs at the era of LHC

Correlation of NP Signals

Take-Away Messages

- Keep track of necessary model/EFT assumptions

- SMEFT can be a powerfull framework

- Interplay between high-energy and high-intensity frontier

The power of EFTs at the era of LHC

Modelbuilding Guidance

Example with relevant phenomenology

- The Strong CP Problem
- Understanding from EFT
- The axion Solution from the EFT perspective.

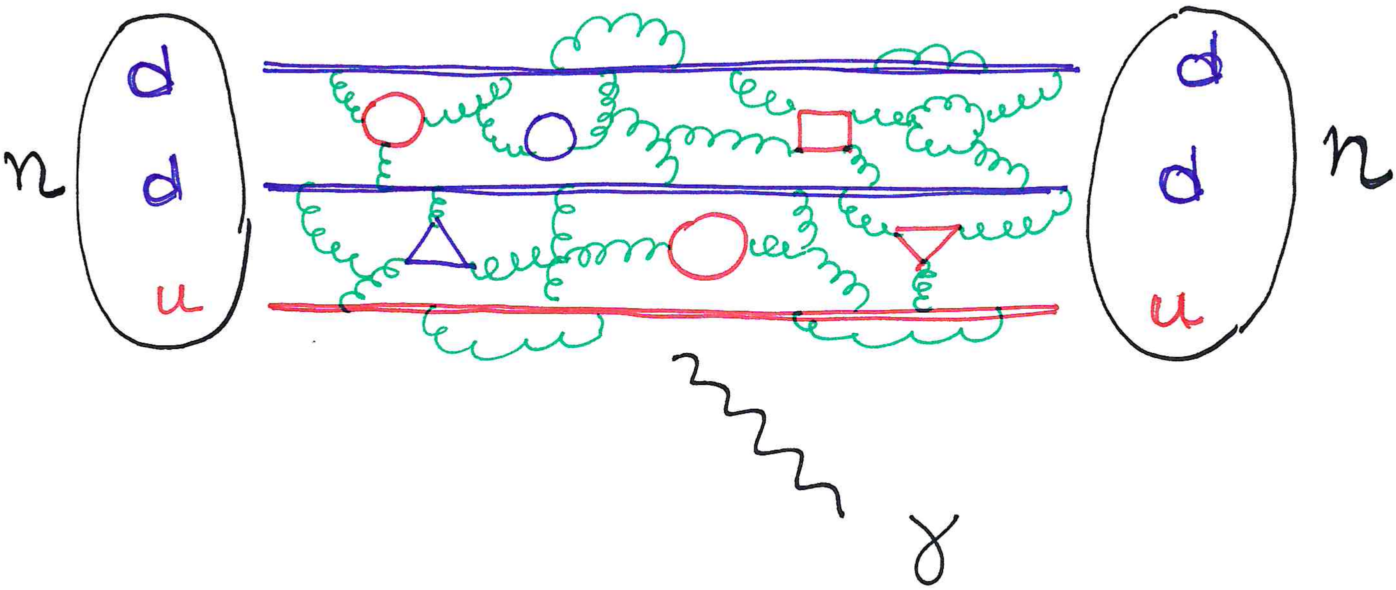
The Strong CP Problem

Why is the neutron EDM at least 13 orders of magnitude smaller than our expectations?

References (excellent)

- TASI lectures by Anson Hook arXiv: 1812.02669
- Online lecture by Gia Dvali @ Leiden University (Axions, Anomalies, and Gravit
- "The QCD axion precisely" arXiv: 1511.02867
- Weinberg Vol II, chapter 23.6
- Coleman, Aspects of symmetry.
- ...

What are our expectations for the neutron EDM ?



Quarks and gluons are not the right degrees of freedom to describe QCD in the IR.

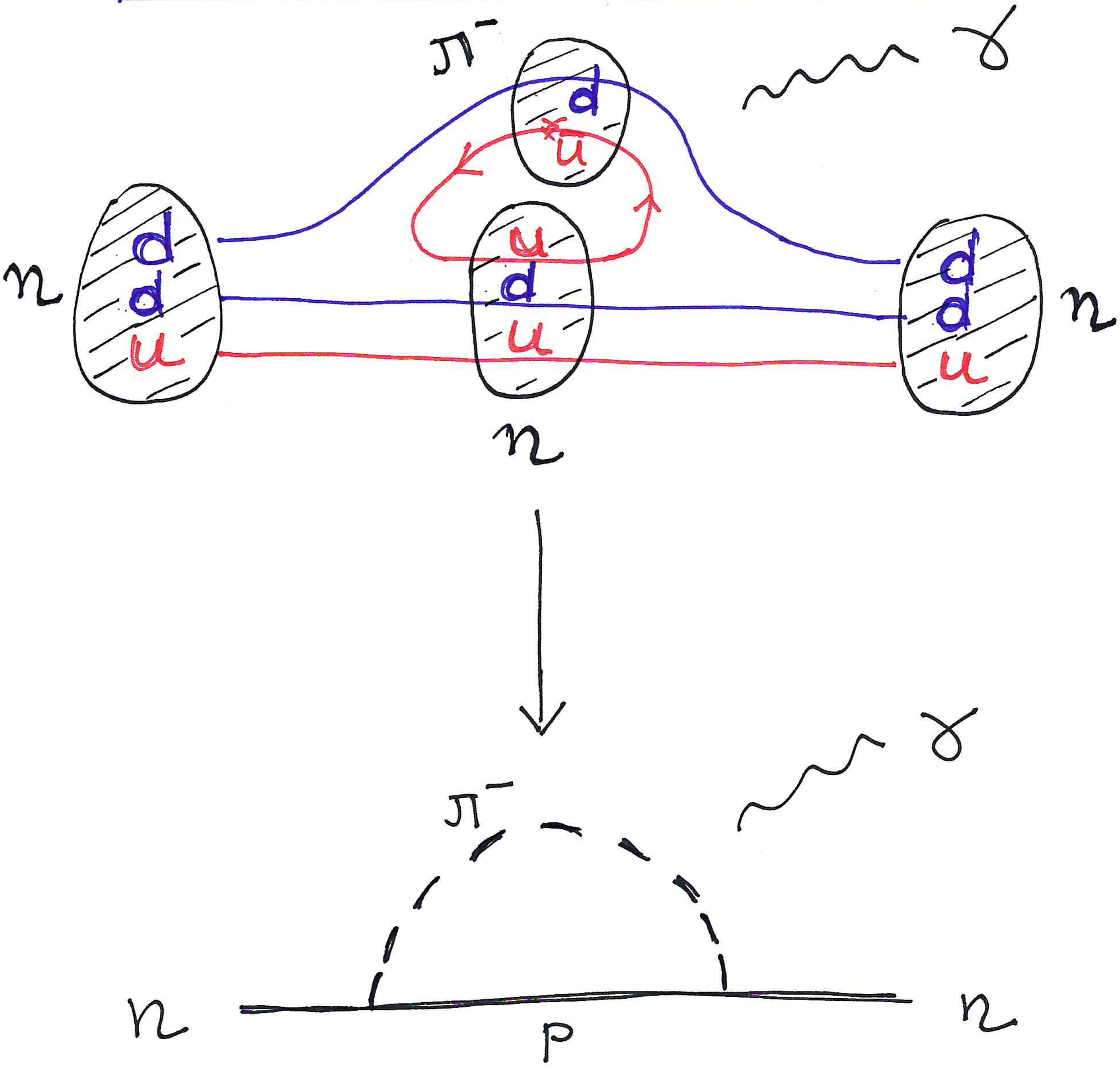
We need something else to estimate d_n.

Proposals?

Pions & Nucleons

$n = udd \qquad \pi^+ = u\bar{d}$

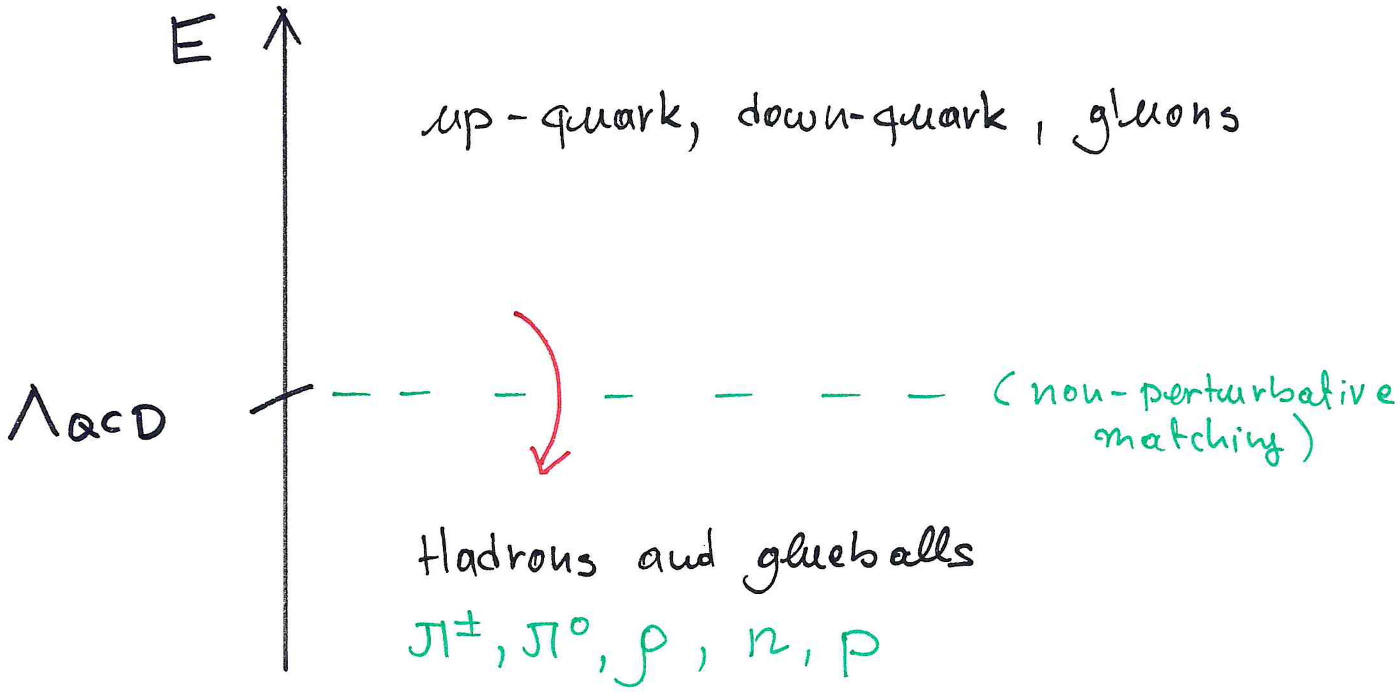
$p = uud \qquad \pi^- = \bar{u}d$



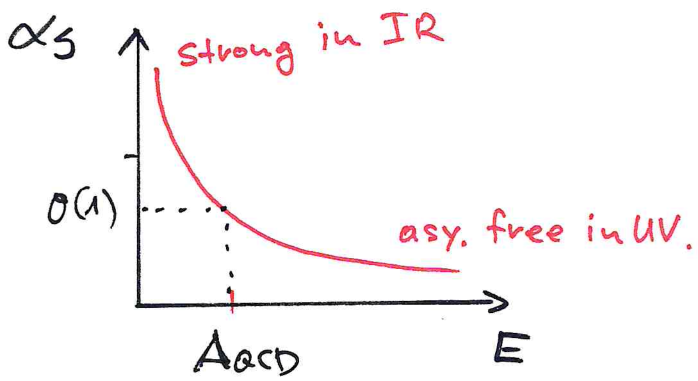
We cannot understand the Strong CP Problem before we understand the EFT of pions and nucleons

Chiral Perturbation Theory

QCD with 2 flavours



Two very interesting phenomena happen at Λ_{QCD}.



i) Confinement

ii) Spontaneous symmetry breaking of the Chiral Symmetry.

QCD
without phases

$$\mathcal{L} = -\frac{1}{4} G G + i \bar{q} \not{D} q - \left(\bar{q}_L M_{LR} q_R + h.c \right)$$

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \qquad M = \begin{bmatrix} m_u & \\ & m_d \end{bmatrix}$$

$q_L = P_L q = (1-\gamma_5)/2 \; q$

$q_R = P_R q = (1+\gamma_5)/2 \; q$

Symmetries

gauge: $SU(3)_c$

global: $SU(2)_L \times SU(2)_R \times U(1)_B \times \boxed{U(1)_A}$

special
later
more

However, M breaks explicitly the $SU(2)_L \times SU(2)_R \times U(1)_A$ global symmetry.

But $m_u, m_d \ll \Lambda_{QCD} \longrightarrow$ treat m_q ~~in~~ as perturbation.

global transformations

$SU(2)_L$	$q_L \longrightarrow U_L\, q_L$
$SU(2)_R$	$q_R \longrightarrow U_R\, q_R$
$U(1)_B$	$\left\{\begin{array}{l} q_L \longrightarrow e^{i\alpha} q_L \\ q_R \longrightarrow e^{i\alpha} q_R \end{array}\right\} \equiv q \to e^{i\alpha} q$
$U(1)_A$	$\left\{\begin{array}{l} q_L \longrightarrow e^{-i\alpha} q_L \\ q_R \longrightarrow e^{+i\alpha} q_R \end{array}\right\} \equiv q \to e^{+i\alpha\gamma} q$

Check yourself :

- $U(1)_B$ preserved
- M_{LR} break $SU(2)_L \times SU(2)_R \times U(1)_A$
- $SU(2)_V$ $[U_L \stackrel{!}{=} U_R]$ preserved if $M_{LR} \sim \mathbb{1}$
- M_{LR} break axial non-abel part $[U_L \stackrel{!}{=} U_R^\dagger]$

Formally restore symmetries by promoting M_{LR} to a <u>spurion</u>.

Spurions for

$$\mathcal{L} = -\frac{1}{4} G G + i \bar{q} \not{D} q - (\bar{q}_L M_{LR} q_R + h.c)$$

	$SU(3)_C$	$SU(2)_L$	$SU(2)_R$	$U(1)_B$	$U(1)_A$
G^a_μ	8	-	-	-	-
q_L	3	2	-	+1	-1
q_R	3	-	2	+1	+1
M_{LR} spurion	-	$\bar{2}$	2	-	-2

The full global symmetry is formally restored.

RGE FLOW

Λ_{QCD} Spontaneous sym. breaking

QCD itself induces

$$\langle \bar{q}^i q^i \rangle \neq 0 \sim \Lambda^3_{QCD}$$

$$SU(2)_L \times SU(2)_R \longrightarrow SU(2)_V$$

Goldstone Theorem

$$SU(2)_L \times SU(2)_R \longrightarrow SU(2)_V$$

3 Goldstone Bosons π^+, π^-, π^0

$$U(1)_B \times U(1)_A \xrightarrow{?} U(1)_B$$

1 Goldstone Boson η' ? (missing/tooheo

The famous $U(1)_A$ Problem (later).

Pion Lagrangian from CCWZ

Callan, Coleman, Wess, Zumino construct

i) Vacuum $\quad U_0 \sim \langle \bar{q}_L^i q_{Ri} \rangle \sim \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

ii) Act with broken generators $(U_L = U_R^\dagger)$

$$U_A U_0 U_A = e^{i\theta^a T^a} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} e^{i\theta^a T^a}$$

iii) Promote trafo params to fields (the GBs)

$\theta^a \longrightarrow \pi^a / f_\pi$

$$\boxed{U(x) = e^{2i\frac{\pi^a}{f_\pi}T^a}}$$

iv) Construct invariants out of $U(x)$

$U_0 \sim \langle \bar{q}^i_L q_{Ri} \rangle \sim (\bar{2}, 2)$ of $SU(2)_L \times SU(2)_R$.

$$\boxed{\begin{aligned} \mathcal{L}^{LO}_\pi &= \frac{f_\pi^2}{4} \mathrm{Tr}[\partial_\mu U^\dagger \partial^\mu U] \\ &+ 2B_0 \frac{f_\pi^2}{4} \mathrm{Tr}[U^\dagger M_{LR} + h.c] \end{aligned}}$$

kinetic term for π's and π-π interact

explicit breaking.

Expand Pion Matrix

$$U(x) = e^{2i \frac{\pi^a}{f_\pi} T^a} = e^{i \frac{\Pi}{f_\pi}}$$

$$\Pi \equiv \begin{bmatrix} \pi^0 & \sqrt{2}\,\pi^+ \\ \sqrt{2}\,\pi^- & -\pi^0 \end{bmatrix}$$

Pion masses

$$m^2_{\pi^0} = m^2_{\pi^\pm} = B_0 f_\pi (m_u + m_d)$$

now let's turn on phases and redo this.

$$\boxed{\begin{array}{c}\text{QCD}\\ \text{with phases and } \vartheta\end{array}}$$

$$\mathcal{L} = \mathcal{L}_{kin} - \left(\bar{q}_L M_{LR} q_R + h.c\right) + \Theta \frac{\alpha_s}{8\pi} G\tilde{G}$$

with $M = \begin{bmatrix} m_u e^{i\theta_u} & \\ & m_d e^{i\theta_d} \end{bmatrix}$

$$G\tilde{G} = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma}$$

breaks CP, we have a priori no reason to neglect it.

[Actually $G\tilde{G}$ is a total derivative, but because QCD is strongly coupled at large distances the presence of Instantons doesn't allow us to neglect it.]

Can we rotate away the phases in the masses?

$$\mathcal{M} = \begin{bmatrix} m_u e^{i\theta_u} & \\ & m_d e^{i\theta_d} \end{bmatrix}$$

Try an axial field redefinition

$$u \longrightarrow e^{i\alpha_u \gamma_5} u$$
$$d \longrightarrow e^{i\alpha_d \gamma_5} d$$

Then

$$\bar{q}_L M_{LR} q_R \longrightarrow \bar{q}_L \begin{bmatrix} m_u e^{i(\theta_u + 2\alpha_u)} & \\ & m_d e^{i(\theta_d + 2\alpha_d)} \end{bmatrix} q_R$$

So naively we can choose α_u and α_d to get rid of θ_u and θ_d.

But that is not entirely correct! Something else happens as well.

$$\boxed{\text{Anomalous } U(1)_A \text{ transformations}}$$

Fermionic measure in path integral.

$$[\mathcal{D}Q\,\mathcal{D}\bar{Q}] \xrightarrow[e^{i\alpha\gamma_5}]{U(1)_A} [\mathcal{D}Q\,\mathcal{D}\bar{Q}]\, e^{i\int d^4x\, \alpha \mathcal{A}}$$

with $\mathcal{A} = \frac{\alpha_s}{4\pi} G\tilde{G}$

Our unphysical field redef. shifts θ_u, θ_d to the $\theta G\tilde{G}$ term!

$$\theta \xrightarrow[\alpha_u,\, \alpha_d]{} \theta + 2(\alpha_u + \alpha_d)$$

$$\theta_u \xrightarrow[\alpha_u]{} \theta_u + 2\alpha_u$$

$$\theta_d \xrightarrow[\alpha_d]{} \theta_d + 2\alpha_d$$

Observables cannot depend on field redefs.

$$e^{-i\theta}\mathcal{M}_u\mathcal{M}_d = \boxed{m_u m_d\, e^{i(\theta_u + \theta_d - \theta)}}$$

is however invariant under the field redef.

$\not{CP}$ Observables must depend on the combination

$$\boxed{m_u m_d \, e^{i(\theta_u+\theta_d-\theta)}}$$

$\longrightarrow$ Only $\bar{\theta} \equiv \theta_u + \theta_d - \theta$ is physical

$\longrightarrow$ We found our first solution to the Strong CP Promblem

$$m_u = 0$$

Unfortunately $m_u \neq 0$.

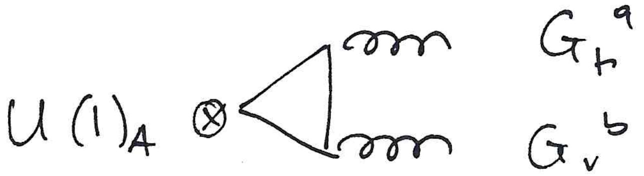
After redefinion of fields to remove the phases from the masses (choice of basis, nothing more)

$$\mathcal{L} = \mathcal{L}_{kin} - \bar{q} M q + \bar{\theta} \frac{\alpha_s}{8\pi} G\tilde{G}$$

$$M = \begin{bmatrix} m_u & \\ & m_d \end{bmatrix}$$

We understand now that $U(1)_A$ was never a good symmetry. It was good classically but broken radiatively

$U(1)_A$ is anomalous



Since $U(1)_A$ was never a good symmetry we do not expect its would-be GB the η' to be real/light GB.

Now let's do CCWZ with the θ-Term!

CCWZ with $\bar{\theta}$

Spurions for $U(1)_A$

$$\boxed{\begin{aligned} q &\longrightarrow e^{i\alpha\gamma_5}\, q \\ M_{LR} &\longrightarrow e^{-2i\alpha}\, M_{LR} \\ \bar{\theta} &\longrightarrow \bar{\theta} - 4\alpha \end{aligned}}$$

leaves $\mathcal{L}$ invariant (formally)

CCWZ including $U(1)_A$

$$\langle \bar{q}_L^i q_{Ri} \rangle \sim \boxed{U(x) = e^{2i\eta'/f_{\eta'}}\, e^{i\Pi/f_\pi}}$$

$\mathcal{L}_{\pi,\eta'}$ from invariants

$$\mathcal{L} = \frac{f_\pi^2}{4} \mathrm{Tr}[\partial_\mu U^\dagger \partial^\mu U] + 2B_0 \frac{f_\pi^2}{4} \mathrm{Tr}[U^\dagger M_{LR} + h.c.]$$

$$\boxed{+\, c\, \underbrace{f_{\eta'}^4}_{\sim \Lambda_{QCD}^4} \det U + h.c.}$$

NEW!

$$\boxed{c f_{\eta'}^4 \det U = c f_{\eta'} e^{4i\eta'/f_{\eta'}}}$$

- breaks $U(1)_A$ but ok because never good

- but our spurion trafo is good. Therefore,
  $U(x) \to e^{2i\alpha} U(x)$

$$\longrightarrow \quad c \overset{!}{=} |c| e^{i\bar{\Theta}}$$

- This is what 't Hooft proved: Instantons induce a term

$$\sim \Lambda_{QCD}^4 e^{i\bar{\Theta}} \det U$$

Using symmetries and EFT we recoverd the interesting result

$$\mathcal{L}_{\eta'} \supset c f_{\eta'}^4 \det U + h.c. = |c| f_{\eta'}^4 \cos\left(\bar{\Theta} + 4\frac{\eta'}{f_{\eta'}}\right)$$

$$\boxed{\mathcal{L}_{\eta'} = |c| f_{\eta'}^4 \cos\left(\bar{\theta} + 4\frac{\eta'}{f_{\eta'}}\right)}$$

- QCD induced a potential for η'

- expand: $m_{\eta'}^2 = 4|c| f_{\eta'}^2 \simeq \Lambda_{QCD}^2$

- Potential minimised when

$$\bar{\theta} + 4\frac{\langle \eta' \rangle}{f_{\eta'}} = 0$$

- If $m_u = m_d = 0$ the only dependence of $\bar{\theta}$ is in $\mathcal{L}_{\eta'}$.

$\rightarrow$ $\boxed{\text{The } \eta' \text{ dynamically sets } \bar{\theta} \rightarrow 0}$

$\rightarrow$ The IR perspective of the $m_u = 0$ solution to the Strong CP Problem.
(η' plays the role of an axion)

- Expand around vaccum

$$\eta' \longrightarrow \eta' - \frac{f_{\eta'}}{4}\bar{\theta}$$

$$U(x) \longrightarrow U_\theta(x) = e^{-i\bar{\theta}/2}\, U(x)$$

- Back in $\mathcal{L}_\pi$

- You will then find that now the π^0 gets a potential. and a vev!

$$\boxed{\langle \pi^0 \rangle = \varphi f_\pi \qquad \text{with} \quad \tan\varphi = \frac{m_d - m_u}{m_d + m_u}\tan\frac{\bar{\theta}}{2}}$$

- Reexpand and find minimum.

After some trigonometric gymnastics we will find the minimum of the QCD potential as a function of $\bar{\theta}$!

$$\boxed{V_{min}(\bar{\theta}) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2 \frac{\bar{\theta}}{2}}}$$

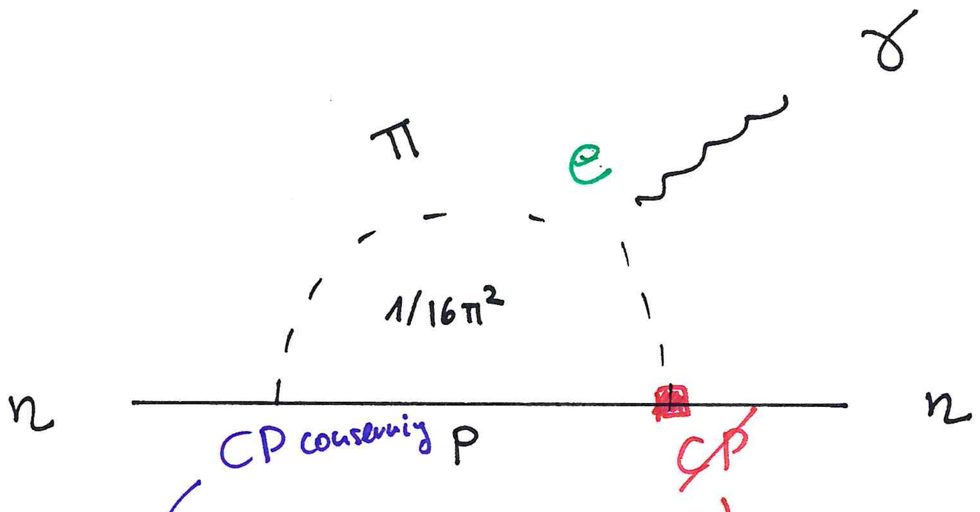
And from the $\bar{\theta}$-dependent pion matrix we can compute the π-n-p $\mathcal{L}$agrangian with $\bar{\theta}$!

Ready to:

i) Estimate $|d_n|$ in QCD

ii) Understand the axion solution!

$|d_n|$ from χPT



$$\simeq \frac{m_n}{f} \simeq g_* \qquad \simeq \bar{\theta}\, \frac{1}{f}\, \frac{m_u m_d}{m_u + m_d} \simeq \bar{\theta}\, \frac{m_\pi}{m_n}\, g^* \; {}^*$$

$$|d_n| \sim \frac{e}{16\pi^2}\, g_*^2\, \bar{\theta}\, \frac{m_\pi^2}{m_n^2}\, \frac{1}{m_n}$$

$$\overset{g^* \simeq 4\pi}{\sim} 10^{-16}\, \bar{\theta}\ \ e\,cm$$

Much too large, $\boxed{\theta < 10^{-10}}$

* see Weinberg II for the reason that the scaling is $\bar{\theta}\, m_\pi / m_n\, g_*$ and not $\bar{\theta}\, m_\pi^2 / m_n^2\, g_*$

Axion Solution

To solve the Strong CP Problem we "only" need one particle with

Axion EFT

$$\boxed{\mathcal{L}_{axion} = \frac{1}{2}\partial_\mu a\, \partial^\mu a + \frac{a}{f_a}\frac{\alpha_s}{4\pi} G\tilde{G}}$$

Why does this solve the Strong CP Problem?

- a couples to QCD via $G\tilde{G}$
- everything we did holds after

$$\bar{\theta} \longrightarrow \bar{\theta} + \frac{a}{f_a}$$

Axion Potential

$$V(a) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u+m_d)^2}\sin^2\left(\frac{a}{2f_a} + \frac{\bar{\theta}}{2}\right)}$$

- a gets a vev $\langle a \rangle = -\bar{\theta} f_a$
- Expand $\longrightarrow$ $\bar{\theta}$ dependence disappears

The axion dynamically sets $\bar{\theta}$ to zero!

Axion EFT

Modelbuilding guidance

- $\mathcal{L}_a$ has a shift symmetry (a is GB)
- Couples to the anomalous $G\tilde{G}$ term.

Axion is a GB of a spontaneously broken U(1) that is anomalous under QCD

$$U(1)_{PQ}$$

PQ = Peccei-Quinn

- f_a associated to PQ breaking scale

- m_a is generated by the QCD potential

$$m_a^2 = \frac{m_u m_d}{m_u + m_d} \frac{m_\pi^2 f_\pi^2}{f_a^2}$$

- All couplings allowed by the shift symmetry are allowed, but only $G\tilde{G}$ is important for the Strong CP Problem.

Axion Models

First model (Peccei and Quinn, Weinberg and Wilczek)
tied PQ-breaking scale to EW scale

$$f_{PQ} \sim v_{ew}$$

Troubles from FCNCs

$$K^+ \longrightarrow \pi^+ \text{ invisible}$$

f_a does not have to be tied to EW scale

$$\boxed{\text{Invisible Axion Models}}$$

KSVZ, DFSZ, ...

Here

$$f_{PQ} \gg v_{ew}$$

$\longrightarrow$ Hiddes PQ dynamics

$\longrightarrow$ lowers axion mass, less pressure from constraits.

KSVZ Model*

$$\mathcal{L} = \mathcal{L}_{SM} + \partial_\mu \phi^\dagger \partial^\mu \phi + \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$
$$+ i \bar{Q} \not{D} Q - y \left(\phi\, \bar{Q}_R Q_L + \phi^\dagger \bar{Q}_L Q_R \right)$$

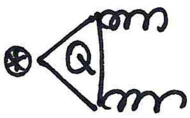
Q: new quark singlet under $SU(2)_L \times U(1)_Y$
ϕ: complex scalar ——— " ———

$U(1)_{PQ}$

$$Q \longrightarrow e^{i\gamma_5 b}\, Q$$
$$\phi \longrightarrow e^{-2ib}\, \phi$$

global $U(1)_A$

But anomalous because Q charged under QCD

$U(1)_{PQ}$ 

every time we do the trofo above we induce $b \frac{\alpha_s}{4\pi} G\tilde{G}$

$U(1)_{PQ}$ spontaneously broken when $\mu^2 > 0$, $\langle \phi \rangle = f_a$

$$\phi = (f_a + h)\, e^{i a / f_a}$$

with $f_a = \mu / \sqrt{2\lambda}$. The ~~Higgs~~ radial mode gets a mass $m_h = 4\mu^2$ and the quark $m_Q = y f_a$.

a, the axion is a GB ~~so~~ gets no mass. from the PQ breaking.

* aka as simple as it gets

Expand Lagrangian around the vacuum

$$\boxed{\mathcal{L}_{Yuk} \supset -y f_a \left(e^{i\frac{a}{f_a}} \bar{Q}_R Q_L + h.c. \right)}$$

- Redefine Q

$$Q \longrightarrow Q' = e^{-i\frac{a}{2f_a}} Q$$

$$\mathcal{L}_{Yuk} \longrightarrow -y f_a \left(\bar{Q}_R Q_L + h.c \right) + \frac{a}{f_a} \frac{\alpha_s}{8\pi} G\tilde{G}$$

- Trivially integrate out Q and h

$$\boxed{\mathcal{L}_{NP} = \frac{1}{2} \partial_\mu a \, \partial^\mu a + \frac{a}{f_a} \frac{\alpha_s}{8\pi} G\tilde{G}} + \text{higher order in } 1/f_a^2$$

We see how simple it is to construct an axion model. But there are also caveats to axion models from the theory perspective.
(axion quality problem): if $U(1)_{PQ}$ is anomalous then why is this ren. Lagrangian such a good approx. Higher order operators that break $U(1)_a$ can spoil the mechanism.

The power of EFTs
at the era of LHC

Modelbuilding Guidance

Example

- EFT as a tool to understand the Strong CP Problem

- EFT exposed minimal setup for a solution

  The axion dynamically sets $\bar{\theta} \to 0$

- Exp. searches constrain the EFT couplings of the axion. (multi channel)
  (FCNCs, Astrophysics, Helioscopes, Haloscopes, ...)

- The axion can be DM if $f_a \sim 10^{12}$ GeV

Conclusions

Why EFTs ?

- Precision
  (important to disentangle even small NP effec

- Signal Correlation
  (correlations with mild assumptions, directs searches)

- Model Building Guidance
  (provide minimal requirements for solving an issue or be phenomenologically viable).