GW's as probes of the early Universe

- Around 300.000 years after the big bang, electrons and protons combine to form hydrogen

  ↳ Matter becomes neutral gas, the Universe becomes transparent

  The photons that were part of the plasma at that time are what we see now in the CMB.

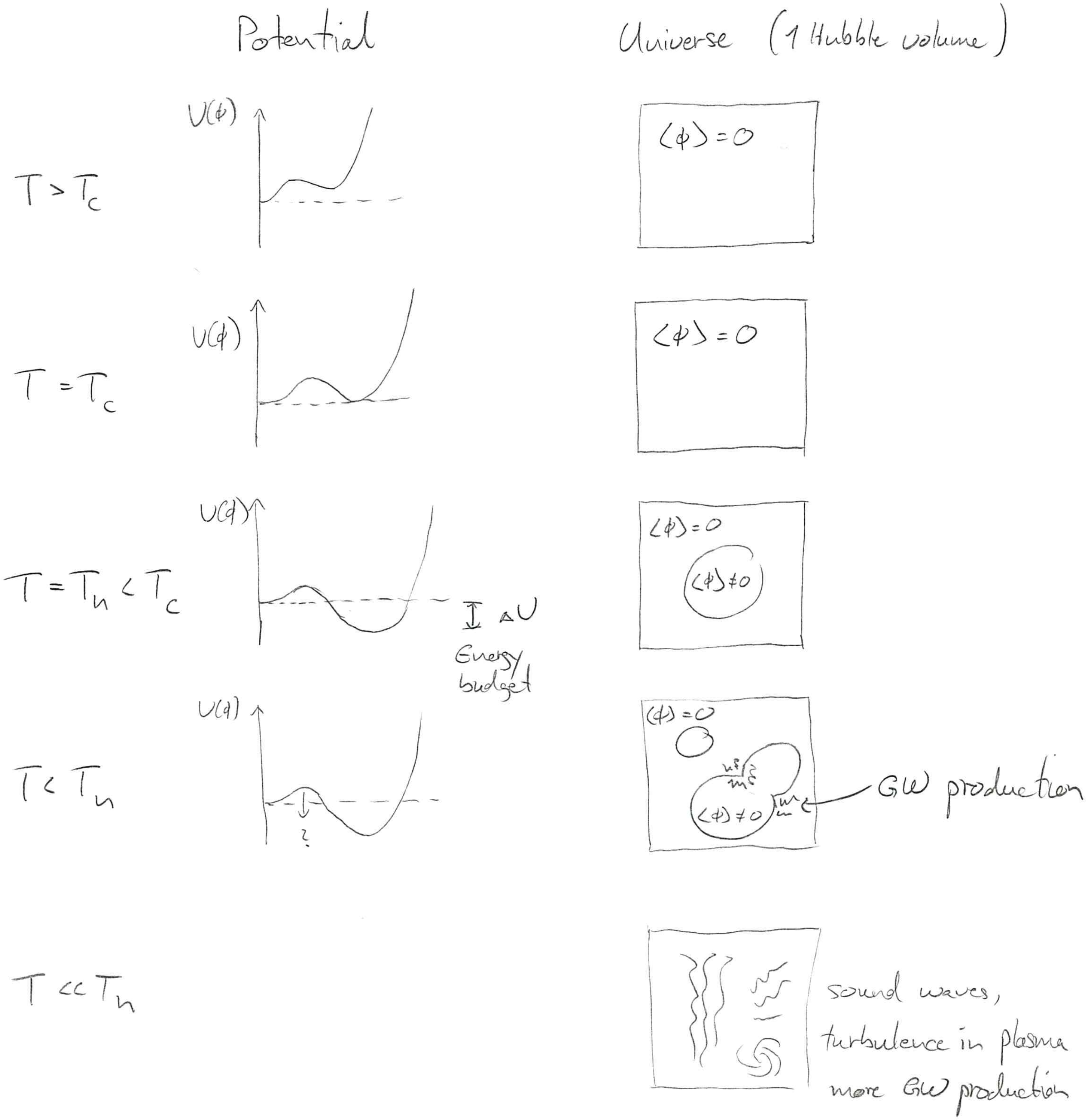
  Earlier times are not observable with telescopes.

- The coupling of matter to gravity is suppressed by $8\pi G = \frac{1}{M_{pl}^2}$

  ↳ GW's produced any time after the big bang propagate freely (even before big bang if you want)

- GW's are a new way to probe the dynamics of the very early Universe → e.g. phase transitions

GW's from PT's



Potential
Universe (1 Hubble volume)
$T > T_c$
$U(\phi)$
$\langle\phi\rangle = 0$
$T = T_c$
$U(\phi)$
$\langle\phi\rangle = 0$
$T = T_n < T_c$
$U(\phi)$
ΔU
Energy budget
$\langle\phi\rangle = 0$
$\langle\phi\rangle \neq 0$
$T < T_n$
$U(\phi)$
$\langle\phi\rangle = 0$
$\langle\phi\rangle \neq 0$
GW production
$T \ll T_n$
sound waves,
turbulence in plasma
more GW production

T_c from $U(\phi, T)$. How to find T_N?

Vacuum decay rate $\Gamma(T) \propto T^4 e^{-S_3/T}$

$S_3(T)$: Action of the $O(3)$ symmetric tunneling "bounce" solution

$\leftrightarrow$ How much energy is needed to cross the barrier

The <u>nucleation temperature</u> T_N is defined by the requirement that one bubble per Hubble volume should be nucleated:

$$\frac{\Gamma(T)}{H^4(T)} \overset{!}{=} 1$$

Now $H \sim \frac{T^2}{M_{pl}} \Rightarrow \frac{\Gamma}{H^4} \sim \frac{M_{pl}^4}{T^4} e^{-S_3/T} \Rightarrow$

$$\frac{S_3}{T} \sim -\log\left(\frac{T^4}{M_{pl}^4}\right) \sim 140 \quad \text{for } T \sim \text{weak scale}$$

Technical details: Coleman, PRD 15, 10 p. 2929 (1977)
(see. Exercises)

$$S_3(\phi_b) = \int d^3x \left(\frac{1}{2}(\nabla\phi_b)^2 + U(\phi_b)\right)$$

where ϕ_b is the <u>bounce</u> solution, i.e. solves $\frac{d^2\varphi}{dr^2} + \frac{2}{r}\frac{d\varphi}{dr} = V'(\varphi)$

with $\varphi \to 0$ at $r \to \infty$, $\frac{d\varphi}{dr} = 0$ at $r = 0$. (and $U = T(\varphi, T)$)

How fast does the transition complete?

$$\beta \equiv -\frac{dS}{dt}\bigg|_{t_N} \quad \rightsquigarrow \quad \frac{\beta}{H}\bigg|_{T_N} = T_i \frac{dS}{dT}\bigg|_{T_N}$$

For large β, $\frac{\Gamma}{H}$ increases rapidly and the PT is fast

Energy budget: $\alpha \simeq \frac{\Delta U}{\rho_{tot}} = \frac{\text{vacuum energy}}{\text{total energy}}$

Bubble wall speed ... difficult. Most PT's of interest have $v_w \to 1$.

Nucleation temperature $T_N \sim \langle\phi\rangle$!

↳ Caveat: For slow, supercooled, vacuum dominated transitions, the PT might complete later ...

How to obtain the GW signal?

Difficult, requires numerical simulations (summary e.g. 1512.06239)
1910.13125
2008.09136

Qualitative:

Peak frequency at time of emission $f_* \sim \frac{1}{\lambda_*}$ ← wavelength

Characteristic length scale = bubble radius at time of collision

$$\lambda_* \sim \frac{1}{H_*}\left(\frac{H_*}{\beta}\right) \cdot v_w \quad (v_w \to \approx 1)$$

$\frac{1}{H_*}$ ↑ size of Hubble patch; $\left(\frac{H_*}{\beta}\right)$ ← how fast is the transition

Now redshift:

$$f_0 \equiv f_{today} = \frac{a_*}{a_0} \cdot f_* \approx \frac{T_0}{T_*} \cdot f_* \qquad \text{(entropy conservation)}$$

$$= \frac{T_0}{T_*} \cdot \frac{1}{H_*} \cdot \left(\frac{\beta}{H_*}\right)$$

$$\approx \frac{T_0}{M_{pl}} \cdot T_* \cdot \left(\frac{\beta}{H_*}\right)$$

$T_0 \sim 3K \sim 10^{-4}\, eV$

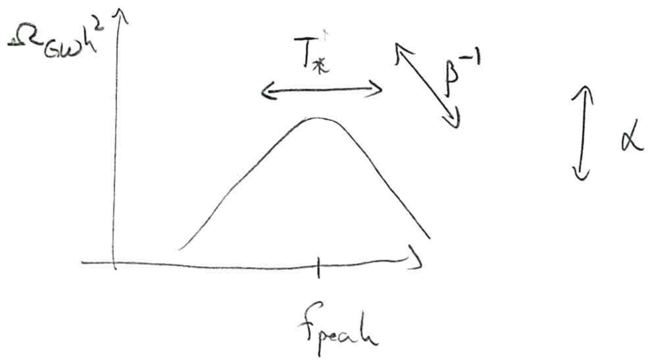
$M_{pl} \sim 10^{18}\, GeV$

$$T_* = \frac{T_*}{100\,GeV} \cdot 100\,GeV \cdot \frac{s}{s}$$

$$= \frac{T_*}{100\,GeV} \cdot 10^{26}\, Hz$$

$$\approx 10^{-5}\, Hz \cdot \left(\frac{T_*}{100\,GeV}\right) \cdot \left(\frac{\beta}{H_*}\right)$$

Signal shape:



What detector?

$10^{-3}\, Hz \rightarrow \lambda \sim 10^{10}\, m \sim 10^{7}\, km = 10\, Mkm$

↳ space → LISA

Sensitivity → SNR