

Precision physics for discoveries

Giulia Zanderighi

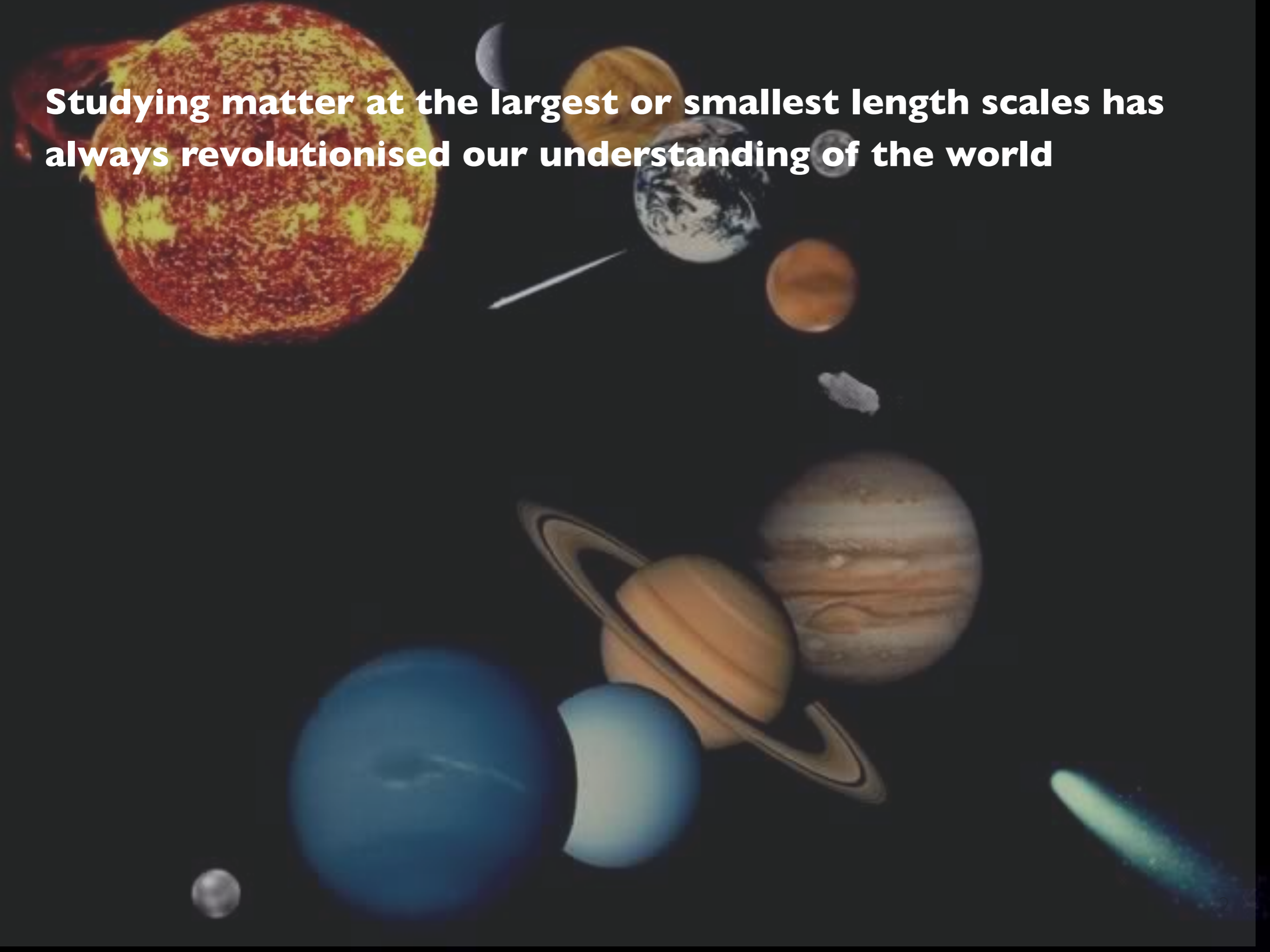
Max Planck Institute for Physics

1st lecture



Next PhD School, June 2021

Studying matter at the largest or smallest length scales has always revolutionised our understanding of the world



A collection of celestial bodies including a large red and yellow star, various planets like Jupiter, Saturn, Uranus, and Earth, and a comet.

Studying matter at the largest or smallest length scales has always revolutionised our understanding of the world

- **In the 19th century, Jupiter, Saturn and Uranus were the biggest planets known, using Kepler's law one could predict their orbit**



Studying matter at the largest or smallest length scales has always revolutionised our understanding of the world

- **In the 19th century, Jupiter, Saturn and Uranus were the biggest planets known, using Kepler's law one could predict their orbit**
- **To a big surprise, the orbit of Jupiter and Saturn agreed well with predictions, but not the one of Uranus!**



Studying matter at the largest or smallest length scales has always revolutionised our understanding of the world

- **In the 19th century, Jupiter, Saturn and Uranus were the biggest planets known, using Kepler's law one could predict their orbit**
- **To a big surprise, the orbit of Jupiter and Saturn agreed well with predictions, but not the one of Uranus!**
- **Having faith in Newton's laws, one can explain the motion of Uranus by assuming that there was a yet undiscovered planet**



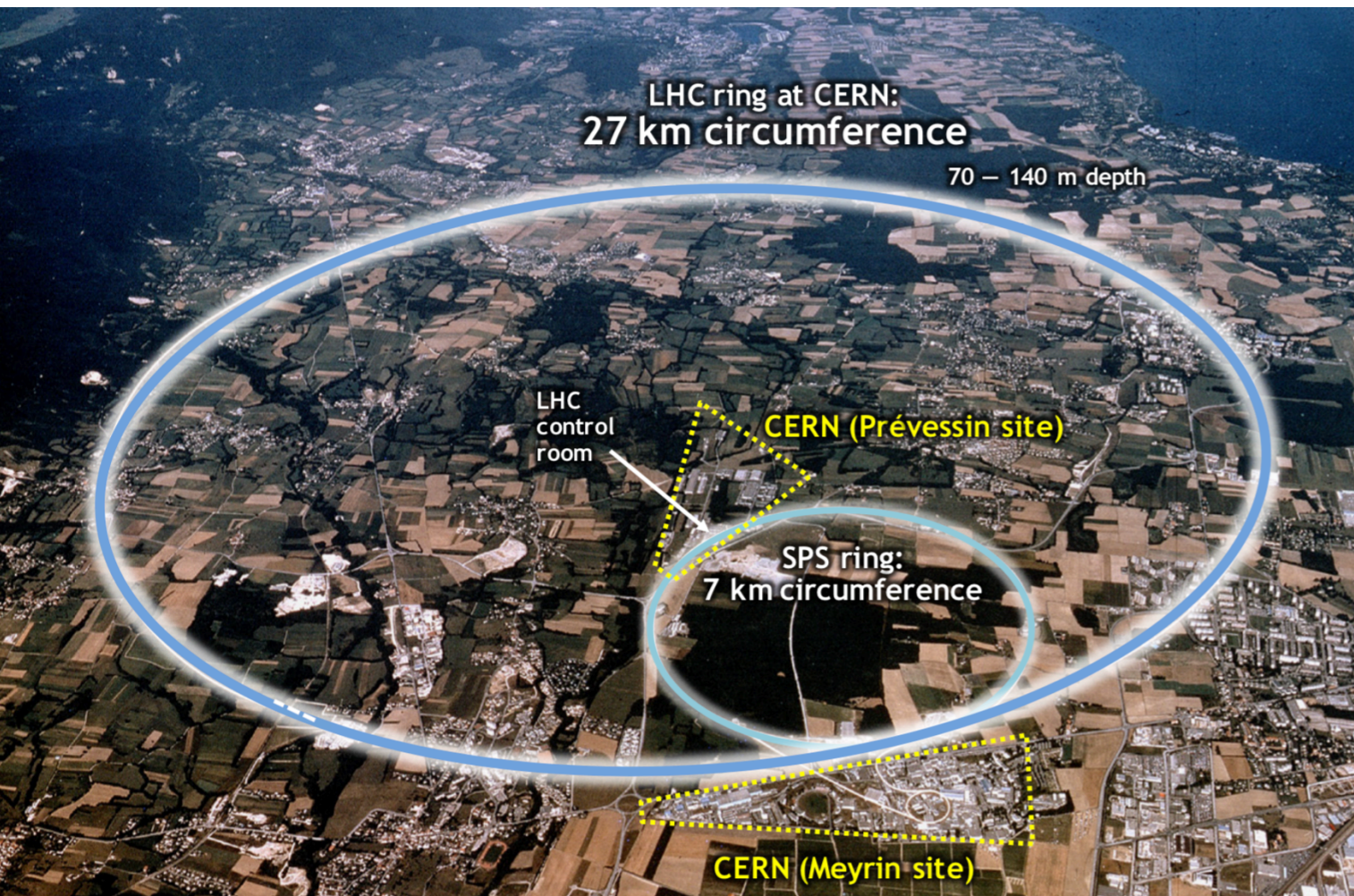
Studying matter at the largest or smallest length scales has always revolutionised our understanding of the world

- **In the 19th century, Jupiter, Saturn and Uranus were the biggest planets known, using Kepler's law one could predict their orbit**
- **To a big surprise, the orbit of Jupiter and Saturn agreed well with predictions, but not the one of Uranus!**
- **Having faith in Newton's laws, one can explain the motion of Uranus by assuming that there was a yet undiscovered planet**
- **Precise calculations and measurements of the orbit of Uranus allowed to precisely know where to aim the telescopes and **Neptune was found****

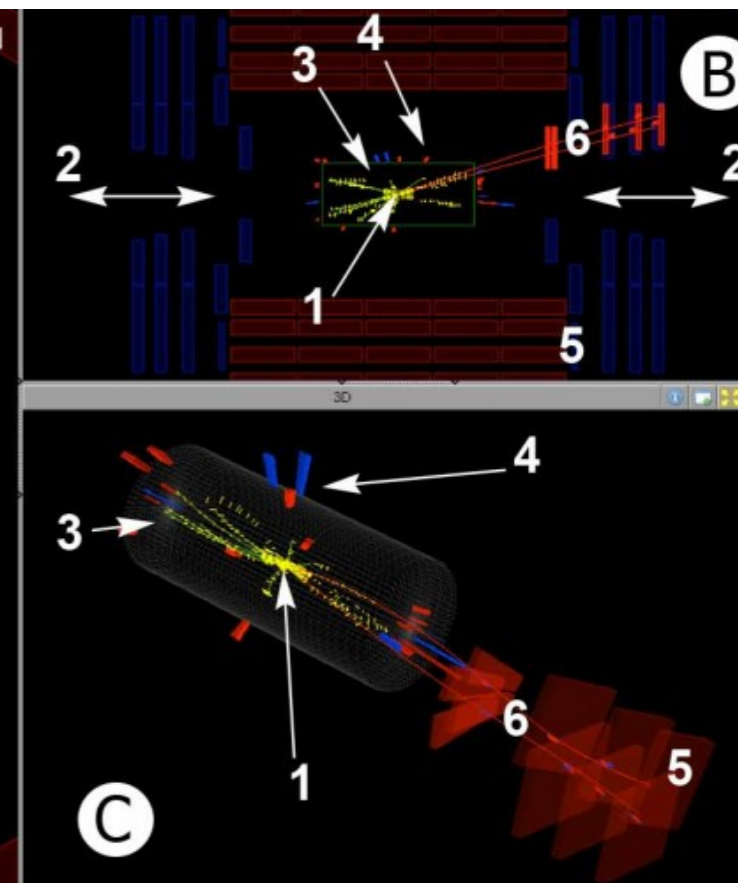
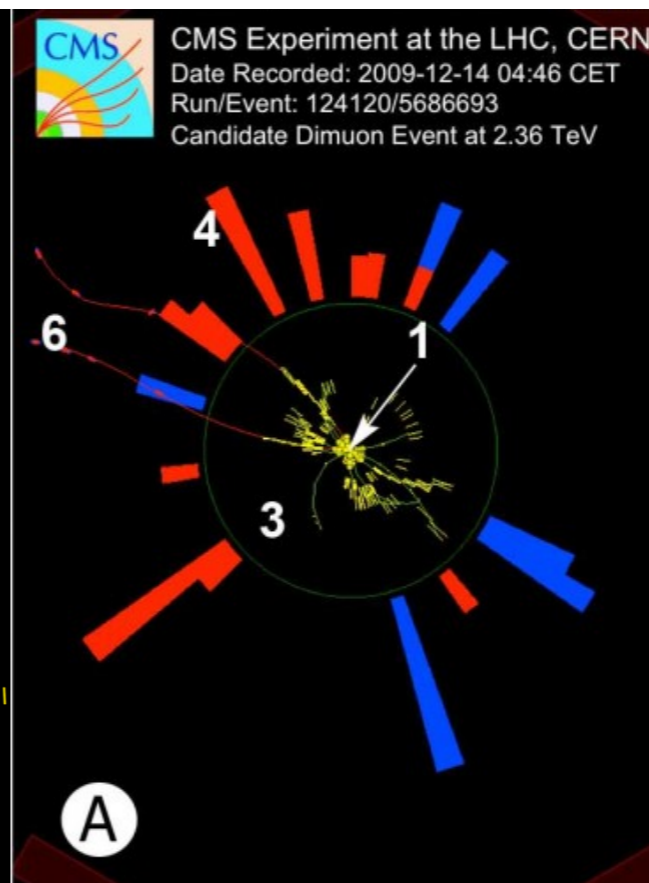
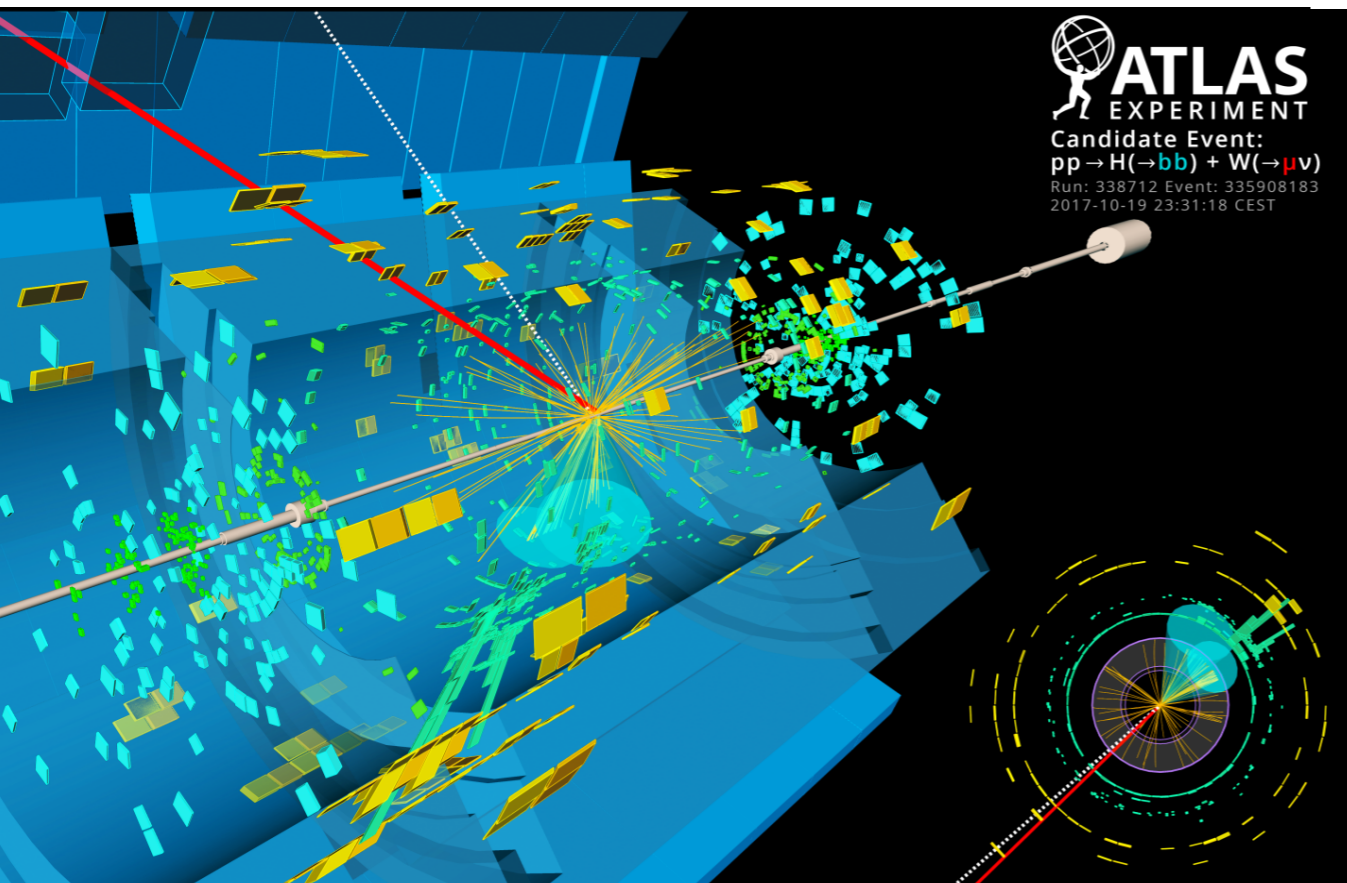


Studying matter at the largest or smallest length scales has always revolutionised our understanding of the world

- **In the 19th century, Jupiter, Saturn and Uranus were the biggest planets known, using Kepler's law one could predict their orbit**
- **To a big surprise, the orbit of Jupiter and Saturn agreed well with predictions, but not the one of Uranus!**
- **Having faith in Newton's laws, one can explain the motion of Uranus by assuming that there was a yet undiscovered planet**
- **Precise calculations and measurements of the orbit of Uranus allowed to precisely know where to aim the telescopes and **Neptune was found****
- **Also precision measurements of the orbit of Mercury gave the **first evidence for General Relativity** (much before any gravitational wave was seen ...)**



The Large Hadron Collider (LHC) experiment probes nature at smaller distances ever explored on Earth in a controlled laboratory. The aim is to improve our current knowledge of matter as it is today encoded in the Standard Model (SM)



Three generations of matter (fermions)

	I	II	III	
mass	2.4 MeV/c ²	1.27 GeV/c ²	171.2 GeV/c ²	0
charge	2/3	2/3	2/3	0
spin	1/2	1/2	1/2	1
name	u up	c charm	t top	γ photon
				125.9 GeV H Higgs Boson
Quarks				
	4.8 MeV/c ²	104 MeV/c ²	4.2 GeV/c ²	0
	-1/3	-1/3	-1/3	0
	1/2	1/2	1/2	1
	d down	s strange	b bottom	g gluon
	<2.2 eV/c ²	<0.17 MeV/c ²	<15.5 MeV/c ²	91.2 GeV/c ²
	0	0	0	0
	1/2	1/2	1/2	1
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	Z⁰ Z boson
Leptons				
	0.511 MeV/c ²	105.7 MeV/c ²	1.777 GeV/c ²	80.4 GeV/c ²
	-1	-1	-1	±1
	1/2	1/2	1/2	1
	e electron	μ muon	τ tau	W[±] W boson

Special role in the SM: only scalar particle. Product of Brout-Englert-Higgs mechanism required to accommodate masses in a gauge invariant way

The SM is *the legacy* of the 20th century of particle physics:

- ▶ It unifies quantum mechanics, field theory and special relativity
- ▶ It unifies electromagnetism and the weak interaction
- ▶ It describes (to the surprise of many) all laboratory data so far

The Higgs sector

Seeds of New Physics in the Higgs Lagrangian:

$$\mathcal{L}(\phi) = (D_\mu \phi)^\dagger (D^\mu \phi) - \mu_0^2 |\phi|^2 + \lambda |\phi|^4 + Y_{ij} \bar{\psi}_L^i \psi_R^j \phi$$

Gauge invariant mass generation of gauge bosons in the SM

The Higgs mass terms. Connected to the naturalness problem

Yukawas give mass to fermions. Connected to flavour/CP problem

The Higgs quartic self-interaction. Connected to the question of the stability of the potential

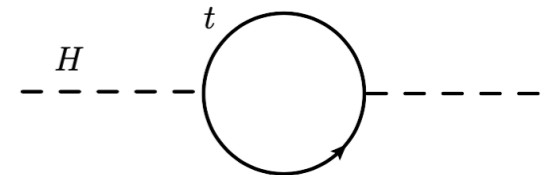
The Higgs naturalness problem

Consider a fermion which couples to the Higgs via

$$\mathcal{L} \subset -\lambda_f H \bar{f} f$$

This gives rise to a correction to the Higgs mass

$$\Delta m_H^2 = N_c i \lambda_f^2 \int \frac{d^4 l}{(2\pi)^4} \frac{\text{tr} [(l + m_t) (l + \not{p} + m_t)]}{(l^2 - m_t^2) ((l + p)^2 - m_t^2)}$$



This is quadratically ultraviolet (UV) divergent:

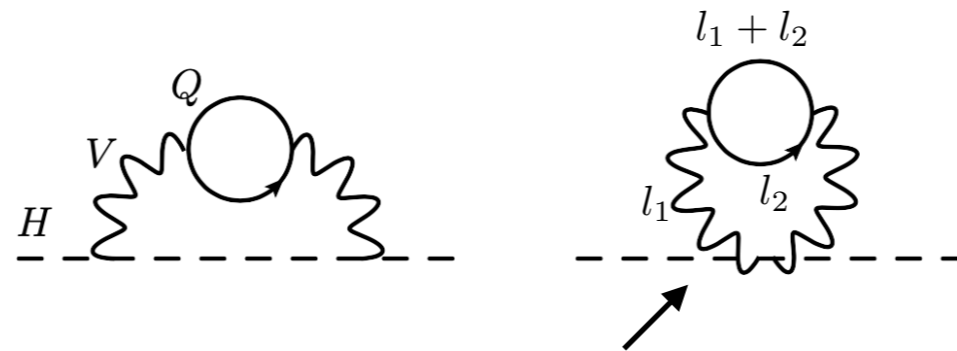
$$\Delta m_H^2 = N_c \frac{i \lambda_f^2}{16\pi^4} \int d^4 l \frac{4l^2}{l^4} = N_c \frac{i \lambda_f^2}{4\pi^4} (-i\pi^2 \Lambda^2) = N_c \frac{G_F m_f^2}{\sqrt{2}\pi^2} \Lambda^2$$

In the SM one finds

$$\frac{\Delta m_H^2}{m_H^2} = \frac{3G_F}{4\sqrt{2}\pi^2} \left(\frac{4m_t^2}{m_H^2} - \frac{2m_W^2}{m_H^2} - \frac{m_Z^2}{m_H^2} - 1 \right) \Lambda^2 \simeq \left(\frac{\Lambda}{500 \text{ GeV}} \right)^2$$

Naturalness problem

Example of two-loop correction that from a heavy fermion Q that couples only indirectly to the Higgs:



$$\Delta m_H^2 \sim g_1^2 g_2^2 C_1 C_2 \int \frac{d^4 l_1}{(2\pi)^4} \frac{d^4 l_2}{(2\pi)^4} \frac{\text{tr}[(\not{l}_1 + \not{l}_2)/\not{l}_2]}{l_1^4 l_2^2 (l_1 + l_2)^2} \sim \frac{g_1^2}{16\pi^2} \frac{g_2^2}{16\pi^2} C_1 C_2 \Lambda^2 \ln \Lambda$$

Even if a new particle does not interact at tree level with the Higgs, as long as it has an interaction with any other SM particle, there will be a quadratic sensitivity of m_H to the UV cutoff scale

Naturalness in the past

First example: the electron Coulomb field $\vec{E} = \frac{e}{4\pi r^2} \frac{\vec{r}}{r}$

Energy stored in the electric field $W_\Lambda = \frac{1}{2} \int_{r>\Lambda^{-1}} d^3r \vec{E}^2 = \frac{1}{2} \alpha \Lambda$

Correction to the electron mass $m_e = M_e + \frac{1}{2} \alpha \Lambda$

Use limits on electron radius $r_e < 10^{-4}$ fm to determine the cut-off Λ

$$\frac{1}{2} \alpha \Lambda = \mathcal{O}(10^7 \text{ keV})$$

but

$$m_e \approx 511 \text{ keV}$$

The non-electrostatic mass term M_e , completely unrelated to the electric field, would need to **cancel it up to five significant digits**

(*) Natural units: $1 \text{ fm} \approx 5 \text{ GeV}^{-1}$

Naturalness in the past

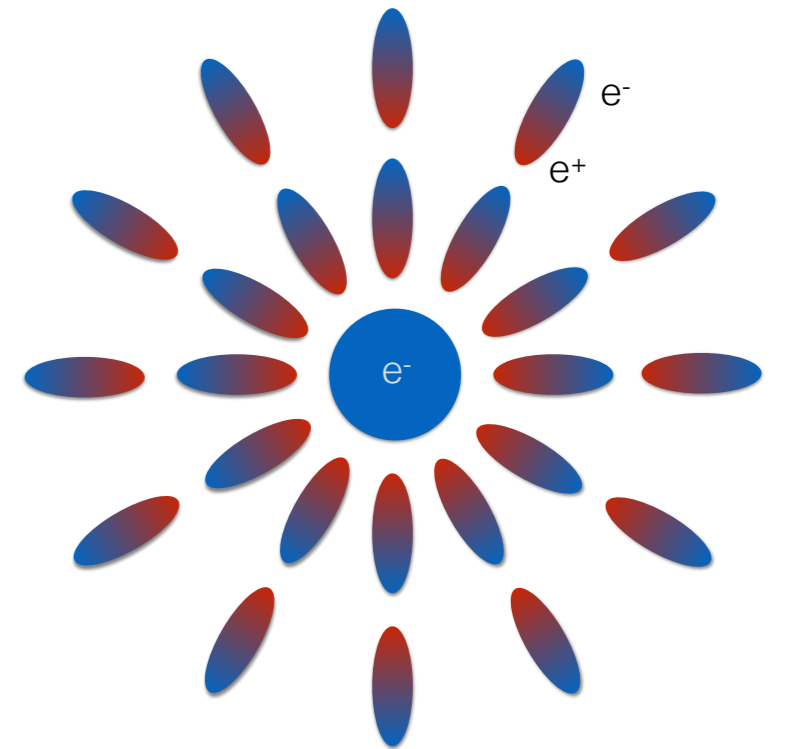
But this is a classical picture probing effects at distances 10^{-4} fm

Quantum effects allow the production of e^+e^- pair that screen the electric field, effectively reducing the electric charge

An explicit calculation gives

$$m_e = M_e \left(1 - \frac{3\alpha}{2\pi} \ln \left(\frac{M_e}{\Lambda} \right) \right)$$

The correction remains small even for a cut-off that reaches the Planck mass (10^{19} GeV)

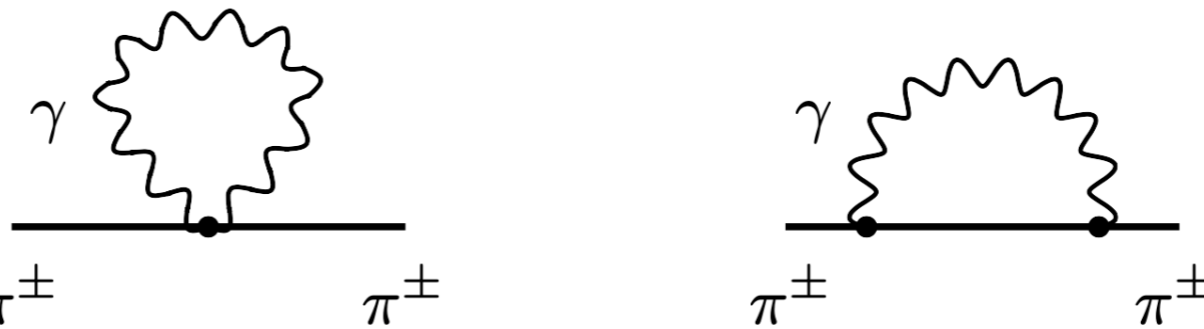


The fine-tuning of the electron mass is solved via the introduction of the positron

Naturalness in the past

Second example: charged-neutral pion mass difference

$$\mathcal{L} \supset ieA_\mu (\pi^+ \partial^\mu \pi^- - \pi^- \partial^\mu \pi^+) + e^2 A_\mu A^\mu \pi^+ \pi^-$$

$$(\Delta m_{\pi^\pm}^2)_{\text{tadpole}} = -i \frac{\alpha}{4\pi^3} 4 \int d^4l \frac{1}{l^2}$$


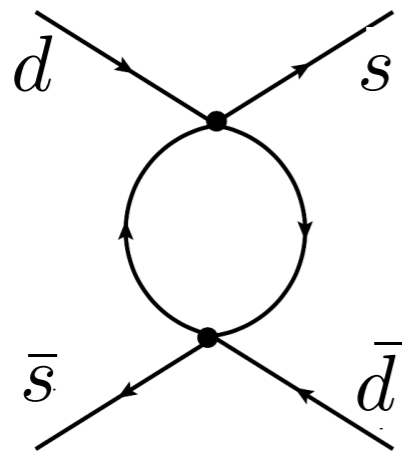
$$\Delta m^2 = m_{\pi^\pm}^2 - m_{\pi^0}^2 \approx \frac{3\alpha}{4\pi} \Lambda^2$$

$$\Delta m_{\text{exp}}^2 \simeq (35.51 \text{ MeV})^2 \quad \longrightarrow \quad \Lambda \simeq 850 \text{ MeV}$$

Fine-tuning explained via the introduction of the ρ vector meson ($m_\rho \simeq 770 \text{ MeV}$) and the axial-vector resonance a_1 ($m_{a_1} \simeq 1250 \text{ MeV}$)

Naturalness in the past

Third example: Kaon transition rates and mixing



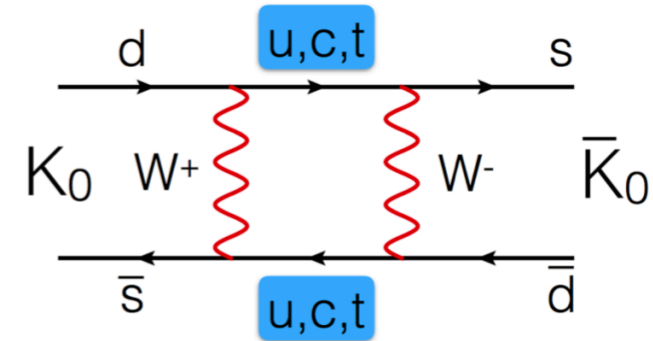
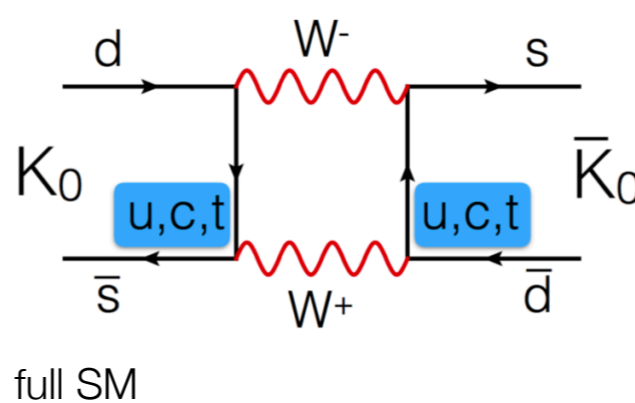
Fermi theory

$$\Delta M_K = M_{K_L} - M_{K_S}$$

$$\frac{\Delta M_K}{M_K} \simeq \frac{G_F^2}{6\pi^2} f_K^2 \sin^2 \theta_c m_c^2$$

GIM suppression

$$\frac{\Delta M_K}{M_K} \simeq 7 \cdot 10^{-15} \quad \text{and} \quad f_K \simeq 0.1 \text{ GeV} \quad \longrightarrow \quad m_c \simeq 1.4 \text{ GeV}$$

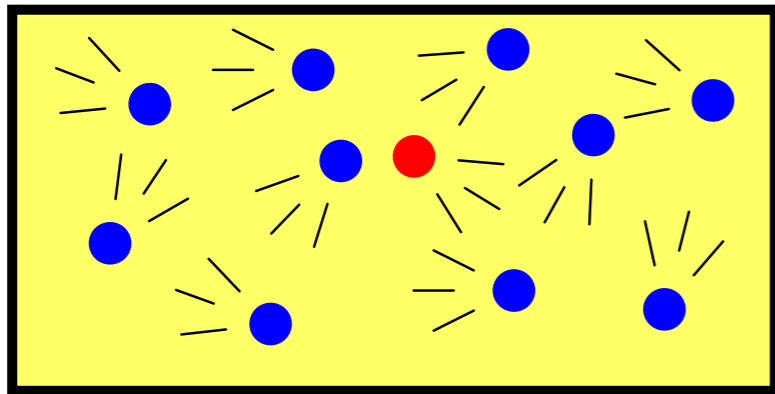


☞ Quadratic divergence in the amplitude used by Gaillard & Lee in 1974 to predict that the charm quark should be lighter than 10 GeV

Naturalness: a guiding principle?

Analogy with thermal fluctuation

t = 0




thermalization

At large t expect

$$E_{\bullet} \sim E_{\bullet}$$

While the observation is

$$E_{\bullet} \sim 10^{-17} E_{\bullet}$$

While no logical inconsistency can be claimed, it just seems hard to believe

- In the analogy: natural explanation could be that red does not really interact with blue because the interaction is screened
- Similarly in the Higgs case, the interaction (or UV sensitivity) could be screened by new forces/particles

Solutions to hierarchy problem

The naturalness problem in the Higgs sector is solved, if e.g.

- there is a natural screening mechanism (a symmetry) protecting the Higgs from the UV sensitivity
- the Higgs Boson is non elementary but composite

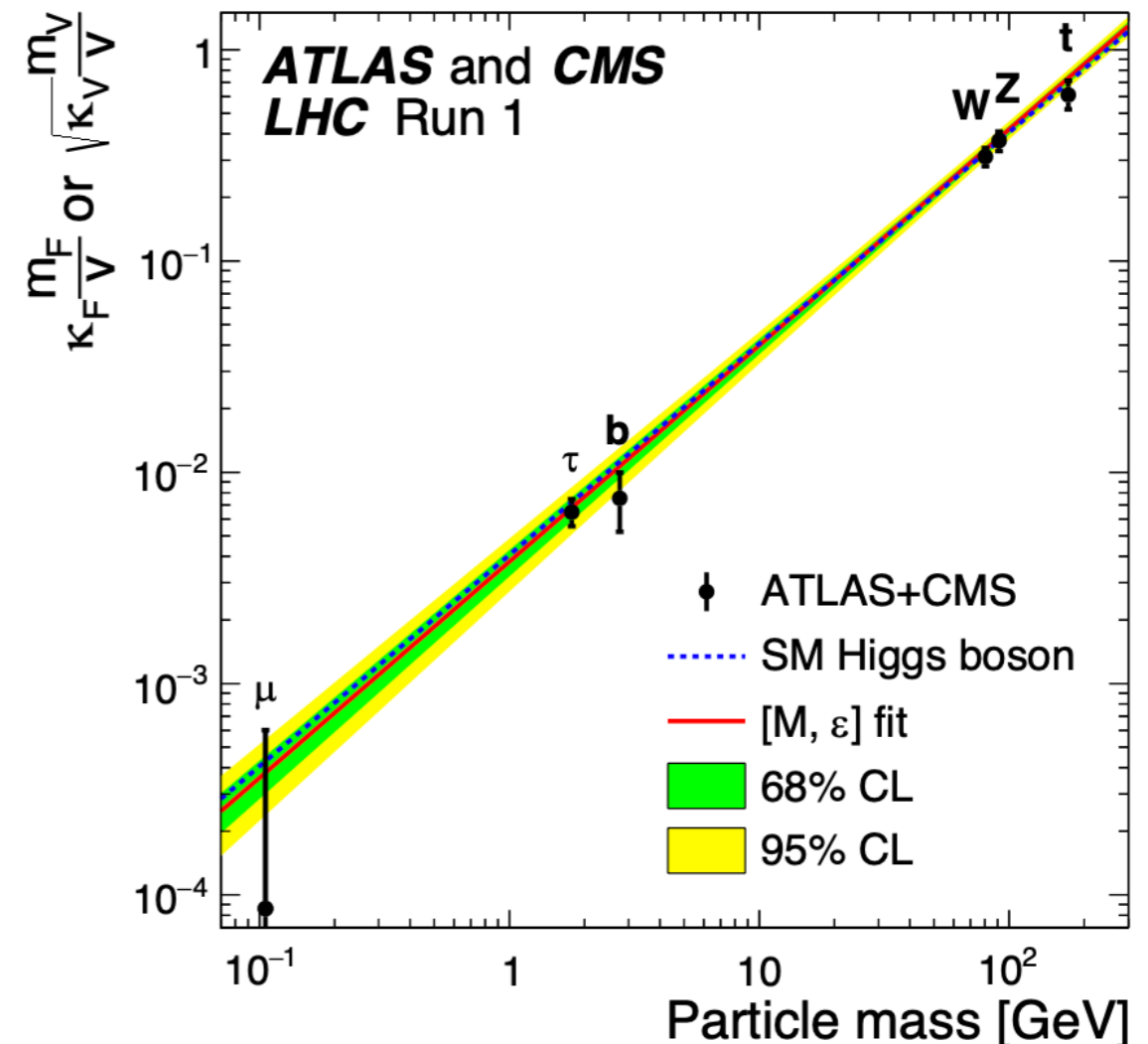
Both scenarios give rise to deviations in the Higgs couplings as predicted in the SM

The LHC might address the naturalness even if no new particles are discovered directly. One just needs to measure very precisely the couplings and see if they differ from the SM pattern

Higgs couplings

So far the Higgs looks SM like, however

- None of the measurements are very precise yet (still room for moderately large deviations)
- Couplings to light states not measured yet — a lot of room for new effects

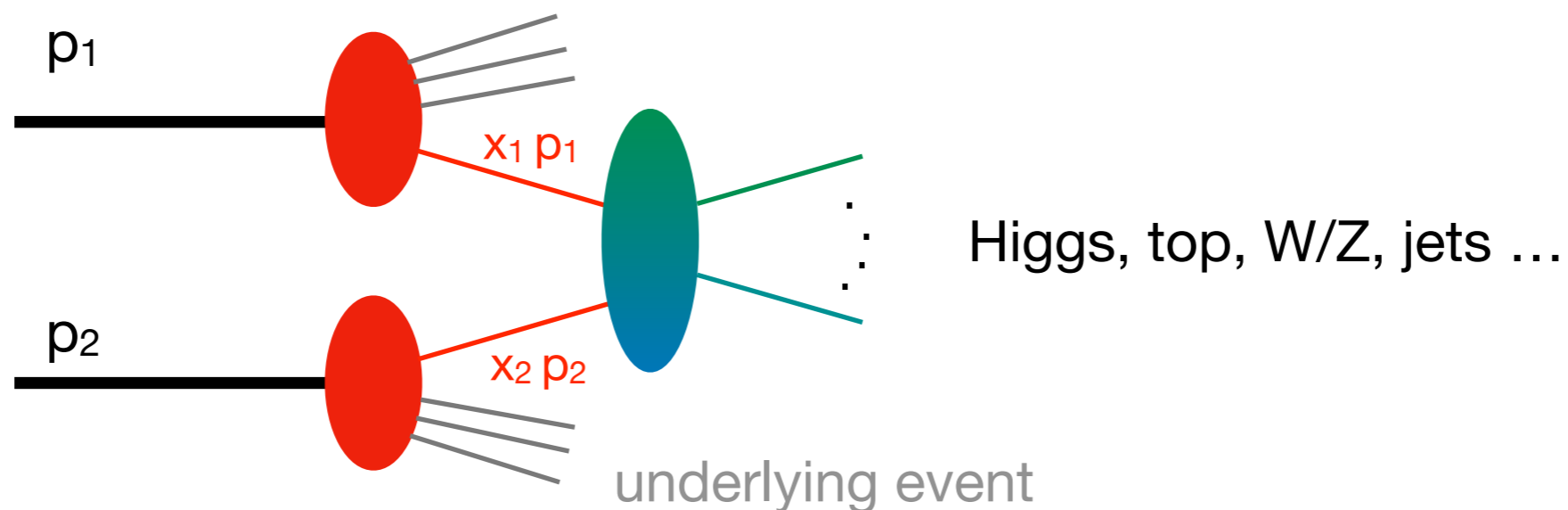


Measuring or constraining these couplings is a major programme that will extend over decades. Theory input crucial to extract couplings and other particle properties in general

Master formula for LHC

For proton-proton collisions, cross sections are convolutions of **parton density functions (PDFs)** with **hard partonic cross sections**

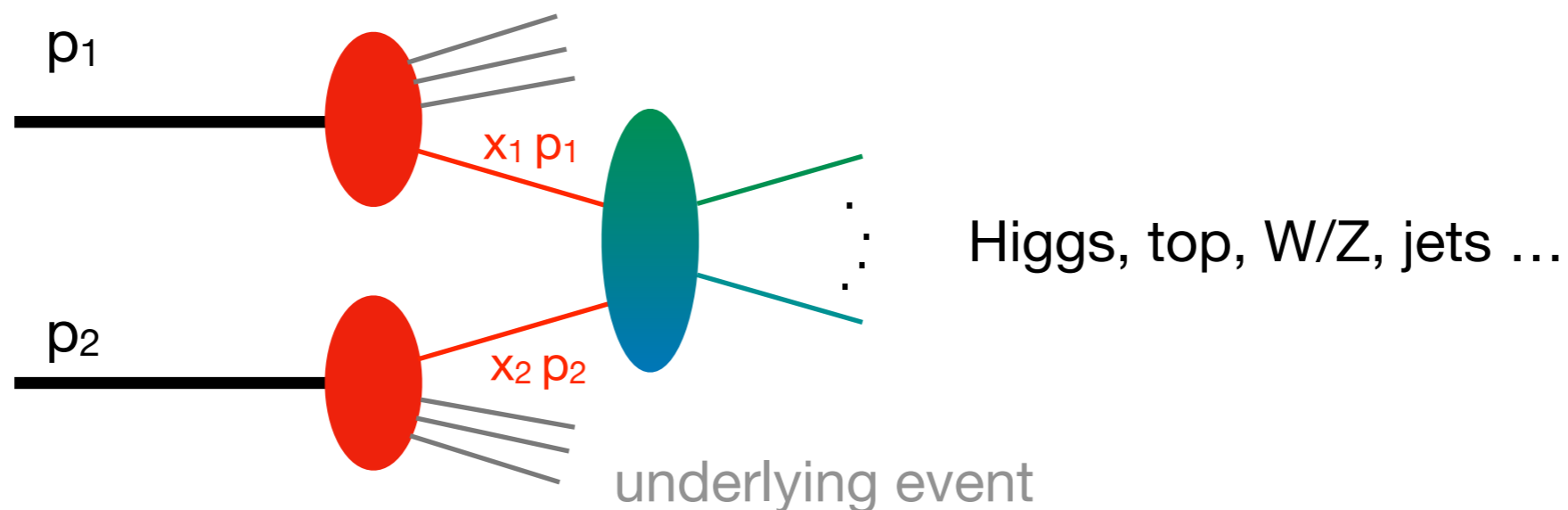
$$\sigma_{\text{had}} = \sum_{ij} \int dx_1 dx_2 f_i(x_1, \mu_F) f_j(x_2, \mu_F) \times \sigma_{ij}(x_1 P_1, x_2 P_2, \mu_F) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{Q}\right)$$



Master formula for LHC

For proton-proton collisions, cross sections are convolutions of **parton density functions (PDFs)** with hard partonic cross sections

$$\sigma_{\text{had}} = \sum_{ij} \int dx_1 dx_2 f_i(x_1, \mu_F) f_j(x_2, \mu_F) \times \sigma_{ij}(x_1 P_1, x_2 P_2, \mu_F) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{Q}\right)$$



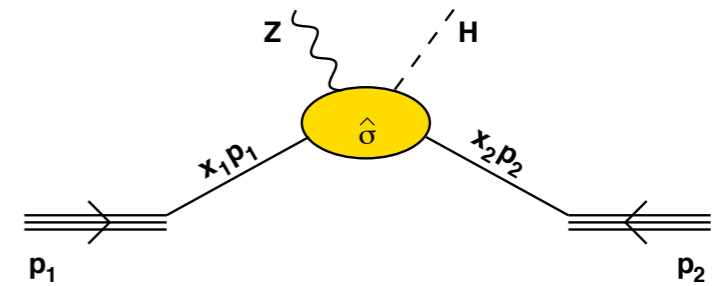
The parton model

Basic idea of the parton model: intuitive picture where in a high transverse momentum scattering partons behave as quasi free in the collision

⇒ cross section is the incoherent sum of all partonic cross-sections

$$\sigma = \int dx_1 dx_2 f_1^{(P_1)}(x_1) f_2^{(P_2)}(x_2) \hat{\sigma}(x_1 x_2 s) \quad \hat{s} = x_1 x_2 s$$

NB: This formula is wrong/incomplete (see later)



$f_i^{(P_j)}(x_i)$: **parton distribution function (PDF)** is the probability to find parton i in hadron j with a fraction x_i of the longitudinal momentum (transverse momentum neglected), **extracted from data**

$\hat{\sigma}(x_1 x_2 s)$: **partonic cross-section** for a given scattering process, **computed in perturbative QCD**

Sum rules

Momentum sum rule: conservation of incoming total momentum

$$\int_0^1 dx \sum_i x f_i^{(p)}(x) = 1$$

Conservation of flavour: e.g. for a proton

$$\int_0^1 dx \left(f_u^{(p)}(x) - f_{\bar{u}}^{(p)}(x) \right) = 2$$

$$\int_0^1 dx \left(f_d^{(p)}(x) - f_{\bar{d}}^{(p)}(x) \right) = 1$$

$$\int_0^1 dx \left(f_s^{(p)}(x) - f_{\bar{s}}^{(p)}(x) \right) = 0$$

In the proton: u, d **valence quarks**, all other quarks are called **sea-quarks**

Sum rules

Momentum sum rule: conservation of incoming total momentum

$$\int_0^1 dx \sum_i x f_i^{(p)}(x) = 1$$

Conservation of flavour: e.g. for a proton

$$\int_0^1 dx \left(f_u^{(p)}(x) - f_{\bar{u}}^{(p)}(x) \right) = 2$$

$$\int_0^1 dx \left(f_d^{(p)}(x) - f_{\bar{d}}^{(p)}(x) \right) = 1$$

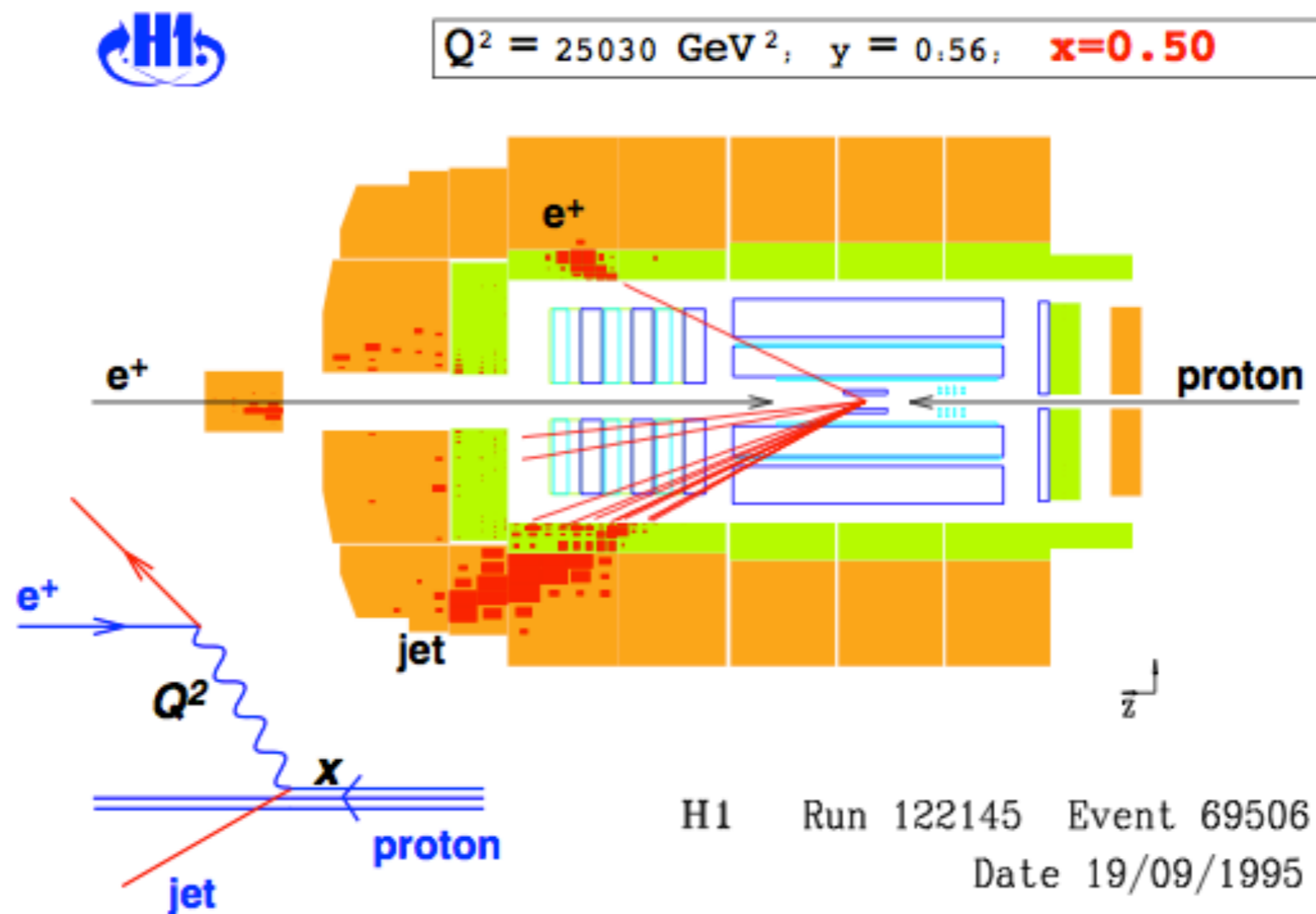
$$\int_0^1 dx \left(f_s^{(p)}(x) - f_{\bar{s}}^{(p)}(x) \right) = 0$$

In the proton: u, d **valence quarks**, all other quarks are called **sea-quarks**

How can parton densities be extracted from data?

Deep inelastic scattering

Clean probe of the structure of the proton with an electron

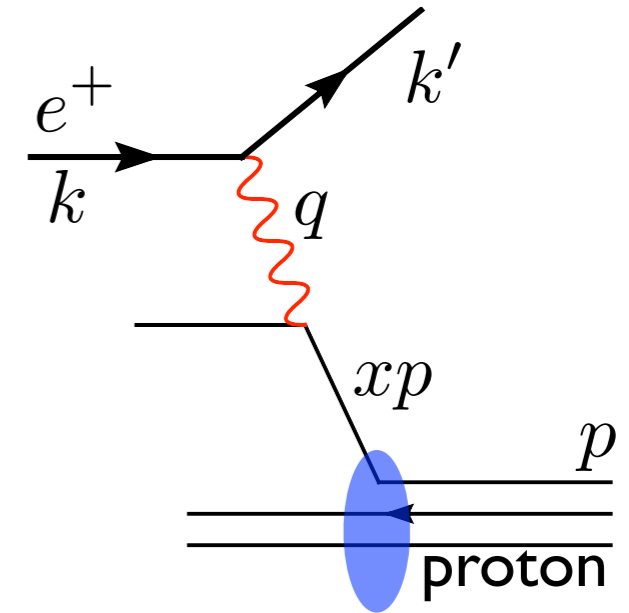


Deep inelastic scattering

Protons made up of point-like quarks.

Different momentum scales involved:

- hard photon virtuality (sets the resolution scale) Q
- hard photon-quark interaction Q
- soft interaction between partons in the proton $m_p \ll Q$



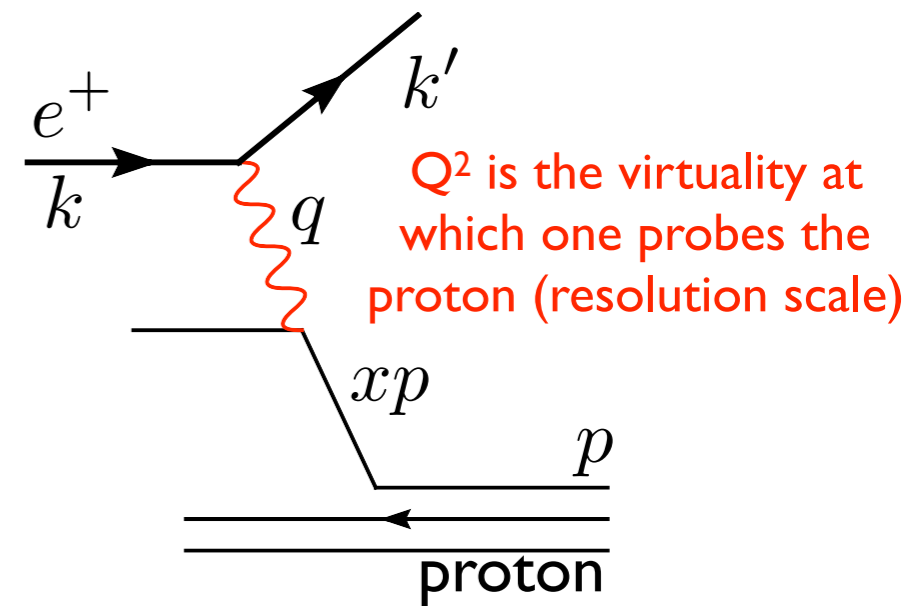
During the hard interaction, partons do not have time to interact among them, they behave as if they were free

\Rightarrow approximate as incoherent scattering on single partons

Deep inelastic scattering

Kinematics:

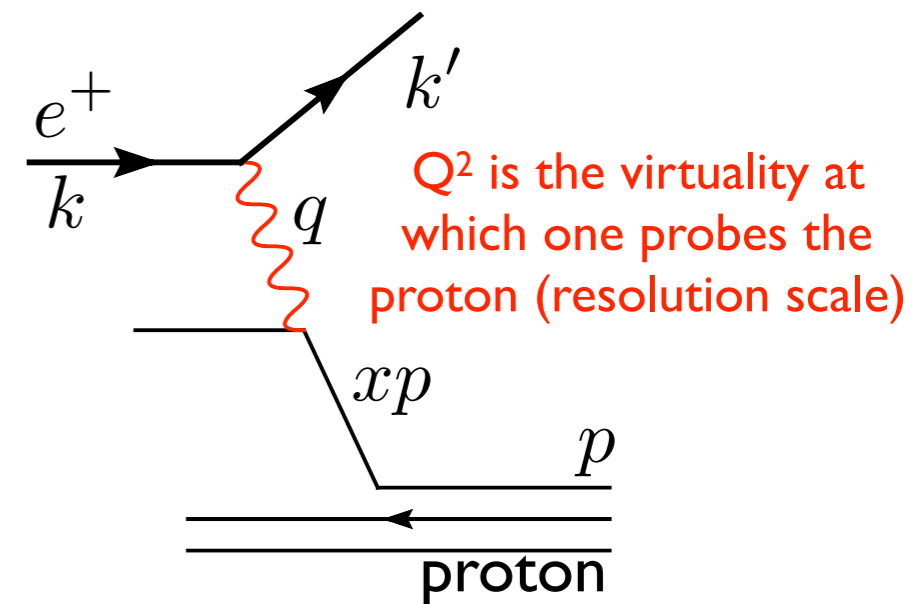
$$Q^2 = -q^2 \quad s = (k + p)^2 \quad x_{Bj} = \frac{Q^2}{2p \cdot q} \quad y = \frac{p \cdot q}{k \cdot p}$$



Deep inelastic scattering

Kinematics:

$$Q^2 = -q^2 \quad s = (k + p)^2 \quad x_{Bj} = \frac{Q^2}{2p \cdot q} \quad y = \frac{p \cdot q}{k \cdot p}$$



Partonic variables:

$$\hat{p} = xp \quad \hat{s} = (k + \hat{p})^2 = 2k \cdot \hat{p} \quad \hat{y} = \frac{\hat{p} \cdot q}{k \cdot \hat{p}} = y \quad (\hat{p} + q)^2 = 2\hat{p} \cdot q - Q^2 = 0$$

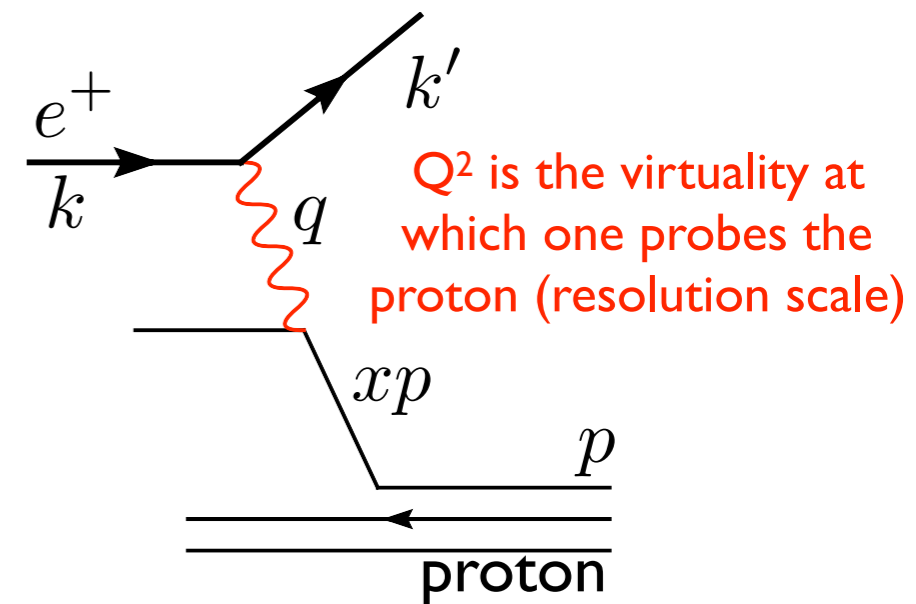
$$\Rightarrow x = x_{Bj}$$

Hence at leading order, the experimentally accessible x_{Bj} coincides with the momentum fraction carried by the quark in the proton

Deep inelastic scattering

Kinematics:

$$Q^2 = -q^2 \quad s = (k + p)^2 \quad x_{Bj} = \frac{Q^2}{2p \cdot q} \quad y = \frac{p \cdot q}{k \cdot p}$$



Partonic variables:

$$\hat{p} = xp \quad \hat{s} = (k + \hat{p})^2 = 2k \cdot \hat{p} \quad \hat{y} = \frac{\hat{p} \cdot q}{k \cdot \hat{p}} = y \quad (\hat{p} + q)^2 = 2\hat{p} \cdot q - Q^2 = 0$$

$$\Rightarrow x = x_{Bj}$$

Hence at leading order, the experimentally accessible x_{Bj} coincides with the momentum fraction carried by the quark in the proton

Partonic cross section:

(apply QED Feynman rules and add phase space)

$$\frac{d\hat{\sigma}}{d\hat{y}} = q_l^2 \frac{\hat{s}}{Q^4} 2\pi \alpha_{em} (1 + (1 - \hat{y})^2)$$

Exercise: show that in the CM frame of the electron-quark system y is given by $(1 - \cos \theta_{e1})/2$, with θ_{e1} the scattering angle of the electron in this frame

Exercise:

- show that the two particle phase space is $\frac{d\phi}{16\pi}$
- show that the squared matrix element is $\frac{16\pi\alpha q_l^2}{Q^4} \hat{s} x p k (1 + (1 - y)^2)$
- show that the flux factor is $\frac{1}{4xpk}$

Hence derive that

$$\frac{d\hat{\sigma}}{d\hat{y}} = q_l^2 \frac{\hat{s}}{Q^4} 2\pi \alpha_{em} (1 + (1 - \hat{y})^2)$$

Deep inelastic scattering

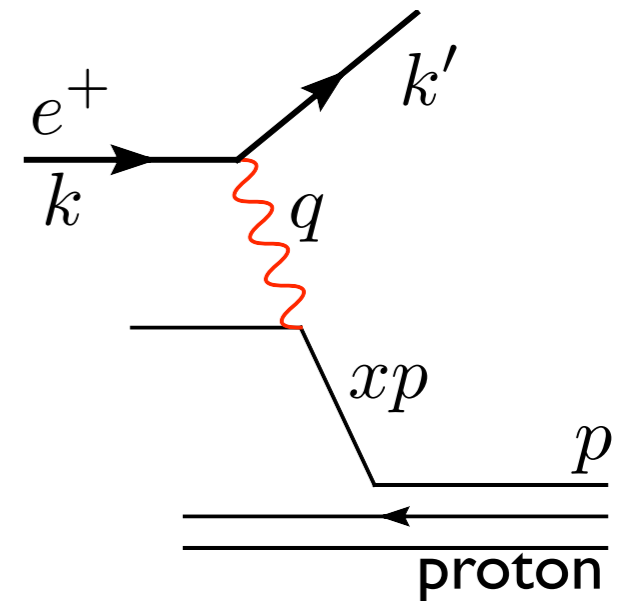
Hadronic cross section (factorization): $\frac{d\sigma}{dy} = \int dx \sum_l f_l^{(p)}(x) \frac{d\hat{\sigma}}{d\hat{y}}$

Deep inelastic scattering

Hadronic cross section (factorization): $\frac{d\sigma}{dy} = \int dx \sum_l f_l^{(p)}(x) \frac{d\hat{\sigma}}{d\hat{y}}$

Using $x = x_{Bj}$

$$\begin{aligned} \frac{d\sigma}{dy dx_{Bj}} &= \sum_l f_l^{(p)}(x) \frac{d\hat{\sigma}}{d\hat{y}} \\ &= \frac{2\pi \alpha_{em}^2 s x_{Bj}}{Q^4} (1 + (1 - y)^2) \sum_l q_l^2 f_l^{(p)}(x_{Bj}) \end{aligned}$$

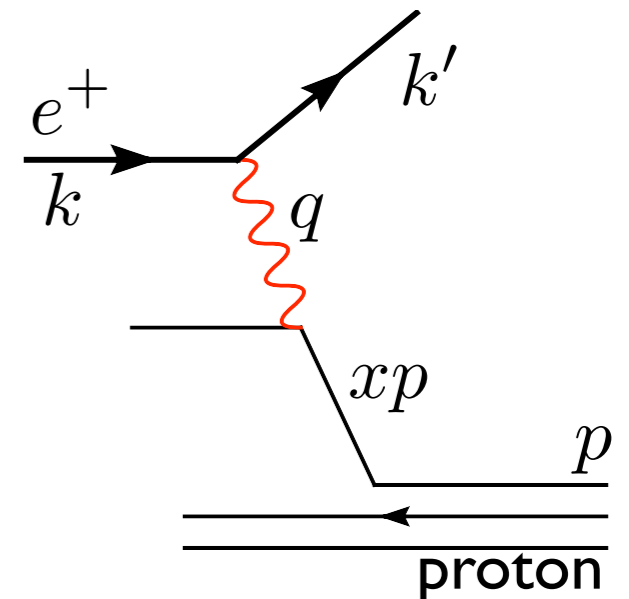


Deep inelastic scattering

Hadronic cross section (factorization): $\frac{d\sigma}{dy} = \int dx \sum_l f_l^{(p)}(x) \frac{d\hat{\sigma}}{d\hat{y}}$

Using $x = x_{Bj}$

$$\begin{aligned} \frac{d\sigma}{dy dx_{Bj}} &= \sum_l f_l^{(p)}(x) \frac{d\hat{\sigma}}{d\hat{y}} \\ &= \frac{2\pi \alpha_{em}^2 s x_{Bj}}{Q^4} (1 + (1-y)^2) \sum_l q_l^2 f_l^{(p)}(x_{Bj}) \end{aligned}$$



1. at fixed x_{Bj} and y the cross-section scales with s
2. the y -dependence of the cross-section is fully predicted and is typical of vector interaction with fermions \Rightarrow **Callan-Gross relation**
3. can access (sums of) parton distribution functions
4. **Bjorken scaling**: pdfs depend on x and not on Q^2 (violated by logarithmic radiative corrections, see later)

The structure function F_2

$$\frac{d\sigma}{dydx} = \frac{2\pi\alpha_{em}^2 s}{Q^4} (1 + (1 - y^2) F_2(x)) \quad F_2(x) = \sum_l xq_l^2 f_l^{(p)}(x)$$

F_2 is called **structure function** (describes structure/constituents of nucleus)

For electron scattering on proton

$$F_2(x) = x \left(\frac{4}{9}u(x) + \frac{1}{9}d(x) \right)$$

NB: use perturbative language of quarks and gluons despite the fact that parton distribution are non-perturbative

Bjorken scaling: the fact the structure functions are independent of Q is a direct evidence for the existence of point-like quarks in the proton (violated by logarithmic corrections)

The structure function F_2

$$\frac{d\sigma}{dydx} = \frac{2\pi\alpha_{em}^2 s}{Q^4} (1 + (1 - y^2) F_2(x)) \quad F_2(x) = \sum_l xq_l^2 f_l^{(p)}(x)$$

F_2 is called **structure function** (describes structure/constituents of nucleus)

For electron scattering on proton

$$F_2(x) = x \left(\frac{4}{9}u(x) + \frac{1}{9}d(x) \right)$$

NB: use perturbative language of quarks and gluons despite the fact that parton distribution are non-perturbative

Question: F_2 gives only a linear combination of u and d. How can they be extracted separately?

Isospin

Neutron is like a proton with u & d exchanged

Isospin

Neutron is like a proton with u & d exchanged

For electron scattering on a proton

$$F_2^p(x) = x \left(\frac{4}{9} u_p(x) + \frac{1}{9} d_p(x) \right)$$

Isospin

Neutron is like a proton with u & d exchanged

For electron scattering on a proton

$$F_2^p(x) = x \left(\frac{4}{9} u_p(x) + \frac{1}{9} d_p(x) \right)$$

For electron scattering on a neutron

$$F_2^n(x) = x \left(\frac{1}{9} d_n(x) + \frac{4}{9} u_n(x) \right) = x \left(\frac{4}{9} d_p(x) + \frac{1}{9} u_p(x) \right)$$

Isospin

Neutron is like a proton with u & d exchanged

For electron scattering on a proton

$$F_2^p(x) = x \left(\frac{4}{9} u_p(x) + \frac{1}{9} d_p(x) \right)$$

For electron scattering on a neutron

$$F_2^n(x) = x \left(\frac{1}{9} d_n(x) + \frac{4}{9} u_n(x) \right) = x \left(\frac{4}{9} d_p(x) + \frac{1}{9} u_p(x) \right)$$

F_2^n and F_2^p allow determination of u_p and d_p separately

Isospin

Neutron is like a proton with u & d exchanged

For electron scattering on a proton

$$F_2^p(x) = x \left(\frac{4}{9} u_p(x) + \frac{1}{9} d_p(x) \right)$$

For electron scattering on a neutron

$$F_2^n(x) = x \left(\frac{1}{9} d_n(x) + \frac{4}{9} u_n(x) \right) = x \left(\frac{4}{9} d_p(x) + \frac{1}{9} u_p(x) \right)$$

F_2^n and F_2^p allow determination of u_p and d_p separately

NB: experimentally get F_2^n from deuteron: $F_2^d(x) = F_2^p(x) + F_2^n(x)$

Sea quarks

Inside the proton via fluctuations, pairs of $u\bar{u}, d\bar{d}, c\bar{c}, s\bar{s}$, etc. are created

An infinite number of pairs can be created as long as they have very low momentum, because of the momentum sum rules.

We saw before that when we say that the proton is made of uud what we mean is

$$\int_0^1 dx (u_p(x) - \bar{u}_p(x)) = 2 \quad \int_0^1 dx (d_p(x) - \bar{d}_p(x)) = 1$$

Sea quarks

Inside the proton via fluctuations, pairs of $u\bar{u}, d\bar{d}, c\bar{c}, s\bar{s}$, etc. are created

An infinite number of pairs can be created as long as they have very low momentum, because of the momentum sum rules.

We saw before that when we say that the proton is made of uud what we mean is

$$\int_0^1 dx (u_p(x) - \bar{u}_p(x)) = 2 \quad \int_0^1 dx (d_p(x) - \bar{d}_p(x)) = 1$$

Photons interact in the same way with $u(d)$ and $\bar{u}(\bar{d})$

How can one measure the difference?

Question: What interacts differently with particle and antiparticle?

Sea quarks

Inside the proton via fluctuations, pairs of $u\bar{u}, d\bar{d}, c\bar{c}, s\bar{s}$, etc. are created

An infinite number of pairs can be created as long as they have very low momentum, because of the momentum sum rules.

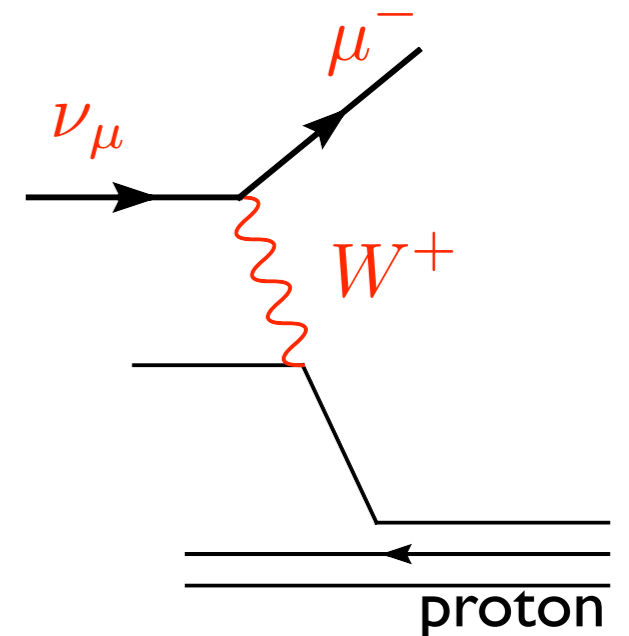
We saw before that when we say that the proton is made of uud what we mean is

$$\int_0^1 dx (u_p(x) - \bar{u}_p(x)) = 2 \quad \int_0^1 dx (d_p(x) - \bar{d}_p(x)) = 1$$

Photons interact in the same way with $u(d)$ and $\bar{u}(\bar{d})$

How can one measure the difference?

Question: What interacts differently with particle and antiparticle? W^+/W^- from neutrino scattering



Check of the momentum sum rule

$$\int_0^1 dx \sum_i x f_i^{(p)}(x) = 1$$

u _v	0,267
d _v	0,111
u _s	0,066
d _s	0,053
s _s	0,033
c _c	0,016
total	0,546

⇒ *half of the longitudinal momentum carried by gluons*

Check of the momentum sum rule

$$\int_0^1 dx \sum_i x f_i^{(p)}(x) = 1$$

u _v	0,267
d _v	0,111
u _s	0,066
d _s	0,053
s _s	0,033
c _c	0,016
total	0,546

⇒ *half of the longitudinal momentum carried by gluons*

$\gamma/W^{+/-}$ don't interact with gluons

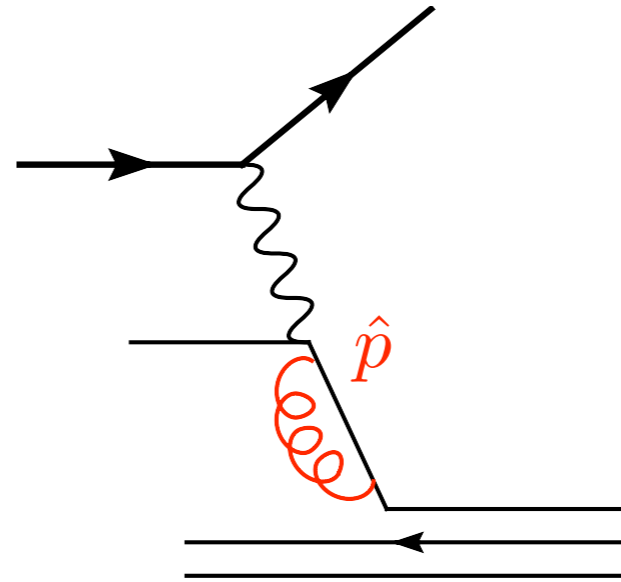
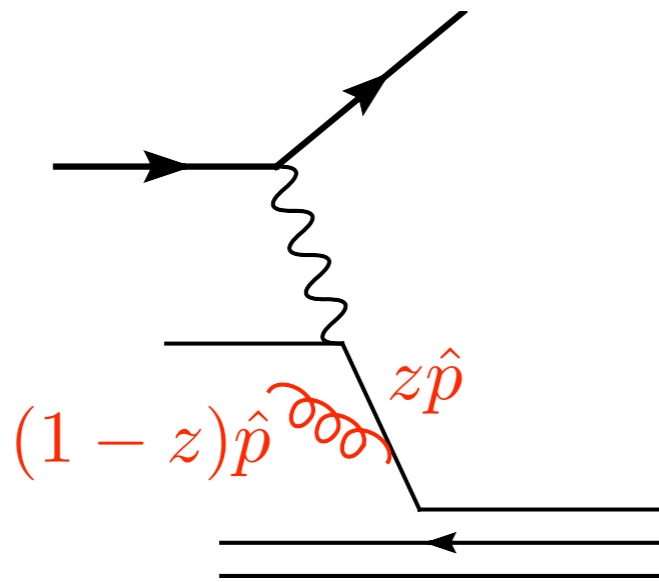
How can one measure gluon parton densities?

We need to discuss radiative effects first

Radiative corrections

To first order in the coupling:

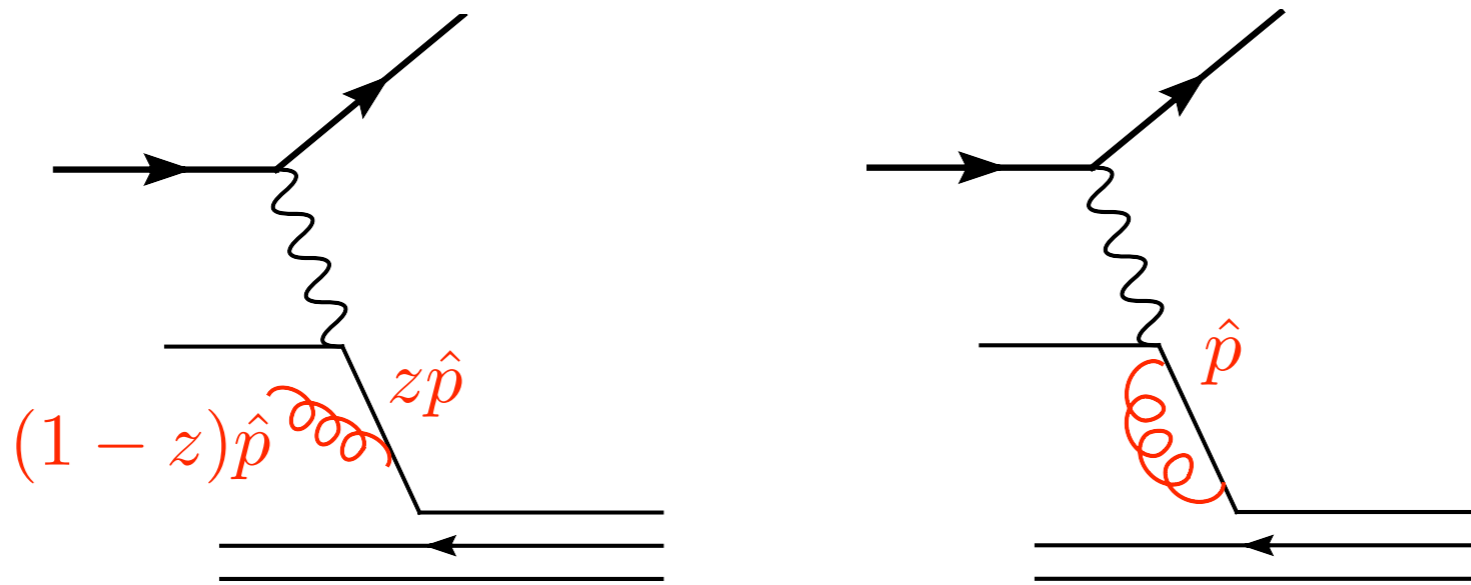
need to consider the emission of one real gluon and a virtual one



Radiative corrections

To first order in the coupling:

need to consider the emission of one real gluon and a virtual one



Adding real and virtual contributions, the partonic cross-section reads

$$\sigma^{(1)} = \frac{C_F \alpha_s}{2\pi} \int dz \frac{dk_{\perp}^2}{k_{\perp}^2} \frac{1+z^2}{1-z} \left(\sigma^{(0)}(z\hat{p}) - \sigma^{(0)}(\hat{p}) \right)$$

Partial cancellation between real (positive), virtual (negative), but real gluon changes the energy entering the scattering, the virtual does not

Radiative corrections

Partonic cross-section:

$$\sigma^{(1)} = \frac{\alpha_s}{2\pi} \int dz \int_{\lambda^2}^{Q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} P(z) \left(\sigma^{(0)}(z\hat{p}) - \sigma^{(0)}(\hat{p}) \right), \quad P(z) = C_F \frac{1+z^2}{1-z}$$

Soft limit: singularity at $z=1$ cancels between real and virtual terms

Collinear singularity: $k_{\perp} \rightarrow 0$ with finite z . **Collinear singularity does not cancel because partonic scatterings occur at different energies**

Radiative corrections

Partonic cross-section:

$$\sigma^{(1)} = \frac{\alpha_s}{2\pi} \int dz \int_{\lambda^2}^{Q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} P(z) \left(\sigma^{(0)}(z\hat{p}) - \sigma^{(0)}(\hat{p}) \right), \quad P(z) = C_F \frac{1+z^2}{1-z}$$

Soft limit: singularity at $z=1$ cancels between real and virtual terms

Collinear singularity: $k_{\perp} \rightarrow 0$ with finite z . **Collinear singularity does not cancel because partonic scatterings occur at different energies**

\Rightarrow naive parton model does not survive radiative corrections

Radiative corrections

Partonic cross-section:

$$\sigma^{(1)} = \frac{\alpha_s}{2\pi} \int dz \int_{\lambda^2}^{Q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} P(z) \left(\sigma^{(0)}(z\hat{p}) - \sigma^{(0)}(\hat{p}) \right), \quad P(z) = C_F \frac{1+z^2}{1-z}$$

Soft limit: singularity at $z=1$ cancels between real and virtual terms

Collinear singularity: $k_{\perp} \rightarrow 0$ with finite z . **Collinear singularity does not cancel because partonic scatterings occur at different energies**

\Rightarrow naive parton model does not survive radiative corrections

Similarly to what is done when renormalizing UV divergences, **collinear divergences** from initial state emissions are **absorbed into parton distribution functions**

The plus prescription

Partonic cross-section:

$$\sigma^{(1)} = \frac{\alpha_s}{2\pi} \int_{\lambda^2}^{Q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} \int_0^1 dz P(z) \left(\sigma^{(0)}(z\hat{p}) - \sigma^{(0)}(\hat{p}) \right)$$

The plus prescription

Partonic cross-section:

$$\sigma^{(1)} = \frac{\alpha_s}{2\pi} \int_{\lambda^2}^{Q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} \int_0^1 dz P(z) \left(\sigma^{(0)}(z\hat{p}) - \sigma^{(0)}(\hat{p}) \right)$$

Plus prescription makes the universal cancelation of singularities explicit

$$\int_0^1 dz f_+(z)g(z) \equiv \int_0^1 f(z) (g(z) - g(1))$$

The plus prescription

Partonic cross-section:

$$\sigma^{(1)} = \frac{\alpha_s}{2\pi} \int_{\lambda^2}^{Q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} \int_0^1 dz P(z) \left(\sigma^{(0)}(z\hat{p}) - \sigma^{(0)}(\hat{p}) \right)$$

Plus prescription makes the universal cancelation of singularities explicit

$$\int_0^1 dz f_+(z)g(z) \equiv \int_0^1 f(z) (g(z) - g(1))$$

The partonic cross section becomes

$$\sigma^{(1)} = \frac{\alpha_s}{2\pi} \int dz \int_{\lambda^2}^{Q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} P_+(z) \sigma^{(0)}(z\hat{p}), \quad P(z) = C_F \left(\frac{1+z^2}{1-z} \right)$$

Collinear singularities still there, but they factorize.

Factorization scale

Schematically use

$$\ln \frac{Q^2}{\lambda^2} = \ln \frac{Q^2}{\mu_F^2} + \ln \frac{\mu_F^2}{\lambda^2}$$

$$\sigma = \sigma^{(0)} + \sigma^{(1)} = \left(1 + \frac{\alpha_s}{2\pi} \ln \frac{\mu_F^2}{\lambda^2} P_+ \right) \times \left(1 + \frac{\alpha_s}{2\pi} \ln \frac{Q^2}{\mu_F^2} P_+ \right) \sigma^{(0)}$$

Factorization scale

Schematically use $\ln \frac{Q^2}{\lambda^2} = \ln \frac{Q^2}{\mu_F^2} + \ln \frac{\mu_F^2}{\lambda^2}$

$$\sigma = \sigma^{(0)} + \sigma^{(1)} = \left(1 + \frac{\alpha_s}{2\pi} \ln \frac{\mu_F^2}{\lambda^2} P_+ \right) \times \left(1 + \frac{\alpha_s}{2\pi} \ln \frac{Q^2}{\mu_F^2} P_+ \right) \sigma^{(0)}$$

So we define

$$f_q(x, \mu_F) = f_q(x) \times \left(1 + \frac{\alpha_s}{2\pi} \ln \frac{\mu_F^2}{\lambda^2} P_{qq}^{(0)} \right) \quad \hat{\sigma}(p, \mu_F) = \left(1 + \frac{\alpha_s}{2\pi} \ln \frac{Q^2}{\mu_F^2} P_{qq}^{(0)} \right) \sigma^{(0)}(p)$$

Factorization scale

Schematically use $\ln \frac{Q^2}{\lambda^2} = \ln \frac{Q^2}{\mu_F^2} + \ln \frac{\mu_F^2}{\lambda^2}$

$$\sigma = \sigma^{(0)} + \sigma^{(1)} = \left(1 + \frac{\alpha_s}{2\pi} \ln \frac{\mu_F^2}{\lambda^2} P_+ \right) \times \left(1 + \frac{\alpha_s}{2\pi} \ln \frac{Q^2}{\mu_F^2} P_+ \right) \sigma^{(0)}$$

So we define

$$f_q(x, \mu_F) = f_q(x) \times \left(1 + \frac{\alpha_s}{2\pi} \ln \frac{\mu_F^2}{\lambda^2} P_{qq}^{(0)} \right) \quad \hat{\sigma}(p, \mu_F) = \left(1 + \frac{\alpha_s}{2\pi} \ln \frac{Q^2}{\mu_F^2} P_{qq}^{(0)} \right) \sigma^{(0)}(p)$$

NB:

- universality, i.e. the PDF redefinition does not depend on the process
- choice of $\mu_F \sim Q$ avoids large logarithms in partonic cross-sections
- PDFs and hard cross-sections don't evolve independently
- the factorization scale acts as a cut-off, it allows to move the divergent contribution into non-perturbative parton distribution functions

Improved parton model

Naive parton model:

$$\sigma = \int dx_1 dx_2 f_1^{(P_1)}(x_1) f_2^{(P_2)}(x_2) \hat{\sigma}(x_1 x_2 s) \quad \hat{s} = x_1 x_2 s$$

After radiative corrections:

$$\sigma = \int dx_1 dx_2 f_1^{(P_1)}(x_1, \mu^2) f_2^{(P_2)}(x_2, \mu^2) \hat{\sigma}(x_1 x_2 s, \mu^2)$$

Intermediate recap

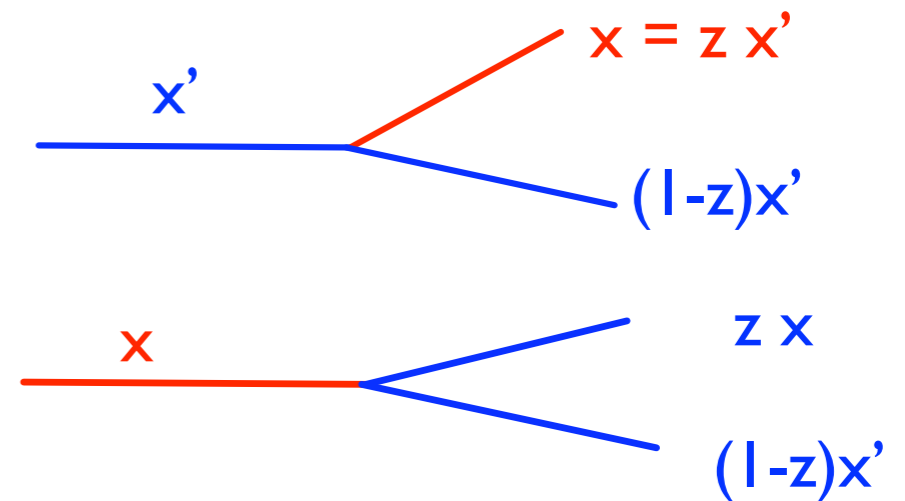
- With initial state parton **collinear singularities don't cancel**
- Initial state emissions with k_{\perp} below a given scale are included in PDFs
- This procedure introduces a scale μ_F , the so-called **factorization scale** which factorizes the low energy (non-perturbative) dynamics from the perturbative hard cross-section
- As for the renormalization scale, the dependence of cross-sections on μ_F is due to the fact that the perturbative expansion has been truncated
- The **dependence on μ_F becomes milder when including higher orders**
- **The redefinition of PDFs is universal and process-independent**

Master formula:
$$\sigma = \int dx_1 dx_2 f_1^{(P_1)}(x_1, \mu^2) f_2^{(P_2)}(x_2, \mu^2) \hat{\sigma}(x_1 x_2 s, \mu^2)$$

Evolution of PDFs

A parton distribution changes when

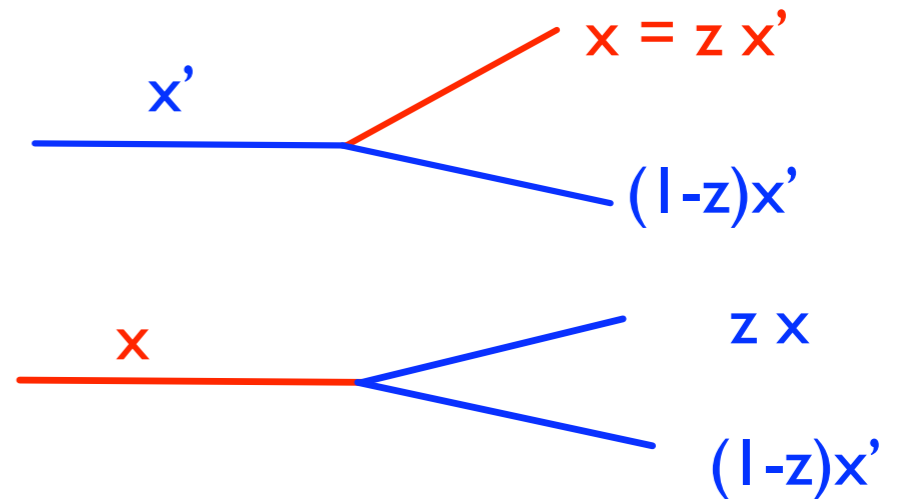
- a different parton splits and produces **it**
- **the parton itself** splits



Evolution of PDFs

A parton distribution changes when

- a different parton splits and produces **it**
- **the parton itself** splits



$$\begin{aligned}
 \mu^2 \frac{\partial f(x, \mu^2)}{\partial \mu^2} &= \int_0^1 dx' \int_x^1 dz \frac{\alpha_s}{2\pi} P(z) f(x', \mu^2) \delta(zx' - x) - \int_0^1 dz \frac{\alpha_s}{2\pi} P(z) f(x, \mu^2) \\
 &= \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P(z) f\left(\frac{x}{z}, \mu^2\right) - \int_0^1 dz \frac{\alpha_s}{2\pi} P(z) f(x, \mu^2) \\
 &= \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P_+(z) f\left(\frac{x}{z}, \mu^2\right)
 \end{aligned}$$

The plus prescription $\int_0^1 dz f_+(z)g(z) \equiv \int_0^1 dz f(z) (g(z) - g(1))$

DGLAP equation

$$\mu^2 \frac{\partial f(x, \mu^2)}{\partial \mu^2} = \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P(z) f\left(\frac{x}{z}, \mu^2\right)$$

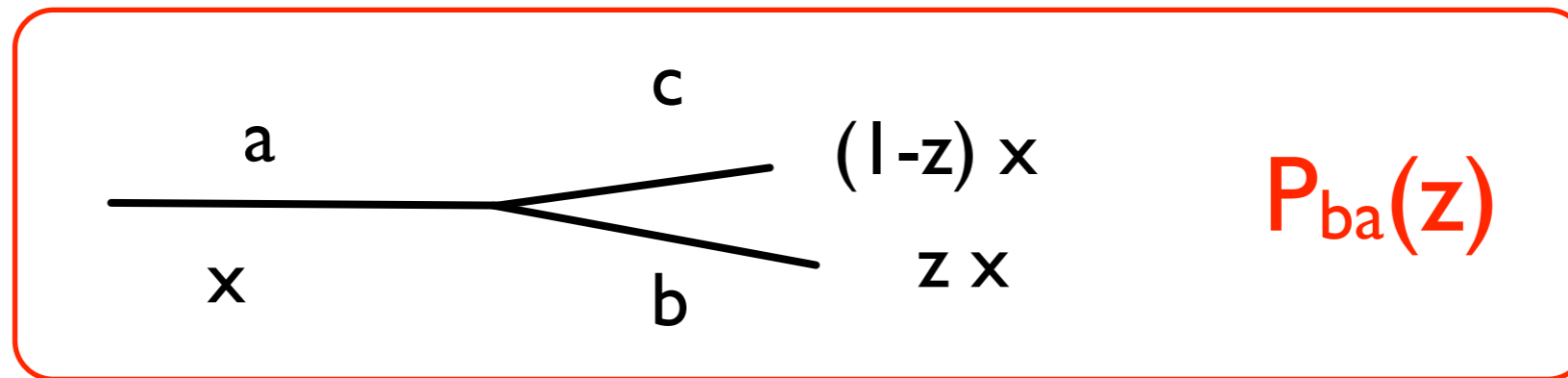
Altarelli, Parisi; Gribov-Lipatov; Dokshitzer '77

Master equation of QCD: we can not compute parton densities, but we can predict how they evolve from one scale to another

Universality of splitting functions: we can measure pdfs in one process and use them as an input for another process

Conventions for splitting functions

There are various partons types. Standard notation:



Accounting for the different species of partons the DGLAP equations become:

$$\mu^2 \frac{\partial f_i(x, \mu^2)}{\partial \mu^2} = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z) f_j\left(\frac{x}{z}, \mu^2\right)$$

This is a system of coupled integro/differential equations

The above convolution in compact notation:

$$\mu^2 \frac{\partial f_i(x, \mu^2)}{\partial \mu^2} = \sum_j P_{ij} \otimes f_j(\mu^2)$$

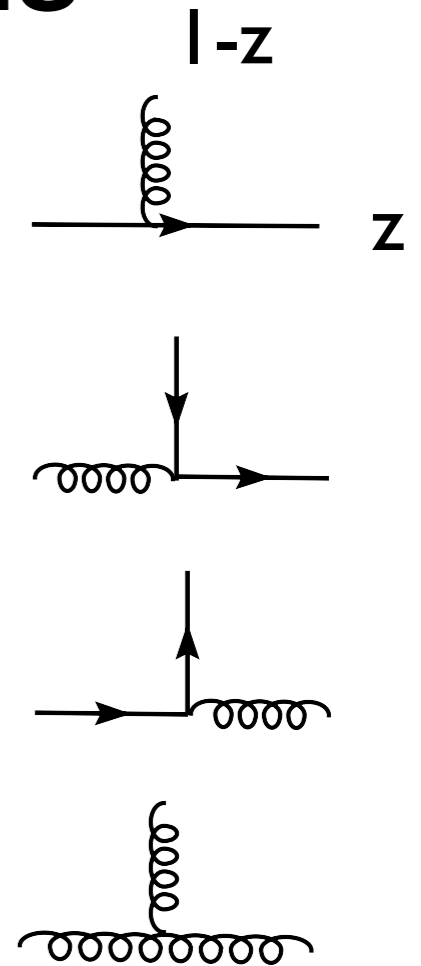
Properties of splitting functions

$$P_{qq}^{(0)} = P_{\bar{q}\bar{q}}^{(0)} = C_F \left[\left(\frac{1+z^2}{1-z} \right)_+ \right]$$

$$P_{qg}^{(0)} = P_{\bar{q}g}^{(0)} = T_R (z^2 + (1-z)^2)$$

$$P_{gq}^{(0)} = P_{g\bar{q}}^{(0)} = C_F \frac{1+(1-z)^2}{z}$$

$$P_{gg}^{(0)} = 2C_A \left[z \left(\frac{1}{(1-z)} \right)_+ + \frac{1-z}{z} + z(1-z) + b_0 \delta(1-z) \right]$$



- P_{qg} and P_{gq} symmetric under $z \leftrightarrow (1-z)$
- P_{qq} and P_{gq} divergence for $z=1$ (soft gluon)
- P_{gq} and P_{gg} divergence for $z=0$ (soft gluon)
- P_{qg} no soft divergence for gluon splitting to quarks

⇒ gluon PDF grows at small x

Sum rules in pQCD

Beyond the naive parton model the probabilistic picture does not hold anymore. *What about basic conservation principles (e.g. sum rules)?*

Exercise: show that e.g.

$$\int_0^1 dx (f_u(x, \mu^2) - f_{\bar{u}}(x, \mu^2)) = \text{constant} \quad \text{if and only if} \quad \int_0^1 dz P_{qq}(z) = 0$$

Sum rules in pQCD

Beyond the naive parton model the probabilistic picture does not hold anymore. *What about basic conservation principles (e.g. sum rules)?*

Exercise: show that e.g.

$$\int_0^1 dx (f_u(x, \mu^2) - f_{\bar{u}}(x, \mu^2)) = \text{constant} \quad \text{if and only if} \quad \int_0^1 dz P_{qq}(z) = 0$$

Solution:

I. Start from DGLAP for u

$$\mu^2 \frac{\partial f_u(x, \mu^2)}{\partial \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi} \int_x^1 \frac{dz}{z} \left(P_{uu}(z) f_u\left(\frac{x}{z}, \mu^2\right) + P_{ug}(z) f_g\left(\frac{x}{z}, \mu^2\right) \right)$$

Sum rules in pQCD

Beyond the naive parton model the probabilistic picture does not hold anymore. *What about basic conservation principles (e.g. sum rules)?*

Exercise: show that e.g.

$$\int_0^1 dx (f_u(x, \mu^2) - f_{\bar{u}}(x, \mu^2)) = \text{constant} \quad \text{if and only if} \quad \int_0^1 dz P_{qq}(z) = 0$$

Solution:

1. Start from DGLAP for u

$$\mu^2 \frac{\partial f_u(x, \mu^2)}{\partial \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi} \int_x^1 \frac{dz}{z} \left(P_{uu}(z) f_u\left(\frac{x}{z}, \mu^2\right) + P_{ug}(z) f_g\left(\frac{x}{z}, \mu^2\right) \right)$$

2. Subtract the same equation for \bar{u} and integrate over x

Sum rules in pQCD

2. Subtract the same equation for \bar{u} and integrate over x

$$\mu^2 \frac{\partial}{\partial \mu^2} \int_0^1 dx (f_u(x, \mu^2) - f_{\bar{u}}(x, \mu^2)) = \frac{\alpha_s(\mu^2)}{2\pi} \int_0^1 dx \int_x^1 \frac{dz}{z} P_{qq}(z) \left(f_u\left(\frac{x}{z}, \mu^2\right) - f_{\bar{u}}\left(\frac{x}{z}, \mu^2\right) \right)$$

Sum rules in pQCD

2. Subtract the same equation for \bar{u} and integrate over x

$$\mu^2 \frac{\partial}{\partial \mu^2} \int_0^1 dx (f_u(x, \mu^2) - f_{\bar{u}}(x, \mu^2)) = \frac{\alpha_s(\mu^2)}{2\pi} \int_0^1 dx \int_x^1 \frac{dz}{z} P_{qq}(z) \left(f_u\left(\frac{x}{z}, \mu^2\right) - f_{\bar{u}}\left(\frac{x}{z}, \mu^2\right) \right)$$

3. Swap x and z integration, replace x with $y = x/z$

$$\mu^2 \frac{\partial}{\partial \mu^2} \int_0^1 dx (f_u(x, \mu^2) - f_{\bar{u}}(x, \mu^2)) = \frac{\alpha_s(\mu^2)}{2\pi} \int_0^1 dz P_{qq}(z) \int_0^1 dy (f_u(y, \mu^2) - f_{\bar{u}}(y, \mu^2))$$

Sum rules in pQCD

2. Subtract the same equation for \bar{u} and integrate over x

$$\mu^2 \frac{\partial}{\partial \mu^2} \int_0^1 dx (f_u(x, \mu^2) - f_{\bar{u}}(x, \mu^2)) = \frac{\alpha_s(\mu^2)}{2\pi} \int_0^1 dx \int_x^1 \frac{dz}{z} P_{qq}(z) \left(f_u\left(\frac{x}{z}, \mu^2\right) - f_{\bar{u}}\left(\frac{x}{z}, \mu^2\right) \right)$$

3. Swap x and z integration, replace x with $y = x/z$

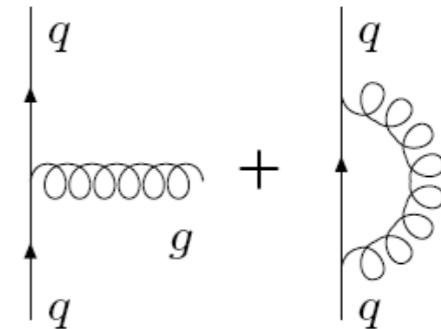
$$\mu^2 \frac{\partial}{\partial \mu^2} \int_0^1 dx (f_u(x, \mu^2) - f_{\bar{u}}(x, \mu^2)) = \frac{\alpha_s(\mu^2)}{2\pi} \int_0^1 dz P_{qq}(z) \int_0^1 dy (f_u(y, \mu^2) - f_{\bar{u}}(y, \mu^2))$$

Conclusion: the integral $\int_0^1 dx (f_u(x, \mu^2) - f_{\bar{u}}(x, \mu^2))$

does not depend on the scale if, and only if $\int_0^1 dz P_{qq}(z) = 0$

Properties of splitting functions

$$P_{qq}^{(0)} = P_{\bar{q}\bar{q}}^{(0)} = C_F \left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2} \delta(1-z) \right]$$



👉 the delta-term is the virtual correction (present only when the flavour does not change)

We have just seen that in order to conserve quark (baryon) number, the integral of the quark distribution can not vary with Q^2 , hence, the splitting functions must integrate to zero

Exercise: use this fact to compute the coefficients of the pure delta terms in P_{qq} and P_{gg} without performing the loop integral!

History of splitting functions

 $P_{ab}^{(0)}$: Altarelli, Parisi; Gribov-Lipatov; Dokshitzer (1977)

 $P_{ab}^{(1)}$: Curci, Furmanski, Petronzio (1980)

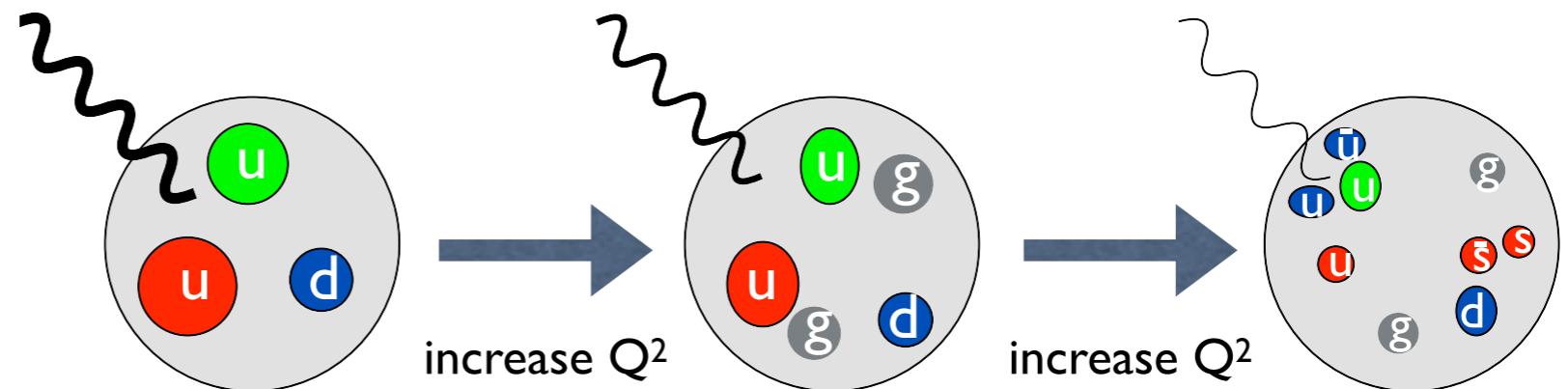
 $P_{ab}^{(2)}$: Moch, Vermaseren, Vogt (2004)

 Essential input for NNLO pdfs determination (state of the art today)

Evolution

So, in perturbative QCD we can not predict values for

- the coupling
- the masses
- the parton densities
- ...



What we can predict is the evolution with the Q^2 of those quantities. These quantities must be extracted at some scale from data.

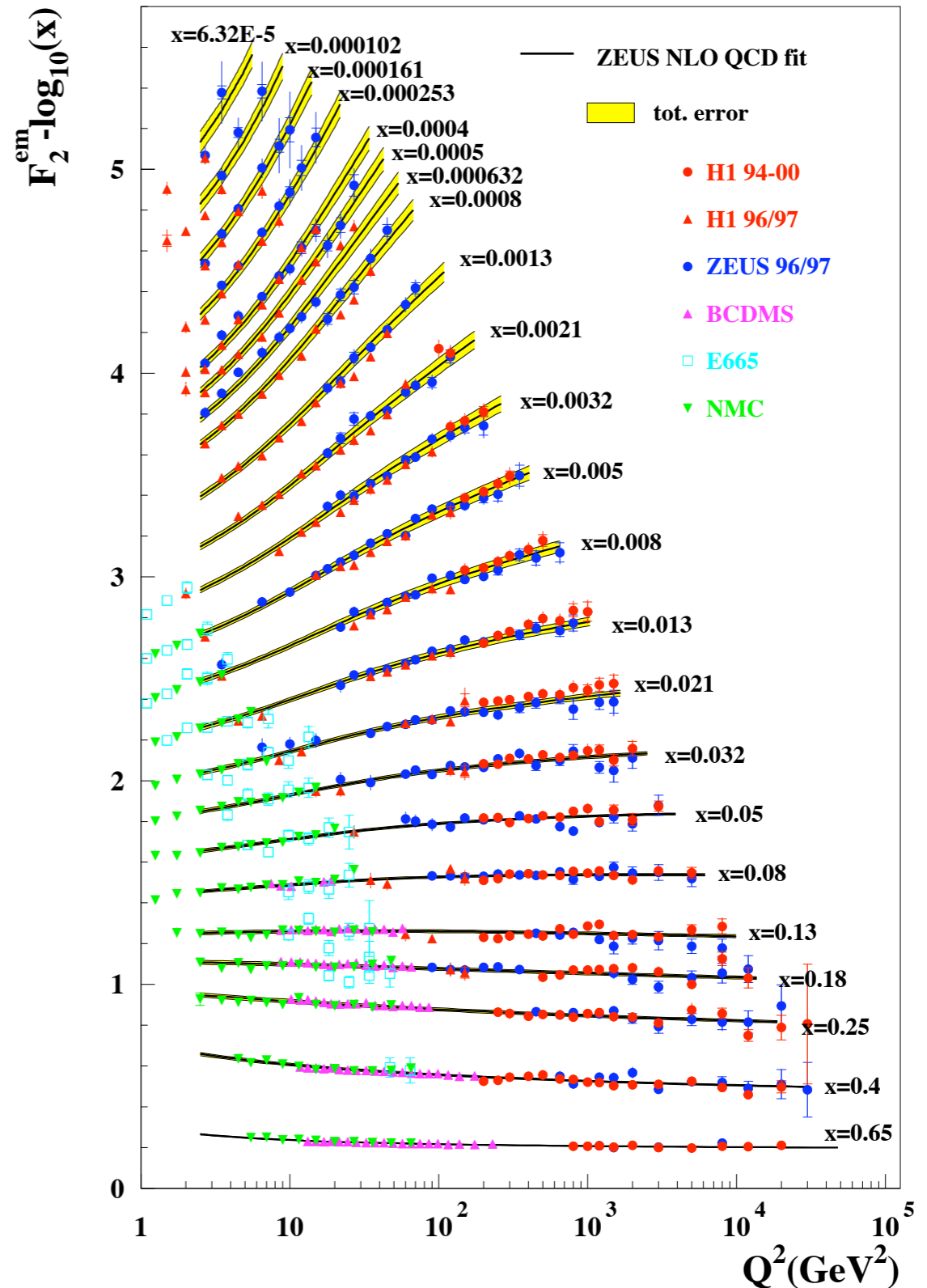
- not only is the coupling scale-dependent, but partons have a scale dependent sub-structure
- we started with the question of how one can access the gluon pdf: Because of the DGLAP evolution, we can access the gluon pdf indirectly, through the way it changes the evolution of quark pdfs. Today also direct measurements using Tevatron jet data and LHC tt and jet data

Recap.

- 📌 **Parton model**: incoherent sum of all partonic cross-sections
- 📌 **Sum rules** (momentum, charge, flavor conservation)
- 📌 Determination of **parton densities** (electron & neutrino scattering)
- 📌 Radiative corrections: **failure of parton model**
- 📌 **Factorization** of initial state divergences into scale dependent parton densities
- 📌 **DGLAP** evolution of parton densities \Rightarrow measure gluon PDF
- 📌 While PDFs lose the naive probabilistic interpretation **basic conservation principle still hold** (momentum sum rules, energy, flavour conservation)

Data: F2

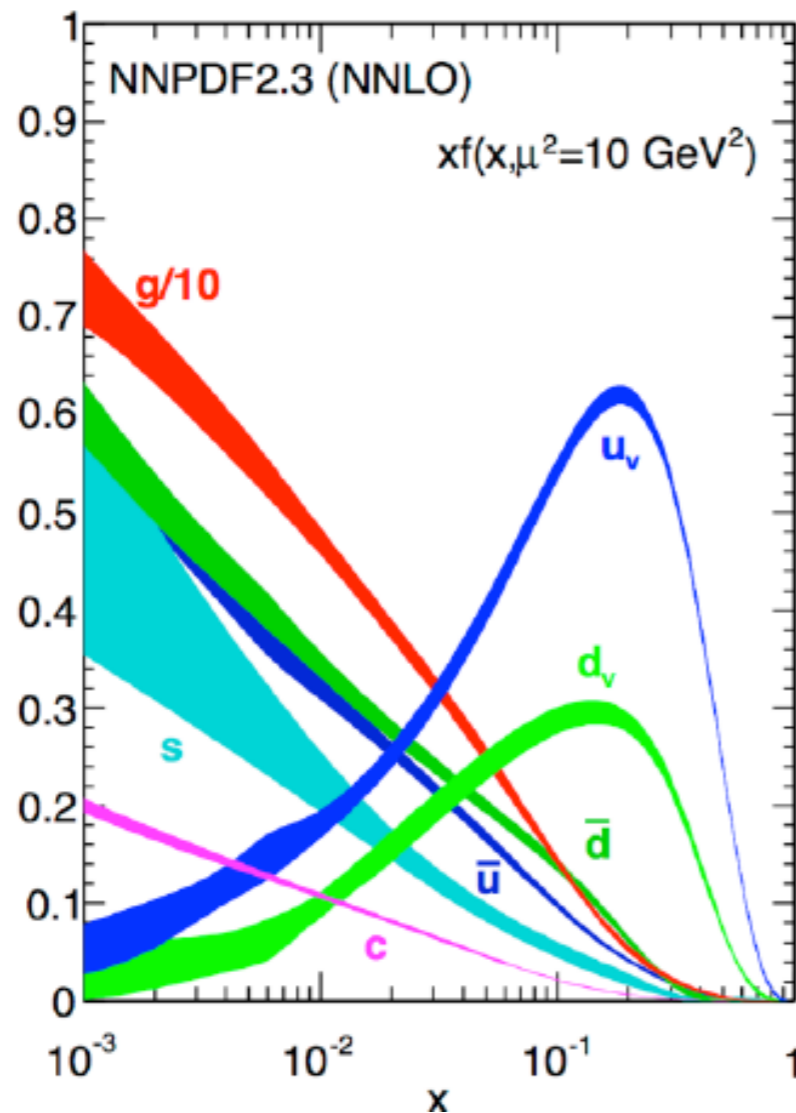
- DGLAP evolution equations allow to predict the Q^2 dependence of DIS data
- gluons crucial in driving the evolution



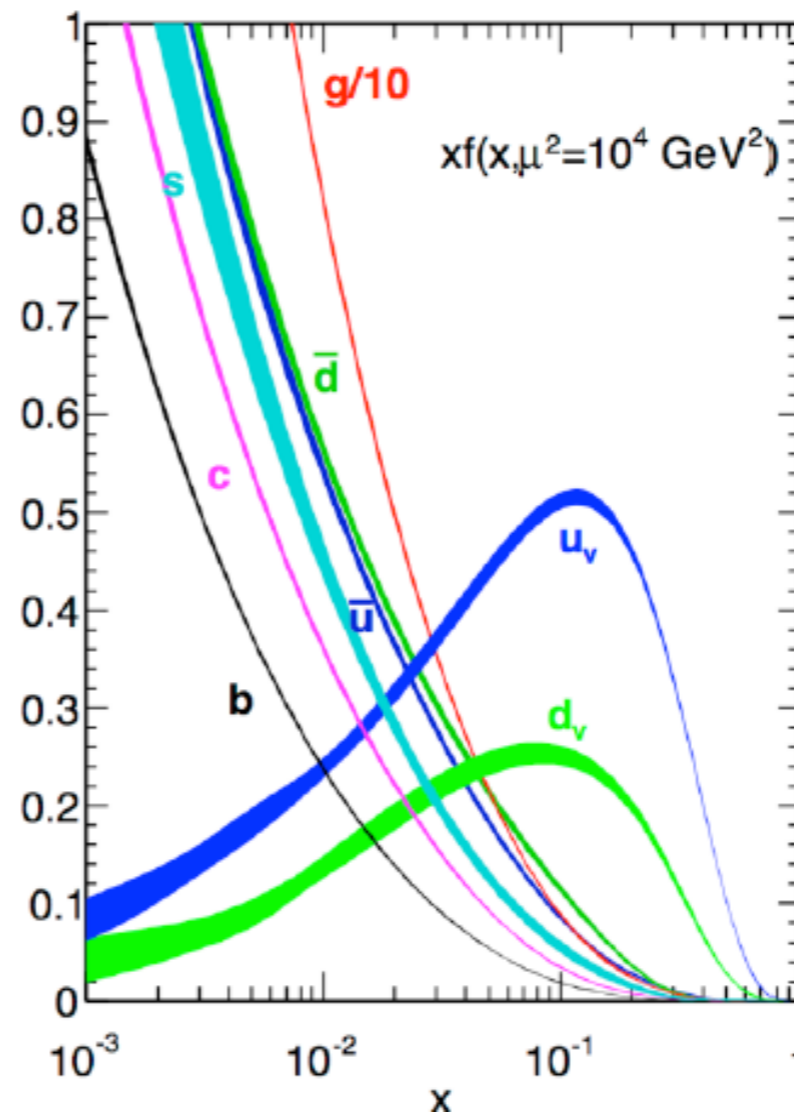
DGLAP Evolution

The DGLAP evolution is a key to precision LHC phenomenology: it allows to measure PDFs at some scale (say in DIS) and evolve upwards to make LHC (7, 8, 13, 14, 33, 100....TeV) predictions

Measure PDFs at 10 GeV



Evolve in Q^2 and make LHC predictions

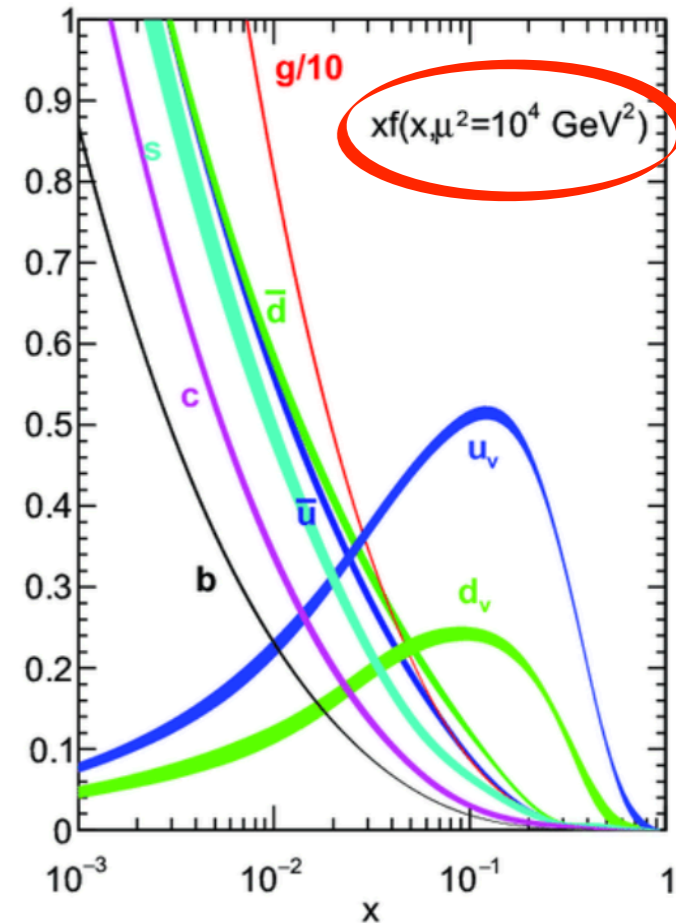
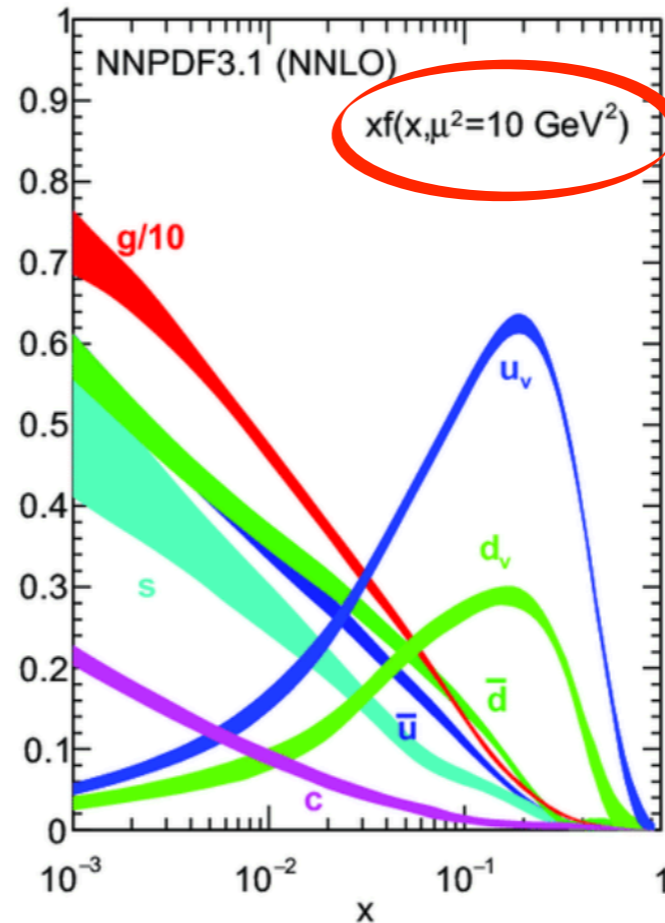


Different PDFs evolve in different ways (different equations + unitarity constraint)

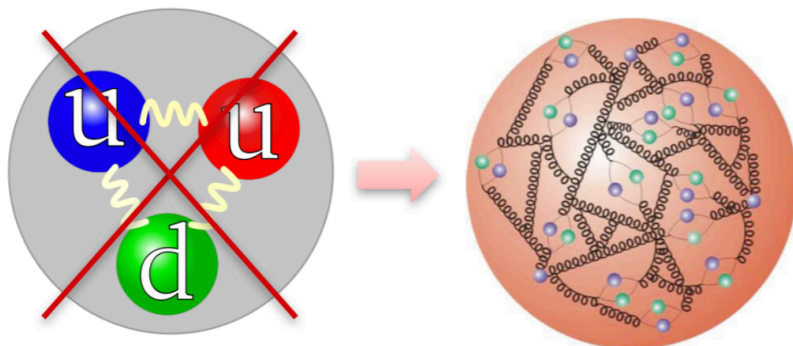
Typical features of PDFs

Features:

- gluon and sea distributions grow at small x
- gluon dominates at small x
- valence distributions peak at $x \sim 0.2$
- largest uncertainties at very small or very large x



Evolution from low to high scale

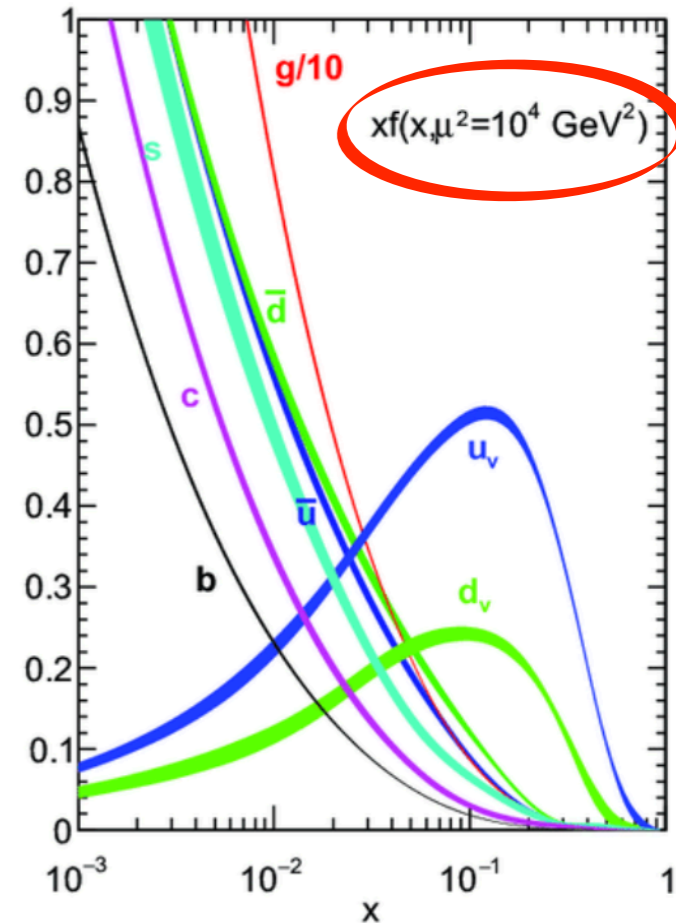
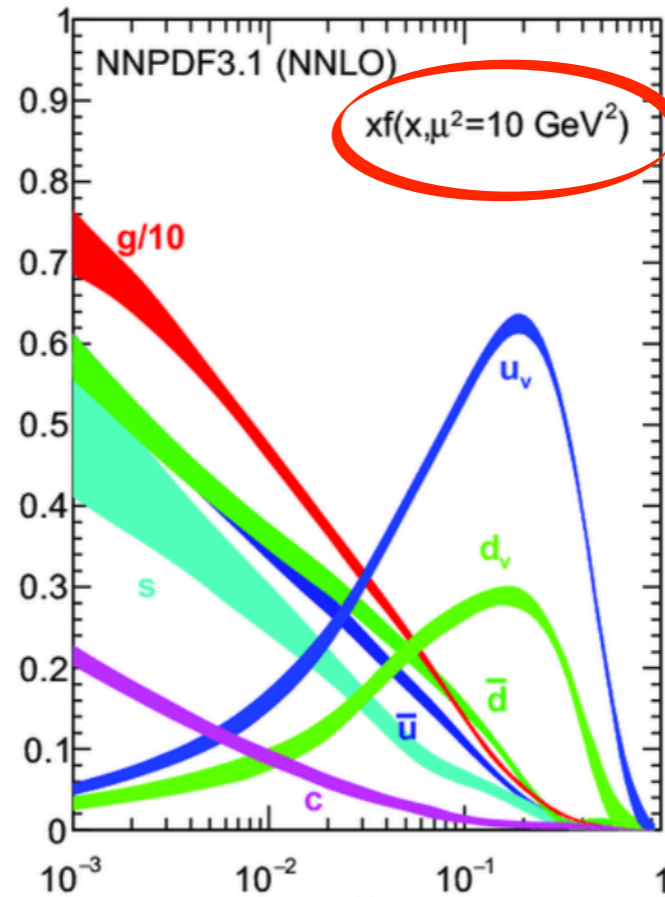


Typical features of PDFs

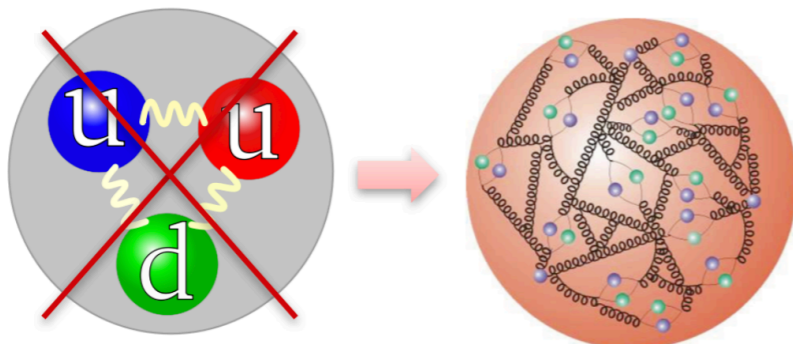
The fact that PDFs grow at small x has important consequences

- Cross sections increase with increasing collider energy s (as opposed to $\sigma \propto 1/s$)
- Higher luminosity also means higher effective energy
- Low- x regime dominated by gluons (e.g. Higgs production)

[e.g. for $m_H=125$ GeV at 13 TeV, $x \sim 0.01$]

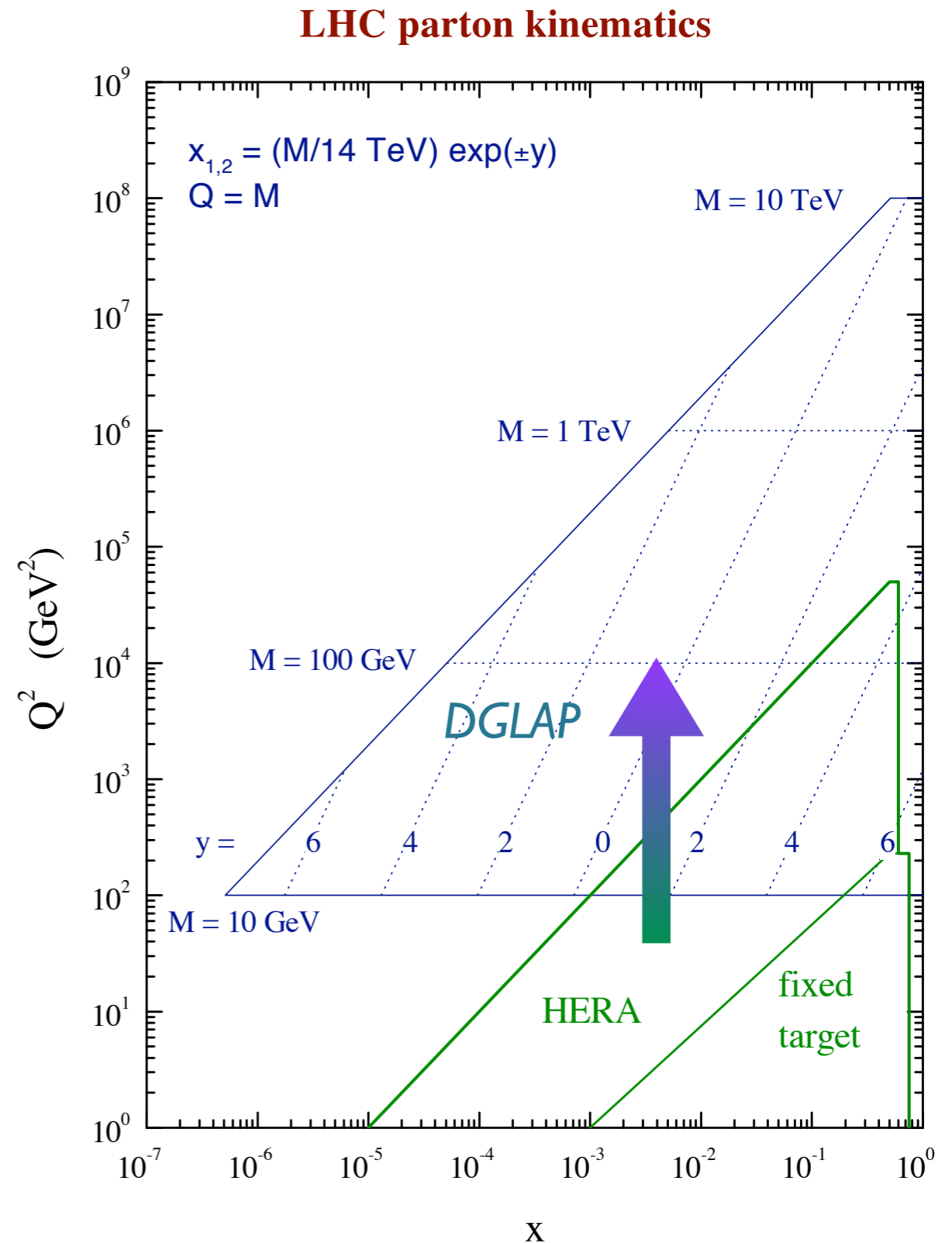


Evolution from low to high scale



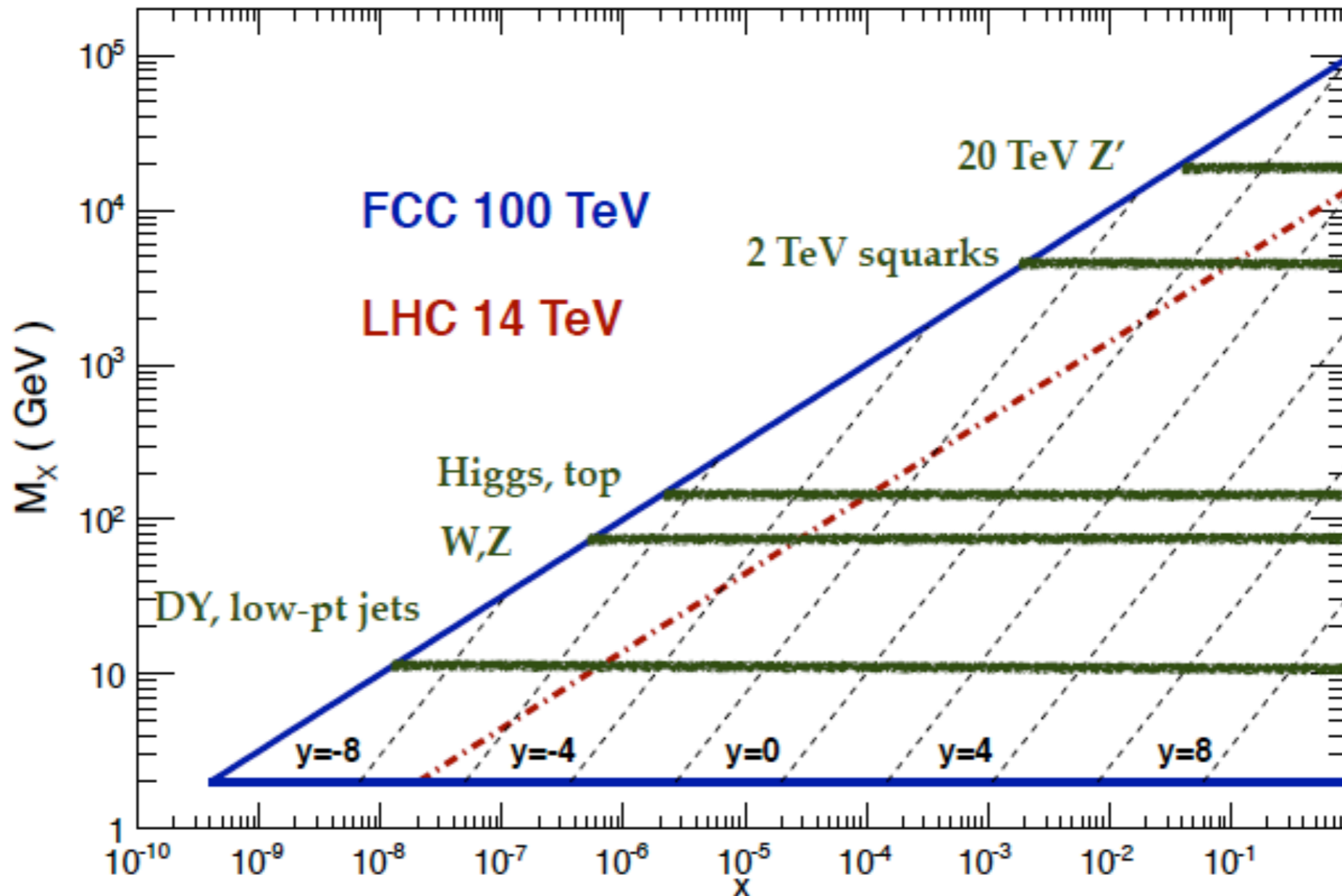
Parton density coverage

- most of the LHC x-range covered by Hera
- need 2-3 orders of magnitude Q^2 -evolution
- rapidity distributions probe extreme x-values
- 100 GeV physics at LHC: small-x, sea partons
- TeV physics: large x



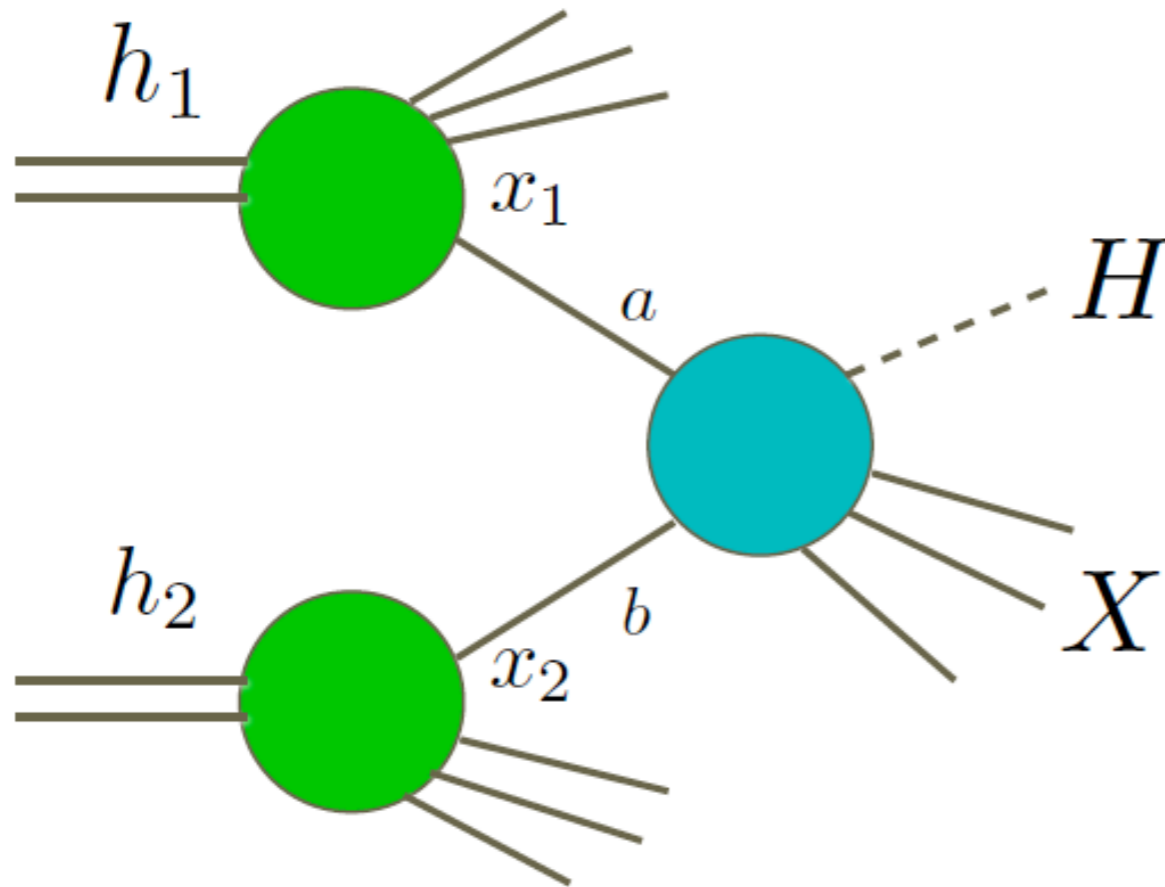
Parton density coverage

Coverage of 14 TeV LHC with respect to 100 TeV FCC



Parton luminosities

Even more interesting that PDFs are parton luminosities for each production channel

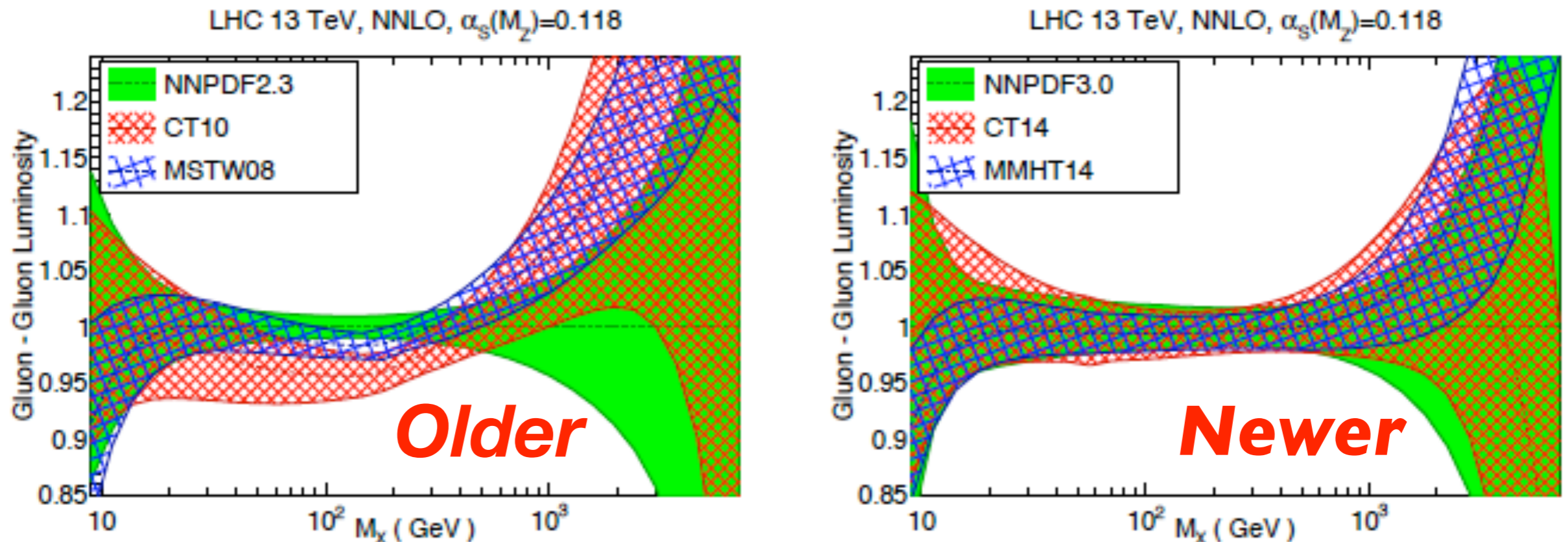


$$\sigma(S) = \sum_{i,j} \int d\tau \left[\frac{1}{S} \frac{dL_{ij}}{d\tau} \right] [\hat{s}\sigma_{ij}]$$

$$\tau \frac{dL_{ij}}{d\tau} = \int_0^1 dx_1 dx_2 x_1 f_i(x_1, \mu_F^2) \times x_2 f_j(x_2, \mu_F^2) \delta(\tau - x_1 x_2)$$

Progress in PDFs: gluon luminosity

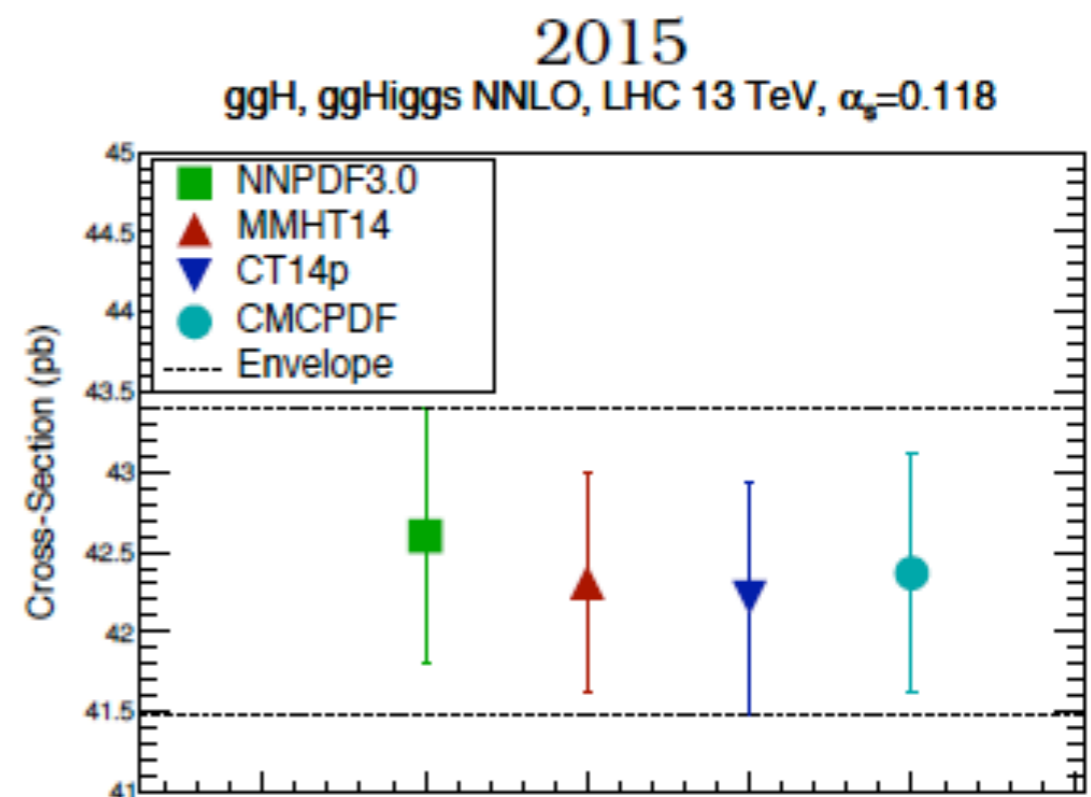
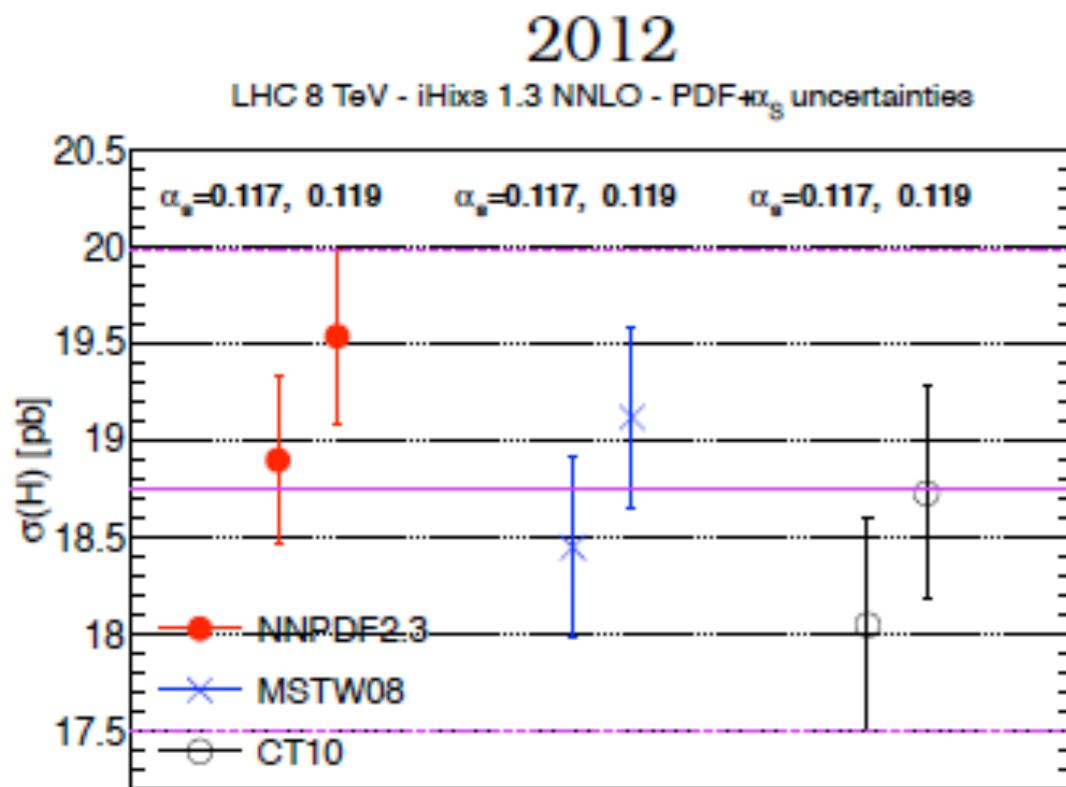
Example: gluon-gluon luminosity as needed for Higgs measurements



- obvious improvement from older sets to newer ones
- agreement at 1σ between different PDFs in the intermediate mass region relevant for Higgs studies (but larger differences at large M , key-region for New Physics searches)

Progress in PDFs: Higgs case

Improved control on gluon distributions results in more consistent Higgs production cross-sections



- PDF uncertainty in the Higgs cross-section down to about 2-3%
- envelope of 3 PDFs (previous recommendation) no longer needed

Progress in PDFs

Recent progress:

- full **NNLO evolution**
- flexible parameterisations or use of **neural network** PDFs (more recently deep learning methods)
- improved treatment of **heavy flavours** near the quark mass
- systematic use of **uncertainties/correlations** (e.g. dynamic tolerance, combinations of PDF + α_s uncertainty)
- exploit wealth for information from **Run I and Run II**
- new PDFs (**photon, leptons, W/Z**)

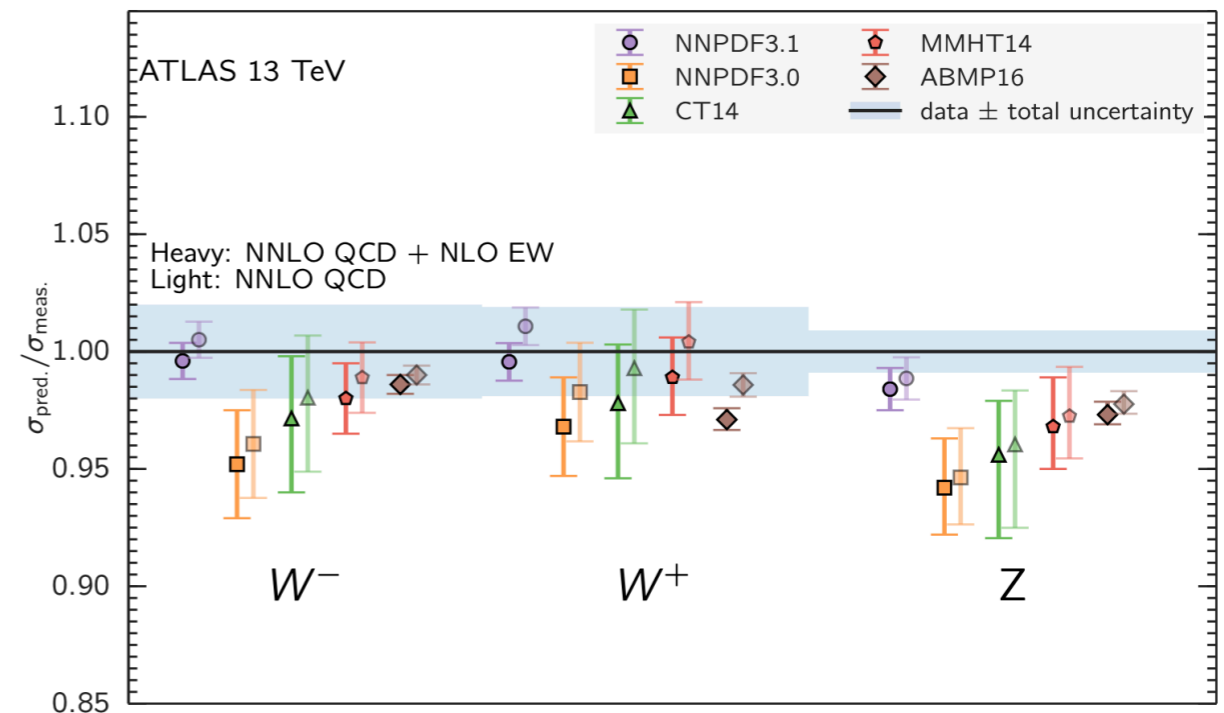
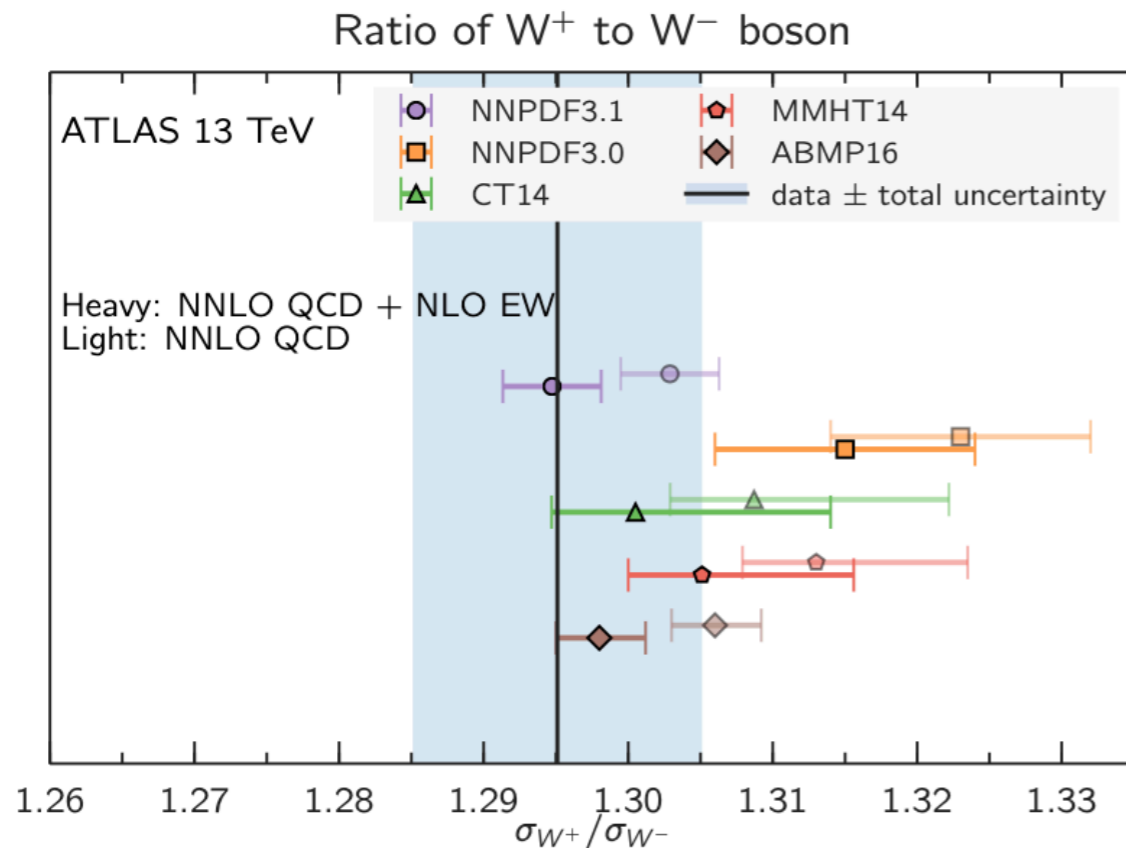
Thanks to this progress, today PDF determinations more precise and reliable (e.g. uncertainties from different groups agree in general well)

Issues under discussion

- which data to include in the fits (and how to deal with incompatible data)
- enhance relevance of some data (reduce effect of inconsistent data sets)
- heavy-quark treatment and masses
- parametrization for PDFs (theoretical bias, reduced in Neural Network PDFs)
- include theoretical improvement (e.g. resummation) for some observables
- unphysical behaviour close to $x=0$ and $x=1$
- meaning of uncertainties
- α_s as external input or fitted with PDFs
- how not to “fit away” New Physics effects in PDFs

Progress in PDFs

Despite the tremendous progress ...



So far PDFs extracted from data.

A major new challenge to compute PDFs via lattice simulations

Photon PDF of the proton

- Protons in LHC beams are fast moving charged particles
- For point-like charged particles the electromagnetic field (the distribution of photons) was computed by Fermi, Weizsäcker and Williams in the 1920-1930s
- But protons are not elementary and made up of quarks/gluons

Ausstrahlung bei Stößen sehr schneller Elektronen.

Von C. F. v. Weizsäcker, zur Zeit in Kopenhagen.

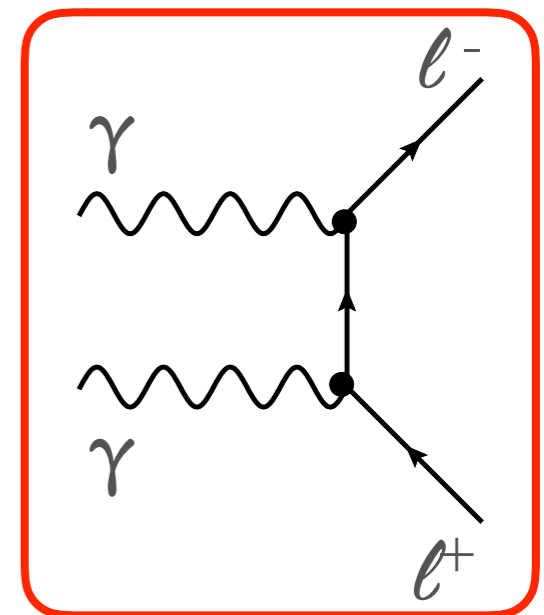
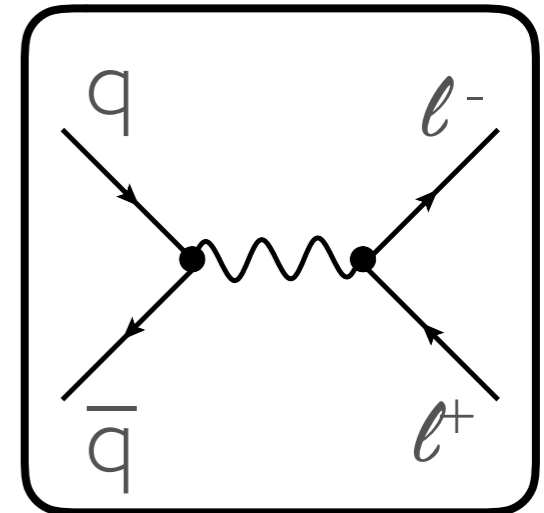
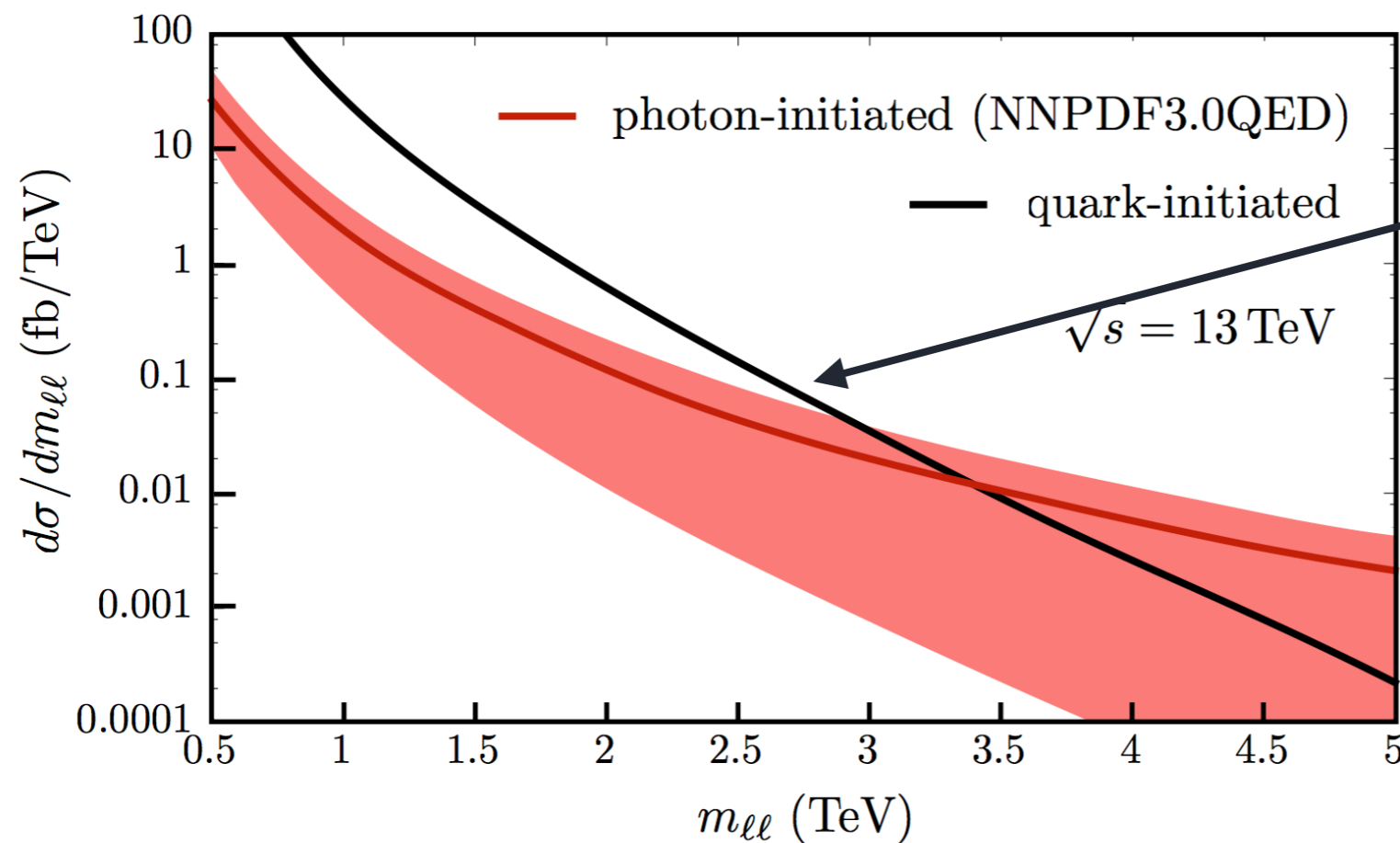
(Eingegangen am 28. Februar 1934.)

Die vorliegende Arbeit ist die Ausarbeitung der Resultate einiger Diskussionen, die vom September 1933 an im Kopenhagener Institut unter Leitung von Herrn Prof. N. Bohr stattfanden, und zu denen vor allem die Herren G. Beck, W. Heisenberg, L. Landau, E. Teller und E. J. Williams wesentliche Beiträge lieferten. — Ich möchte diese Gelegenheit gern benutzen, um Herrn Prof. Bohr für die schöne und fruchtbare Zeit, die ich in seinem Institut zubringen durfte, meinen herzlichsten Dank auszudrücken.

A fundamental question is what is the electromagnetic field associated to fast moving protons. This is the photon parton distribution function (PDF) of the proton

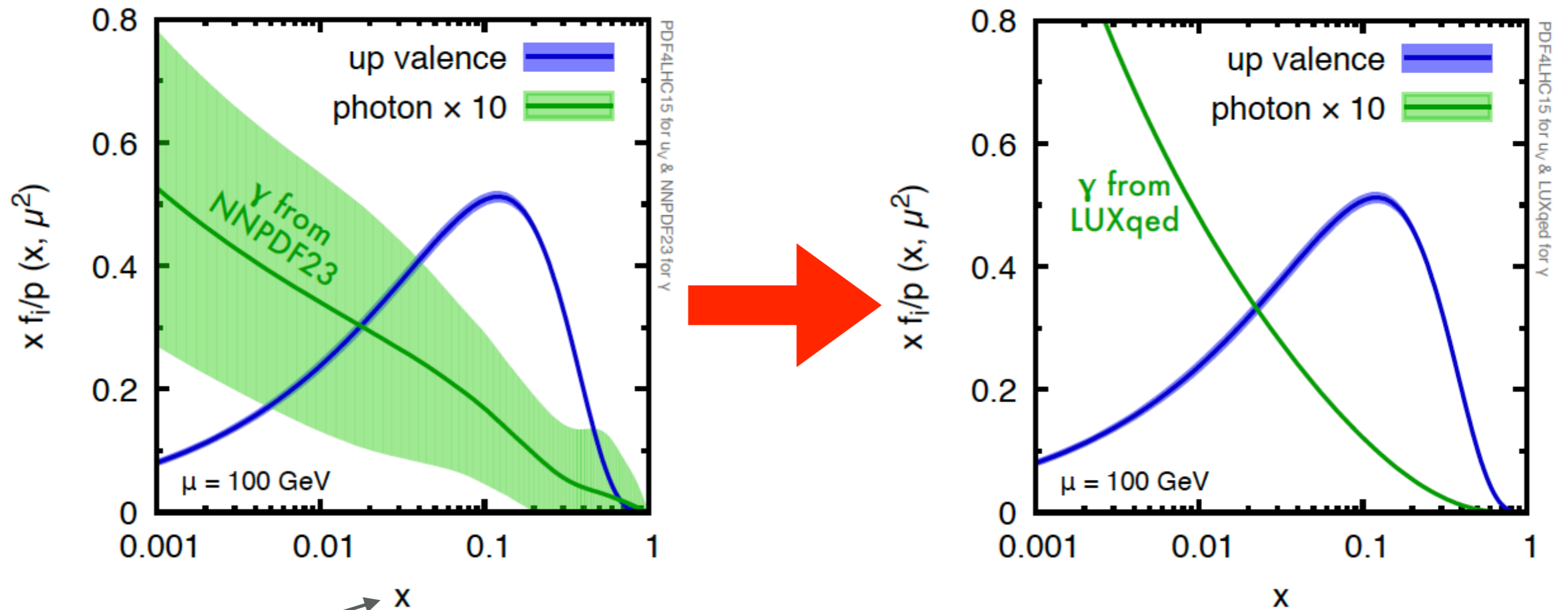
Does the photon PDF matter?

Example: Drell-Yan production



Poor knowledge of photon PDF impacts both New Physics searches and precision physics

The LUX photon PDF



Momentum fraction x carried by photon

A. Manohar, P. Nason, G. Salam, GZ PRL 117 (2016) 242002

By looking at the problem in a new way we reduced the uncertainty drastically (100% \rightarrow 1%)

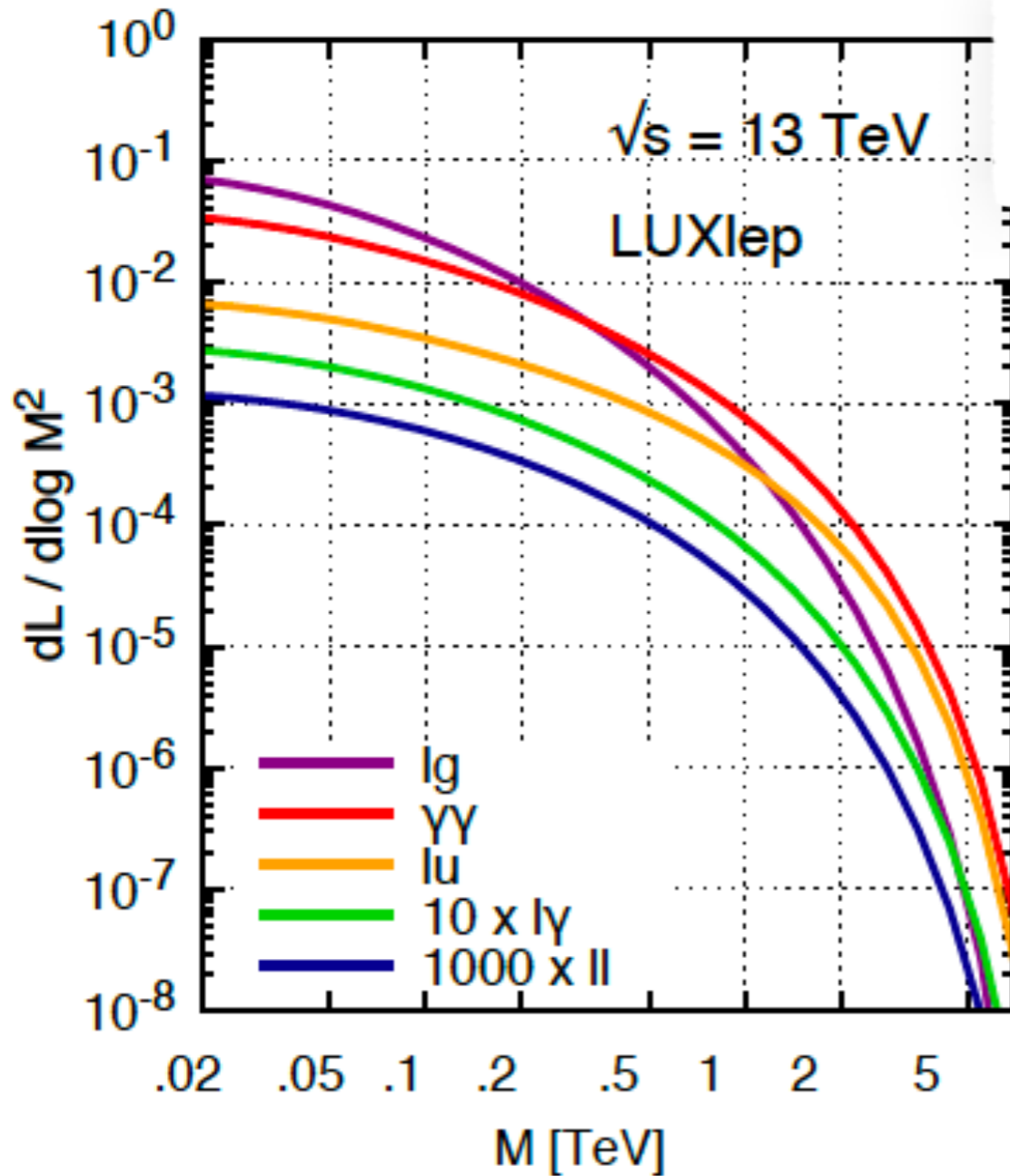
Key idea: view Deep Inelastic Scattering as a lepton probing the photon field in the proton

Leptons in the proton

- So far, processes explored at the LHC are mainly induced by incoming quarks, gluons or photons in the protons
- Studies of processes with incoming leptons relegated to future colliders (CLIC, FCC-ee, CEPC, ...). Timescale 30y
- But, because of quantum fluctuations, **protons in LHC beams also contain leptons**
- Lepton luminosities are very small, but give rise to **unique signatures at the LHC**
- Possibility to study **scattering processes that are beyond the capability of future colliders** (e.g. μ - τ scattering)

Leptons in the proton

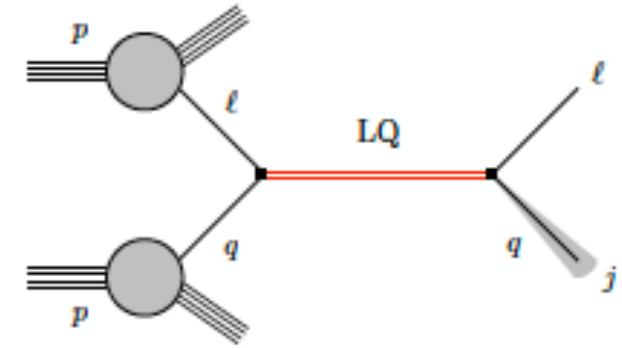
Lepton luminosities



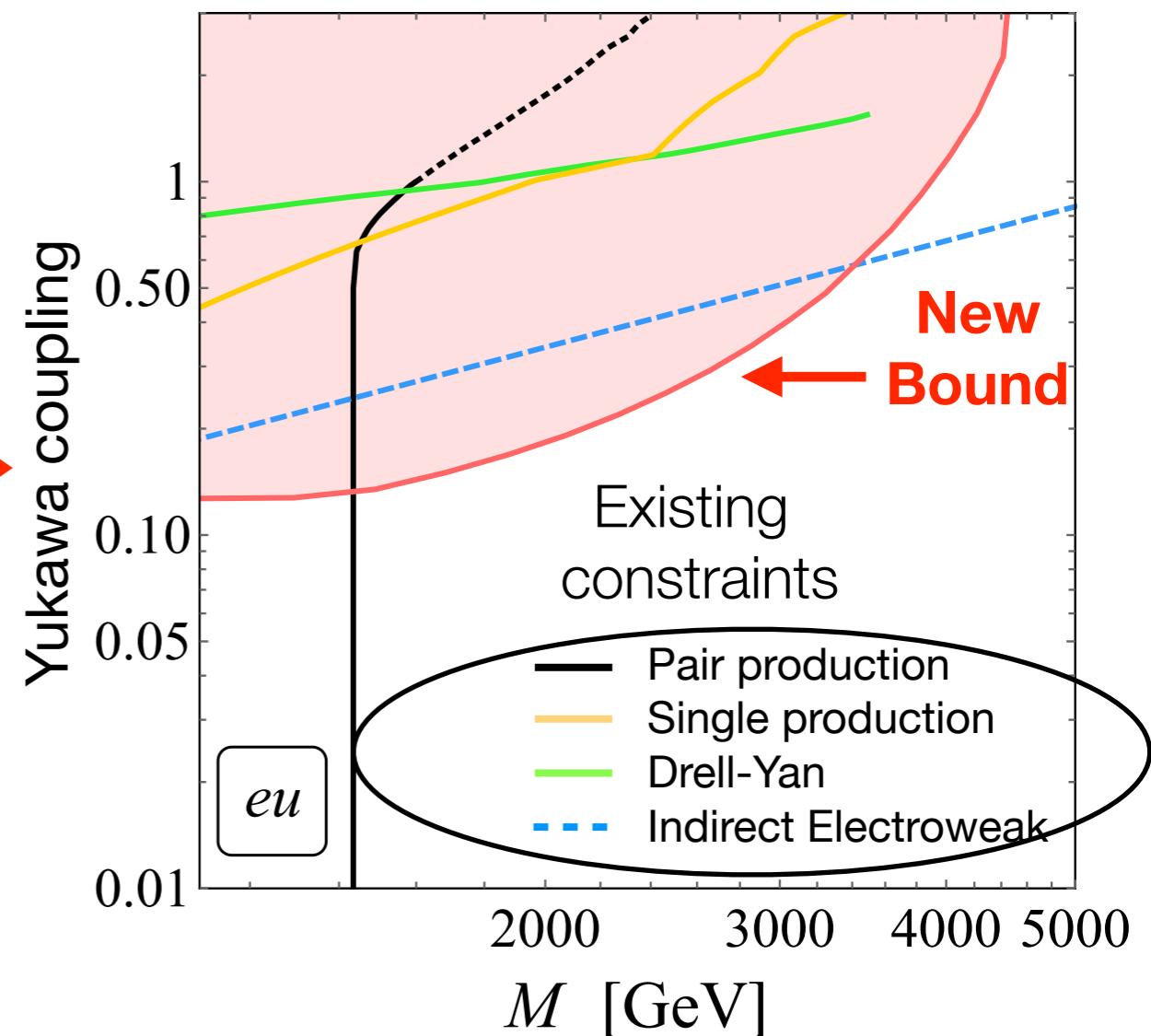
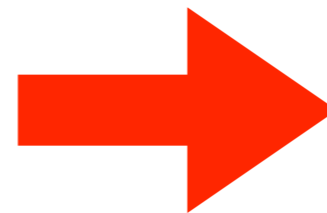
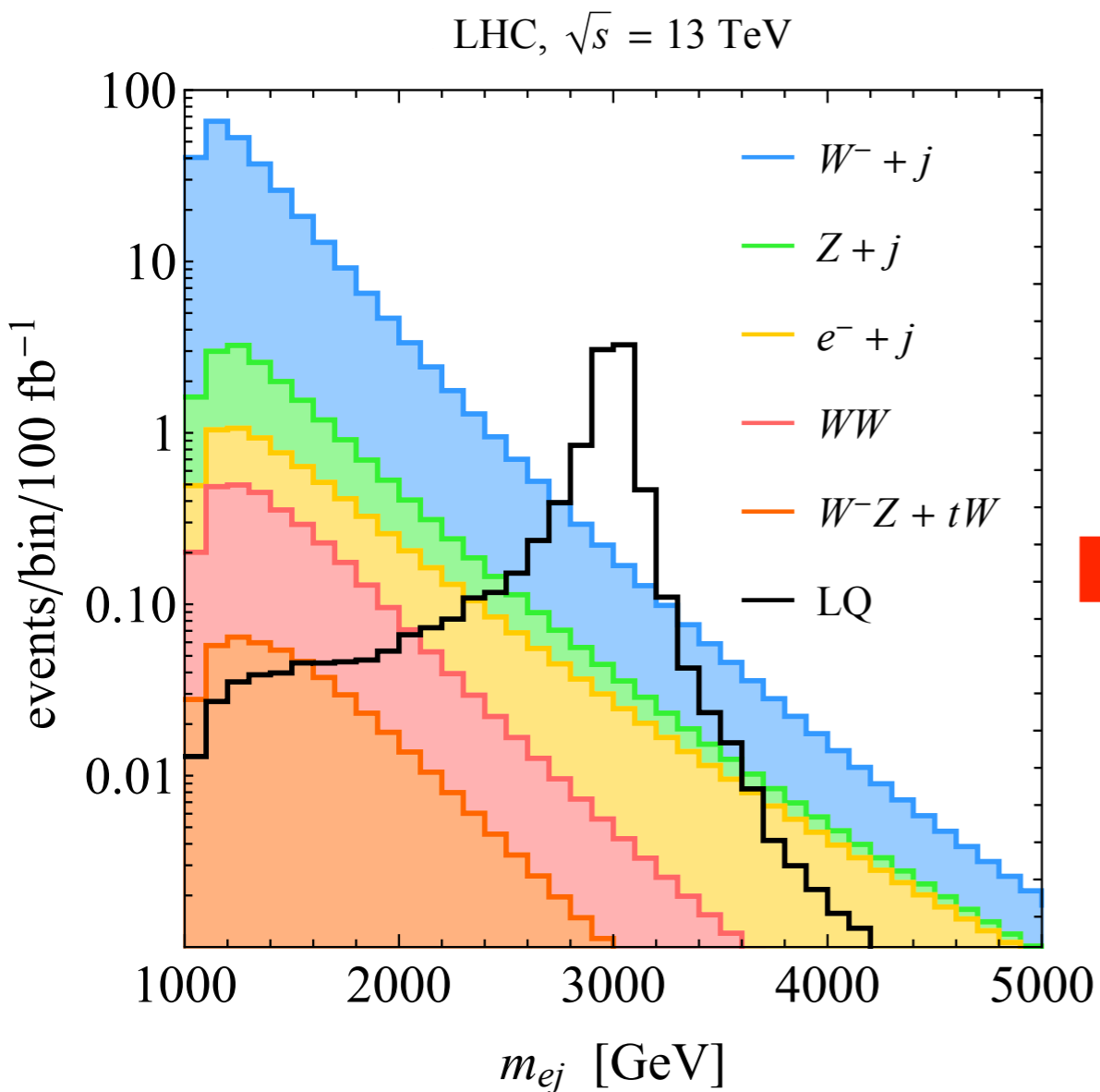
$$\mathcal{L}_{ij} \equiv M^2 \int_0^1 dz dy f_i(z, M^2) f_j(y, M^2) \delta(M^2 - szy)$$

Buonocore, Nason, Tramontano, GZ 2005.06477

Leptoquarks



New constraints on Leptoquarks



Buonocore, Haisch, Nason, Tramontano, GZ 2005.06475