Essential renormalization group

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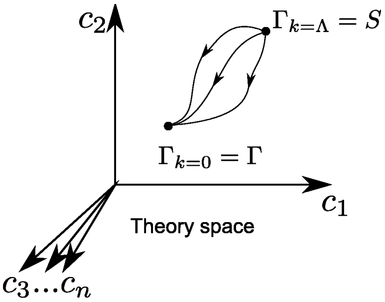
Non-perturbative functional renormalisation group

In Wilson's renormalisation group (RG) the key idea consists on the fact that physical systems can have effective descriptions at different scales k.



The modern formulation of non-perturbative functional renormalisation group is based on an exact flow equation for the Effective Average Action (EAA) Γ_k, which is a one parameter family of functionals interpolating between bare action S and effective action Γ .

IR cutoff and flow equation for EAA

- The EAA obtains a dependence on the RG scale k from the presence of a momentum dependent IR cutoff $\mathcal{R}_k$, which implements the coarse-graining procedure cutting off low momentum modes in the functional integral.
- The EAA is given by the following integro-differential equation

$$\mathrm{e}^{-\Gamma_k[\chi]} = \int (\mathrm{d}\hat{\chi})\; \mathrm{e}^{-S[\hat{\chi}]+(\hat{\chi}-\chi)\cdot\frac{\delta}{\delta\chi}\Gamma_k[\chi]-\frac{1}{2}(\hat{\chi}-\chi)\cdot\mathcal{R}_k\cdot(\hat{\chi}-\chi)} \,,$$

and it satisfies the following exact differential equation

$$k\partial_k\Gamma_k = \frac{1}{2}\mathrm{Tr}\left[\left(\frac{\delta^2\Gamma_k}{\delta\chi\delta\chi}+\mathcal{R}_k\right)^{-1}\cdot k\partial_k\mathcal{R}_k\right] .$$

How to solve the flow equation

- Practically, the EAA is treated as an EFT.
- Depending on the symmetry of our theory we can choose a generic basis of operator $\{\mathcal{O}_i\}_i$ such that

$$\Gamma_k[\chi] = \sum_i g_i(k)\mathcal{O}_i[\chi]\,,$$

  where we truncate the series in order to perform practical calculations.
- The set $\{g_i\}_i$ span the theory space and can be classified into essential couplings and inessential ones depending on the subspace of the theory space.

Examples of expansions

- $\mathbb{Z}_2$-symmetric scalar theory at $O\left(\partial^4\right)$ in derivative expansion

$$\Gamma_k = \int \mathrm{d}^d x \left\{ V_k(\chi^2) + \frac{1}{2} z_k(\chi^2) \left(\partial_\mu \chi \, \partial_\mu \chi\right) + W_k^a(\chi^2) \left(\Delta\chi\right)^2 \right.$$
$$\left. + W_k^b(\chi^2) \chi \Delta \chi \left(\partial_\mu \chi \, \partial_\mu \chi\right) + W_k^c(\chi^2) \left(\partial_\mu \chi \, \partial_\mu \chi\right)^2 + O\left(\partial^6\right) \right\} .$$

  At $O\left(\partial^6\right)$ in derivative expansion there are 8 more terms.
- Pure gravity at second order in curvature expansion

$$\Gamma_k = \int \mathrm{d}^d x \sqrt{g} \left\{ \frac{1}{16\pi G_k} \left(2\Lambda_k - R\right) + a_k R^2 + b_k R_{\mu\nu} R^{\mu\nu} - c_k E \right\} ,$$

  where $\bar{E} := \bar{R}^2_{\mu\nu\alpha\beta} - 4 \bar{R}_{\mu\nu} \bar{R}^{\mu\nu} + \bar{R}^2$. In $d = 4$ and at third order in curvature expansion there are 9 more terms.

Issues with the propagator

Increasing the order of the truncation the complexity of the flow is not the only problem, in fact the propagator $\mathcal{G}_k$ evaluated on shell has the following form

$$\mathcal{G}_k^{-1} \equiv \frac{\delta^2 \Gamma_k}{\delta\chi\delta\chi} = c_0 + c_1\Delta + c_2\Delta^2 + c_3\Delta^3 + \dots .$$

Calculating the quantum contributions, new poles appears inside the propagator and this implies more complex calculations and possible issues with unitarity.

Generalising to frame covariant flow for EAA

Let's introduce a second source of k dependence, that takes into account the liberty to perform field reparameterisation along the flow parameterised by a k diffeomorphism $\hat{\phi}_k[\hat{\chi}]$ of configuration space which we integrate over.
Therefore, at each RG step we perform simultaneously

- integration of UV modes,
- reparametrization of the fields,

and in particular, we can choose $\hat{\phi}_k[\hat{\chi}]$ to impose renormalization conditions on inessential couplings fixing their running to a specific value.

Generalising to frame covariant flow for EAA

The exact RG flow equation obeyed by the frame covariant EAA is given by the following integro-differential equation

$$\mathrm{e}^{-\Gamma_k[\phi]} = \int (\mathrm{d}\hat{\chi})\ \mathrm{e}^{-S[\hat{\chi}]+(\hat{\phi}_k-\phi)\cdot\frac{\delta}{\delta\phi}\Gamma_k[\phi]-\frac{1}{2}(\hat{\phi}_k-\phi)\cdot\mathcal{R}_k\cdot(\hat{\phi}_k-\phi)}\,,$$

and it satisfies the following exact differential equation

$$\left(k\partial_k+\Psi_k[\phi]\cdot\frac{\delta}{\delta\phi}\right)\Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\left(\frac{\delta^2\Gamma_k}{\delta\phi\delta\phi}+\mathcal{R}_k\right)^{-1}\left(k\partial_k+2\cdot\frac{\delta}{\delta\phi}\Psi_k[\phi]\right)\cdot\mathcal{R}_k\,,$$

where

$$\Psi_k[\phi] := \mathrm{e}^{\Gamma_k[\phi]}\int(\mathrm{d}\hat{\chi})\ k\partial_k\hat{\phi}_k[\hat{\chi}]\ \mathrm{e}^{-S[\hat{\chi}]+(\hat{\phi}_k-\phi)\cdot\frac{\delta}{\delta\phi}\Gamma_k[\phi]-\frac{1}{2}(\hat{\phi}_k-\phi)\cdot\mathcal{R}_k\cdot(\hat{\phi}_k-\phi)}.$$

Examples in minimal essential scheme

- $\mathbb{Z}_2$-symmetric scalar theory at $O\left(\partial^4\right)$ in derivative expansion

$$\Gamma_k = \int \mathrm{d}^d x \left\{ V_k(\phi^2) + \frac{1}{2}\cancelto{1}{z_k(\phi^2)} (\partial_\mu \phi\, \partial_\mu \phi) + \cancelto{0}{W_k^a(\phi^2)} (\Delta\phi)^2 \right.$$
$$\left. + \cancelto{0}{W_k^b(\phi^2)} \phi\Delta\phi\, (\partial_\mu \phi\, \partial_\mu \phi) + W_k^c(\phi^2)\, (\partial_\mu \phi\, \partial_\mu \phi)^2 + O\left(\partial^6\right) \right\}.$$

- Pure gravity at second order in curvature expansion

$$\Gamma_k = \int \mathrm{d}^d x \sqrt{g} \left\{ \frac{1}{16\pi G_k} (2\Lambda_k - R) + \cancelto{0}{a_k} R^2 + \cancelto{0}{b_k} R_{\mu\nu} R^{\mu\nu} - c_k E \right\}.$$

Propagator in minimal essential scheme

Increasing the order of the truncation, the complexity of the flow increases slowly with respect to the standard case.
Moreover, the propagator evaluated on shell has the following form

$$\mathcal{G}_k^{-1} \equiv \frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi} = c_0 + c_1 \Delta \, .$$

This form has no issue with unitarity and in principle can give the possibility to Wick rotate.

Thanks!!!

Some references

- “Essential renormalisation group,” A. Baldazzi, R. B. A. Zinati and K. Falls, [arXiv:2105.11482 [hep-th]].
- in preparation, A. Baldazzi and K. Falls