UNIVERSITÀ DEGLI STUDI DI TORINO

XI NExT PhD Workshop: Probing fundamental physics at colliders and beyond 2021

A NEW SUBTRACTION SCHEME IN PERTURBATIVE QCD

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Motivation

- Future high-precision phase of the LHC = growing experimental data accuracy
- Better control on SM expectation to the identify any deviations from it

How? → Improve theoretical prediction of the main multi-particle processes in colliders

to match

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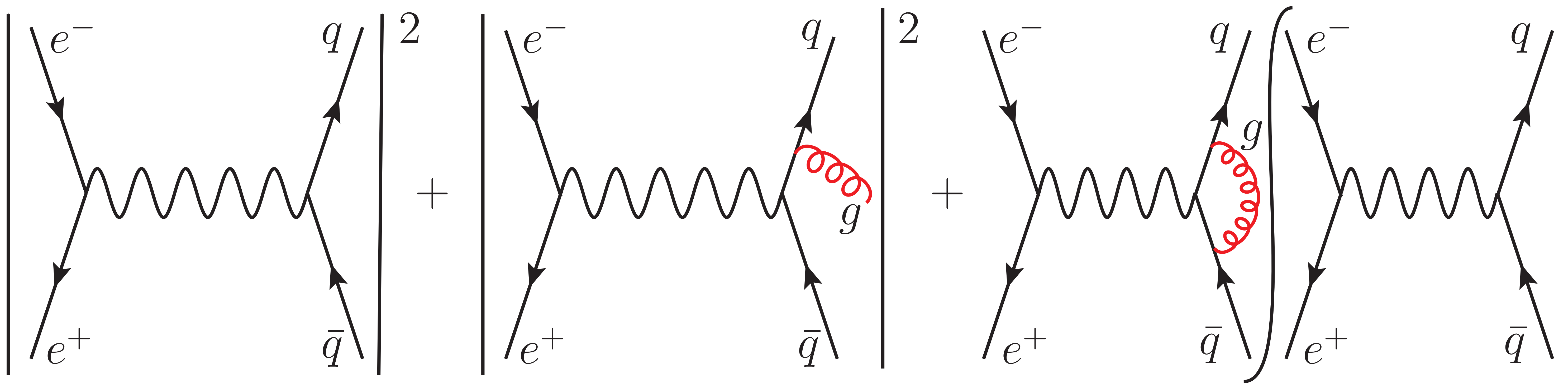
Problem Higher orders corrections in perturbative QCD manifest **IR singularities**

LO *NLO*



→ *Finite sum*

KLN Theorem

$d = 4 - 2\epsilon$

Implicit **phase space singularities**

Explicit **loop singularities**

General solution? **Subtraction procedure**

Well established schemes at NLO:

- **Catani-Seymour (CS)** *[9602277]*
- **Frixione-Kunst-Signer (FKS)** *[9512328]*
- Nagy-Soper *[0308127]*

➔ **What about NNLO?**
Many proposed schemes but a general and efficient method is still missing due to the increasing complexity of the emerging singularities

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Local Analytic Sector Subtraction

Why a new scheme?

- Generality
- Locality
- Simplicity of analytical calculation
- Structure minimality

} **Ideal features for a fully general scheme**

Local Analytic Sector Subtraction (massless partons, $d = 4 - 2\epsilon$)

- **NLO**

$$\frac{d\sigma^{\rm NLO}}{dX} = \lim_{d\to 4}\left\{ \int d\Phi_n\, V_n\, \delta_n + \int d\Phi_{n+1}\, R_{n+1}\delta_{n+1} \right\}$$

Counterterm $$\frac{d\sigma^{\rm NLO}_{ct}}{dX} = \int d\Phi_{n+1}\, \overline{K}_{n+1}$$

Integrated counterterm $$I_n = \int d\Phi_{\rm rad}\, \overline{K}_{n+1}$$

$$\frac{d\sigma^{\rm NLO}}{dX} = \int d\Phi_n \underbrace{\left(V_n + I_n \right)}_{\text{finite in } \epsilon} \delta_n + \int d\Phi_{n+1} \underbrace{\left(R_{n+1}\, \delta_{n+1} - \overline{K}_{n+1}\, \delta_n \right)}_{\text{integrable in } d=4}$$

Local Analytic Sector Subtraction (massless partons, $d = 4 - 2\epsilon$)

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Key features

- **Phase-space partition (FKS) :** identify each singular contribution exactly

Sector functions $\mathcal{W}_{ij}$:

$$R = \sum_{i,j} R\,\mathcal{W}_{ij} = R\,\mathcal{W}_{31} + R\,\mathcal{W}_{32} + \ldots$$

Sums rules

$$\sum_{i\in\mathcal{F}} \sum_{j\neq i} W_{ij} = 1$$

$$\sum_{j\neq i} S_i\, W_{ij} = 1$$

$$C_{ij}\,(W_{ij} + W_{ji}) = 1$$

$$S_i\, C_{ij}\, W_{ij} = C_{ij}\, S_i\, W_{ij} = 1$$

$\longrightarrow$

$$W_{ij} = \frac{\sigma_{ij}}{\sigma} \qquad \sigma_{ij} = \frac{1}{\epsilon_i\, \omega_{ij}} = \frac{s_{qj}}{s_{ij}}$$

$$\sigma = \sum_{i\in\mathcal{F}} \sum_{j\neq i} \sigma_{ij}$$

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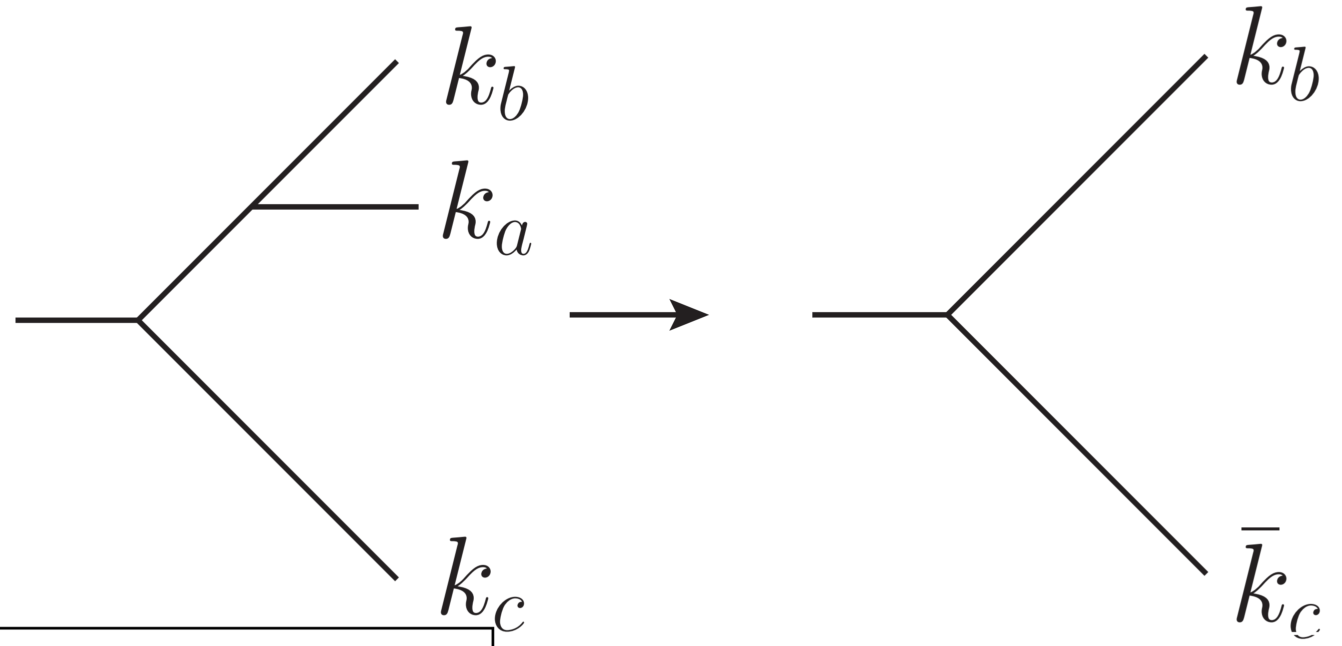
Key features

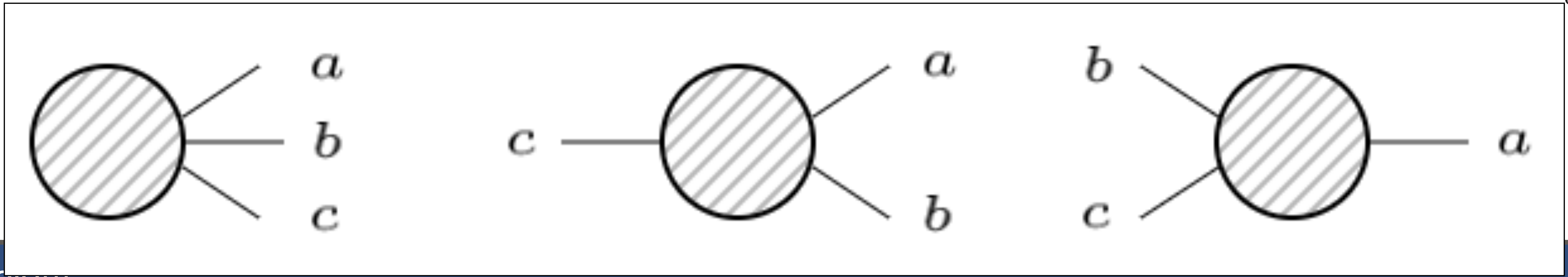
- **Phase-space partition (FKS) :** identify each singular contribution exactly
- **Momenta mapping (CS) :** phase-space factorisation and
  on-shell momentum conserving Born-level kinematics

Phase space factorisation: $d\Phi_{n+1} = d\overline{\Phi}_n\, d\overline{\Phi}_{\text{rad}}$

Mapped kinematics: $\{\bar{k}\}^{(abc)} = \left\{ \{k\}_{\not a \not b \not c}, \bar{k}_b^{(abc)}, \bar{k}_c^{(abc)} \right\}$

$$\bar{k}_b^{(abc)} + \bar{k}_c^{(abc)} = k_a + k_b + k_c$$





Local Analytic Sector Subtraction (massless partons, $d = 4 - 2\epsilon$)

$$\frac{d\sigma^{\rm NLO}}{dX} = \int d\Phi_n \left(V_n + I_n \right) \delta_n + \int d\Phi_{n+1} \left(R_{n+1}\,\delta_{n+1} - \overline{K}_{n+1}\,\delta_n \right)$$

Key features

- **Phase-space partition (FKS) :** identify each singular contribution exactly
- **Momenta mapping (CS) :** phase-space factorisation and
  on-shell momentum conserving Born-level kinematics
- **Candidate counterterm** with remapped kinematics

$$\overline{K}_{ij} = \left[\overline{S}_i + \overline{C}_{ij} - \overline{S}_i\,\overline{C}_{ij} \right] R\,W_{ij}$$

Consistency relations

$$S_i\,\overline{S}_i\,R = S_i\,R$$
$$S_i\,\overline{S}_i\,\overline{C}_{ij}\,R = S_i\,\overline{C}_{ij}\,R$$
$$C_{ij}\,\overline{C}_{ij}\,R = C_{ij}\,R$$
$$C_{ij}\,\overline{S}_i\,\overline{C}_{ij}\,R = C_{ij}\,\overline{S}_i\,R$$

ensure cancellation
in each sector

$$RW_{ij} - \overline{K}_{ij} \longrightarrow \textit{finite}$$

Local Analytic Sector Subtraction (massless partons, $d = 4 - 2\epsilon$)

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$$\overline{K}_{ij} = \left[\overline{S}_i + \overline{C}_{ij} - \overline{S}_i\, \overline{C}_{ij} \right] R\, W_{ij}$$

$$\overline{K} = \sum_{i \in \mathcal{F}} \overline{S}_i\, R + \sum_{i \in \mathcal{F}} \sum_{\substack{j \in \mathcal{F} \\ j > i}} \overline{C}_{ij} \left(1 - \overline{S}_i - \overline{S}_j \right) R + \sum_{i \in \mathcal{F}} \sum_{j \in \mathcal{I}} \overline{C}_{ij} \left(1 - \overline{S}_i \right) R$$

Soft | Hard-Collinear with final j | Hard-Collinear with initial j

Local Analytic Sector Subtraction (massless partons, $d = 4 - 2\epsilon$)

$$\frac{d\sigma^{\rm NLO}}{dX} = \int d\Phi_n \left(V_n + I_n\right)\delta_n + \int d\Phi_{n+1}\left(R_{n+1}\,\delta_{n+1} - \overline{K}_{n+1}\,\delta_n\right)$$

Key features

- **Phase-space partition (FKS) :** identify each singular contribution exactly
- **Momenta mapping (CS) :** phase-space factorisation and
  on-shell momentum conserving Born-level kinematics
- **Candidate counterterm** with remapped kinematics
- **Simple analytic calculation** of the counterterm

$$\overline{\mathbf{S}}_i\, R\big(\{k\}\big) \propto \sum_{c,d\neq i} \frac{s_{cd}}{s_{ic}\,s_{id}} B_{cd}\big(\{k\}^{(icd)}\big)$$

$$\overline{\mathbf{C}}_{ij}\, R\big(\{k\}\big) \propto \frac{1}{s_{ij}} P^{\mu\nu}_{ij} B_{\mu\nu}\big(\{k\}^{(ijr)}\big)$$

$\longrightarrow$

$$I^s \propto \sum_{c,d} \int d\Phi_{\rm rad}\left(s^{(icd)}_{cd}; y, z, \phi\right) \frac{s_{cd}}{s_{ic}\,s_{id}} B_{cd}\left(\{\bar{k}\}^{(icd)}\right)$$

$$\propto \sum_{c,d} B_{cd}\left(\{\bar{k}\}^{(icd)}\right) \frac{(4\pi)^{\epsilon-2}\,\Gamma(1-\epsilon)\,\Gamma(2-\epsilon)}{\epsilon^2\,\Gamma(2-3\epsilon)}$$

What about NNLO?

$$\frac{d\sigma^{NNLO}}{dX} = \int d\Phi_n \underbrace{\left(VV + I^{(2)} + I^{(RV)}\right)}_{\text{finite in d=4, finite in PS}} \delta_n(X)$$

$$+ \int d\Phi_{n+1} \underbrace{\left[\underbrace{\left(RV + I^{(1)}\right)}_{\text{finite in d=4, div. in PS}} \delta_{n+1}(X) - \underbrace{\left(\overline{K}^{(RV)} + I^{(12)}\right)}_{\text{finite in d=4, div. in PS}} \delta_n(X) \right]}_{\text{finite in d=4, finite in PS}}$$

$$+ \int d\Phi_{n+2} \underbrace{\left[RR\,\delta_{n+2}(X) - \overline{K}^{(1)}\,\delta_{n+1}(X) - \left(\overline{K}^{(2)} - \overline{K}^{(12)}\right) \delta_n(X) \right]}_{\text{finite in d=4, finite in PS}}$$

Results

- NLO scheme recently extended to ISR for massless QCD
  + ongoing code implementation

- NNLO scheme:
  main singular limits and analytic integrations are available

Future plans

- Generalisation of the NNLO subtraction to initial-state massless radiation

- Numerical implementation of the NNLO Local Sector Subtraction

- Extension to the treatment of the massive case

Thanks for your attention!