Jet substructure: theory

Understanding QCD dynamics through jet substructure

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LBNL

CERN,
Jets and EW Bosons WG
10/26/2020

Jet substructures and characteristic scales

Single prong observables

- Jet angularities
- Energy-energy correlations
- Jet shape

Multi-prong observables

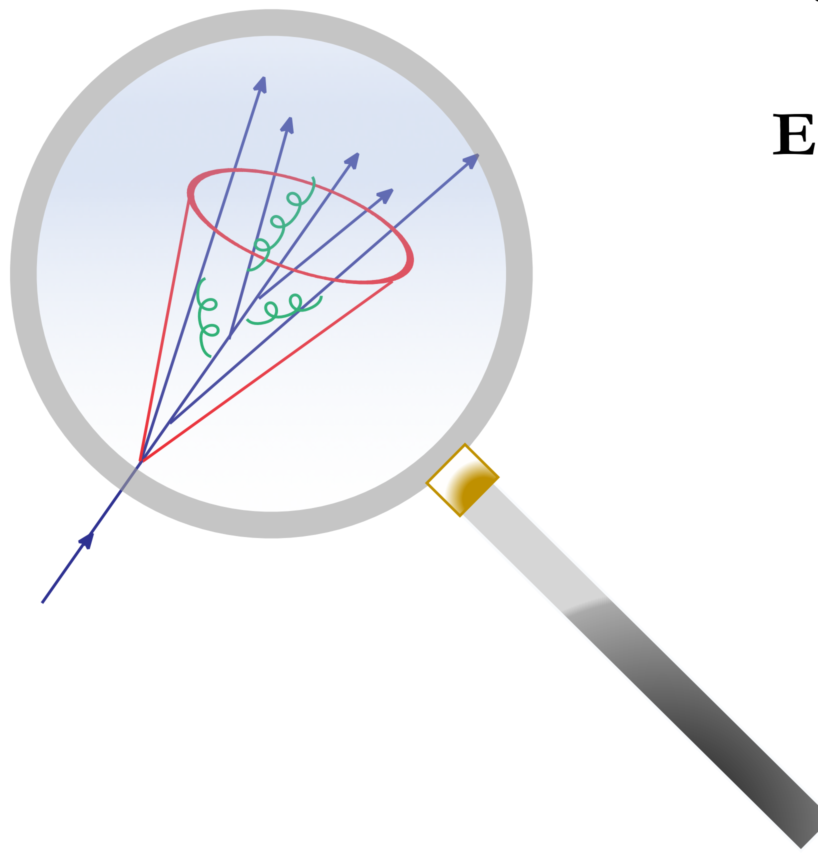
- N-subjettiness
- D_2

IRC unsafe /NP sensitive

- Hadron in jet
- Multiplicities
- Jet charge

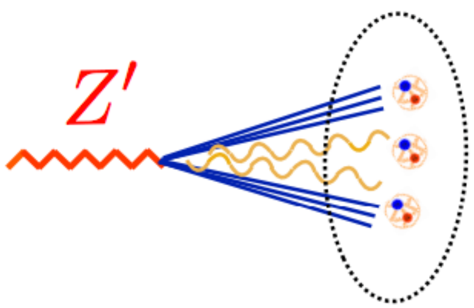
Groomed observables

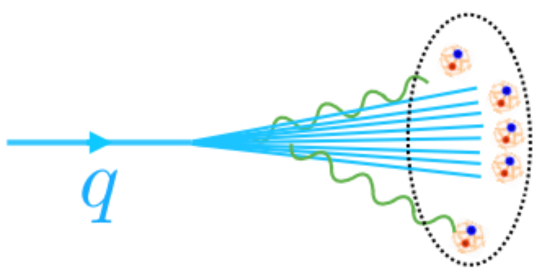
- All of the above
- Observables characterizing grooming (SD): $z_g, \quad \theta_g = R_g/R$

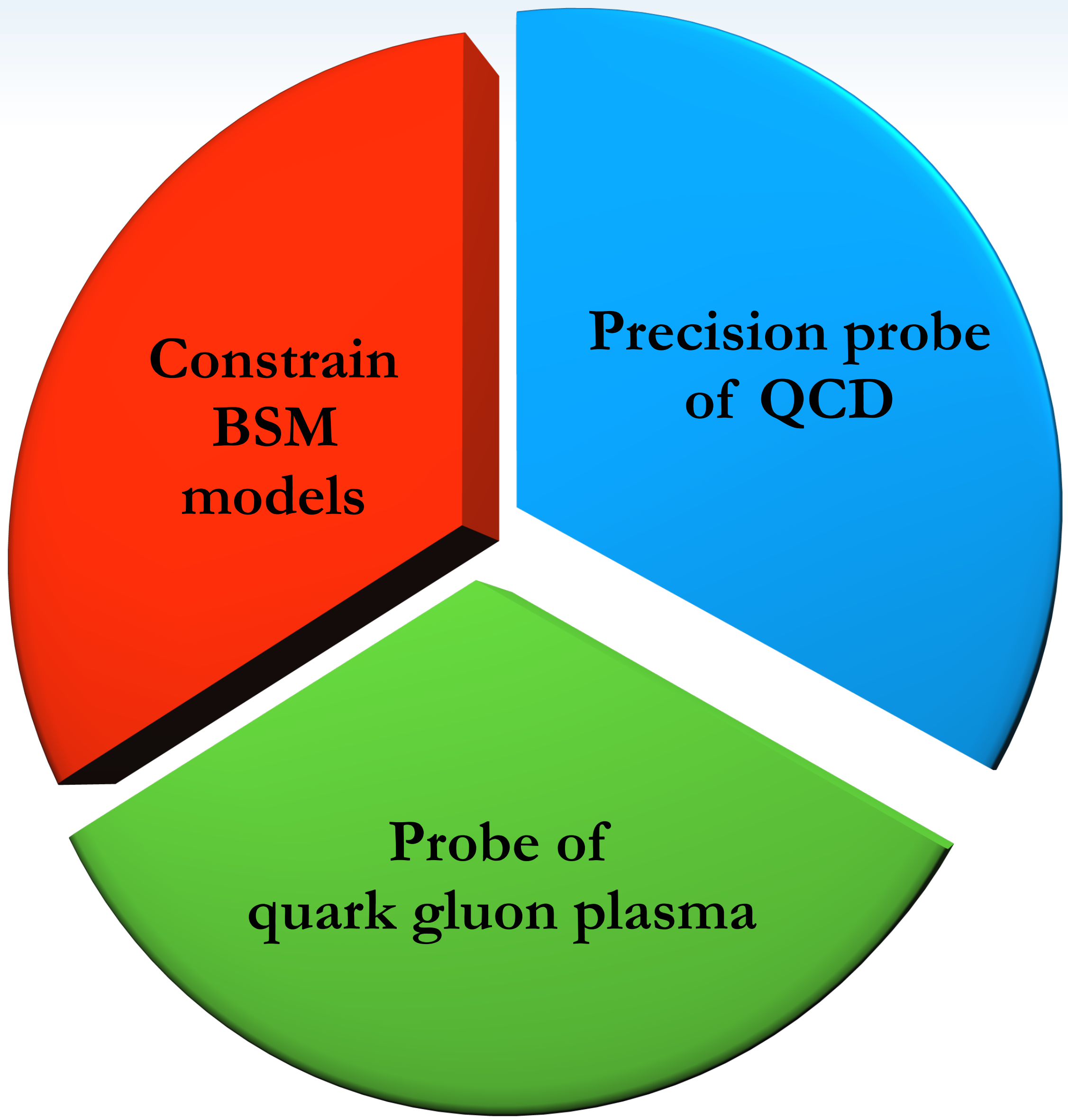


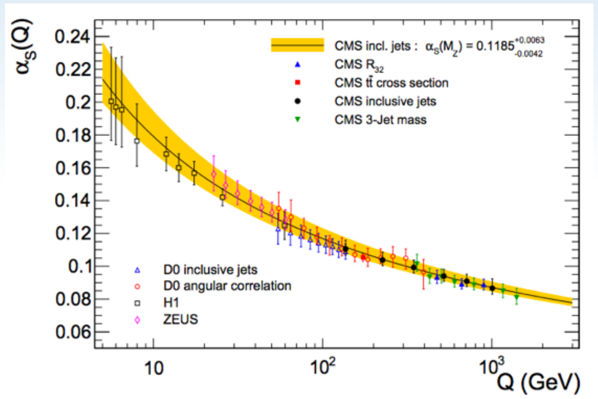
- Hard — p_T
- Jet algorithm — $p_T R$
- Energy profile — $p_T r$
- Jet mass — m_J^2/p_T
- Grooming — $z_{\rm cut} p_T R$
- $\vdots$
- Temperature — T
- Hadronization — Λ

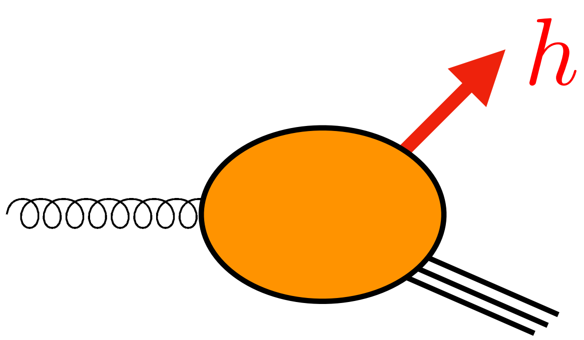
Jet substructures and characteristic scales

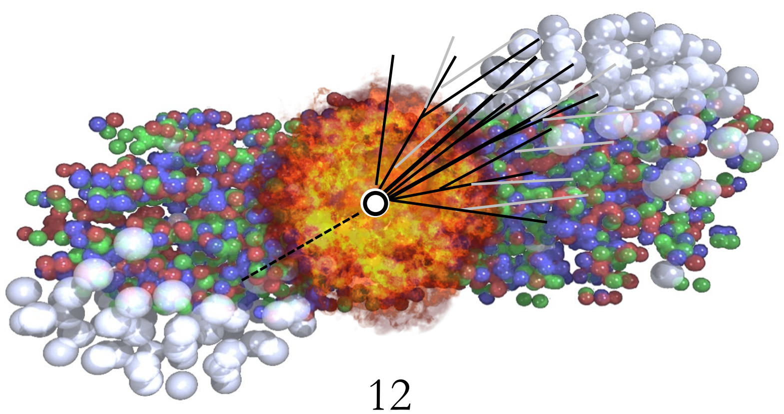












Lund diagram to map different splitting

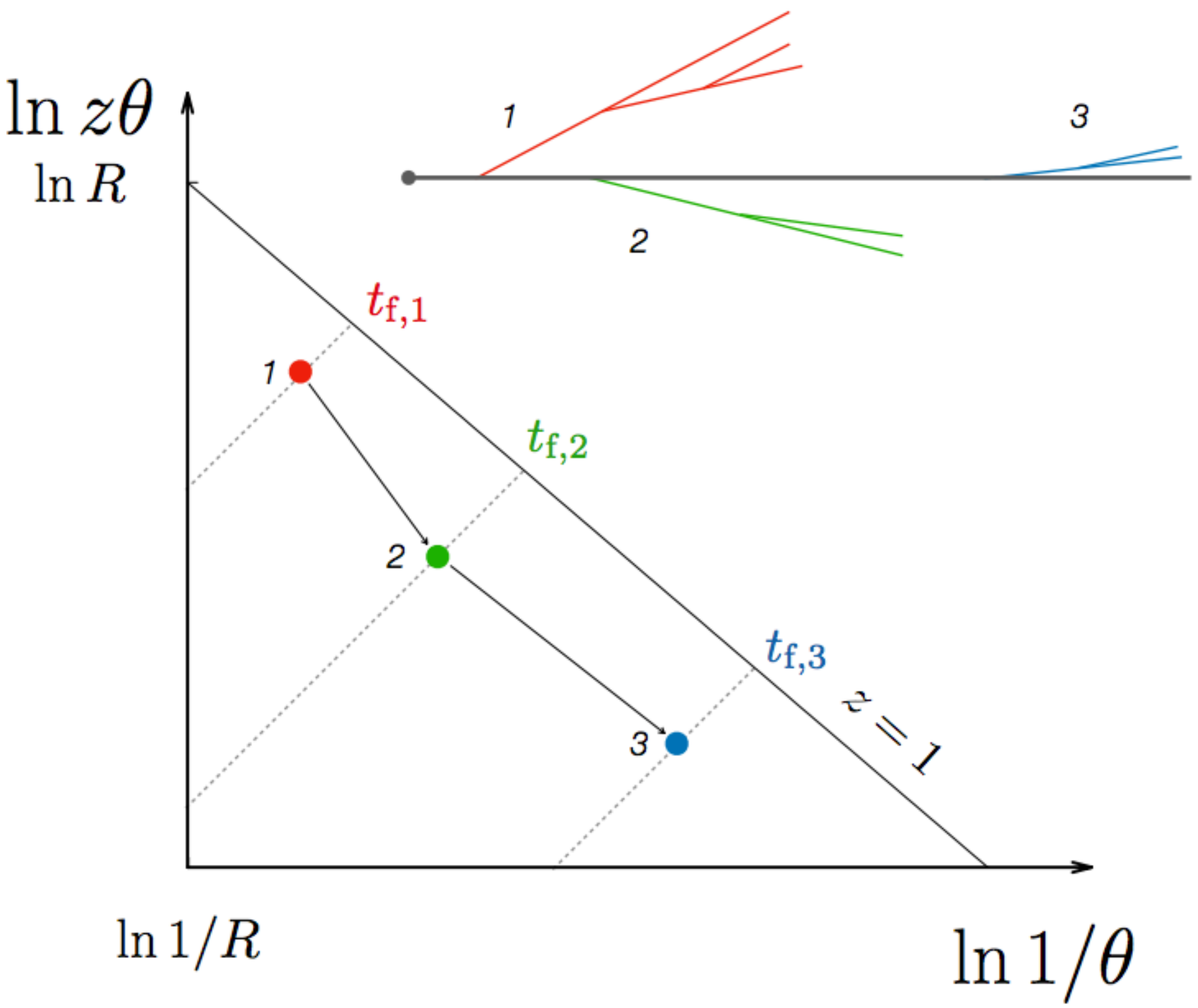
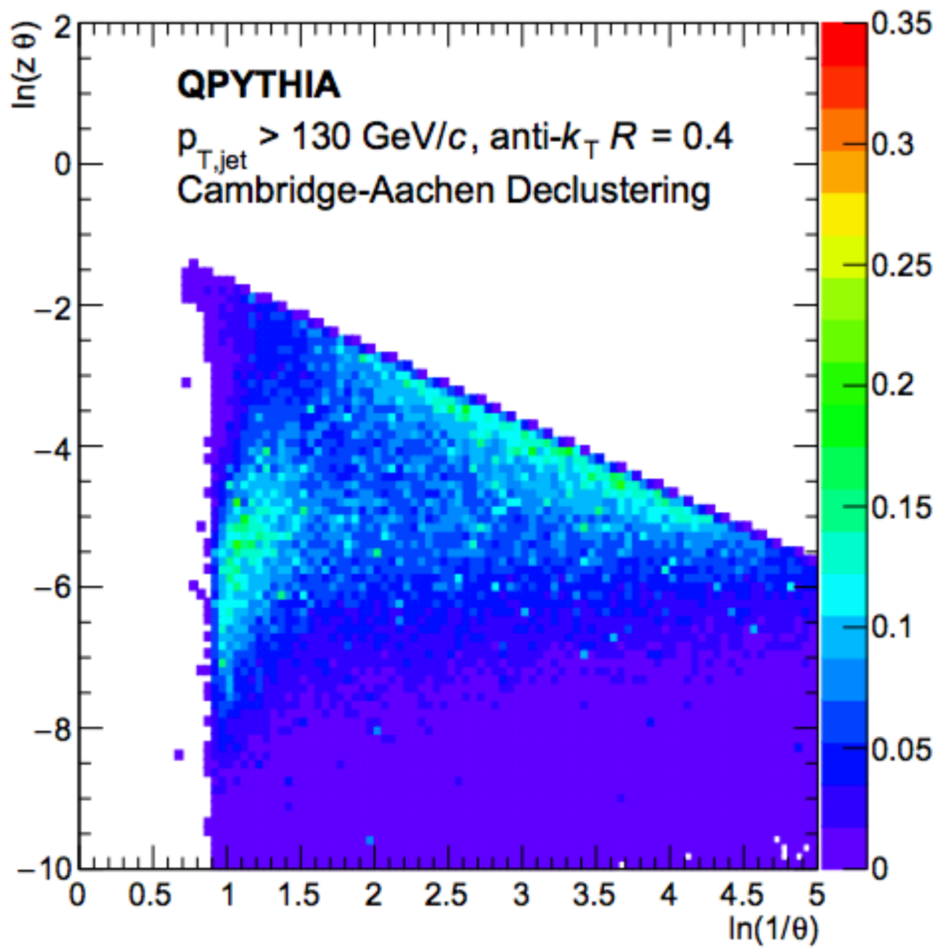


Fig. from 5th Heavy Ion Jet Workshop Report, 2018

- **Lund diagram is useful to visualize collinear and soft splittings.**
- **Map splitting history into Lund Plane.**



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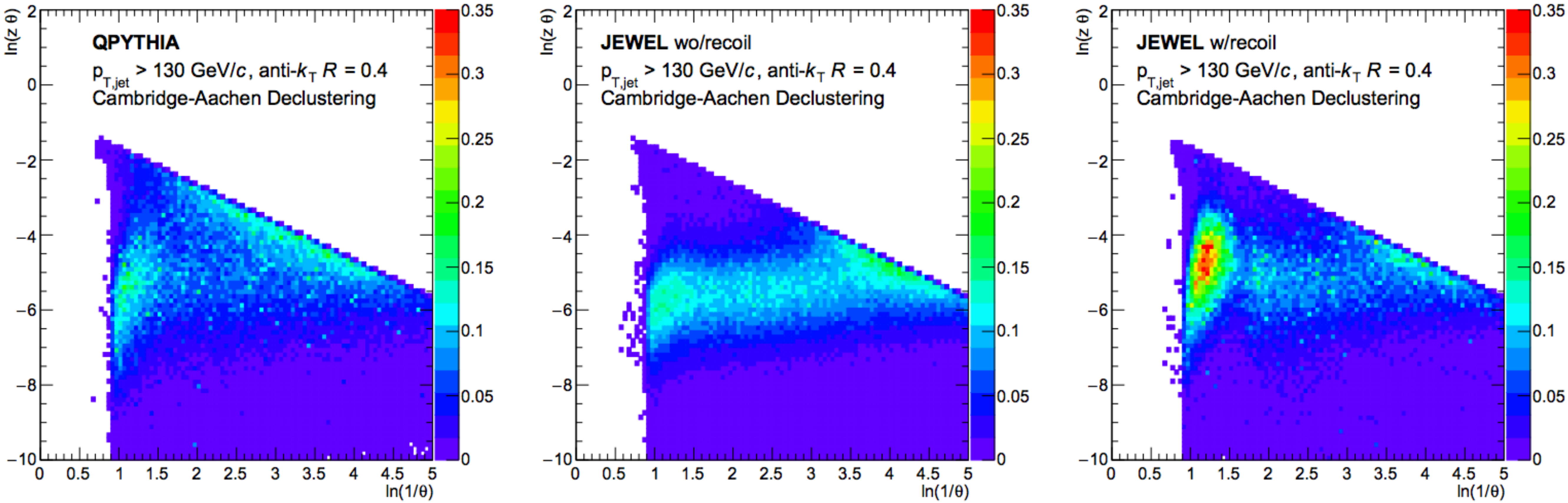
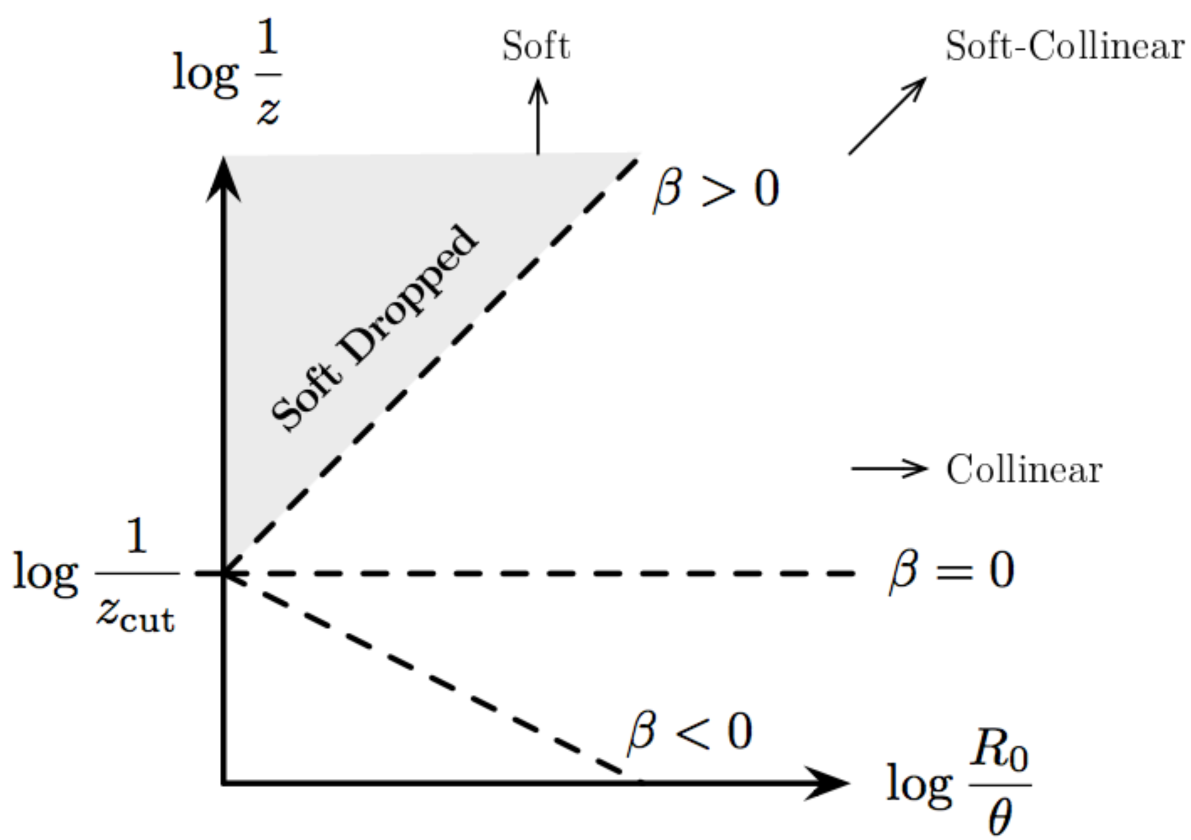


- **Devise jet substructure observables to probe different kinematical regions to discriminate different jet quenching models and tune Monte Carlo (with / without medium).**

5th Heavy Ion Jet Workshop Report, 2018

Soft Drop Groomed Jet Observables

Fig. from Larkoski, Marzani, Soyez, Thaler `14



- **Different grooming parameters removes different region of phase space**

Soft Drop Condition

$$z > z_{\rm cut} \left(\frac{\Delta R_{12}}{R} \right)^{\beta}$$

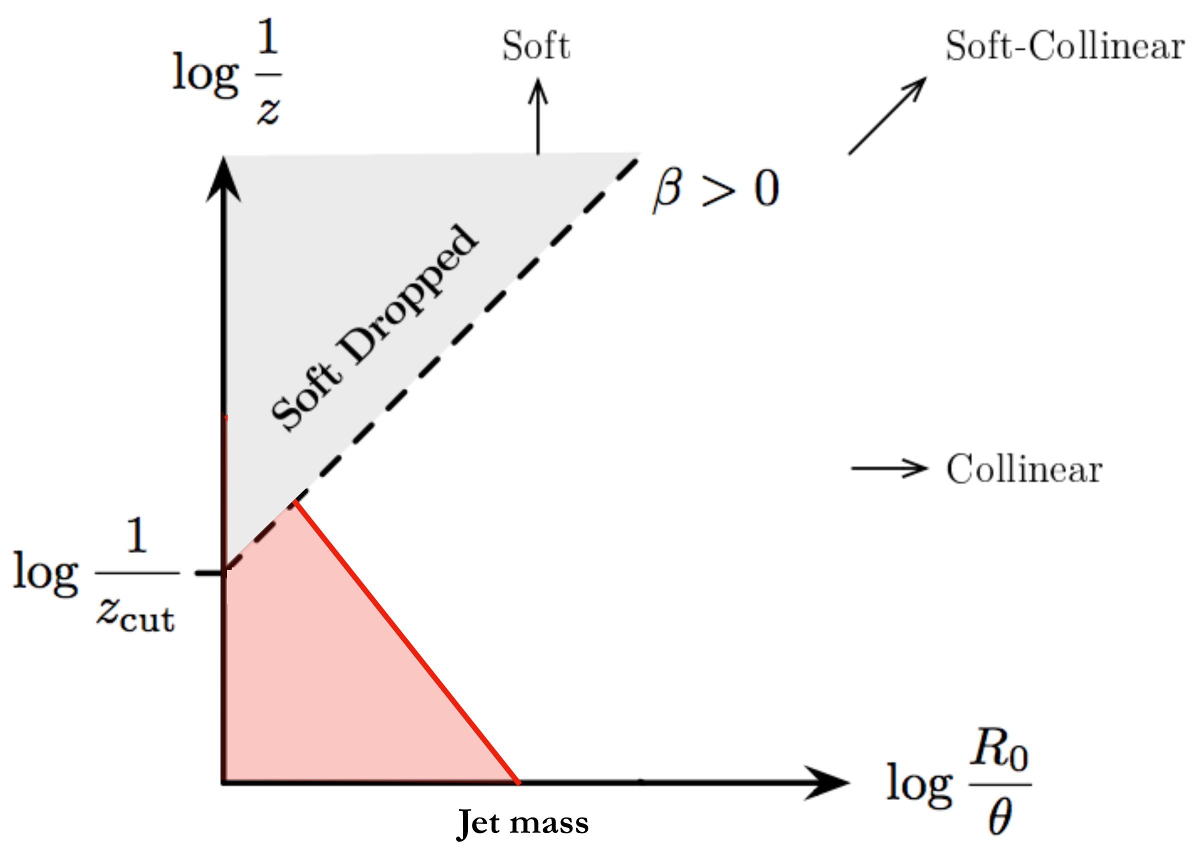
- **Soft drop grooming algorithms:**

1. Reorder emissions in the identified jet according to their relative angle using C/A jet algorithm.

2. Recursively remove soft branches until soft drop condition is met.

Larkoski, Marzani, Soyez, Thaler `14
Frye, Larkoski, Schwartz, Yan `16

Soft Drop Groomed Jet Observables

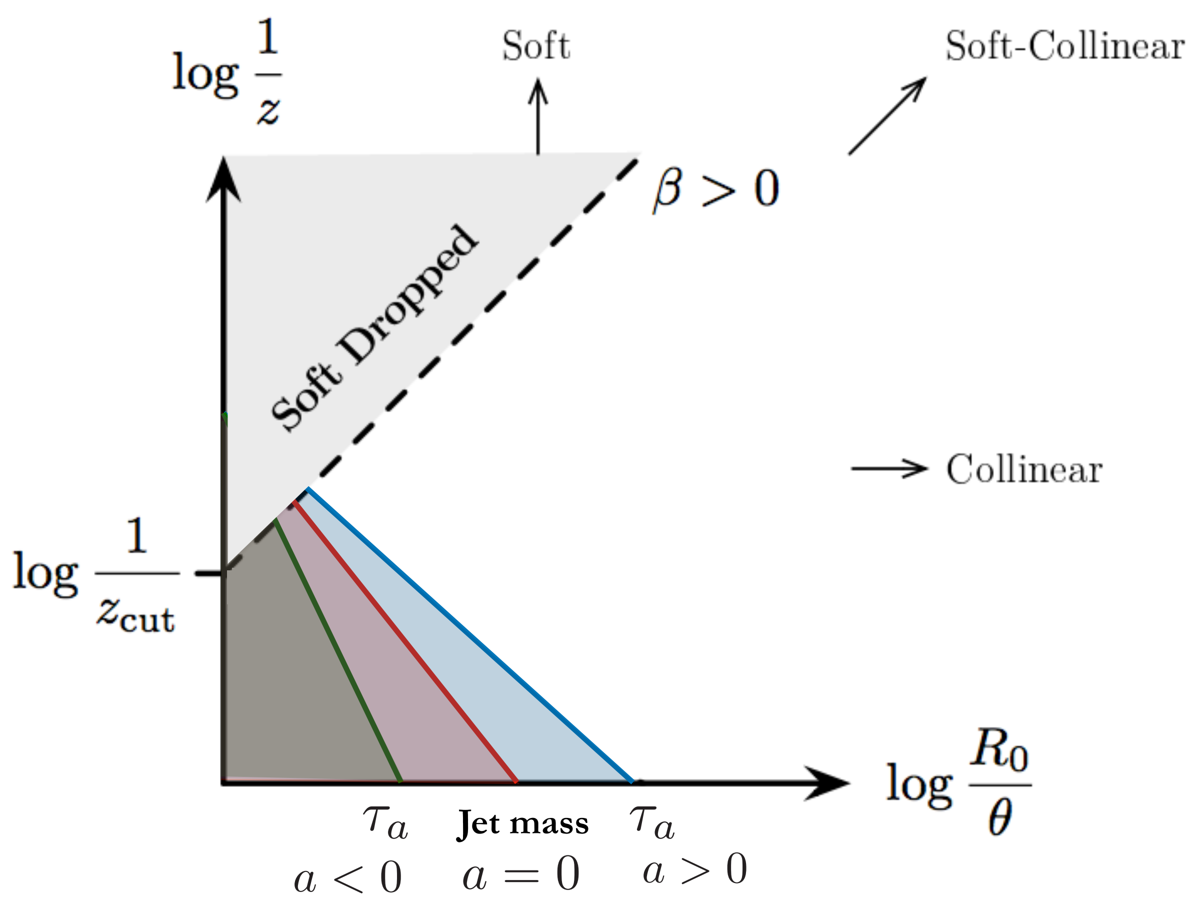


- Different grooming parameters removes different region of phase space

I. Usual jet observables

- Jet mass

Soft Drop Groomed Jet Observables



- **Different grooming parameters removes different region of phase space**

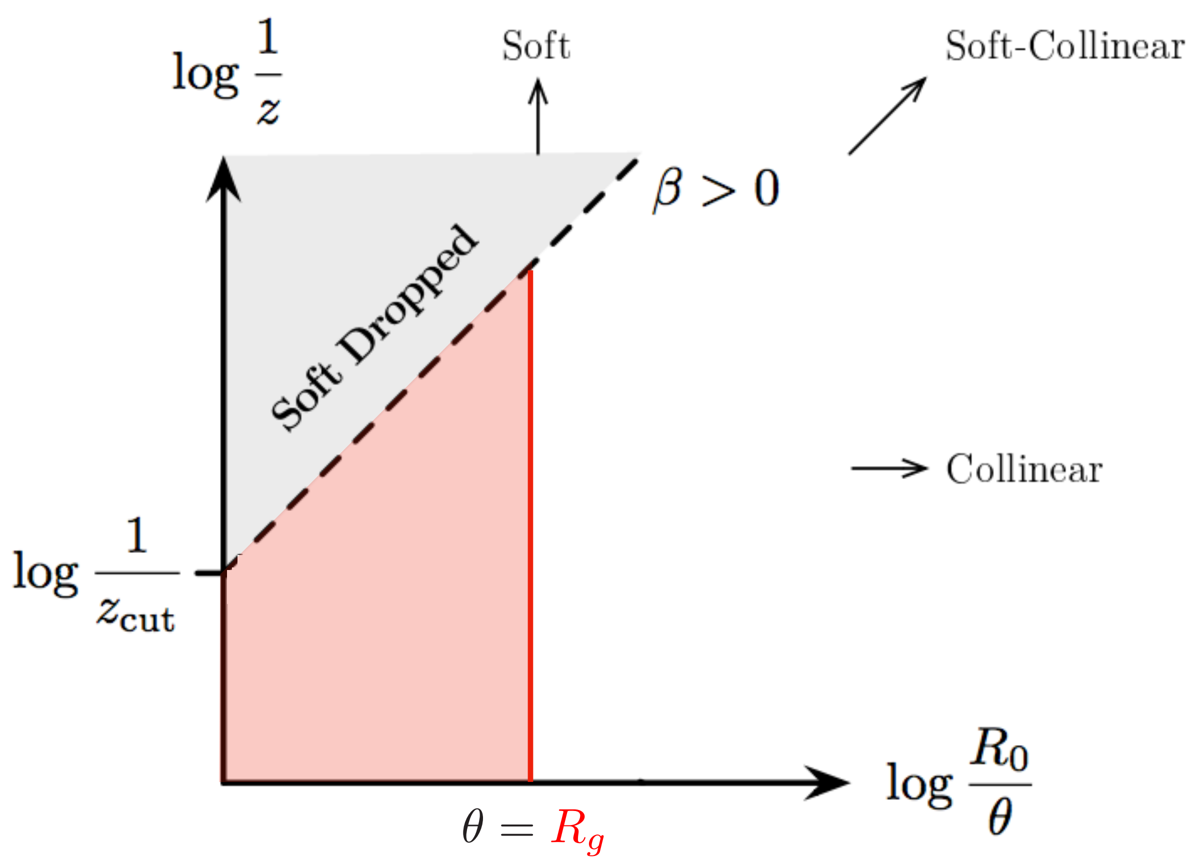
I. Usual jet observables

- **Jet mass** $(a = 0)$
- **Jet angularity** τ_a

$$\tau_a = \frac{1}{p_T} \sum_{i \in J} p_{T,i} \left(\Delta R_{iJ}\right)^{2-a}$$

- ...

Soft Drop Groomed Jet Observables



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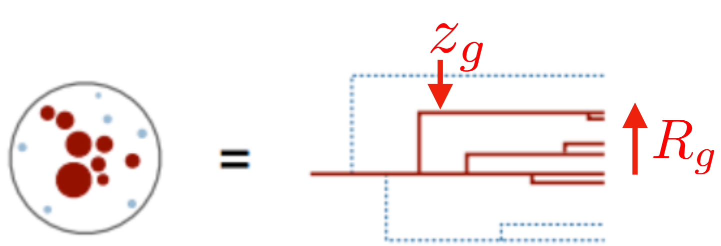
II. Observables unique to soft drop

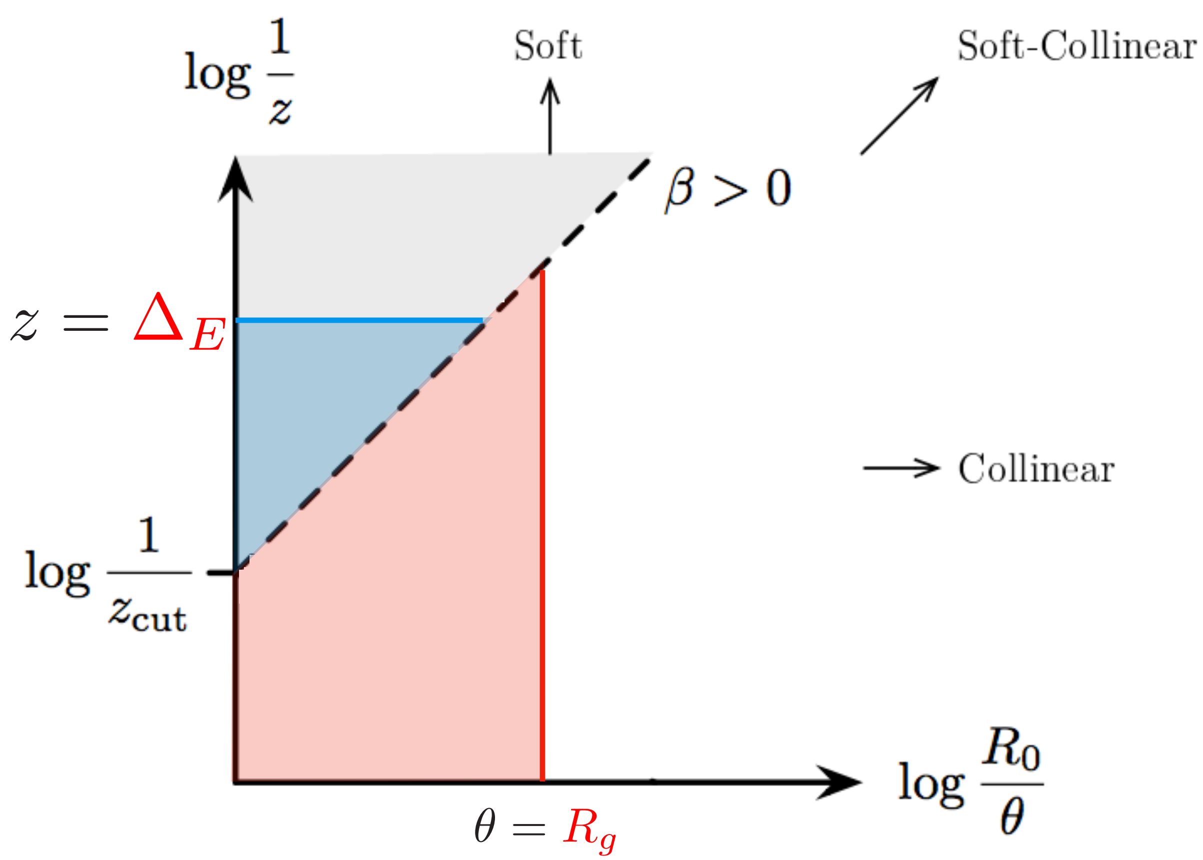
- **At the time soft drop condition is met**

$$z > z_{\rm cut} \left(\frac{\Delta R_{12}}{R} \right)^{\beta}$$



$$z_g = z \qquad R_g = \Delta R_{12}$$

Soft Drop Groomed Jet Observables



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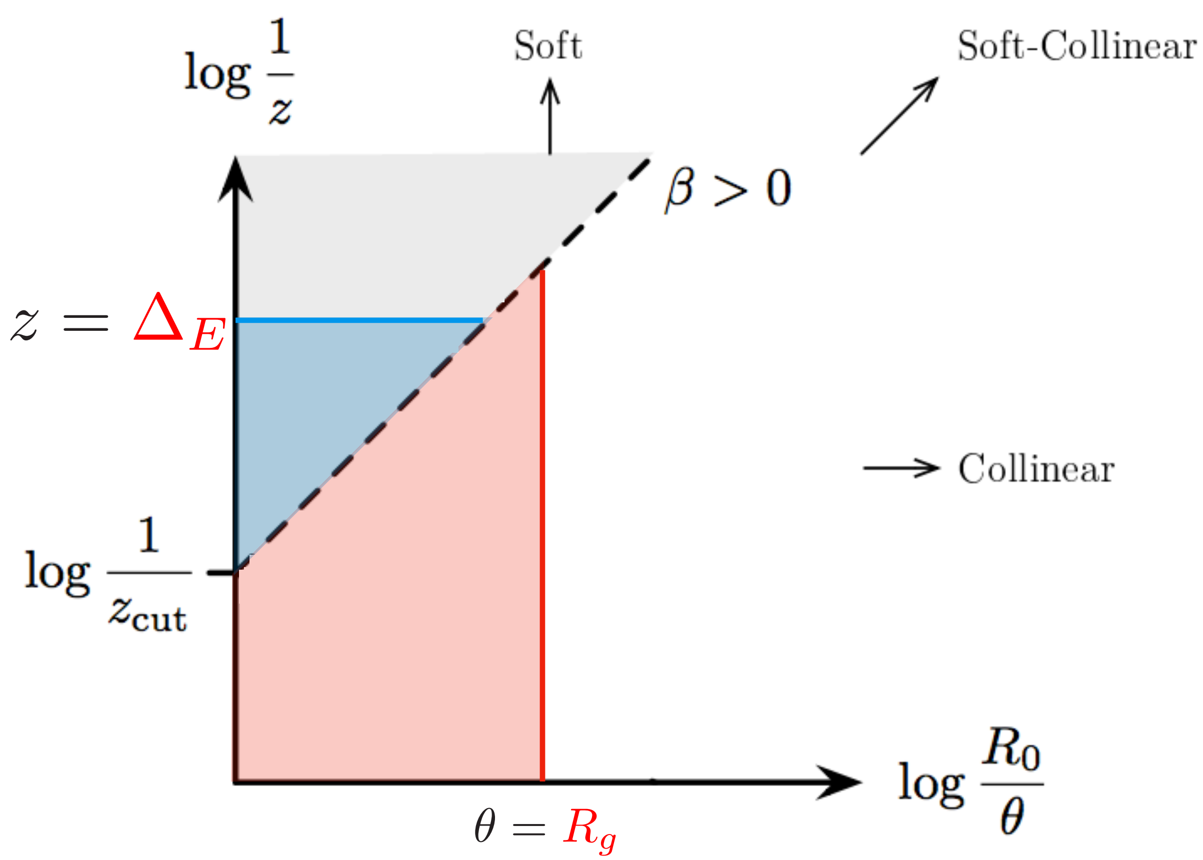
III. Soft-sensitive observables unique to groomed jets

- Energy drop $\Delta_E = \frac{p_T - p_T^{\rm gr}}{p_T}$
- Angle between standard and groomed jet axes
- …

Soft Drop Groomed Jet Observables



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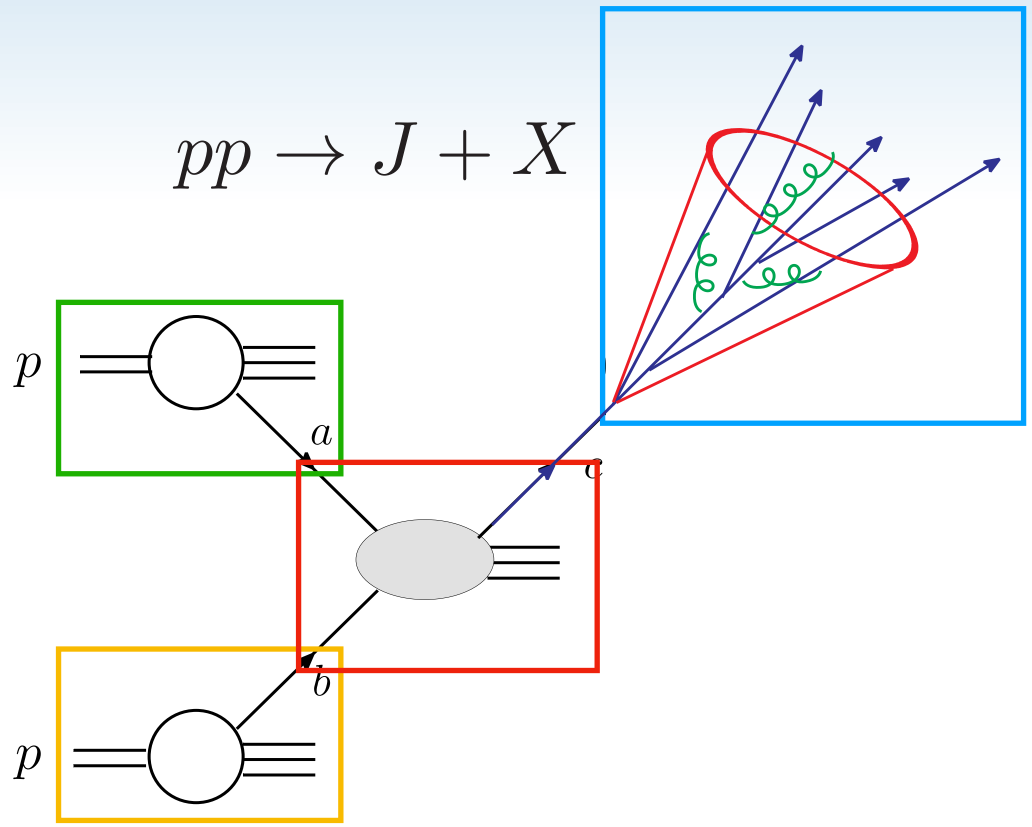
QCD factorization for inclusive jet

$pp \to J + X$



- Jet dynamics isolated to jet function J_c
- Similar factorization holds for AA collision?

$J_c \to J_c^{\text{med}}$ *Qiu, Ringer, Sato, Zurita `19*

Factorization for pp

Inclusive Jet $\frac{d\sigma^{pp\to JX}}{dp_T d\eta} = \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab}^c \otimes J_c$

Λ_{QCD} p_T $p_T R$

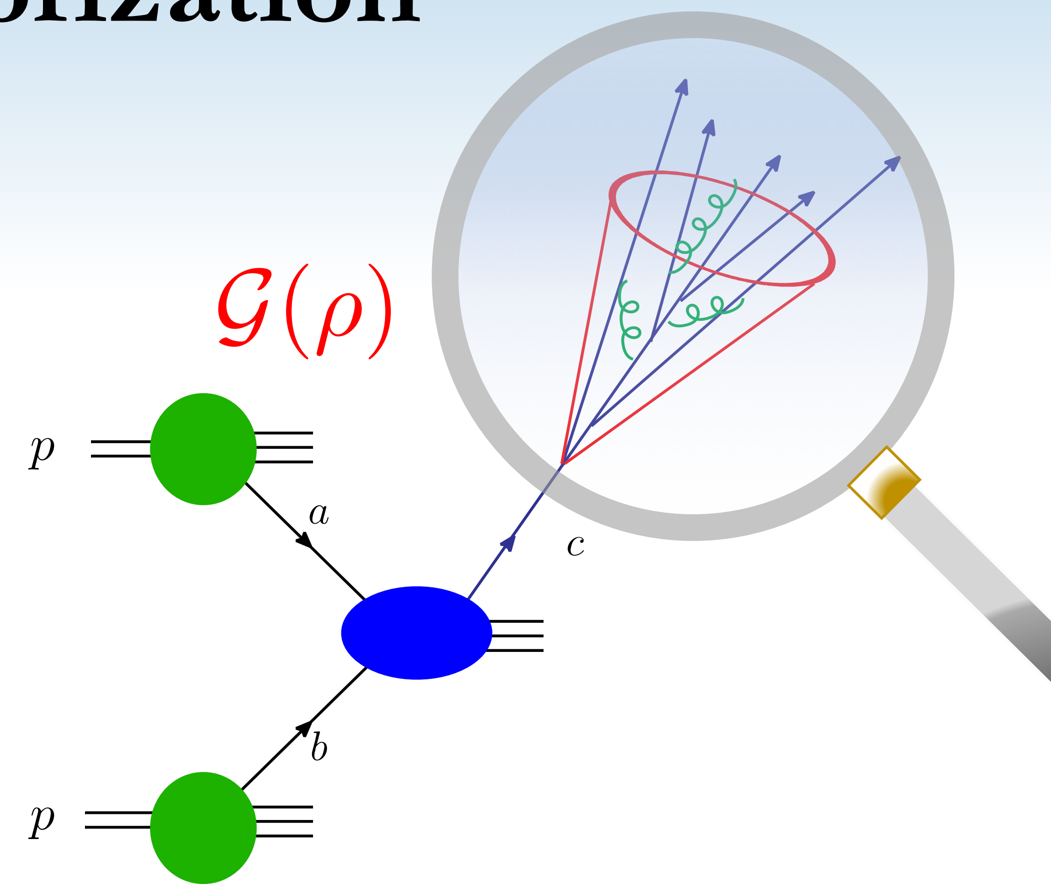
RG evolution

$$\mu \frac{d}{d\mu} J_i = \sum_j P_{ji} \otimes J_j$$

Dasgupta, Dreyer, Salam, Soyez `15
Kaufmann, Mukherjee, Vogelsang `15
Kang, Ringer, Vitev `16
Dai, Kim, Leibovich `16

Jet substructure factorization

Inclusive jet production $pp \to \text{jet} + X$



$$\frac{d\sigma^{pp\to \text{jet}X}}{dp_T d\eta} = \sum_{a,b,c} \underset{\Lambda_{\text{QCD}}}{f_a \otimes f_b} \otimes \underset{p_T}{H_{ab}^c} \otimes \underset{p_T R}{\boxed{J_c}}$$

Jet substructure ρ

$$\frac{d\sigma^{pp\to \text{jet}(\rho)X}}{dp_T d\eta d\rho} = \sum_{a,b,c} \underset{\Lambda_{\text{QCD}}}{f_a \otimes f_b} \otimes \underset{p_T}{H_{ab}^c} \otimes \underset{p_T R}{\boxed{\mathcal{G}_c(\rho)}}$$

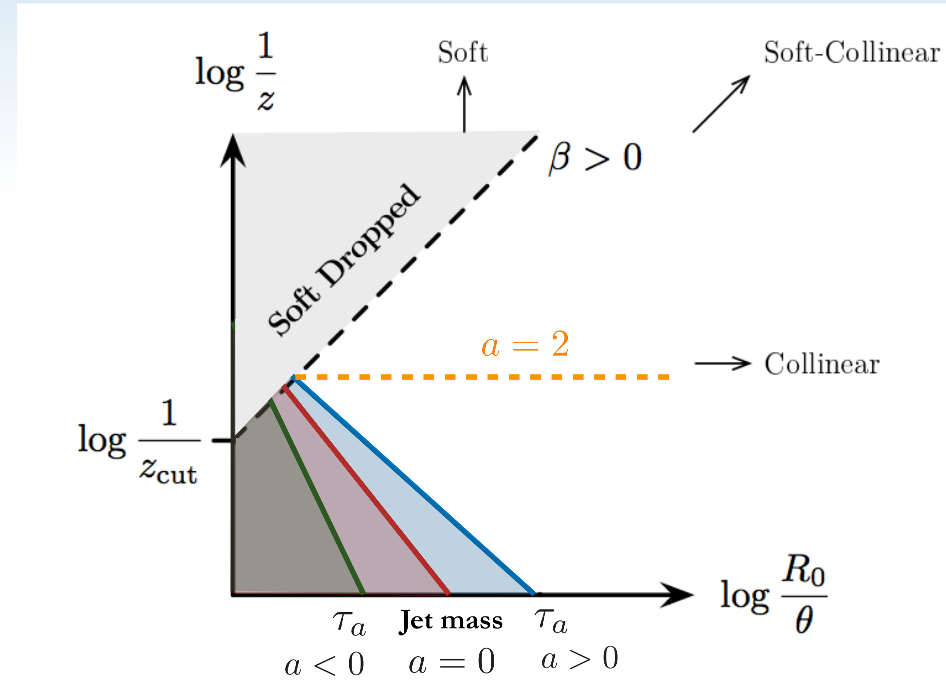
and other scale(s) depending on ρ

Jet angularity τ_a

$$\frac{d\sigma^{pp\to \text{jet}(\tau_a)X}}{dp_T\, d\eta\, d\tau_a} = \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab}^c \otimes \mathcal{G}_c(\tau_a)$$

Λ_{QCD} $\quad p_T \quad p_T R$

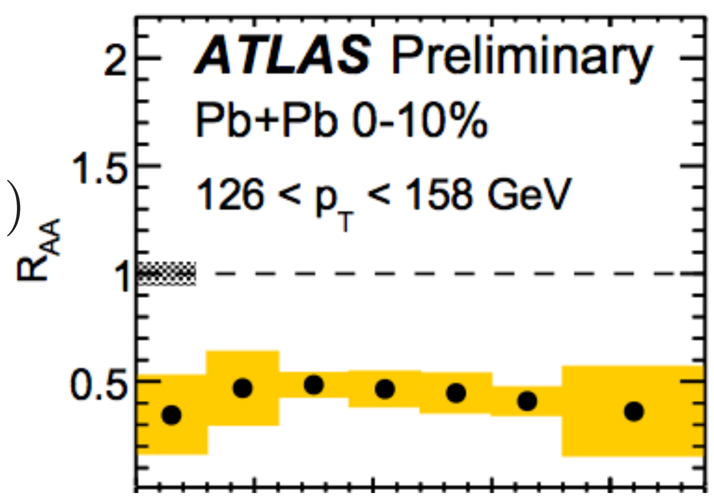
$$\tau_a = \frac{1}{p_T}\sum_{i\in J} p_{T,i}(\Delta R_{iJ})^{2-a}$$



- A generalized class of IR safe observables for $-\infty < a < 2$
- Parameter a gives varying sensitivity to collinear radiations.

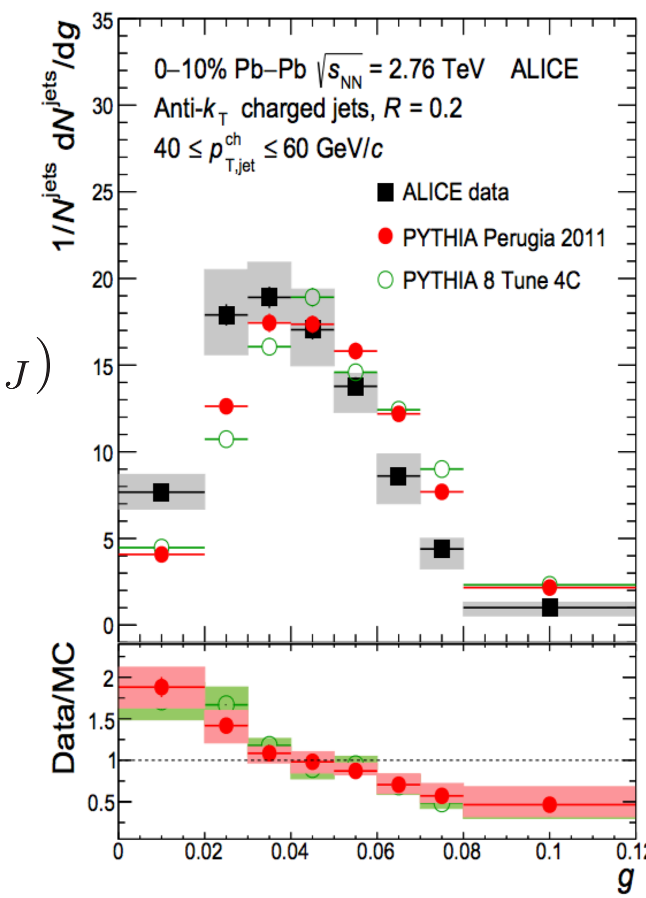
$a = 0$

$$\tau_0^{pp} = \frac{m_J^2}{p_T^2} + \mathcal{O}((\tau_0^{pp})^2)$$



$a = 1$

$$g(\text{broadening}) = \frac{1}{p_T}\sum_{i\in J} p_{T,i}\,(\Delta R_{iJ})$$



- Analysis for different a currently in progress.

ALICE Preliminary

Jet angularity τ_a

$$\frac{d\sigma^{pp\to \text{jet}(\tau_a)X}}{dp_T\, d\eta\, d\tau_a} = \sum_{a,b,c} \underset{\Lambda_{\rm QCD}}{f_a \otimes f_b} \otimes \underset{p_T}{H_{ab}^c} \otimes \underset{p_T R}{\boxed{\mathcal{G}_c(\tau_a)}}$$

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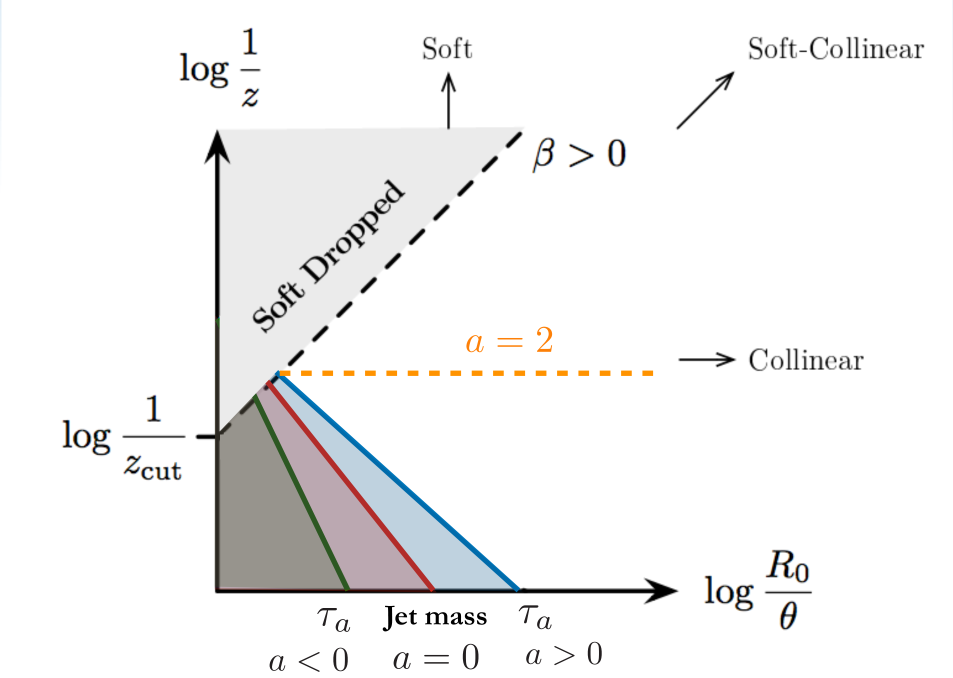
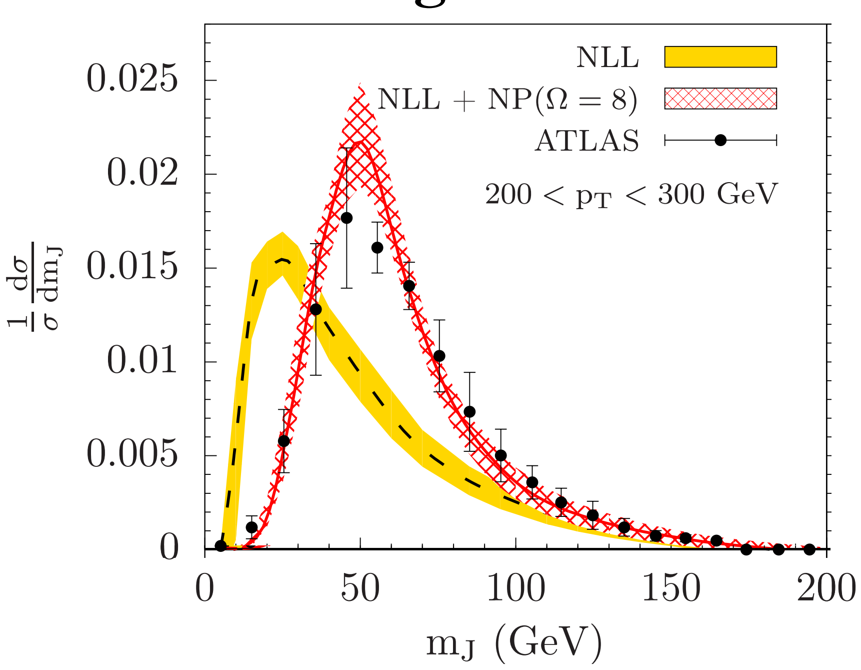


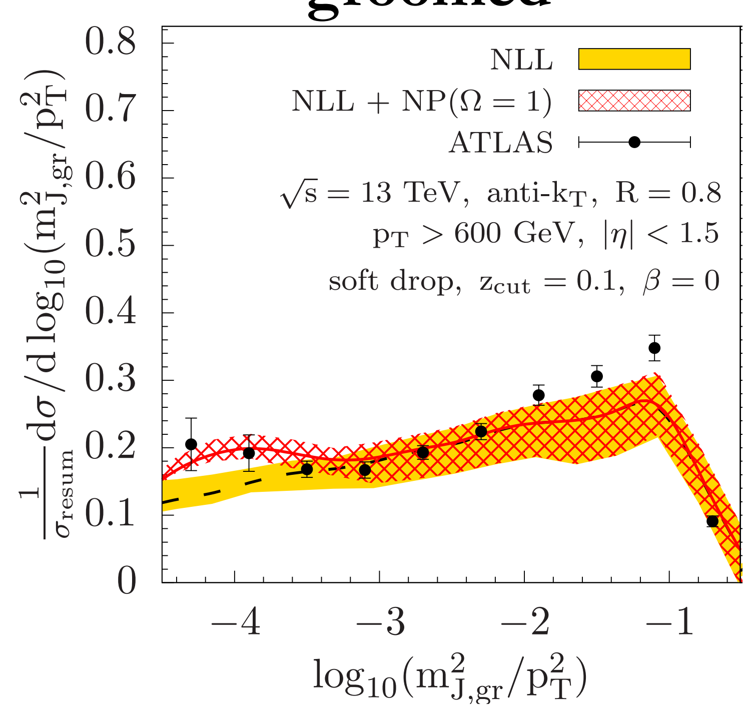
- **A generalized class of IR safe observables for** $-\infty < a < 2$
- **Parameter** a **gives varying sensitivity to collinear radiations.**

Perturbative result $\otimes$ NP shape function **(Parametrized by Ω)**

ungroomed



groomed



- **Groomed jet mass** $(a = 0)$ **is more robust to NP effects**
- **Nonperturbative corrections to groomed jet mass**

Hoang, Mantry, Pathak, Stewart `19

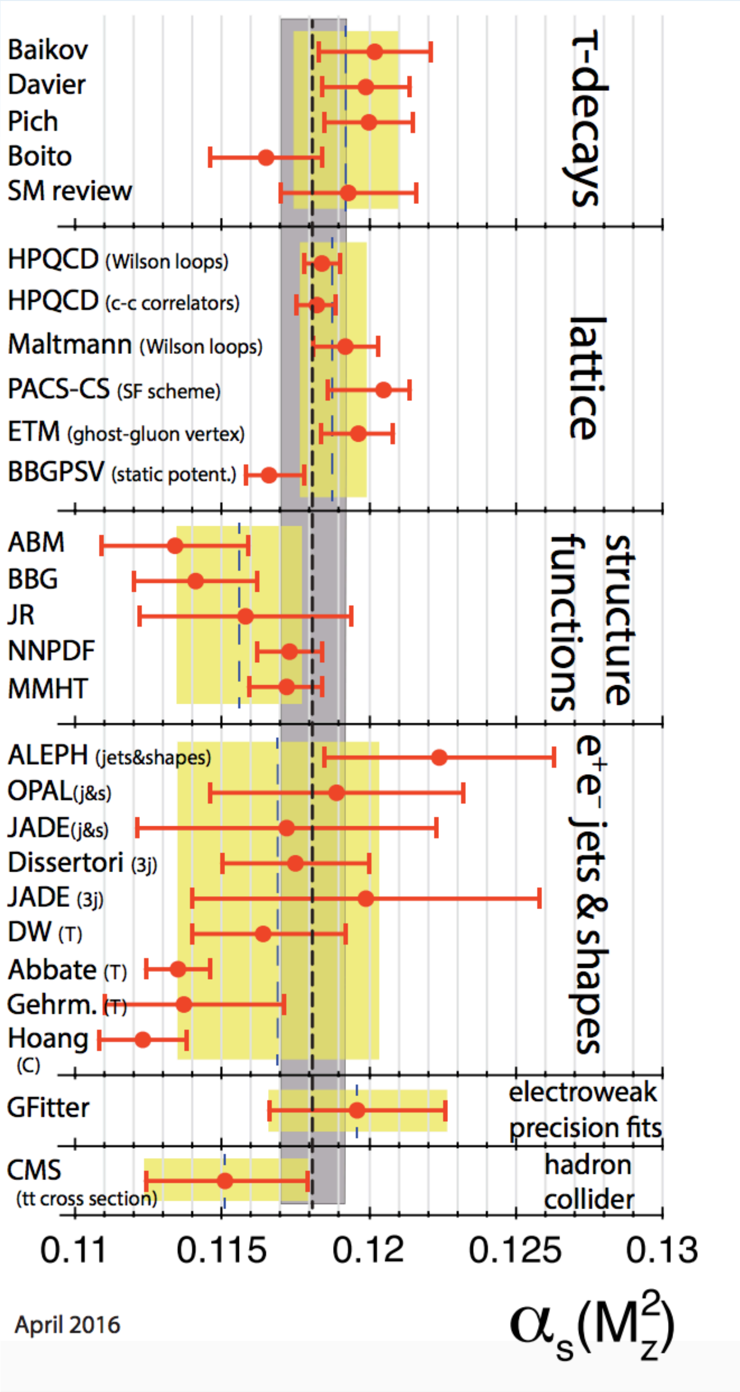
Kang, KL, Liu, Ringer `18

α_s extraction

- World Average with 0.9% total uncertainty

$$\alpha_s(m_Z) = 0.1181 \pm 0.0011$$

- Most precise input: lattice determination
- Next precise input: e^+e^- event shape determination: thrust and C-parameter.
  - $3-4\sigma$ tension with lattice.



- High-quality of data pouring out of the LHC.
  - Can we carry out an α_s extraction using pp data?

α_s extraction

- **Key challenge:** (for e^+e^- event shape extraction or ungroomed jet substructure at pp) degeneracy between the extractions of α_s and fitting Ω^{had}

Also, contaminations from MPI

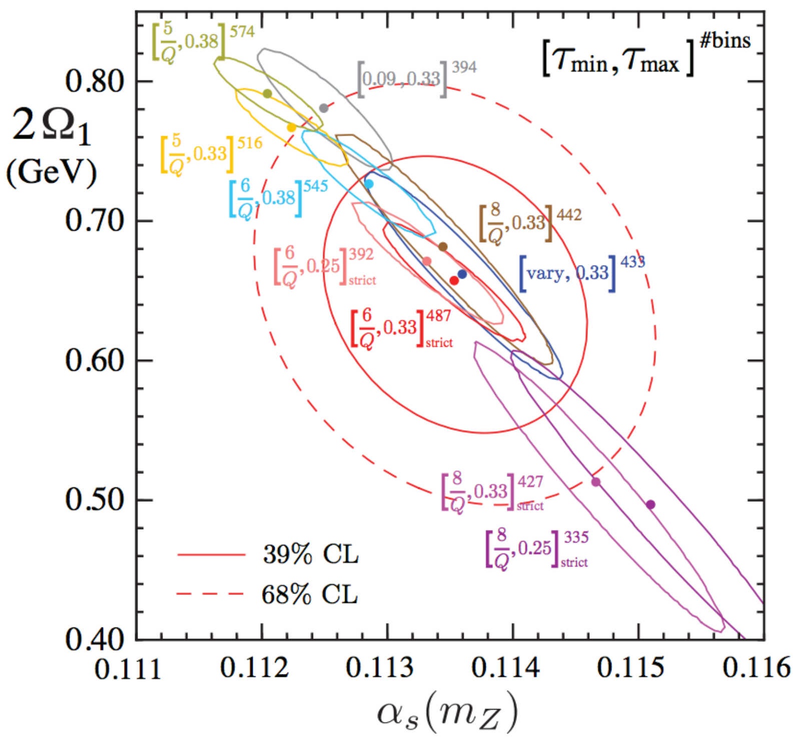
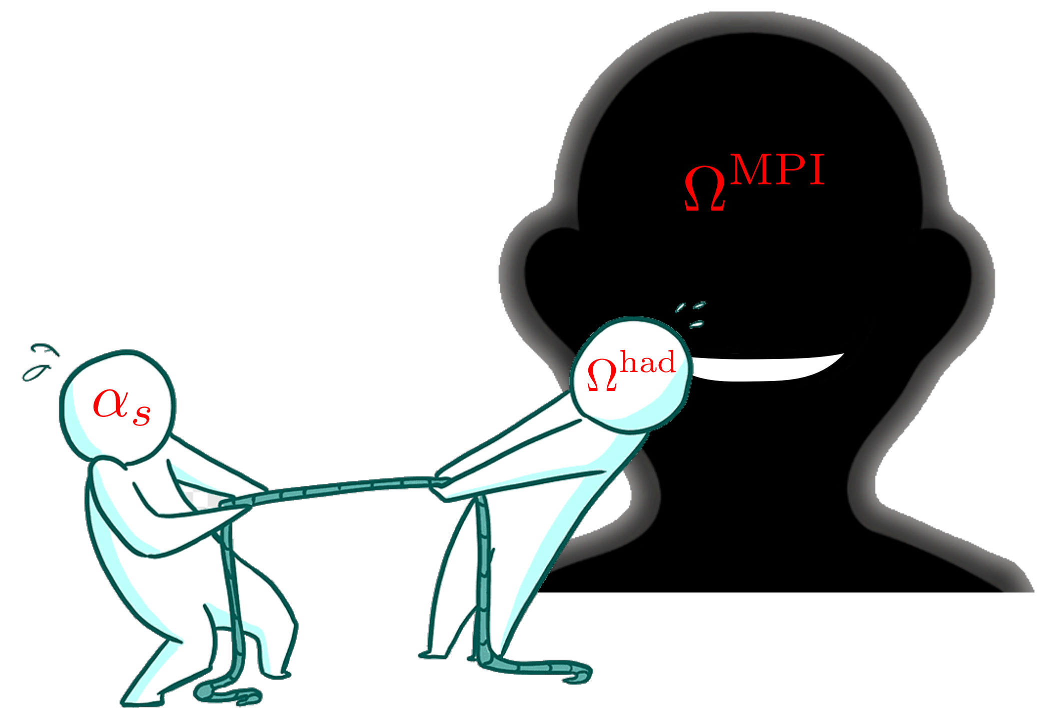


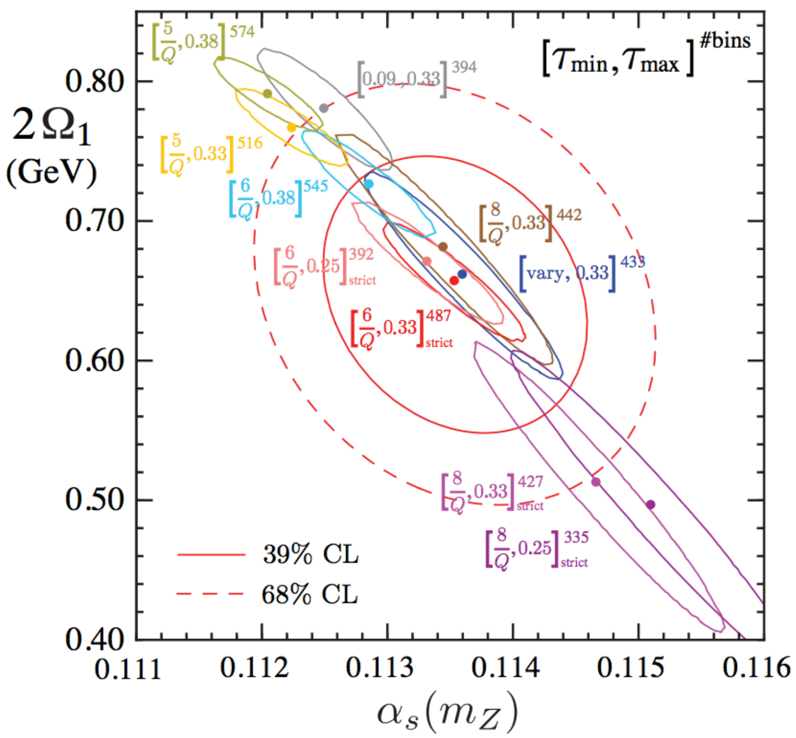
Fig. from
Abbate, Fickinger, Hoang, Mateu, Stewart `10



α_s extraction

- **Key challenge:** (for e^+e^- event shape extraction or ungroomed jet substructure at pp)

degeneracy between the extractions of α_s and fitting $\Omega^{\rm had}$



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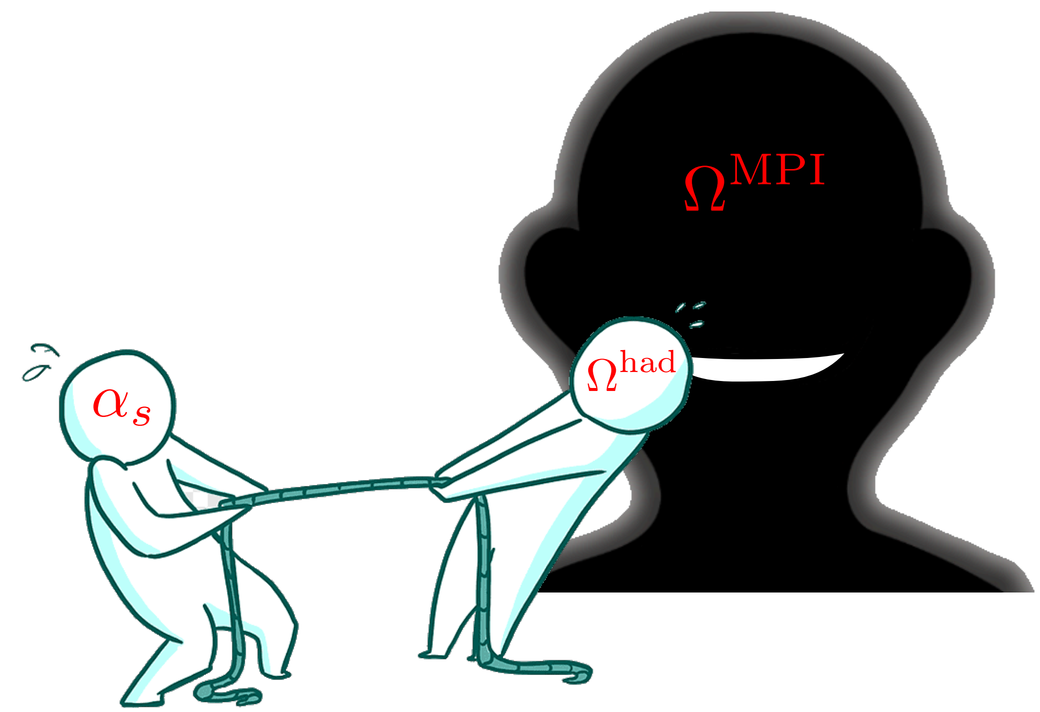


- **Desire:**

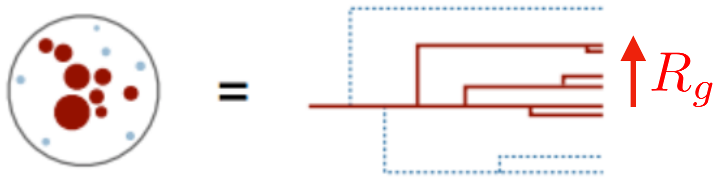
- reduced sensitivity to non-perturbative effects (hadronizations and MPI)
- additional handle(s) to lift degeneracy between extractions of α_s and fitting $\Omega^{\rm had}$

Groomed jet angularities!

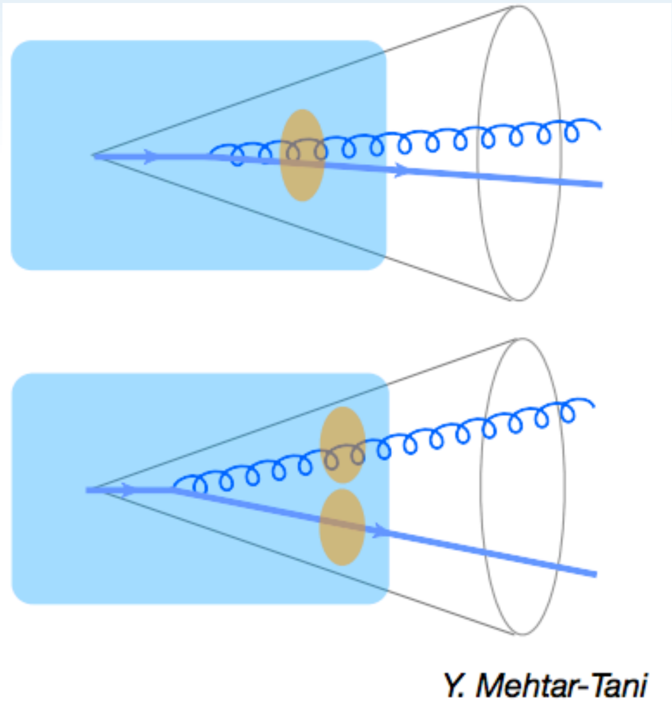
- Proof-of-principle Monte Carlo study shows groomed angularity to be an ideal observable.
  - Currently feasible to determine with 10% uncertainty. *Les Houches `17*

Groomed jet radius

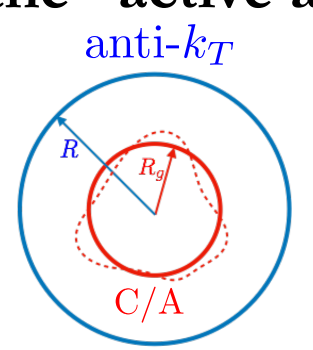
- Dynamically determines the size of the hard-collinear splitting



  - What is the transverse resolution length of the medium?



- Also gives the "active area" of the groomed jet



Active area $\sim \pi R_g^2$

  - Can serve as a proxy for the sensitivity to pileup.
  - Complicated by clustering effects

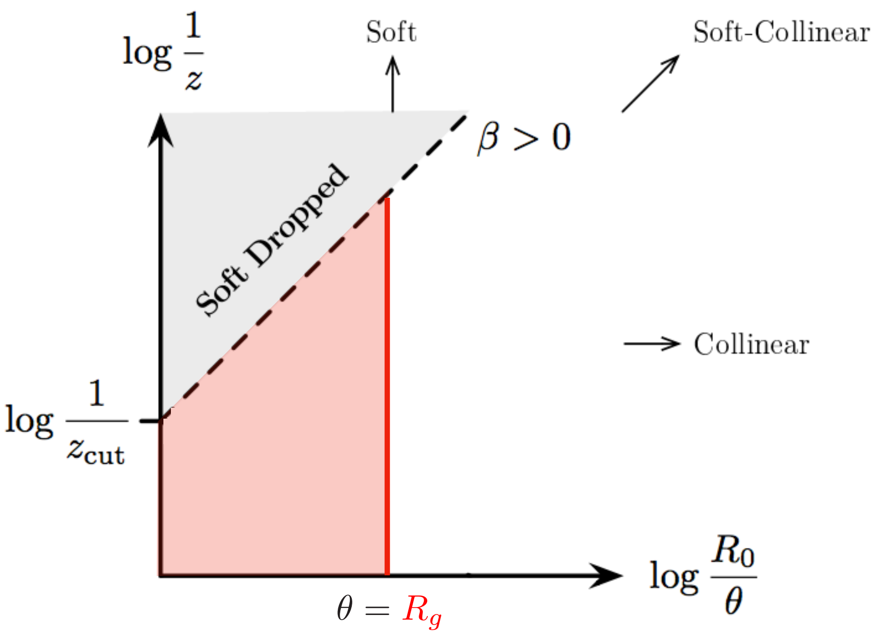
- Factorization formalism necessary for studying many groomed jet observables

  - To regulate divergence of the Sudakov safe observables
  - Sometimes the observables are invariably coupled to R_g measurements.

ex: Δ_E , energy drop

Kang, KL, Liu, Neill, Ringer `19
Larkoski, Marzani, Soyez, Thaler `14

Groomed jet radius

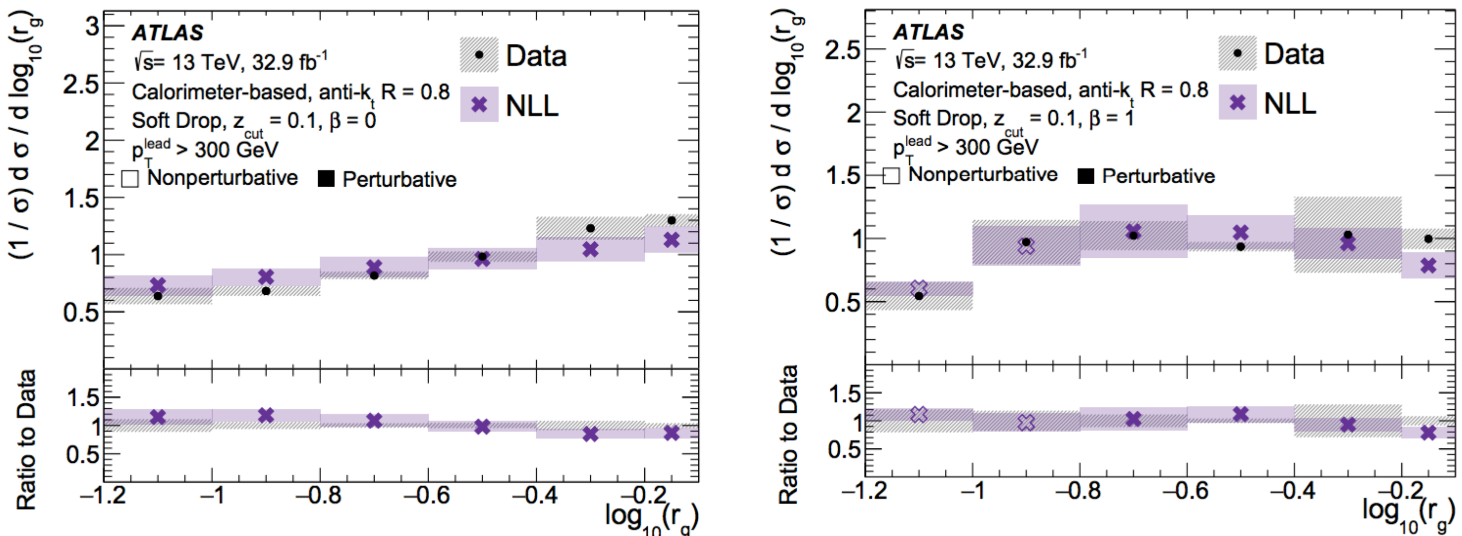


Kang, KL, Liu, Neill, Ringer `19
Larkoski, Marzani, Soyez, Thaler `14

- **Factorization derived to NLL**

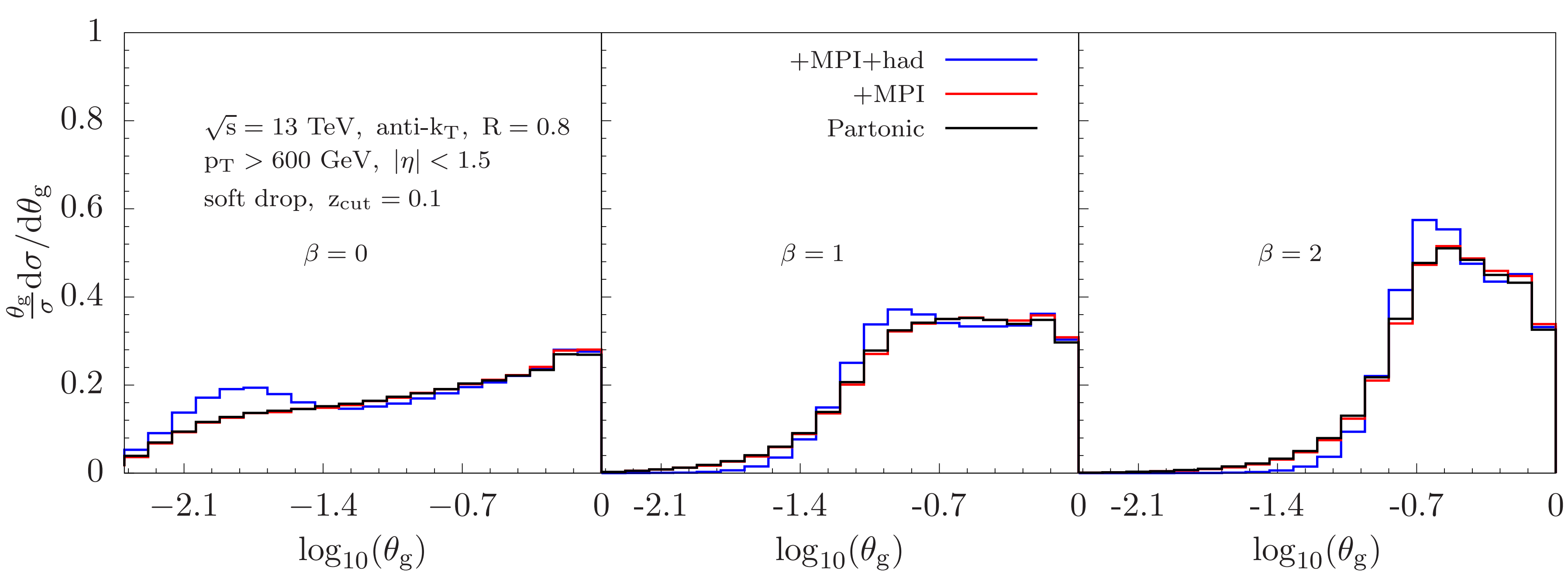
Kang, KL, Liu, Neill, Ringer `19

- **Data comparison**

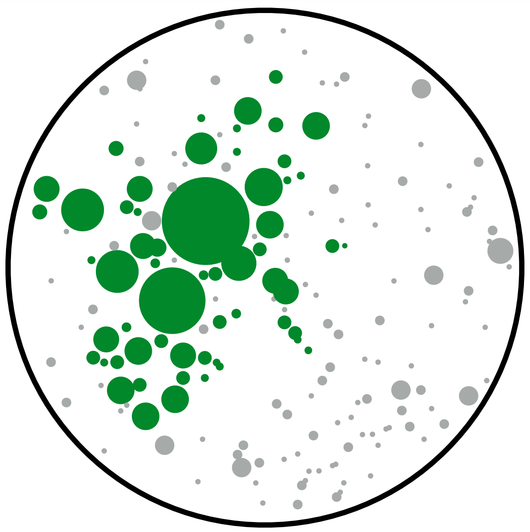


ATLAS `19

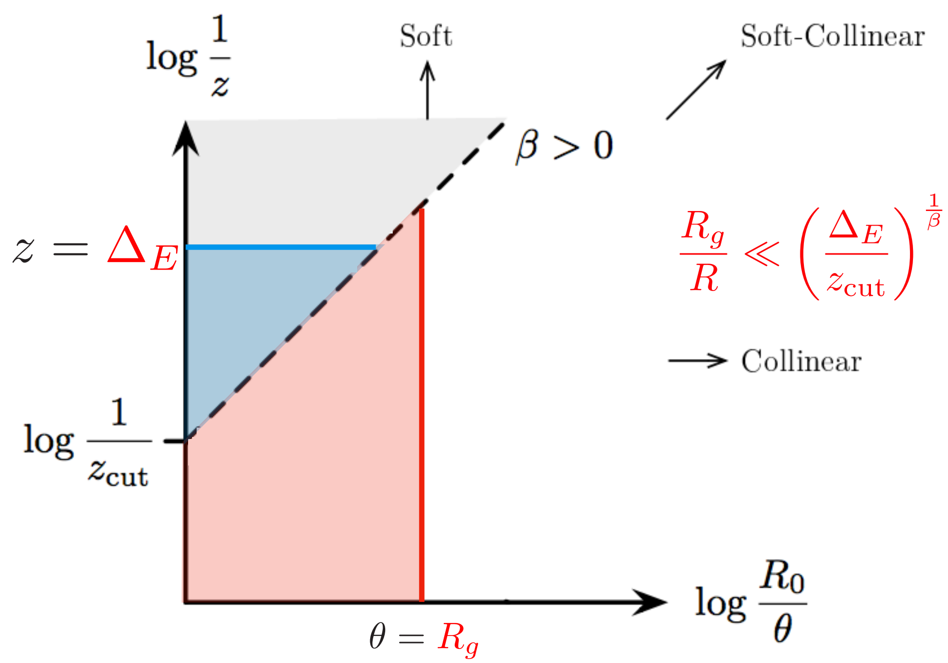
- **Small MPI effects**

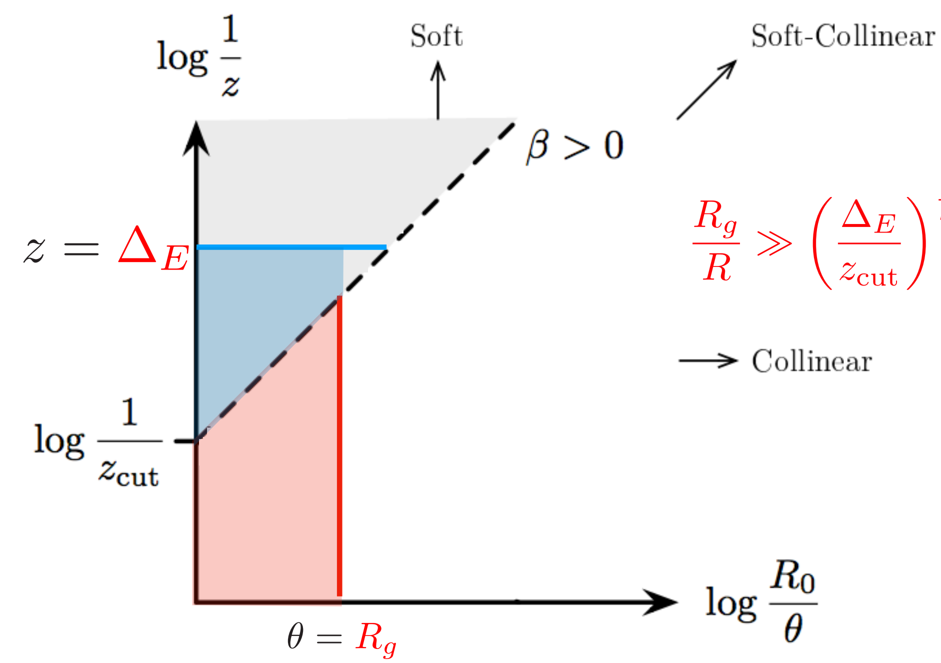


Energy drop



$$\Delta_E = \frac{p_T - p_T^{\rm gr}}{p_T}$$





Cal, KL, Ringer, Waalewijn `20
Larkoski, Marzani, Soyez, Thaler `14

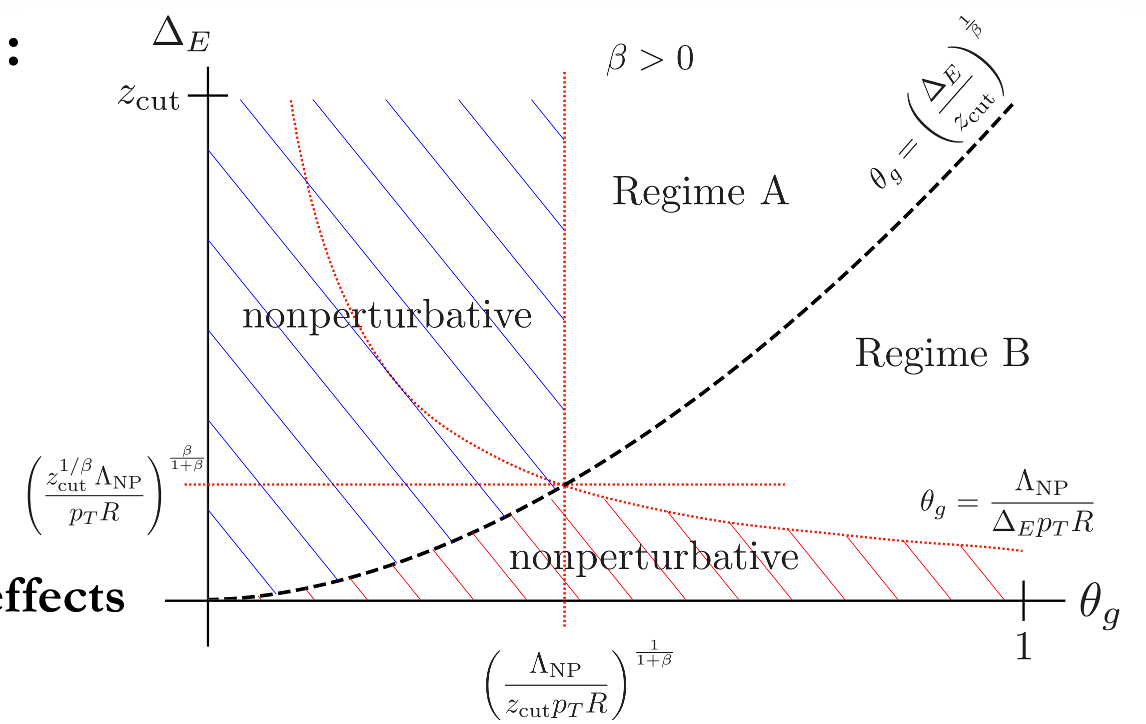
- Energy drop measurements in soft drop groomed jet is coupled with R_g measurements.
  - Factorizations for trimmed jets, iterated soft drop groomed jets *Cal, KL, Ringer, Waalewijn `20*
- All energy drop dependent “modes” are only sensitive to soft modes only.
  - Does the medium modify the soft and collinear radiations differently?

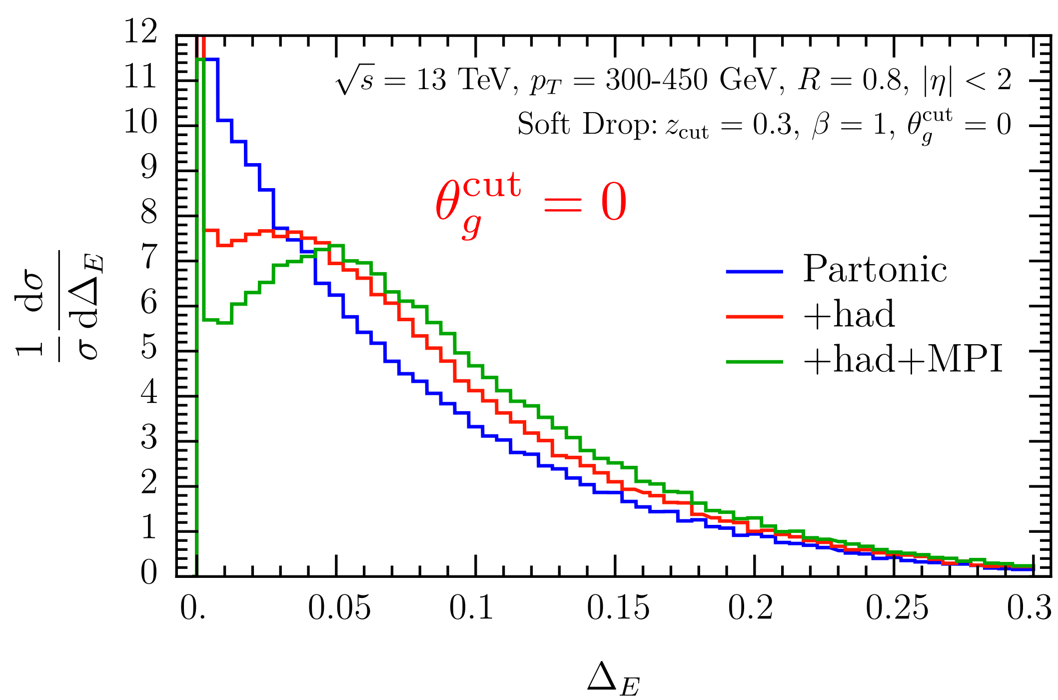
Energy drop

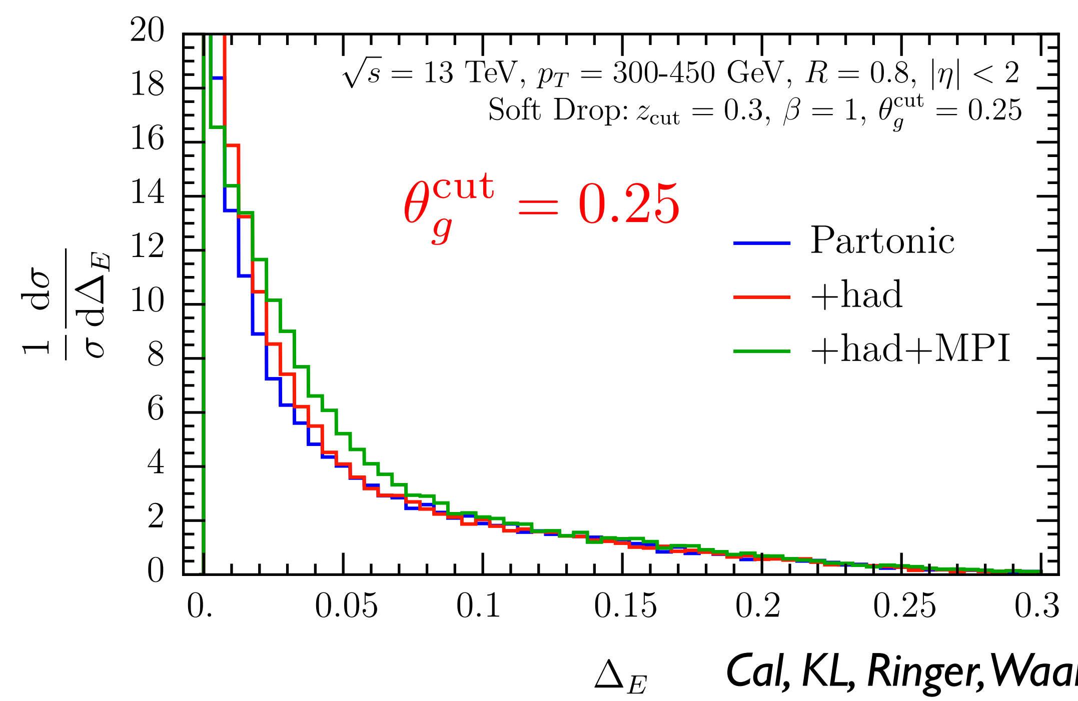
- **Factorization framework differential in both Δ_E, R_g:**

$$\frac{d\sigma}{dp_T\, d\eta\, d\Delta_E\, dR_g}$$

- **Integrate over regions of R_g : $\theta_g^{\rm cut} < \theta_g < 1 \quad \left(\theta_g = \frac{R_g}{R}\right)$**
  - **Experimental tracking efficiency may require $\theta_g^{\rm cut}$**
  - **Different $\theta_g^{\rm cut}$ gives varying sensitivity to non-perturbative effects**
    - **May be useful for MC tuning**







Cal, KL, Ringer, Waalewijn `20

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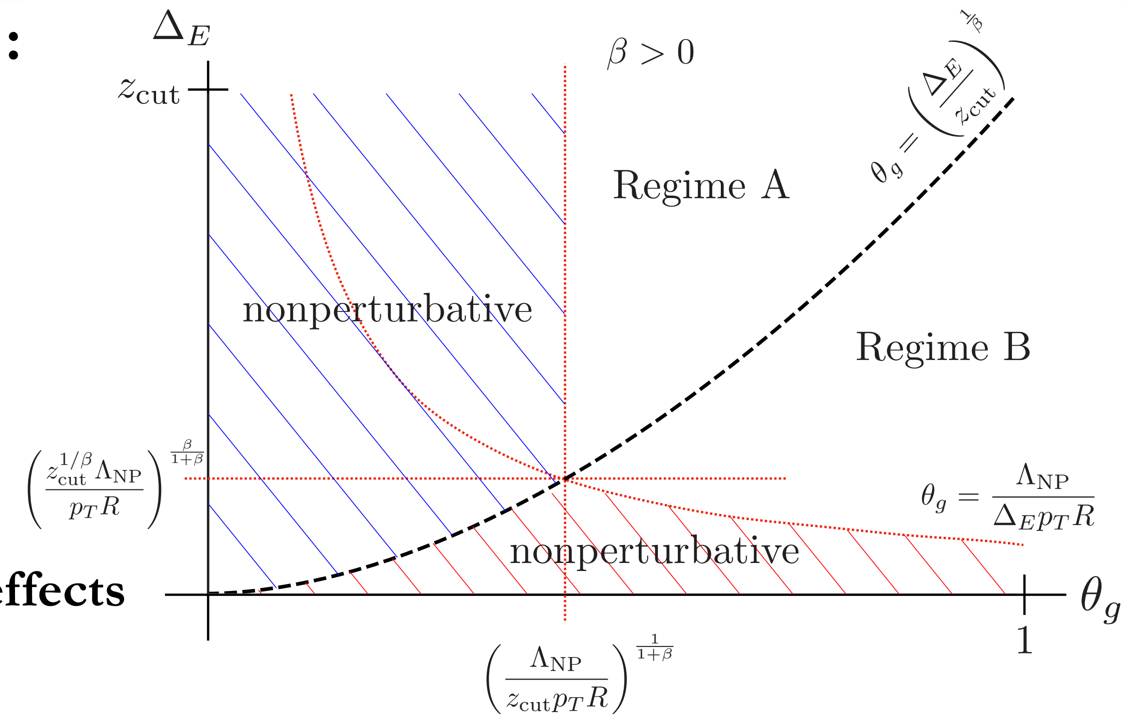
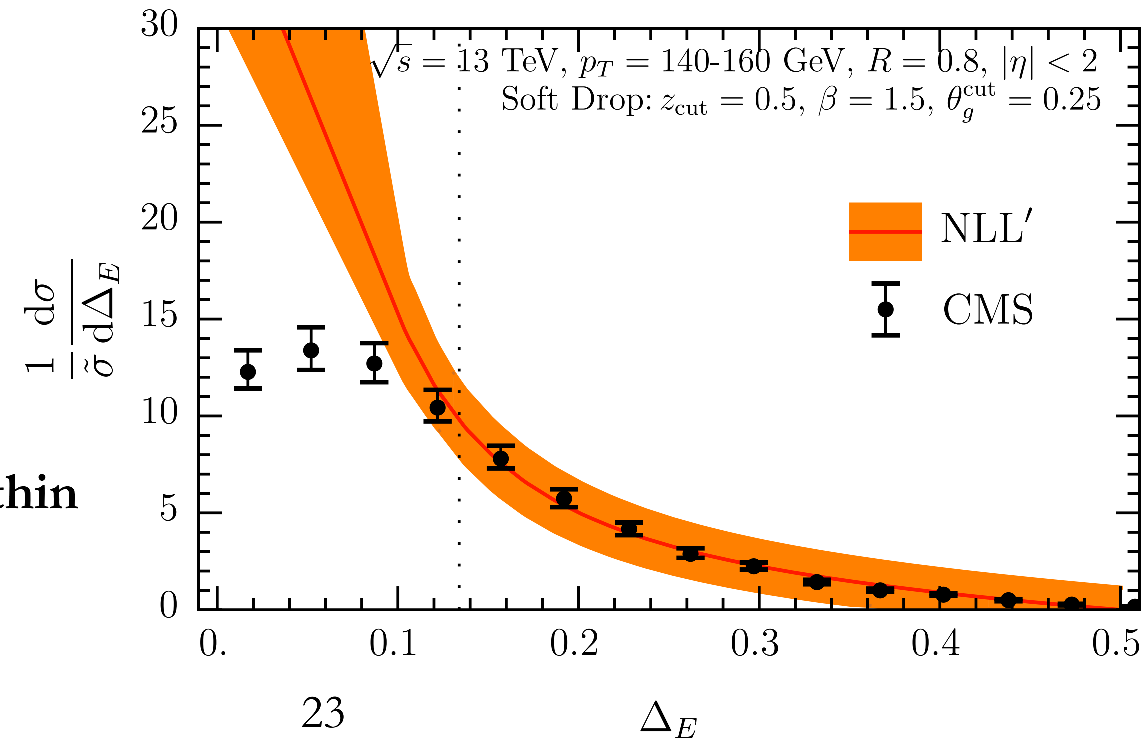


- **Good agreement within perturbative region**



CMS `17

Cal, KL, Ringer, Waalewijn `20

Fragmenting Jet Function (FJF)

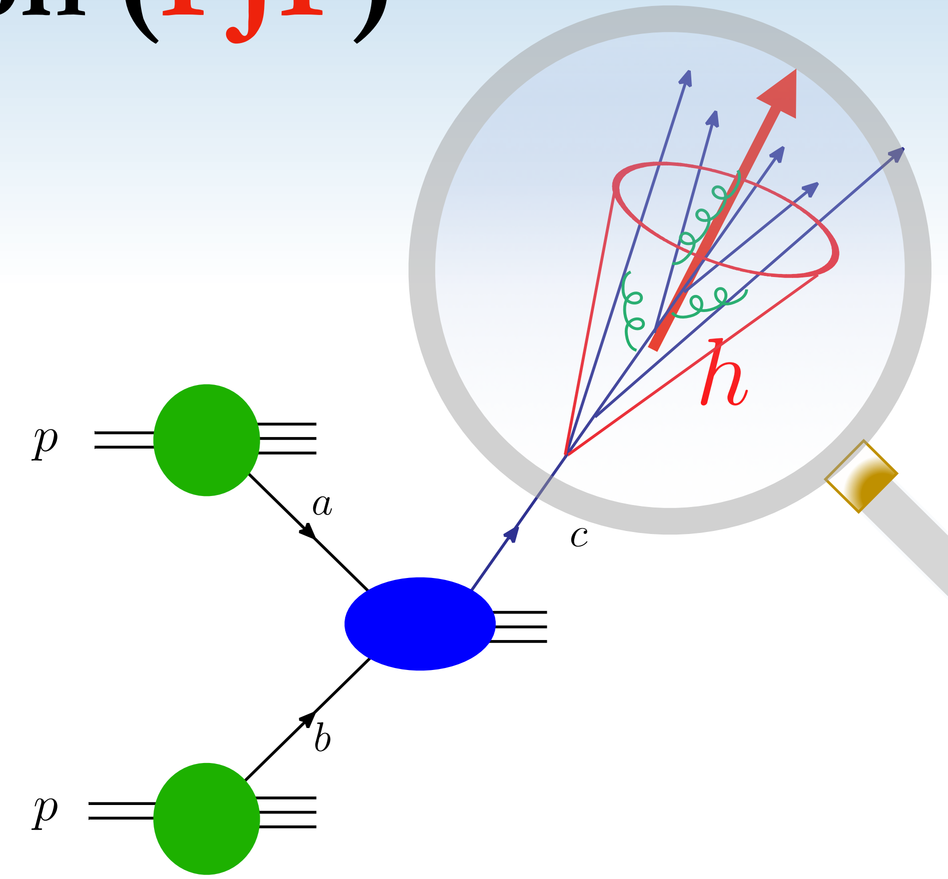
Unpolarized case:

$$\frac{d\sigma^{pp\to \text{jet}(h)X}}{dp_T\, d\eta\, dz_h} = \sum_{a,b,c} f_{a/A} \otimes f_{b/B} \otimes H_{ab}^c \otimes \mathcal{G}_c^h(z_h)$$

$\Lambda_{\rm QCD}$ (under $f_{a/A} \otimes f_{b/B}$), p_T (under H_{ab}^c), $p_T R$, $\Lambda_{\rm QCD}$ (under $\mathcal{G}_c^h(z_h)$)

where $z = p_T^J / p_T^c$

$z_h = p_T^h / p_T^J$



$$\frac{d\sigma^{pp\to hX}}{dp_T\, d\eta} = \sum_{a,b,c} f_{a/A} \otimes f_{b/B} \otimes H_{ab}^c \otimes \boxed{D_c^h}$$

$\Lambda_{\rm QCD}$ (under $f_{a/A} \otimes f_{b/B}$), p_T (under H_{ab}^c), $\Lambda_{\rm QCD}$ (under D_c^h)

where $z = p_T^h / p_T^c$

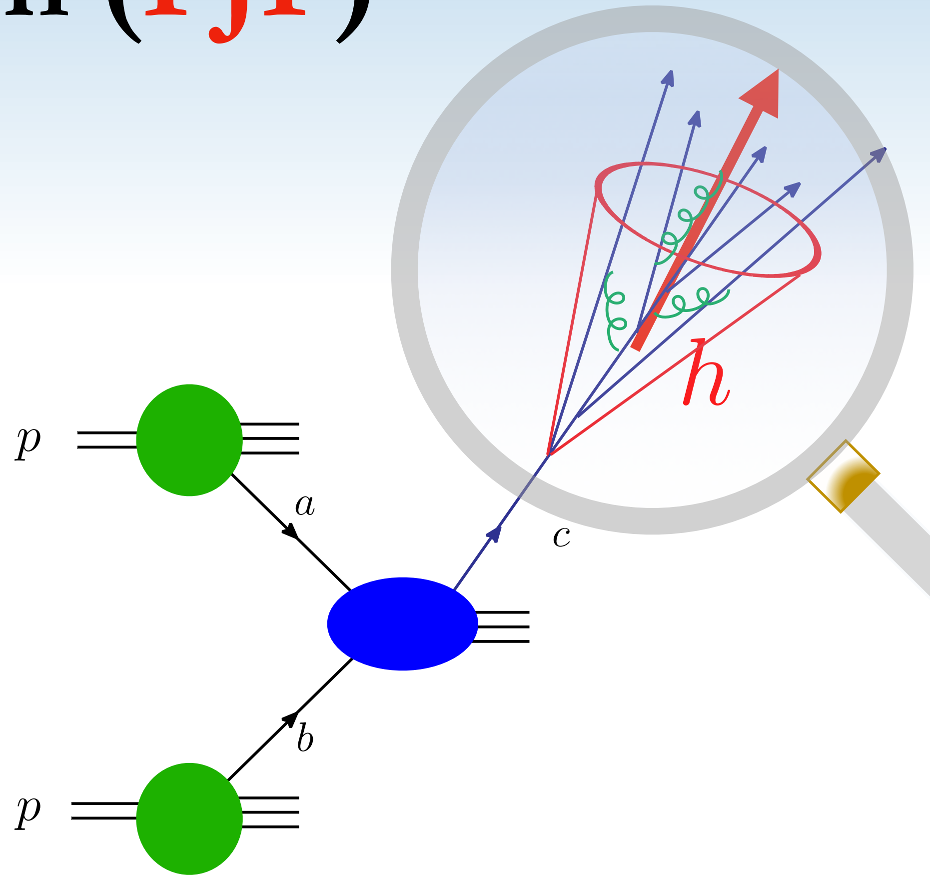
Procura, Stewart `10
Arleo, Fontannaz, Guillet, Nguyen `14
Kaufmann, Mukherjee, Vogelsang `15
Kang, Ringer, Vitev `16
Dai, Kim, Leibovich `16

Fragmenting Jet Function (FJF)

Unpolarized case:

$$\frac{d\sigma^{pp\to \text{jet}(h)X}}{dp_T d\eta dz_h} = \sum_{a,b,c} f_{a/A} \otimes f_{b/B} \otimes H_{ab}^c \otimes \mathcal{G}_c^h(z_h)$$

$\Lambda_{\rm QCD}$ $\quad p_T \quad$ $p_T R$, $\Lambda_{\rm QCD}$



where $z = p_T^J / p_T^c$

$z_h = p_T^h / p_T^J$

IR sensitivity and require matching:

$$\mathcal{G}_c^h(z, z_h, p_T R, \mu) = \sum_j \mathcal{J}_{ij}(z, z_h, p_T R, \mu) \otimes D_j^h(z_h, \mu)$$

$p_T R$ $\quad \Lambda_{\rm QCD}$

matching coefficients **collinear FFs**

- **Collinear FJFs can be related to collinear FFs**

Procura, Stewart `10
Arleo, Fontannaz, Guillet, Nguyen `14
Kaufmann, Mukherjee, Vogelsang `15
Kang, Ringer, Vitev `16
Dai, Kim, Leibovich `16

Polarized TMDFJF

TMD Fragmentation Functions (TMDFF)

Quark polarization (columns) / Hadron polarization (rows)

	U	L	T
U	$D^{h/q}$		$H^{\perp h/q}$
L		$G^{h/q}$	$H_L^{\perp h/q}$
T	$D_T^{\perp h/q}$	$G_T^{h/q}$	$H^{h/q}$ $H_T^{\perp h/q}$

TMD Fragmenting Jet Functions (TMDFJF)

Quark polarization (columns) / Hadron polarization (rows)

	U	L	T
U	$\mathcal{D}^{h/q}$		$\mathcal{H}^{\perp h/q}$
L		$\mathcal{G}^{h/q}$	$\mathcal{H}_L^{\perp h/q}$
T	$\mathcal{D}_T^{\perp h/q}$	$\mathcal{G}_T^{h/q}$	$\mathcal{H}^{h/q}$ $\mathcal{H}_T^{\perp h/q}$

Polarized TMD hadron in jet production

Actually not really good notations:

when fragmenting parton is unpolarized, $\mathcal{G}_c^h(z_h) \to \mathcal{G}_c^h(z_h, j_\perp) F_U$

" longitudinally polarized, $\Delta_L \mathcal{G}_c^h(z_h) S_L \to \Delta_L \mathcal{G}_c^h(z_h, j_\perp) F_L$

" transversely polarized, $\Delta_T \mathcal{G}_c^h(z_h) S_{hT}^i \to \Delta_T \mathcal{G}_c^h(z_h, j_\perp) F_T^i$

F_U, F_L and F_T^i can depend on $j_\perp, S_{h\perp}, \Lambda_h$ depending on the polarization of the final hadron.

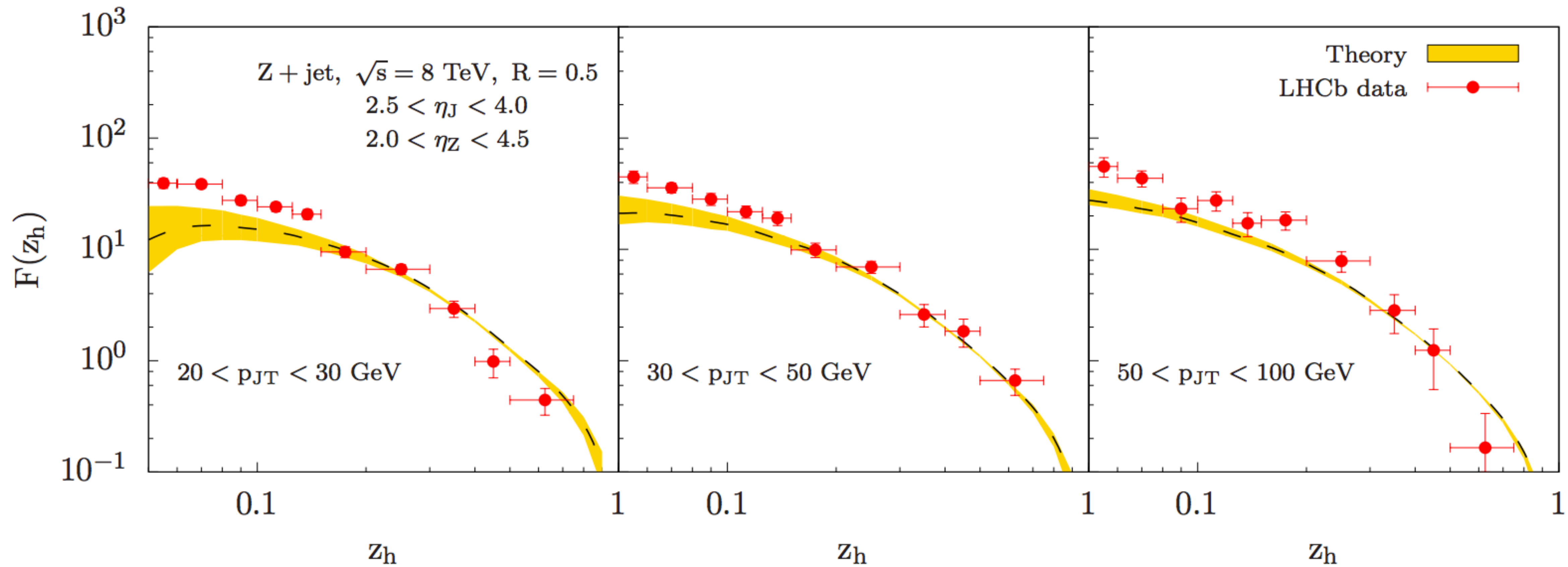
Kang, KL, Zhao, `20

Study of FJF

	$e^+e^- \to hX$	$pp \to hX$	$pp \to \text{jet}(h)X$
Gluon sensitivity	✗	✓	✓
Differential in z_h	✓	✗	✓

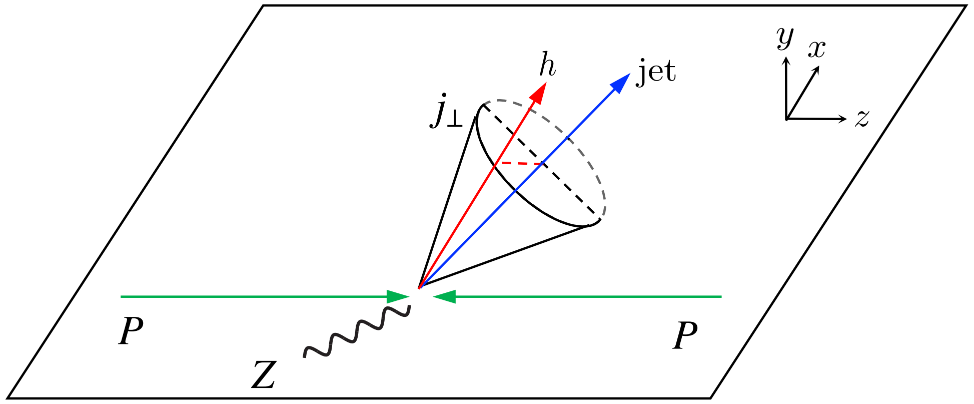
Also large amounts of data available for pp!

Z-tagged jet



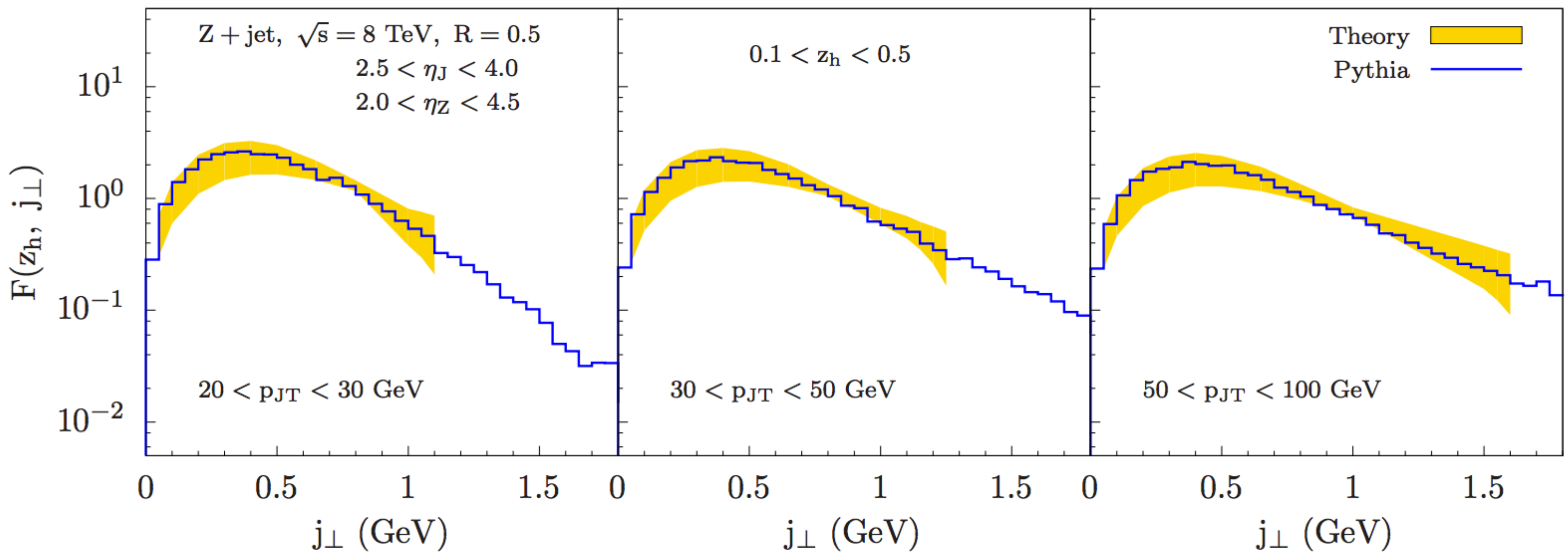
- **LHCb collaboration measured collinear FFs and $j_\perp$**

Agrees well in the $0.1 < z_h < 0.5$ region



Kang, KL, Terry, Xing `19

Z-tagged jet

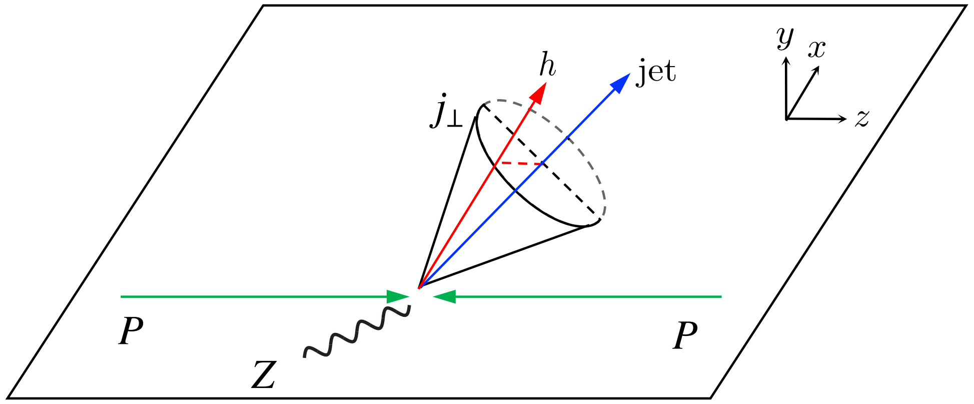


- **LHCb collaboration measured collinear FFs and** $j_\perp$

  Agrees well in the $0.1 < z_h < 0.5$ **region**

- **LHCb collaboration measured** $j_\perp$ **recently.**

  but with entire $0 < z_h < 1$



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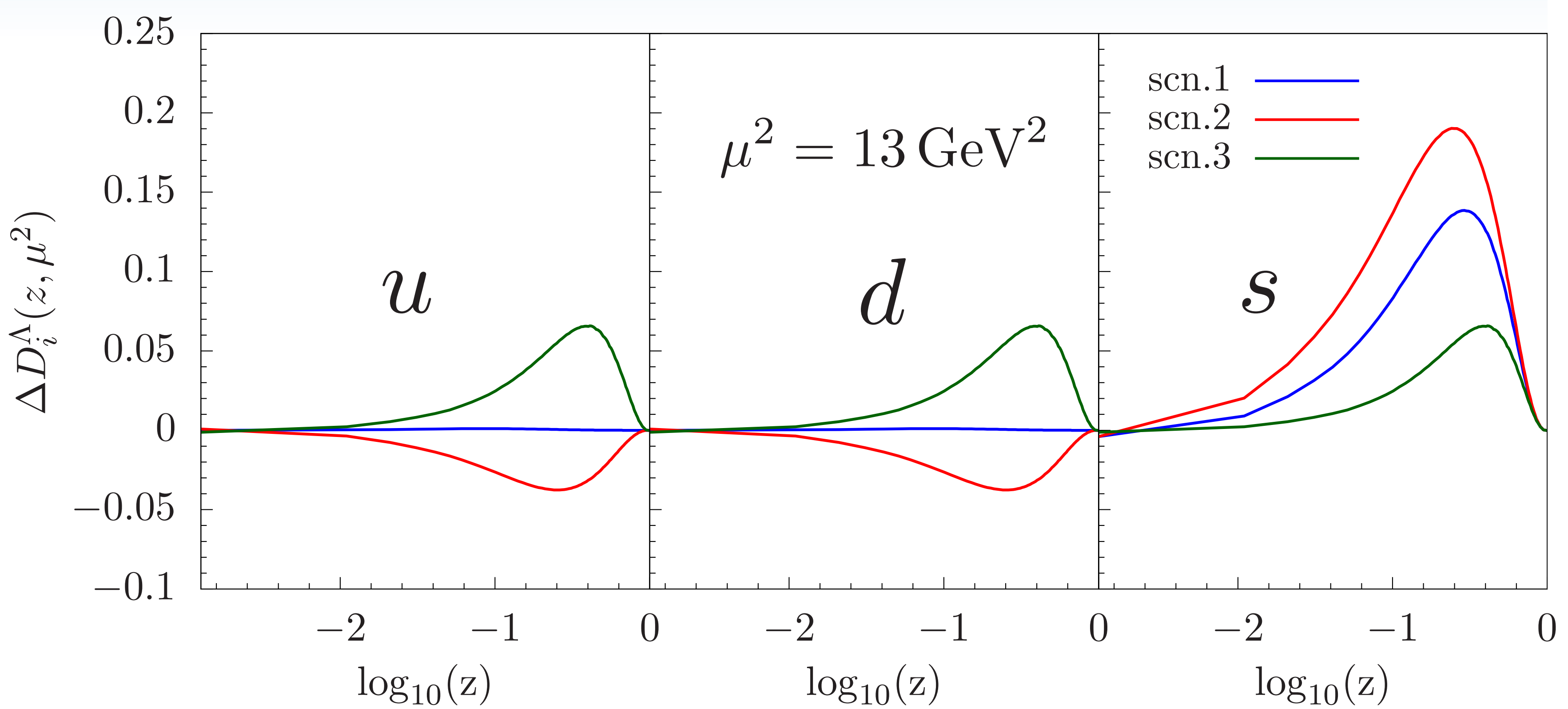
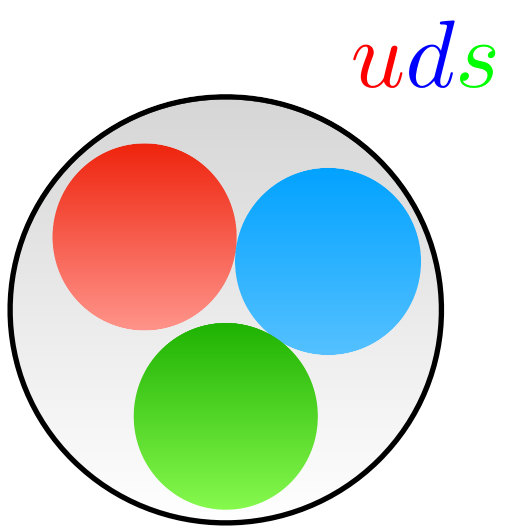
Longitudinally polarized Λ



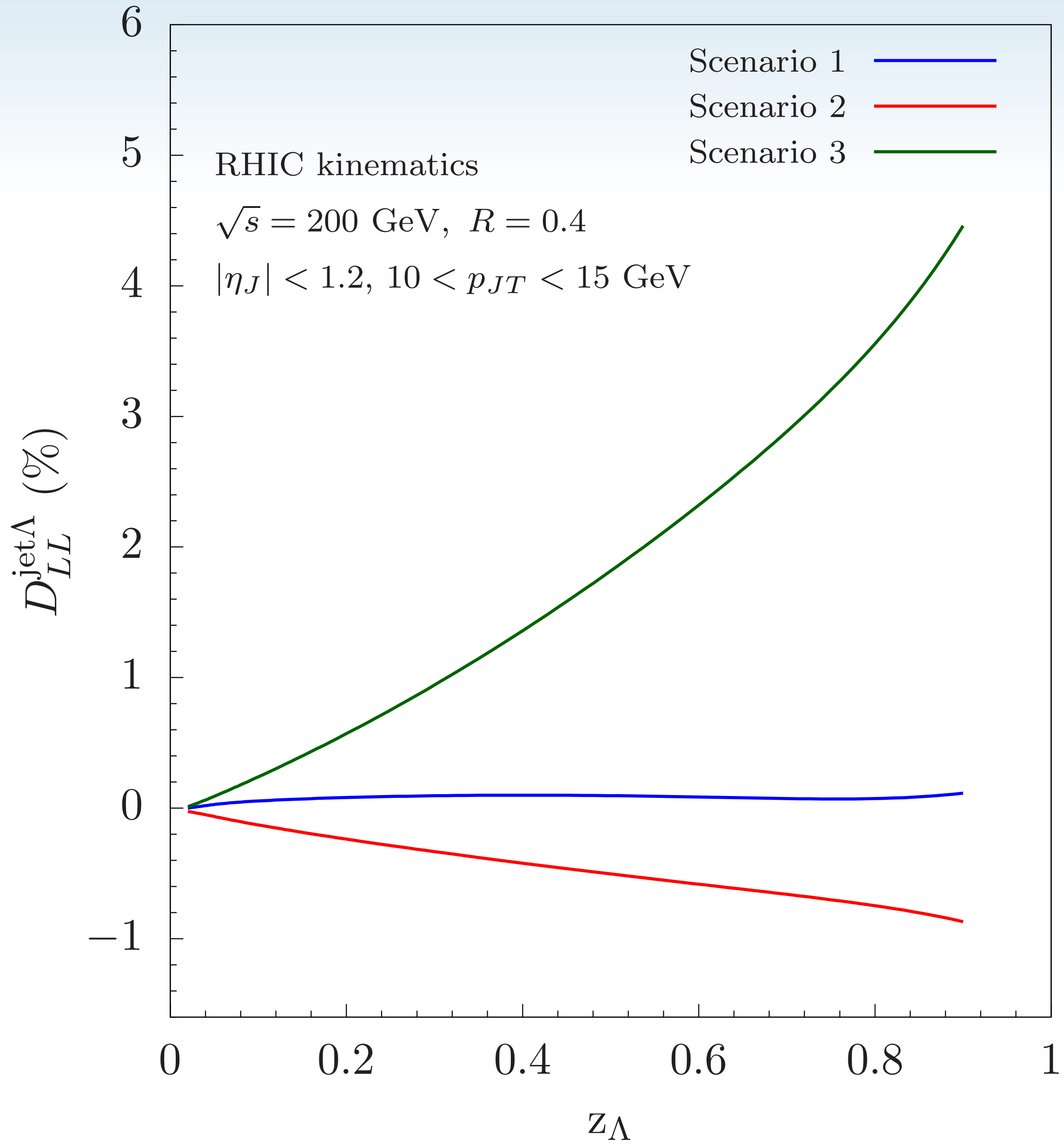


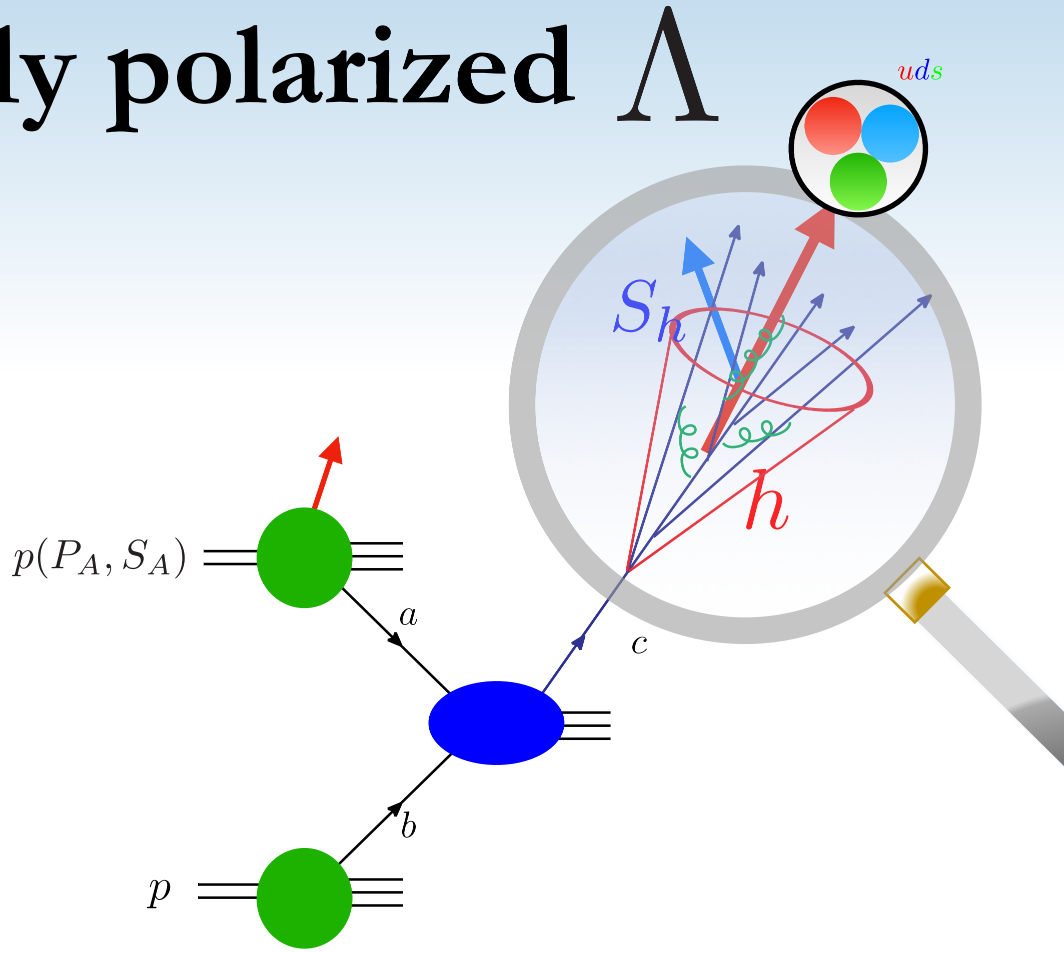
Three different scenarios

- **Scenario 1:** Only strange quarks have contribution to the fragmentation.
- **Scenario 2:** Negative distributions of up and down quarks are assumed.
- **Scenario 3:** Same fragmentation for up, down, and strange quarks.

Burkardt, Jaffe, `93
De Florian, Stratmann, Vogelsang, `98

FJF to study longitudinally polarized Λ





Gives shape expected from the scenarios

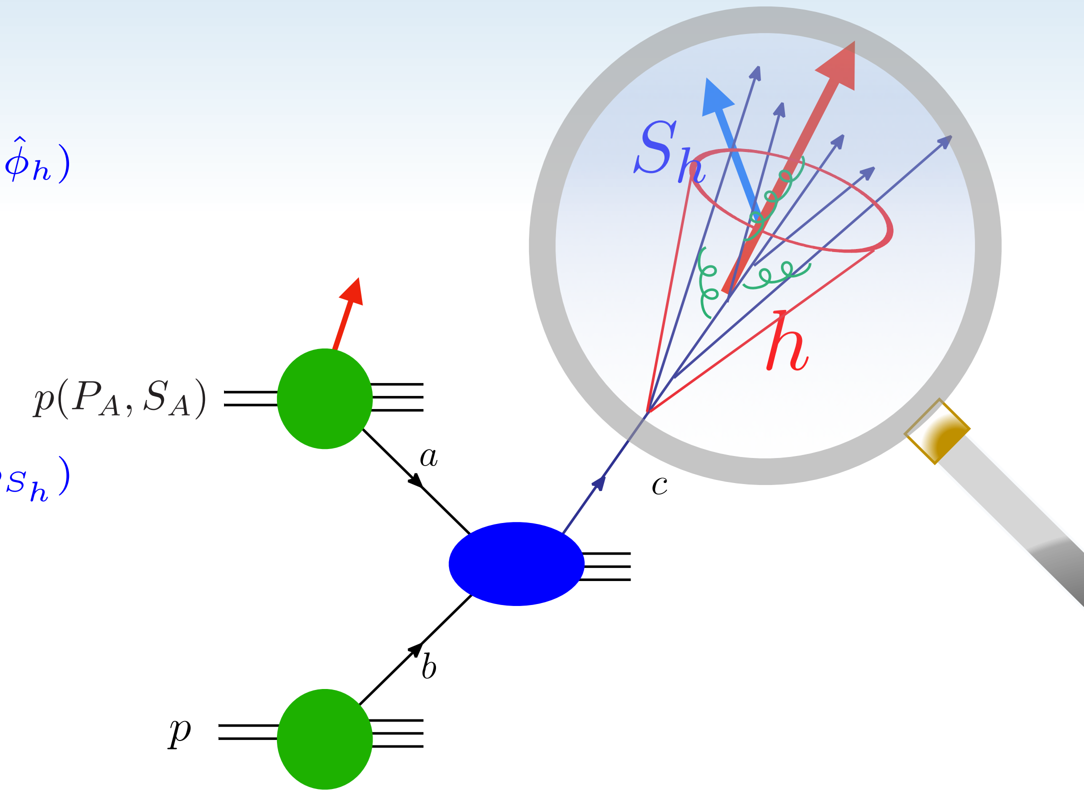
$$D_{LL}^{\text{jet}\Lambda} = \frac{d\Delta_{LL}\sigma}{d\sigma}$$

- **Scenario 1:** Only strange quarks have contribution to the fragmentation.
- **Scenario 2:** Negative distributions of up and down quarks are assumed.
- **Scenario 3:** Same fragmentation for up, down, and strange quarks.

Kang, KL, Zhao, `20

Azimuthal angular dependence

$$\frac{d\sigma^{p(S_A)+p/e\to(\text{jet}\,h(S_h))X}}{dp_{JT}d\eta_J dz_h d^2\boldsymbol{j}_\perp} = F_{UU,U} + |\boldsymbol{S}_T|\sin(\phi_{S_A}-\hat\phi_h)F_{TU,U}^{\sin(\phi_{S_A}-\hat\phi_h)}$$
$$+\Lambda_h\left[\lambda F_{LU,L} + |\boldsymbol{S}_T|\cos(\phi_{S_A}-\hat\phi_h)F_{TU,L}^{\cos(\phi_{S_A}-\hat\phi_h)}\right]$$
$$+|\boldsymbol{S}_{h\perp}|\Big\{\sin(\hat\phi_h-\hat\phi_{S_h})F_{UU,T}^{\sin(\hat\phi_h-\hat\phi_{S_h})} + \lambda\cos(\hat\phi_h-\hat\phi_{S_h})F_{LU,T}^{\cos(\hat\phi_h-\hat\phi_{S_h})}$$
$$+|\boldsymbol{S}_T|\Big(\cos(\phi_{S_A}-\hat\phi_{S_h})F_{TU,T}^{\cos(\phi_{S_A}-\hat\phi_{S_h})}$$
$$+\cos(2\hat\phi_h-\hat\phi_{S_h}-\phi_{S_A})F_{TU,T}^{\cos(2\hat\phi_h-\hat\phi_{S_h}-\phi_{S_A})}\Big)\Big\},$$



$$F_{S_A S_B, S_h}$$

Polarization of A, B, h

- **Different structures come with different characteristic angular dependence.**

Kang, KL, Zhao, `20

Thank you!