Recent results on $t\bar{t}$ cross sections at NNLL/NNLO

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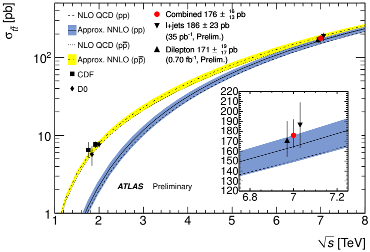
TH-LPCC Summer Institute,
CERN, August 2011

M. Beneke, PF, S. Klein, C. Schwinn work in preparation

The top-pair production cross section

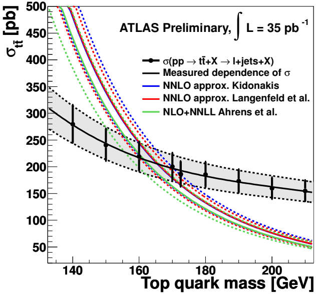
Total $t\bar{t}$ cross section measured at Tevatron with $\Delta\sigma/\sigma \sim 15\%$... **LHC is catching up quickly!**

- Atlas (0.7 fb^{-1}): 176^{+16}_{-13} ($\Delta\sigma/\sigma \sim 16\%$)
- CMS (36 pb^{-1}): 158^{+19}_{-19} ($\Delta\sigma/\sigma \sim 24\%$)



Precise measurements of $\sigma_{t\bar{t}}$ phenomenologically relevant for

- constraining **gluon PDF** in the proton (see Thorne's and Guffanti's talks)
- theoretically clean extraction of the **top quark mass**



this requires that theoretical uncertainties are under control: $\Delta\sigma^{\text{th}} \lesssim \Delta\sigma^{\text{exp}}$

$$\sigma^{\text{NLO}}_{t\bar{t}} = 162^{+25}_{-26}\ \text{pb}$$

$\Delta\sigma^{\text{NLO}}/\sigma^{\text{NLO}} \sim 30\%$ (scale+PDF+α_s)

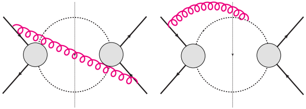
$\Rightarrow$ **need better prediction than NLO!**

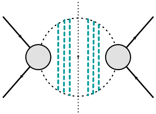
Soft-gluon and Coulomb corrections

Total NLO **partonic cross sections** enhanced near **threshold**, $\beta \equiv \sqrt{1 - 4m_t^2/\hat{s}} \to 0$

- **Threshold logarithms**: $\sim \alpha_s^n \ln^m \beta$
  $\Leftrightarrow$ **soft-gluon emission**

- **Coulomb corrections**: $\sim (\alpha_s/\beta)^n$
  $\Leftrightarrow$ **potential interactions** of non-relativistic particles





resummation of threshold logs and Coulomb singularities leads to improved predictions and reduced theoretical uncertainties

Counting scheme: $\alpha_s/\beta \sim \alpha_s \ln\beta \sim 1$

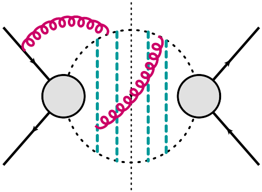
$$\hat{\sigma}_{pp'} \propto \hat{\sigma}^{(0)} \sum_{k=0} \left(\frac{\alpha_s}{\beta}\right)^k \exp\Big[\underbrace{\ln\beta\, g_0(\alpha_s \ln\beta)}_{\text{(LL)}} + \underbrace{g_1(\alpha_s \ln\beta)}_{\text{(NLL)}} + \underbrace{\alpha_s g_2(\alpha_s \ln\beta)}_{\text{(NNLL)}} + \dots\Big]$$

$$\times \left\{1\ \text{(LL,NLL)};\ \alpha_s, \beta\ \text{(NNLL)};\ \alpha_s^2, \alpha_s\beta, \beta^2\ \text{(NNNLL)}; \dots\right\}$$

Factorisation of pair production near threshold

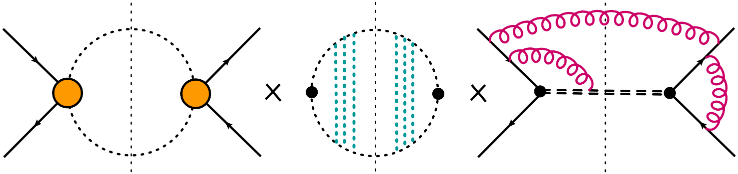
- **non-relativistic H, H' and Coulomb gluons:** $E \sim m_H\beta^2$, $|\vec{p}| \sim m_H\beta$
- **soft gluons:** $q_s \sim m_H\beta^2$

potential and soft modes have the same energy and can "talk" to each other



Effective-theory description of pair production near threshold $\hat{s} \sim (m_H + m_{H'})^2$
[Beneke, PF, Schwinn, '09/'10] $\Rightarrow$ factorization of **hard**, **soft** and **Coulomb** contributions

$$\hat{\sigma}_{pp'}(\hat{s},\mu_f) = \sum_i H_i(M,\mu_f) \int d\omega \sum_{R_\alpha} J_{R_\alpha}(E - \frac{\omega}{2})\, W_i^{R_\alpha}(\omega,\mu_f)$$



Soft/hard resummation in momentum space

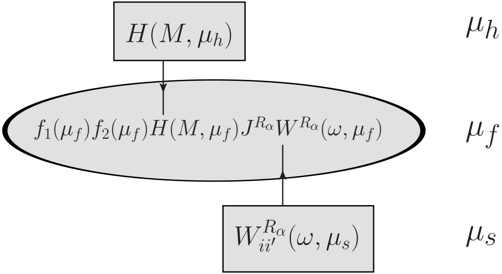
IR structure of QCD amplitudes and scale-invariance of the hadronic cross section determine **RG evolution equations for the soft function $W_i^{R_\alpha}$ and the hard function $H_i^{R_\alpha}$** (generalisation of DY result [Becher, Neubert, Xu '07] to **arbitrary** R_α)

$$\frac{d}{d\ln\mu_f} W_i^{R_\alpha}(\omega,\mu_f) = -2\left[(C_r + C_{r'})\,\Gamma_{\rm cusp}\ln\left(\frac{\omega}{\mu_f}\right) + 2\gamma_{H,s}^{R_\alpha} + 2\gamma_s^{r} + 2\gamma_s^{r'}\right] W_i^{R_\alpha}(\omega,\mu_f)$$
$$-2\,(C_r + C_{r'})\,\Gamma_{\rm cusp}\int_0^\omega d\omega'\,\frac{W_i^{R_\alpha}(\omega',\mu_f) - W_i^{R_\alpha}(\omega,\mu_f)}{\omega-\omega'}$$

and similar for hard function $H_i(M,\mu_f)$

Resummation strategy

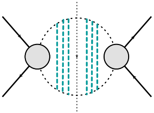
- Solve evolution equation in momentum space
- Evolve the function H_i from the hard scale μ_h to μ_f
- Evolve soft function $W_i^{R_\alpha}$ from a low scale μ_s to μ_f.



Resummation of Coulomb corrections

Resummation of Coulomb effects well understood from **PNRQCD** and quarkonia physics:

$$J_{R_\alpha}(E) = 2\mathrm{Im}\left[G^{(0)}_{C,R_\alpha}(0,0;E)\Delta_{\rm nc}(E) + G^{(1)}_{C,R_\alpha}(0,0;E) + \ldots\right]$$

$G^{(0)}_{C,R_\alpha} \quad \Leftrightarrow$ 

$$= -\frac{(2m_{\rm red})^2}{4\pi}\left\{\sqrt{-\frac{E}{2m_{\rm red}}} + \alpha_s(-D_{R_\alpha})\left[\frac{1}{2}\ln\left(-\frac{8\,m_{\rm red}E}{\mu_f^2}\right) - \frac{1}{2} + \gamma_E + \psi\left(1 - \frac{\alpha_s(-D_{R_\alpha})}{2\sqrt{-E/(2m_{\rm red})}}\right)\right]\right\}$$

Includes bound-states below threshold (E<0)

- NLO Coulomb potential: $G^{(1)}_{C,R_\alpha} = \mathcal{O}(\alpha_s^2/\beta, \alpha_s^2 \ln\beta/\beta) + \ldots$
- Non-Coulomb potentials: $\Delta_{\rm nc} = 1 + \mathcal{O}(\alpha_s \ln\beta) + \ldots$

$t\bar{t}$ production at NNLL/NNLO

All ingredients for **NNLL resummation** of $t\bar{t}$ cross section known

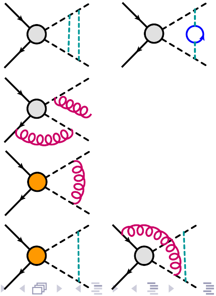
- 1-loop colour-separated hard functions $H_i^{(1)}$ [Czakon, Mitov '09]
- 2-**loop soft anomalous dimension** [Beneke, PF, Schwinn '09; Czakon, Mitov, Sterman '09]

$$\gamma_{H,s}^{R_\alpha,(1)} = C_{R_\alpha}\left[-C_A\left(\frac{98}{9}-\frac{2\pi^2}{3}+4\zeta_3\right)+\frac{20}{9}n_f\right]$$

- NLO Coulomb and non-Coulomb potentials [Beneke, Signer, Smirnov '99]

Can be used to construct approx. NNLO containing all terms singular in β
[Beneke, PF, Czakon, Mitov, Schwinn '09; HATHOR Aliev et al. '10]

$$\begin{aligned}\hat{\sigma}^{\rm NNLO}_{\rm approx.} = \;& \frac{k_{\rm LO}^2}{\beta^2}+\frac{1}{\beta}\left[k_{\rm NLO,1}\ln\beta+k_{\rm NLO,0}\right]+k_{\rm n\text{-}C}\ln\beta \\ &+c_{S,4}^{(2)}\ln^4\beta+c_{S,3}^{(2)}\ln^3\beta+c_{S,2}^{(2)}\ln^2\beta+c_{S,1}^{(2)}\ln\beta \\ &+H^{(1)}\left[c_{S,2}^{(1)}\ln^2\beta+c_{S,1}^{(1)}\ln\beta\right] \\ &+\frac{k_{\rm LO}}{\beta}\left[c_{S,2}^{(1)}\ln^2\beta+c_{S,1}^{(1)}\ln\beta+c_{S,0}^{(1)}+H^{(1)}\right]\end{aligned}$$



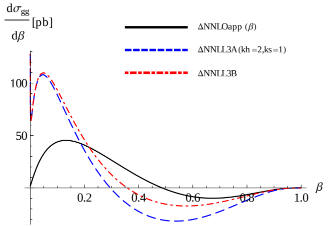
N^3LO approximations

Resummed cross sections can be re-expanded to even higher order in α_s
$\Rightarrow$ information on **residual theoretical uncertainties**

$$
\begin{aligned}
f_{gg(1)}^{(3,0)} &= 147456.\ln^6\beta - 59065.6\ln^5\beta - 286099.\ln^4\beta + 349463.\ln^3\beta \\
&+ \frac{1}{\beta}\left[121278.\ln^4\beta + 103557.\ln^3\beta - 164944.\ln^2\beta + 56418.5\ln\beta\right] \\
&+ \frac{1}{\beta^2}\left[22166.\ln^2\beta + 39012.1\ln\beta - 2876.61\right] + \tilde{f}_{gg(1)}^{(3,0)} + \mu_f - \text{dep. terms}
\end{aligned}
$$

Not all singular terms known exactly at $\mathcal{O}(\alpha_s^5)$ $\Leftrightarrow \tilde{f}_{gg(1)}^{(3,0)}$

- **$\text{N}^3\text{LO}_\text{A}$**: keep all terms generated by resummation at $\mathcal{O}(\alpha_s^5)$
- **$\text{N}^3\text{LO}_\text{B}$**: keep only terms known exactly $\rightarrow \tilde{f}_{pp'}^{(3,0)} = 0$



Matching and scales

Resumed cross sections matched to fixed-order results:

- $\text{NNLL}_1 = \text{NNLL} - \text{NNLL}(\alpha_s) + \text{NLO}$
- $\text{NNLL}_2 = \text{NNLL} - \text{NNLL}(\alpha_s^2) + \text{NNLO}_{\text{approx}}$

NNLL_1, NNLL_2 (and $\text{N}^3\text{LO}_\text{A}$) depend on several scales, μ_f, μ_h, μ_s, μ_C.

What are reasonable choices for μ_s, μ_h and μ_C?

- **Hard scale**: $\mu_h = 2m_t$
- **Coulomb scale**: set by typical **virtuality of a Coulomb gluons** $\sqrt{|q^2|} \sim m_t\beta \sim m_t\alpha_s$

$$\Rightarrow \mu_C = \max\{2m_t\beta, C_F m_t \alpha_s(\mu_C)\}$$

$\hookrightarrow$ twice inverse Bohr radius of first bound state

Soft scale determination: Method 1

Fixed soft scale chosen such that one-loop soft corrections to the **hadronic cross section** are minimised [Becher, Neubert, Xu '07]

$$\frac{d}{d\bar{\mu}_s} \sum_{p,p'} \int_{\tau_0}^{1} d\tau L(\tau, \bar{\mu}_s) \frac{\hat{\sigma}^{(1)}_{pp',\text{soft}}(\tau s, \bar{\mu}_s)}{\sigma^{(0)}_{N_1 N_2}(s, \bar{\mu}_s)} = 0$$

choice motivated by requirement of **good convergence** at the scale μ_s...

The choice of fixed scale presents problems:

- **Kinematic ambiguities**: $E = \sqrt{s} - 2m_t \Leftrightarrow m_t \beta^2$

$$\begin{aligned} \ln(m_t\beta^2/\mu_s): \quad & \tilde{\mu}_s = 35\text{GeV}(\text{Tevatron}), \quad 58\,\text{GeV (LHC7)}, \quad 65\,\text{GeV (LHC14)}, \\ \ln(E/\mu_s): \quad & \tilde{\mu}_s = 52\,\text{GeV (Tevatron)}, \quad 99\,\text{GeV (LHC7)}, \quad 120\,\text{GeV (LHC14)}, \end{aligned}$$

- A fixed soft scale does not correctly reproduce all the threshold logs at NNLO

Soft scale determination: Method 2

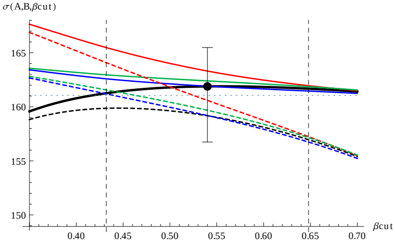
Running soft scale ($\mu_s \sim q_s \sim m_t\beta^2$) above a certain **cutoff** $\beta_{\mathbf{cut}}$

$$\mu_s^< = k_s m_t \beta_{\text{cut}}^2 \qquad\qquad \mu_s^> = k_s m_t \beta^2$$

β_{cut} chosen such that resummation ambiguities are small for $\beta < \beta_{\text{cut}}$ and pertubation theory is not spoiled by large logs for $\beta > \beta_{\text{cut}}$:

$$\hat{\sigma}_{t\bar{t}}(A_<, B_>, \beta_{\text{cut}}) = \hat{\sigma}_{t\bar{t}}^{A<}\ \theta(\beta_{\text{cut}} - \beta) + \hat{\sigma}_{t\bar{t}}^{B>}\ \theta(\beta - \beta_{\text{cut}})$$

- $A_< = \text{NNLL}_1, \text{NNLL}_2$
- $B_> = \text{NNLL}_2, \text{NNLO}_{\text{approx}}, \text{N}^3\text{LO}_\text{A}, \text{N}^3\text{LO}_\text{B}$



β_{cut} determined by minimisation of the envelope of the 8 curves

β_{cut} :	0.35 (Tevatron),	0.54 (LHC7),	0.55 (LHC14)
$\mu_s^<$:	42.5GeV (Tevatron),	101GeV (LHC7),	105GeV (LHC14)

Estimate of theoretical uncertainties

Resummation uncertainties

- β_{cut}: vary β_{cut} by $\pm 20\%$ and take width of envelope of the 8 curves
- $\mu_s = k_s m_t \beta^2$: choose default value as $k_s = 2$ and vary between $k_s = 1$ and $k_s = 4$
- power-suppressed corrections: consider difference between $E = m_t \beta^2$ and $E = \sqrt{s} - 2m_t$

Scale uncertainties

- for NLO and NNLO vary $m_t/2 < \mu_f, \mu_R < 2m_t$ with $1/2 < \mu_R/\mu_f < 2$
- for NNLL vary μ_f, μ_h, μ_C in the interval $[\tilde{\mu}_i/2, 2\tilde{\mu}_i]$

PDF+α_s uncertainty

- MSTW2008 with 90% CL sets
- $\alpha_s(M_Z) = 0.1171 \pm 0.0034$

$O(\alpha_s^2)$ constant

- choose $C^{(2)}_{pp'} = 0$ as default in $\text{NNLO}_{\text{approx}}$
- vary by $\pm {C^{(1)}_{pp'}}^2$, where $C^{(1)}_{pp'}$ is the $\mathcal{O}(\alpha_s)$ constant for the partonic channel pp'

NNLL/NNLO total $t\bar{t}$ cross section

$m_t = 173.3$ GeV, $\mu_f = m_t$, MSTW2008NLO/NNLO

Beneke, PF, Klein, Schwinn, **PRELIMINARY!**

$\sigma_{t\bar{t}}$[pb]	Tevatron	LHC@7	LHC@14
NLO	$6.68^{+0.36+0.51}_{-0.75-0.45}$	$158.1^{+18.5+13.9}_{-21.2-13.1}$	$884^{+107+65}_{-106-58}$
NLL_2	$7.31^{+0.39+0.57}_{-0.53-0.54}$	$172.8^{+19.7+15.9}_{-14.7-14.6}$	$954^{+107+73}_{-71-66}$
NNLO_{app}	$7.06^{+0.25+0.69}_{-0.33-0.53}$	$161.1^{+11.4+15.2}_{-10.9-14.5}$	891^{+71+64}_{-63-63}
NNLL_2	$7.22^{+0.29+0.71}_{-0.46-0.55}$	$162.6^{+5.7+15.4}_{-5.9-14.7}$	896^{+29+65}_{-24-64}
BST	$0.014^{+0.011+0.005}_{-0.009-0.004}$	$0.67^{+0.33+0.12}_{-0.31-0.10}$	$3.1^{+1.7+0.5}_{-1.6-0.4}$

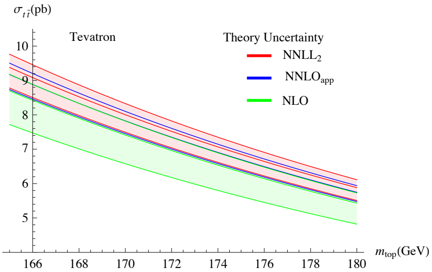
Experiment ($m_t = 172.5$):

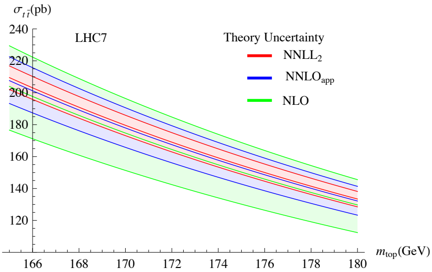
- **CDF**: 7.5 ± 0.48
- **D0**: $7.56^{+0.63}_{-0.56}$
- **Atlas**: 176^{+16}_{-13}
- **CMS**: 158 ± 19

Theory:

- Ahrens et al, 2011, NNLO($m_t = 173.1$):
  Tevatron: $6.63^{+0.007+0.63}_{-0.41-0.43}$
  LHC: 155^{+8+14}_{-9-14}
- Kidonakis, 2011, NNLO($m_t = 173$):
  Tevatron: $7.08^{+0.00+0.36}_{-0.24-0.27}$
  LHC: 163^{+7+9}_{-5-9}

Residual theoretical uncertainties





- strong reduction of theoretical uncertainties from NLO to NNLO
- reduction of theoretical uncertainties from NNLO to NNLL at LHC (but not at Tevatron: $q\bar{q}$ VS gg???)

Tevatron : $\Delta\sigma^{\text{th, NNLL}}/\sigma \sim 10.5\%$

$+ \sim 18\%$ from PDF+α_s uncertainty

LHC : $\Delta\sigma^{\text{th, NNLL}}/\sigma \sim 7\%$

Including estimate of unknown $O(\alpha_s^2)$ constant ($\sim (C^{(1)})^2$):

Tevatron : $\Delta\sigma^{\text{th, NNLL}}/\sigma \sim 12.9\%$

LHC : $\Delta\sigma^{\text{th, NNLL}}/\sigma \sim 11.7\%$

Summary

- Fixed-order prediction for $t\bar{t}$ production improved by threshold resummations
  - $\Rightarrow$ factorization of **hard**, **soft** and **Coulomb** modes in PNRQCD+SCET
  - $\Rightarrow$ **simultaneous resummation** of threshold logarithms and Coulomb singularities in momentum space
- Size of threshold term at Tevatron and LHC
  - $\Rightarrow$ **NNLO** corrections $\sim 13\%$ at Tevatron and $\sim 8\%$ at LHC
  - $\Rightarrow$ **NNLL** corrections beyond NNLO small at both Tevatron and LHC ($\lesssim 2\%$)
- Remaining theoretical uncertainties
  - $\Rightarrow$ resummation ambiguities: $\sim \pm 4\%$
  - $\Rightarrow$ scale uncertainty: $\sim 3 - 6\%$
  - $\Rightarrow$ unknow constant term at $O(\alpha_s^2)$: $\sim 1 - 3\%$
  - $\Rightarrow$ PDF+α_s uncertainty: $\sim \pm 9\%$

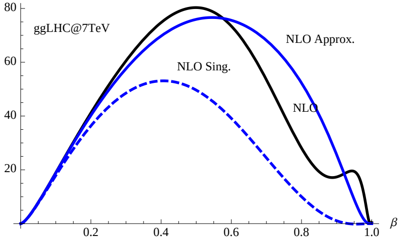
Backup slides

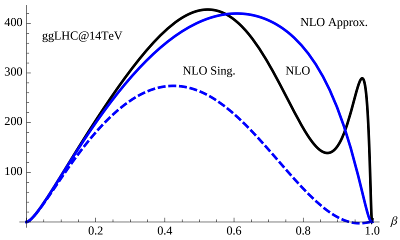
Contribution of threshold-enhanced terms

At LHC $\sqrt{s} \gg 2m_t \Rightarrow$ **How good is the threshold approximation?**
can study the approximation at the NLO level...

Plot $8\beta m_t^2/(s(1-\beta^2)^2)\mathcal{L}_{gg}(\beta)\hat{\sigma}_{tt}(\beta)$:

- **NLO**: exact NLO result
- **NLO sing.**: only singular terms in β
- **NLO approx.**: singular terms + O(1) term ($\Leftrightarrow H_i^{(1)}$)



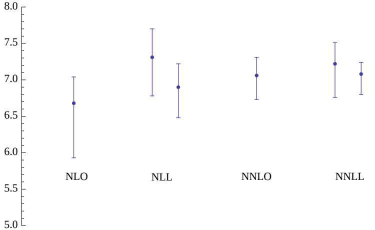


NLO sing. is good approximation only up to $\beta \sim 0.3$
However: expect NNLO approximation to be better (more singular terms at $O(\alpha_s^2)$...)

Fixed soft scale VS Running soft scale

$\sigma_{t\bar{t}}$[pb]	Tevatron	LHC@7	LHC@14
NLO	$6.68^{+0.36(5.3\%)}_{-0.75(11\%)}$	$158.1^{+18.5(12\%)}_{-21.2(13\%)}$	$884^{+107(12\%)}_{-106(12\%)}$
NLL_2	$7.31^{+0.39(5.3\%)}_{-0.53(7.3\%)}$	$172.8^{+19.7(11\%)}_{-14.7(12\%)}$	$954^{+107(11\%)}_{-71(7.4\%)}$
NLL_2(fixed μ_s)	$6.90^{+0.32(4.6\%)}_{-0.42(6.1\%)}$	$157.6^{+23.3(15\%)}_{-20.2(13\%)}$	$879^{+136(15\%)}_{-113(13\%)}$
NNLO_{app}	$7.06^{+0.25(3.5\%)}_{-0.33(4.7\%)}$	$161.1^{+11.4(7.1\%)}_{-10.9(6.8\%)}$	$891^{+71(8.0\%)}_{-63(7.1\%)}$
NNLL_2	$7.22^{+0.29(4.0\%)}_{-0.46(6.4\%)}$	$162.6^{+5.7(3.5\%)}_{-5.9(3.6\%)}$	$896^{+29(3.2\%)}_{-24(2.7\%)}$
NNLL_2(fixed μ_s)	$7.08^{+0.16(2.3\%)}_{-0.28(3.6\%)}$	$157.4^{+10.3(6.5\%)}_{-3.6(2.3\%)}$	$868^{+69(7.9\%)}_{-21(2.4\%)}$

Tevatron



LHC@7TeV