Conformal Field Theories
as
Building Blocks of Nature

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Suppose LHC fails...

to find physics beyond the Standard Model Higgs...

We will need a good exit strategy

Exit strategy 1

go anthropic

However, likely to become crowded...

Exit strategy 2 (this talk):

become a condensed matter physicist

Particle physics = condensed matter physics of the vacuum

But beware the opinion of condensed matter physicists:

perturbation theory is boring

This is a talk about

strongly coupled,

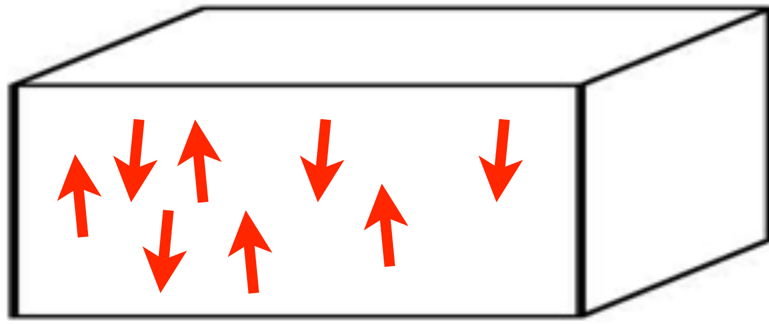
conformally invariant dynamics

And how it can be relevant for

- condensed matter
- statistical mechanics
- quantum gravity
- Beyond the Standard Model (jokes aside)

Scale Invariance

Ferromagnet (Ising Model)



$T \approx T_c$

Magnetization $\vec{M}(x)$

High temperature
T>Tc

$$\langle M(x) \rangle = 0$$

$$\langle M(x)M(0) \rangle \sim \frac{1}{|x|^{2\Delta}} \times \exp(-|x|/\xi(T))$$

Critical point T→Tc

$$\xi(T) \to \infty$$

$$\langle M(x)M(0) \rangle \sim \frac{1}{|x|^{2\Delta}}$$

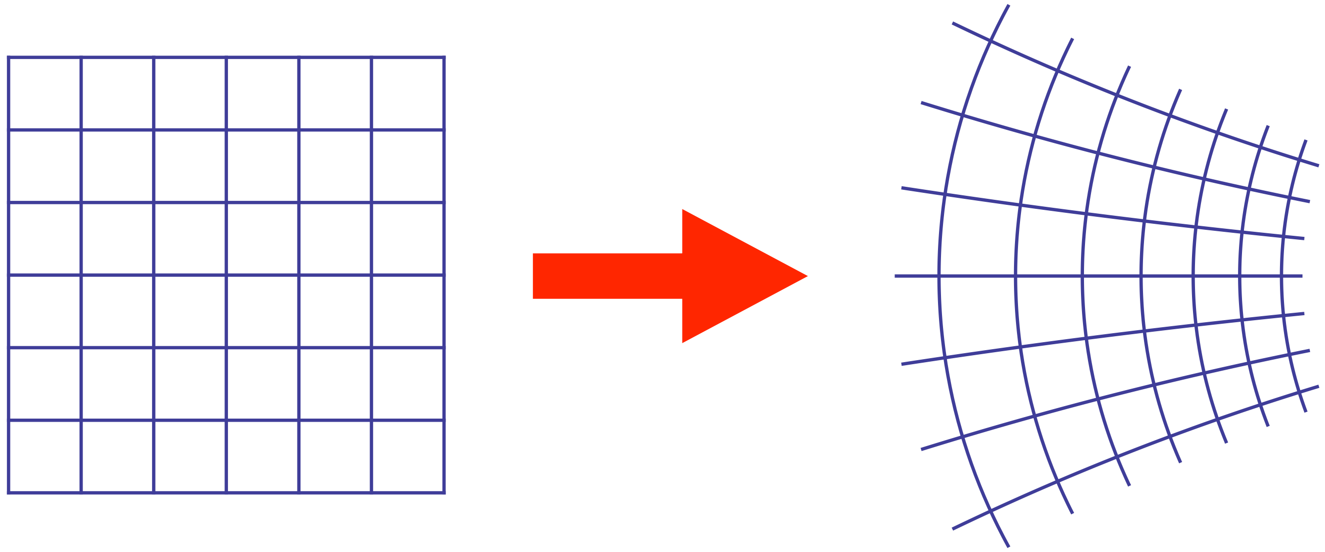
Scale invariance $x \to \lambda x$

Conformal Invariance

- emergent at the critical point

Conformal transformation

$$\delta_\kappa x_a = 2(\kappa \cdot x)x_a - x^2 \kappa_a$$



(preserves orthogonality of coordinate grid; locally looks like dilation)

Why this extra symmetry?

- not yet fully understood

Generically but *not always* true

Polchinski 1988, Dorigoni, S.R. 2009,
El-Showk, Nakayama, S.R. 2011, Antoniadis, Buican 2011
Fortin, Grinstein, Stergiou 2011

Power of conformal symmetry

e.g. *constrains 3-point correlation functions*

ε(x) energy density field in 3D Ising model

2-point correlator $\langle \epsilon(x)\epsilon(0)\rangle \sim \frac{1}{|x|^{2\Delta}}$ (scale inv.)

$\Delta = 1.412(1)$ (experiment+theory+numerics)

3-point correlator

scale ⇒ $\langle \epsilon(x)\epsilon(y)\epsilon(0)\rangle \sim \sum_{a+b+c=3\Delta} \frac{f_{abc}}{|x-y|^a |x|^b |y|^c}$

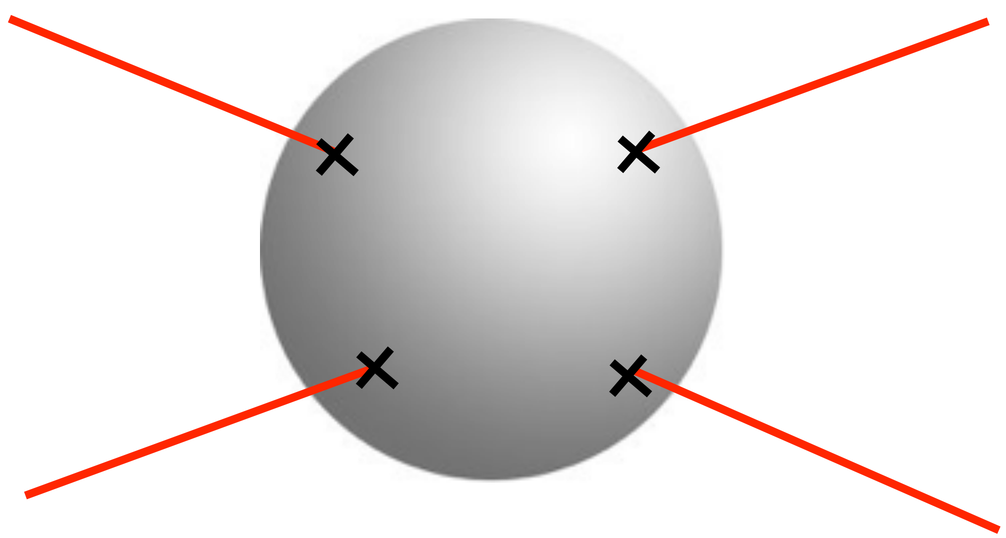
conformal ⇒ $\langle \epsilon(x)\epsilon(y)\epsilon(0)\rangle \sim \frac{1}{|x-y|^\Delta |x|^\Delta |y|^\Delta}$

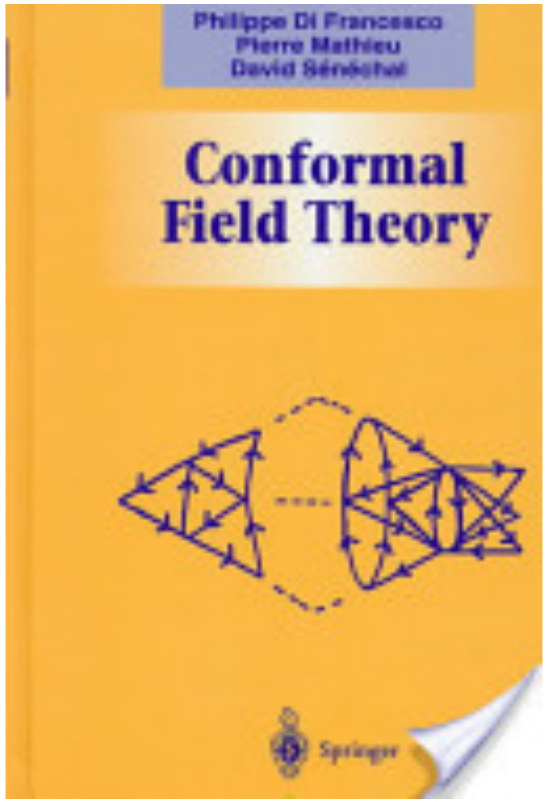
[Polyakov 1970]

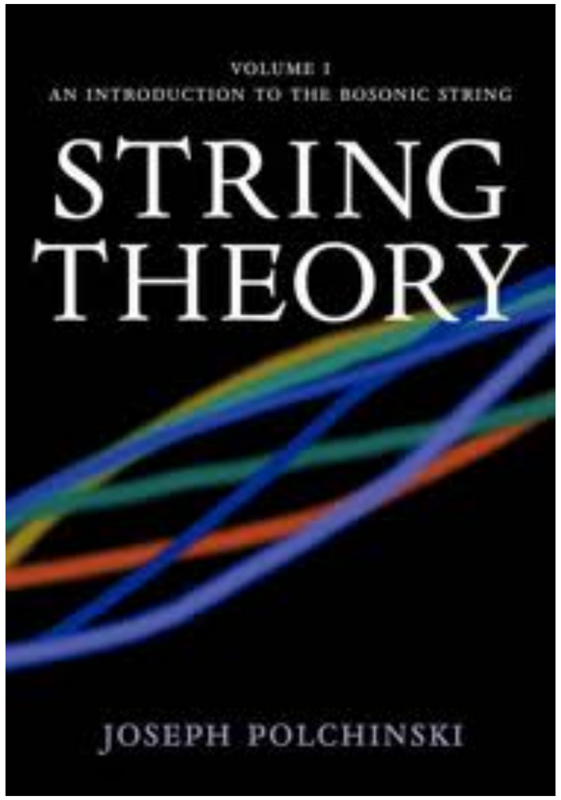
A success story - 2D CFT

[Belavin,Polyakov,Zamolodchikov 1984]

- Conformal symmetry is infinite-dimensional $z \to f(z)$
- Lots of exactly solvable models (2D Ising,...)
- Applications to worldsheet perturbative string theory







911pp

422 pp+552 pp

More 2D CFT than most of us can absorb in a lifetime

In contrast, no standard textbook on CFTs in D ≥ 3

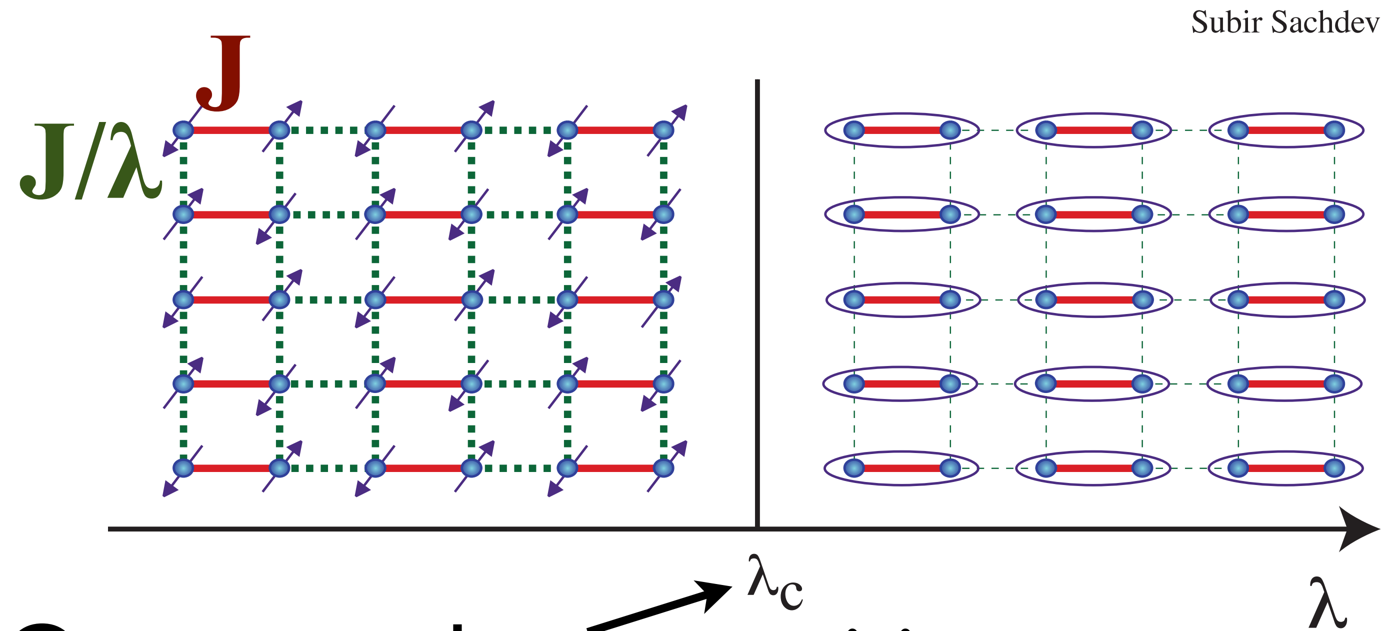
Physics reasons to think about CFTs in D$\geq$3

- Statistical Mechanics
- Quantum criticality
- Quantum Gravity in AdS
- Hierarchies in particle physics

Quantum Criticality

Condensed Matter systems at T=0: $\mathcal{H}=\mathcal{H}$(control params)

Example:
anisotropic 2D
antiferromagnet



Quantum phase transition
(Néel-dimer; 3D Ising universality class)

In general:

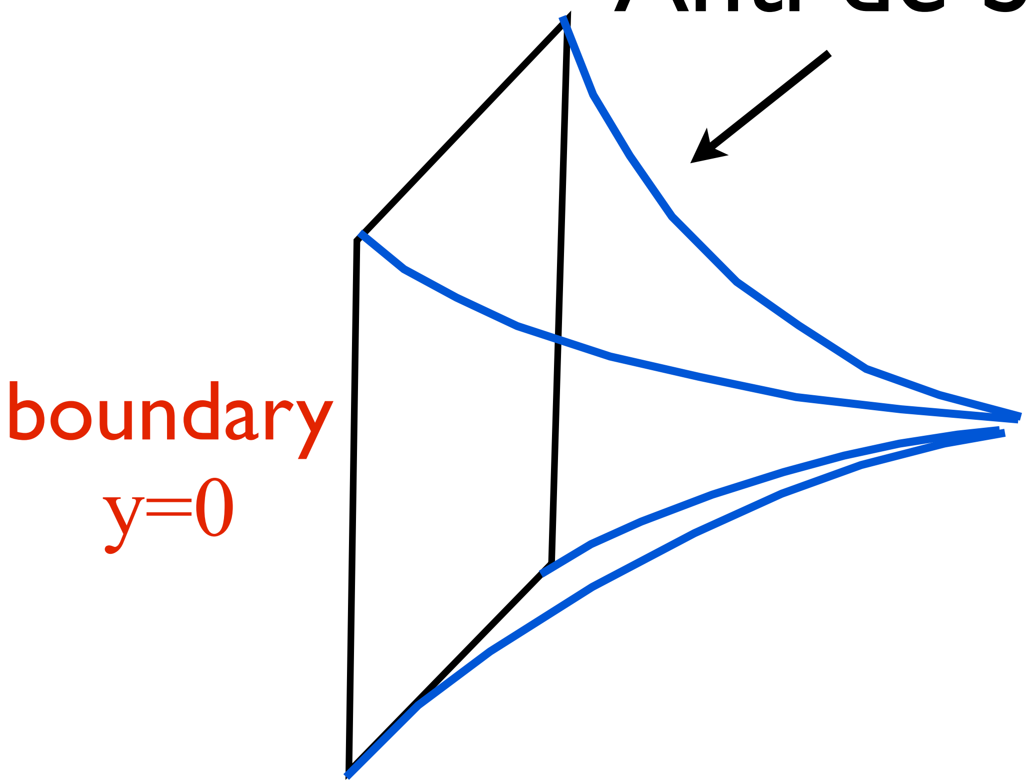
D-dim quantum system at T=0 ⇔ (D+1)-dim QFT

- Request for more D ≥ 3 CFTs
- Also with fermionic excitations

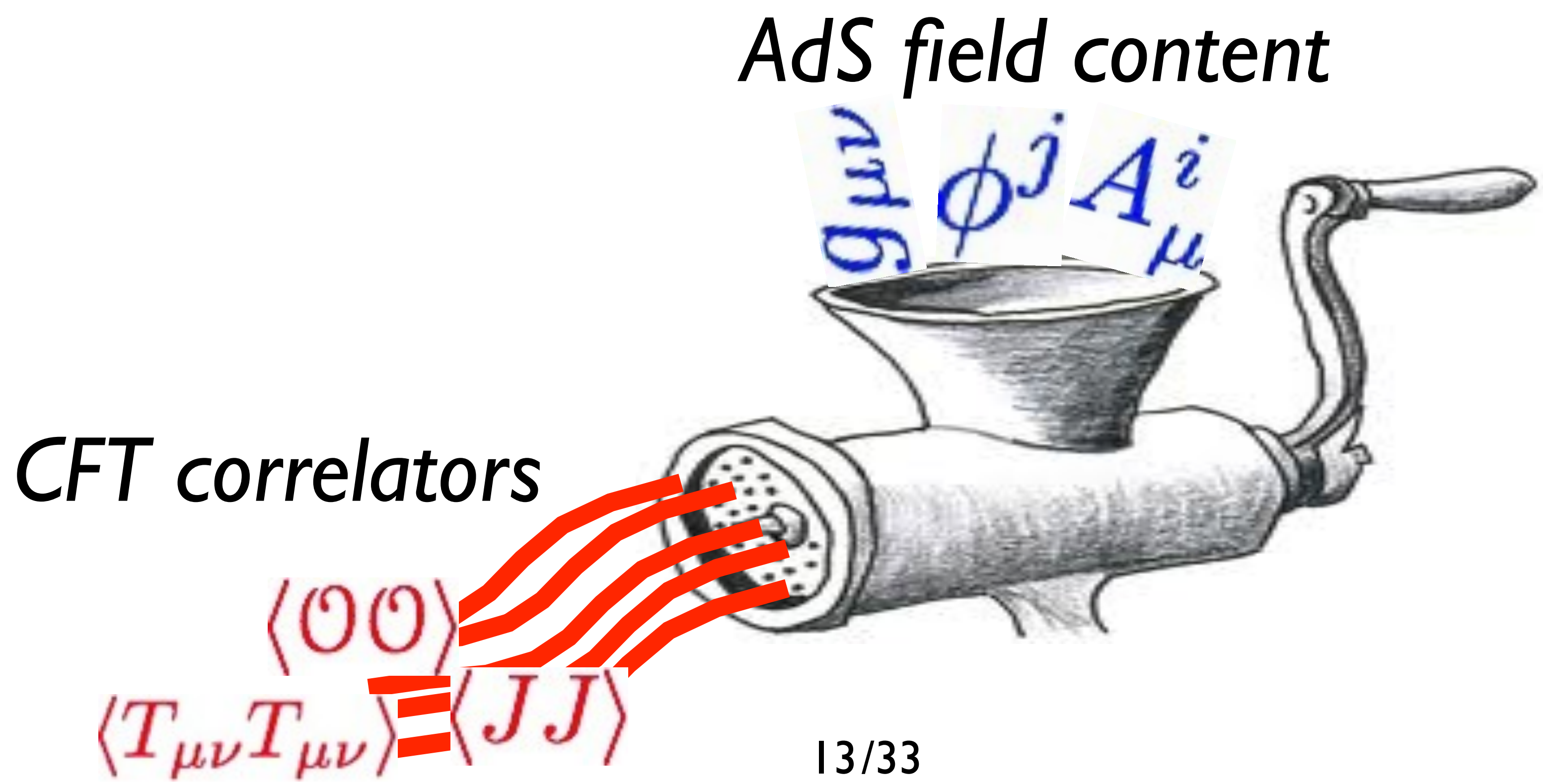
AdS/CFT

[Maldacena; Gubser, Klebanov, Polyakov; Witten]

Anti de Sitter $ds^2 = \frac{R^2}{y^2}(dx_\mu^2 + dy^2)$



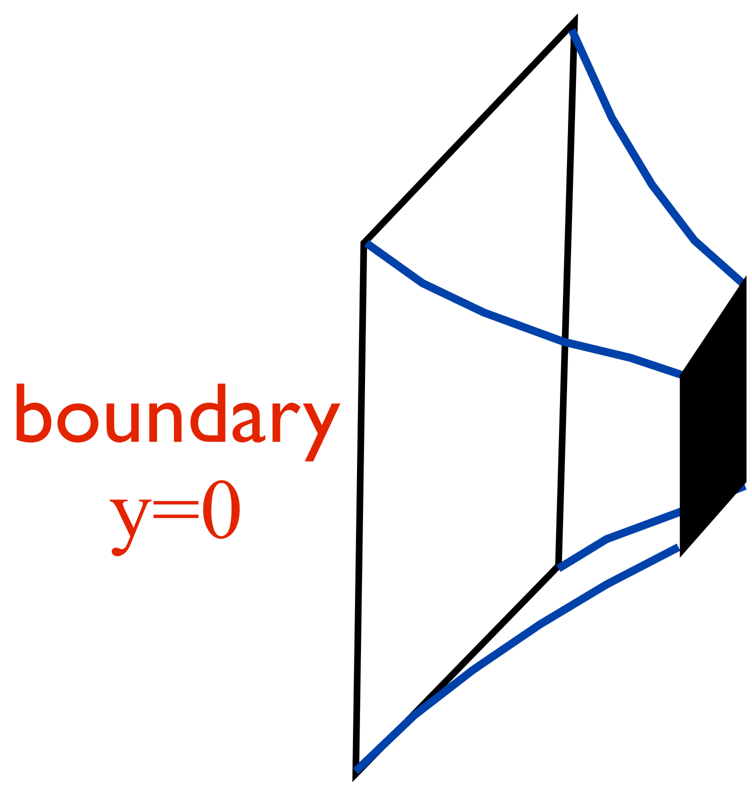
Gravity+other fields in AdS_{D+1}
$\Leftrightarrow$ CFT on D-dim boundary



Advantages of AdS/CFT

- Flexibility of the operator content (e.g. Bose/Fermi)

- Easy to go to T>0: put Black Hole in AdS



Black hole horizon

$$T = T_{\text{Hawking}}$$

$\Rightarrow$ can study transport properties

Limitations of AdS/CFT

- Factorization of operator dimensions

$$\mathcal{O}_1(x) \times \mathcal{O}_2(0) \to \mathcal{O}_1\mathcal{O}_2 \qquad \Delta \approx \Delta_1 + \Delta_2$$

Cf. 3D Ising model:

$$M \times M \to \varepsilon \qquad \begin{aligned} \Delta_M &= 0.52 \\ \Delta_\varepsilon &= 1.4 \not\approx 2\Delta_M \end{aligned}$$

⇒ not every CFT has an AdS dual

UV completion issue

Gravity theory in AdS is not UV complete

→ UV complete in string theory (as for $\mathcal{N}$=4 SYM)

Or better: Think of it as an *Effective Field Theory*,

valid up to some high energy scale, $M_{\mathrm{Pl}} \gg R^{-1}$

→ Field theory on the boundary is 'not quite' CFT

'Effective CFT' describing operators up to $\Delta=\Delta_c \approx RM_{Pl} >> 1$

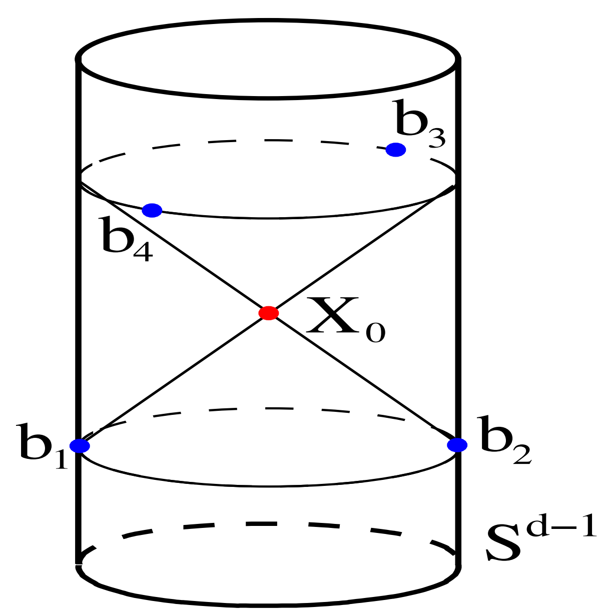
[Fitzpatrick, Katz,Poland,Simmons-Duffin '10]

UV-completing theory of gravity in AdS ⟷ Completing Effective CFT on the boundary

Quantum Gravity problem in AdS is mapped into a better-defined problem about boundary CFTs

Example 1

Graviton 2→2 S-matrix can be extracted from CFT $T_{\mu\nu}$ 4-point function *(if you know it)*



[Gary,Giddings,Penedones '09]

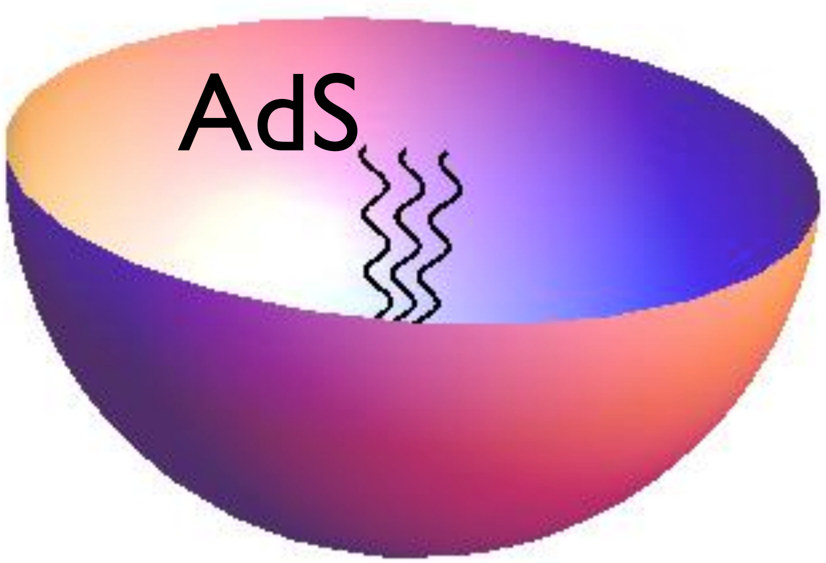
UV-completing theory of gravity in AdS

$\leftrightarrow$

Completing Effective CFT on the boundary

Example 2

Can get constraints on Quantum Gravity spectrum from CFT consistency [Helleman '09]



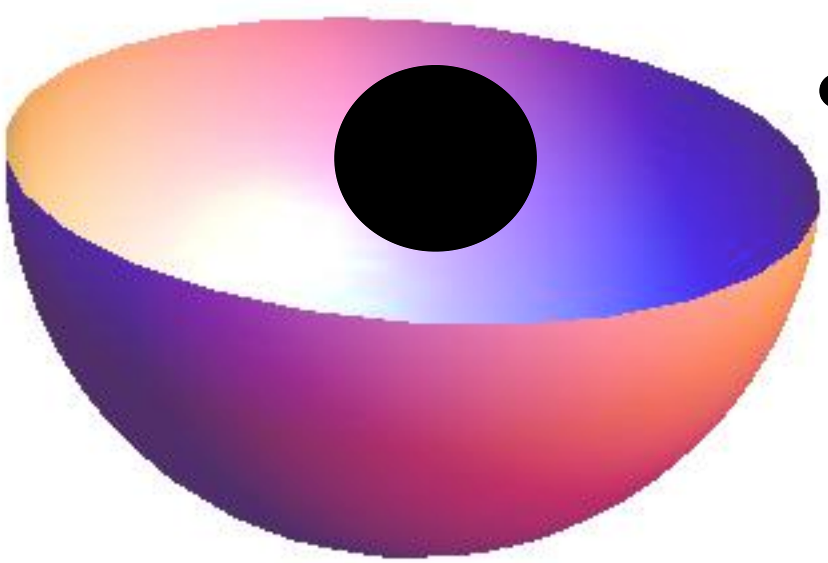
CFT boundary

any theory of quantum gravity must contain gravitons (dual to $T_{\mu\nu}$ in CFT)

UV-completing theory of gravity in AdS ⟷ Completing Effective CFT on the boundary

Example 2

Can get constraints on Quantum Gravity spectrum from CFT consistency [Hellerman '09]



- Can show that CFT must have more operators (not just $T_{\mu\nu}$)
- These are interpreted as dual to quantum black holes (mass ~M_{Pl})

Reasons to think about CFTs in D$\geq$3

- Quantum criticality
- Quantum Gravity in AdS
- Hierarchies in particle physics

Is Standard Model a CFT?

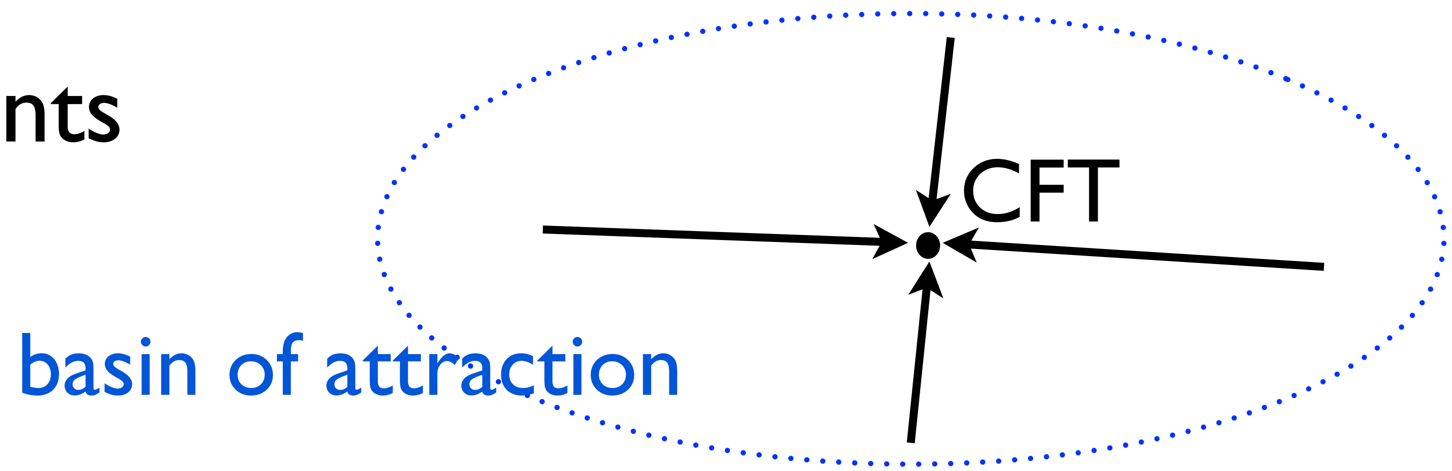
- Standard Model contains massive particles
- in particular several **very** massive particles
  W,Z,top,Higgs? - masses O(100 GeV)

↳ not a CFT

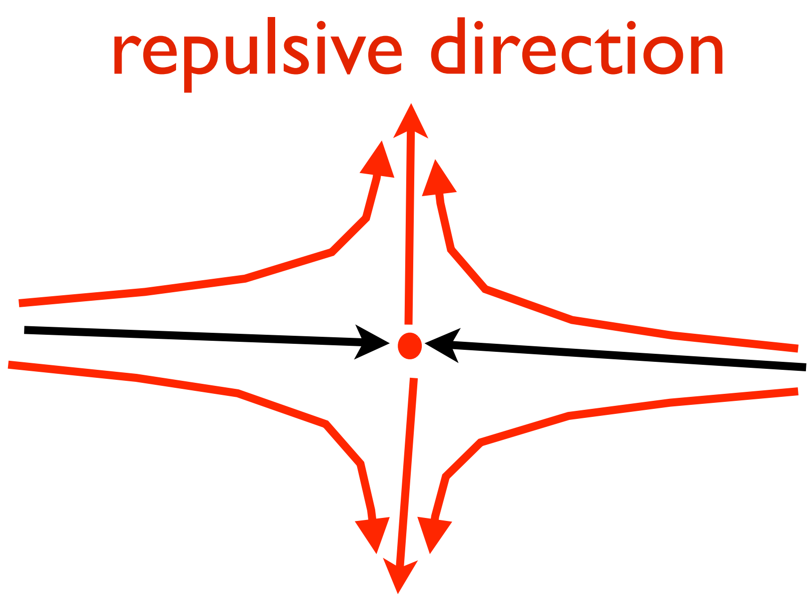
But at E >> 100 GeV (e.g. at LHC)
Standard Model ≈ CFT
(free theory perturbed by slowly running weak couplings)

Two types of CFTs

1) **Stable** IR fixed points
of RG flows



2) **Unstable** IR fixed points



⇔ scalar operator of dimension <D

Life in such a fixed point needs an 'experimentalist' adjusting the control knobs

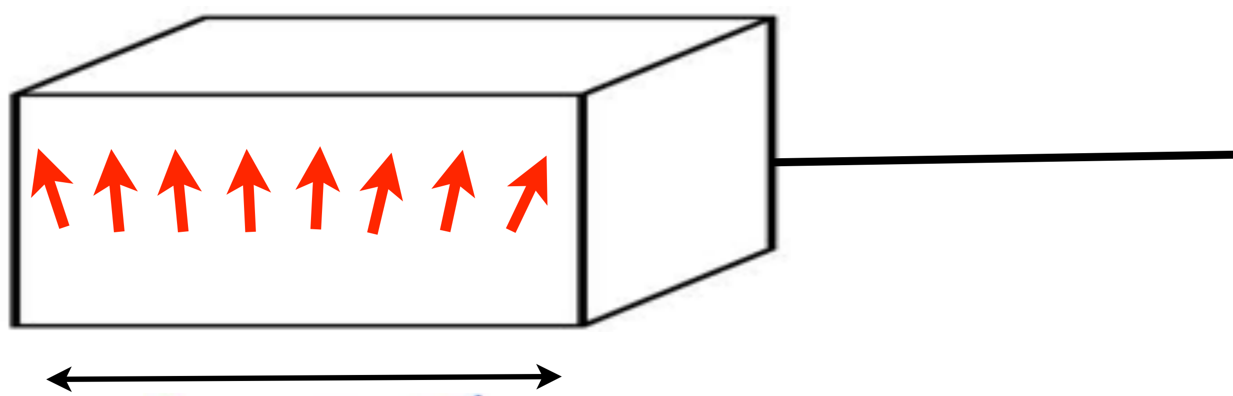
E.g. for 3D Ising $\Delta_\varepsilon = 1.4 < 3 \Rightarrow$ need temperature adjustment

In Standard Model, there is one such **repulsive** scalar operator

$$\mathcal{O} = |H|^2, \qquad \Delta = 2 < 4$$

$\Rightarrow$ Standard Model as a CFT is **unstable**

Hierarchy Problem



$$\xi \sim m_H^{-1}$$

Experimenter

Conformal Technicolor [Luty,Okui '04]

(Walking TC on steroids)

Imagine that:

At energies E>>TeV the Higgs sector
is *a strongly coupled CFT* such that

1) $|H|^2$ has dimension **above** 4

↳ no hierarchy problem

2) H has dimension **close** to $\Delta_{\text{free}}=1$

↳ Yukawa couplings $y\bar{\psi}\psi H$
are near-dimensionless

How close?

Depends on assumptions about theory of flavor.

$\Delta_H<1.5$ under most optimistic assumptions

Do we know such a CFT?

No

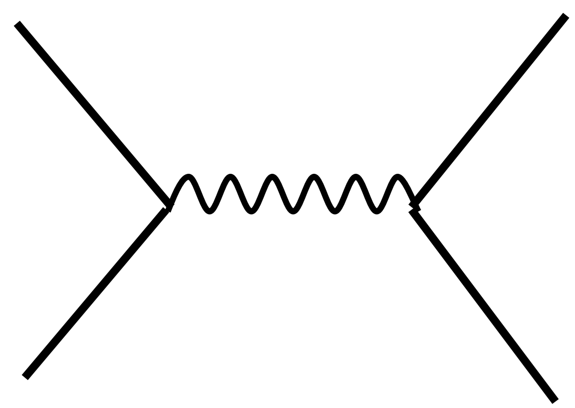
Do we know of any reason which could preclude the existence of such a CFT?

Yes: Crossing symmetry constraint

Scattering amplitudes in weakly coupled theory

(Feynman diagrams)

$$\mathcal{M}(e^+e^- \to e^+e^-) =$$



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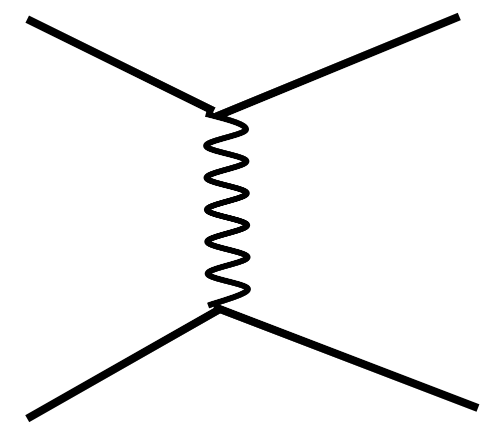
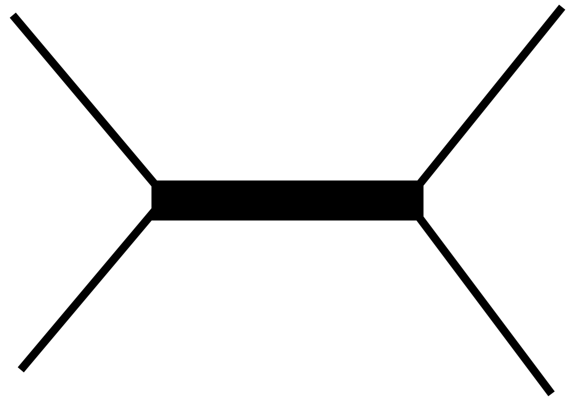


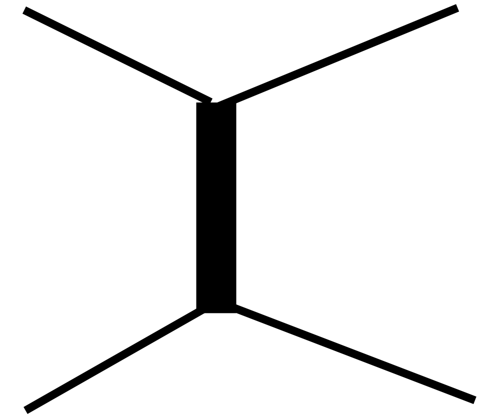
Correlation functions in CFT

(conformal block expansion)

$$\langle H H^\dagger H H^\dagger \rangle = \sum_{\mathcal{O}}$$

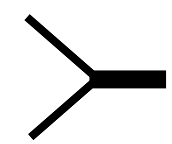


$$= \sum_{\mathcal{O}}$$



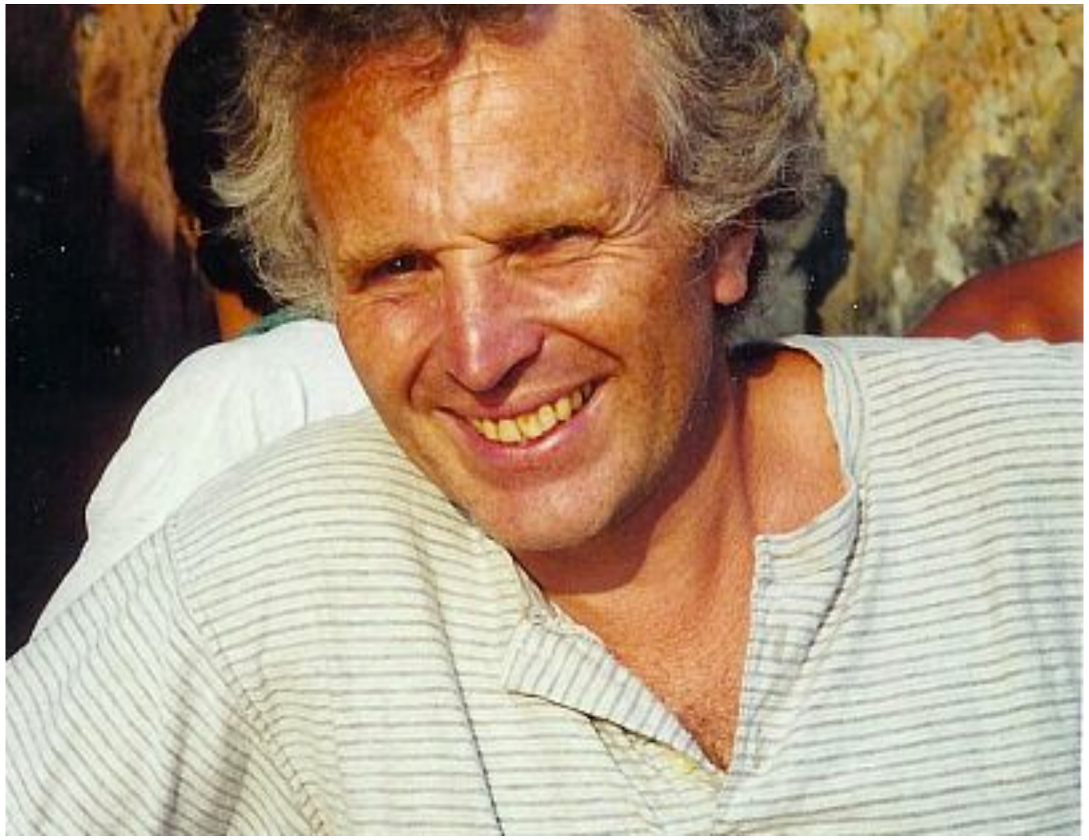
crossing symmetry (duality)

Functional equation for 'CFT data'

(≡ dimensions of operators O and 'couplings' )

Bootstrap hypothesis : this equation should be enough to fix the CFT [Polyakov 1974]

Baron von Münchhausen (1720-1797)



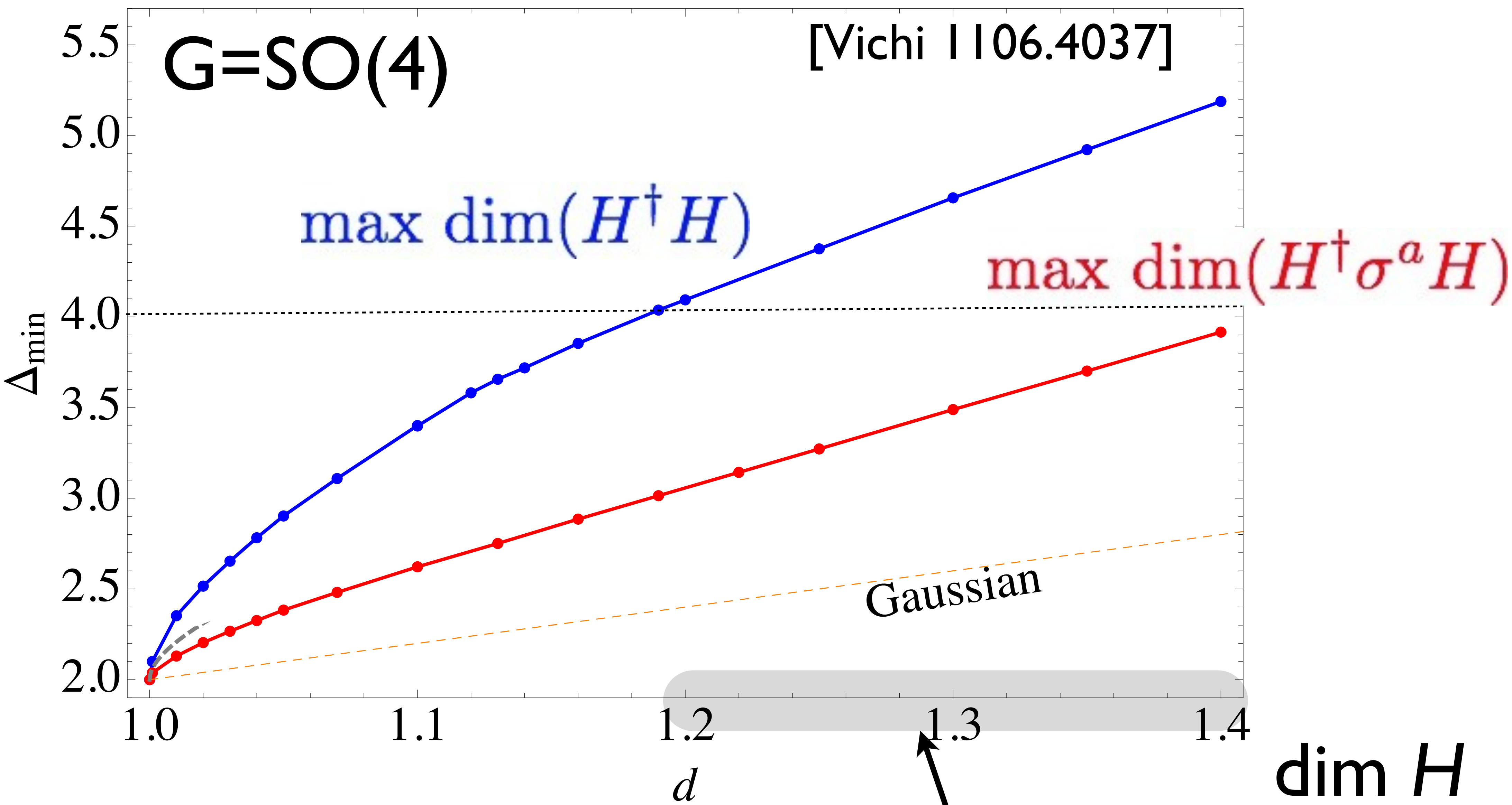
Sasha Polyakov

Swamp = Space of CFT data

Hair = Conformal Block Expansion

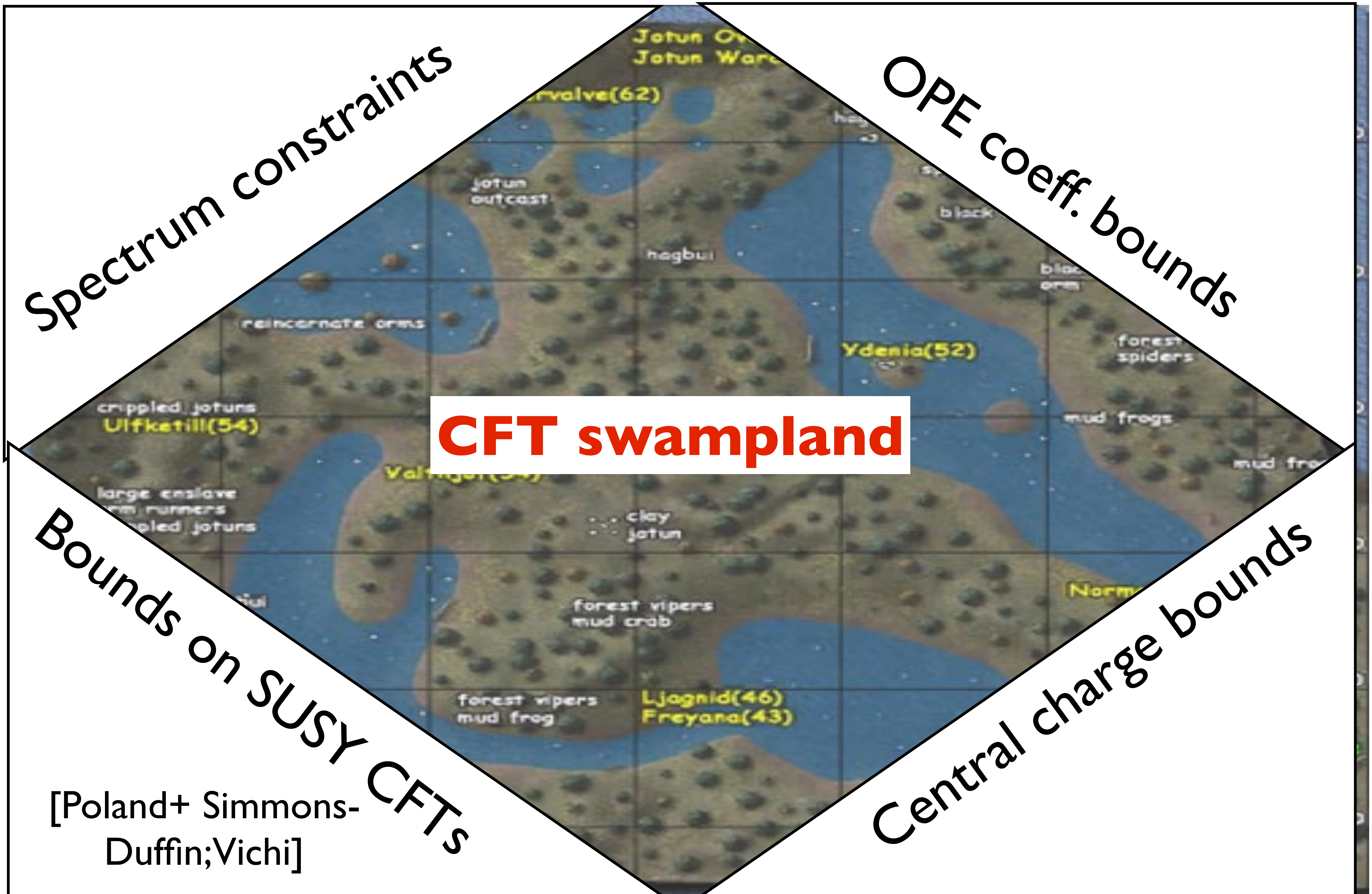
Spectrum constraints

[Rattazzi, S.R., Tonni, Vichi 2008,…]



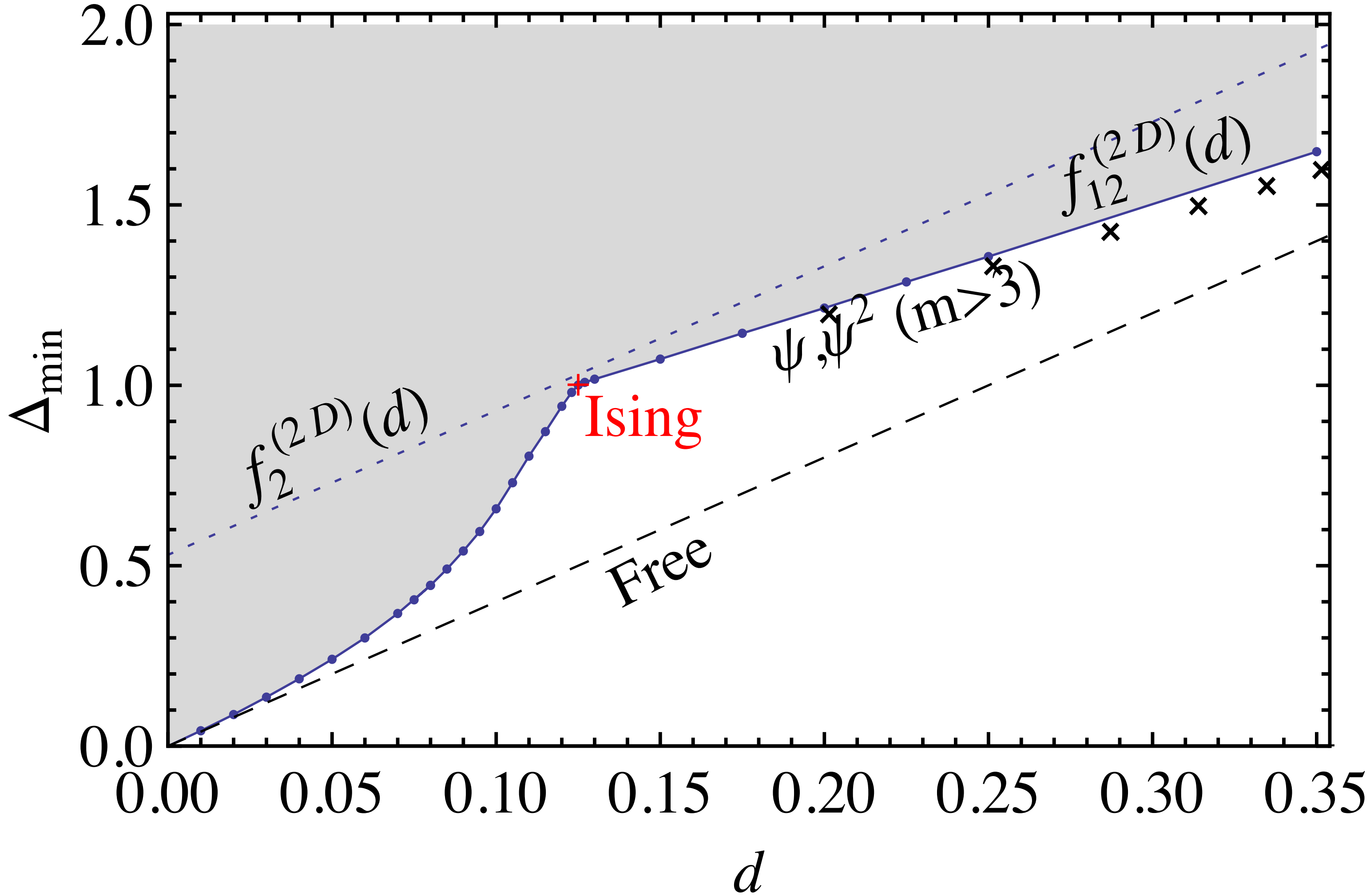
allowed range for Conformal Technicolor

Conformal bootstrap applications explored so far



Bootstrap as a means to discover new CFTs?

The same bound as before but in 2D: [S.R., Vichi 2009]



- can the same be done in 3D? (alternative way to 3D Ising critical exponents)
- some SUSY plots in 4D show similar kinks; new SCFTs?

Physics demands that we continue studying CFTs, especially in D$\geq$3

AdS/CFT...

Recently, many general results about CFTs
just from prime principles
without any simplifying assumptions

Bootstrap
holds a great promise
for further progress in D$\geq$3 CFT