Implications for the CMSSM from CMS and Dark Matter Searches - A Bayesian Approach

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The BayesFITS Group (A. Fowlie, A. Kalinowski, M. Kazana, S. Tsai,...)

Outline

- TH vs EXPT
- Bayesian analysis of the CMSSM
- likelihood
- (preliminary) results
- summary

A conjecture

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...or abandoned

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  inclusive, least model-dep.

- “razor”

  hemisphere algorithm to cluster events into effective di-jet system

- α_T

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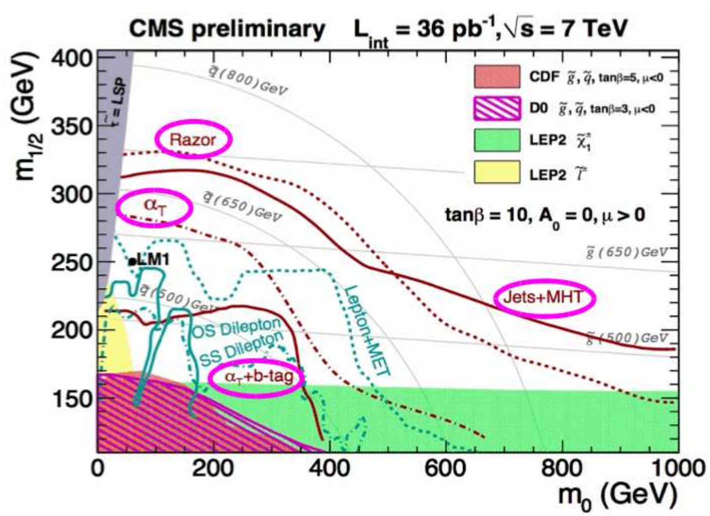

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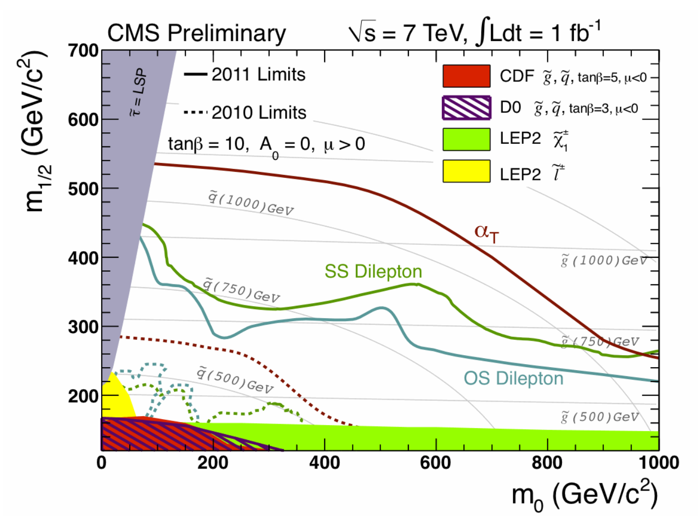

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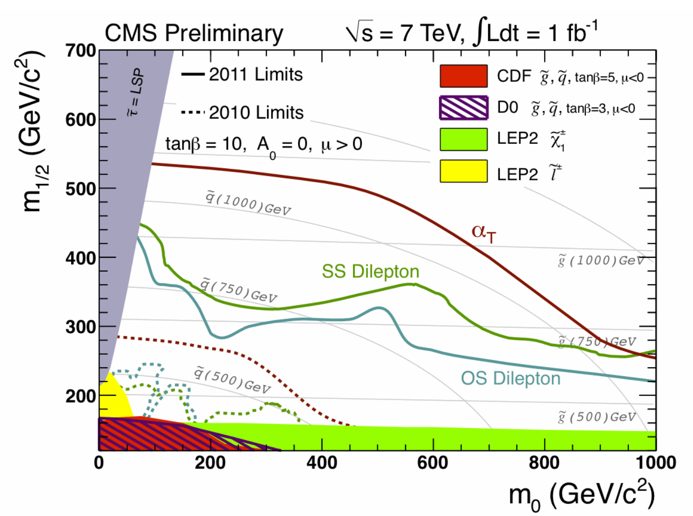
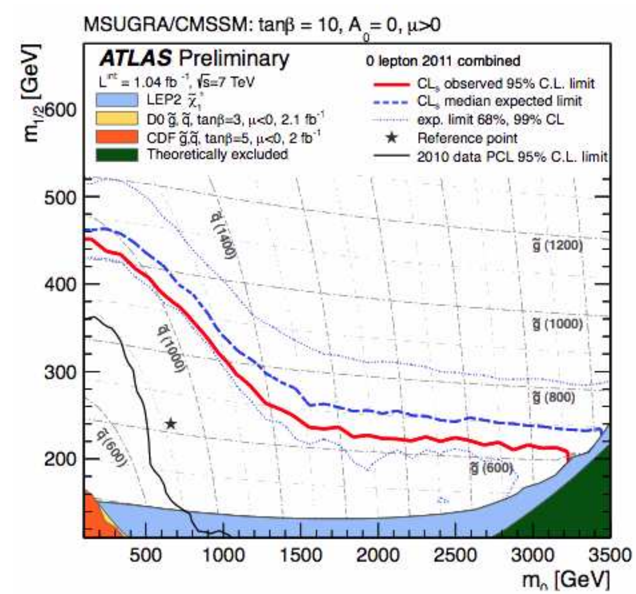


ATLAS:

- jets + missing transverse momentum
- ...



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Statistical approaches: construct the likelihood!

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- relevant SM param's $\boxed{\psi = M_t, m_b(m_b)^{\overline{MS}}, \alpha_s^{\overline{MS}}, \alpha_{\rm em}(M_Z)^{\overline{MS}}}$

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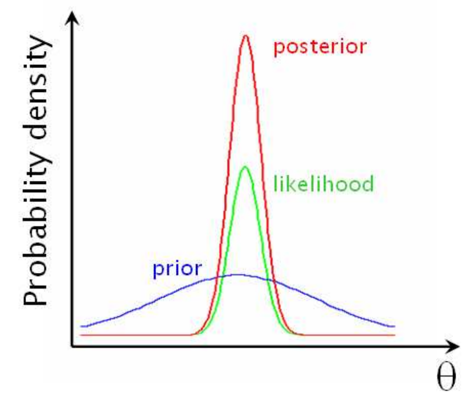

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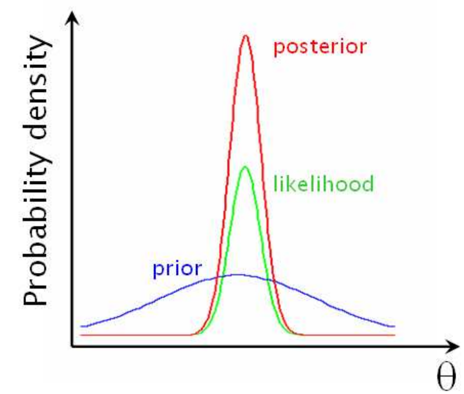
- Bayes' theorem: posterior pdf

$$p(\theta, \psi | d) = \frac{p(d|\xi)\pi(\theta,\psi)}{p(d)}$$

- $p(d|\xi) = \mathcal{L}$: likelihood
- $\pi(\theta, \psi)$: prior pdf
- $p(d)$: evidence (normalization factor)



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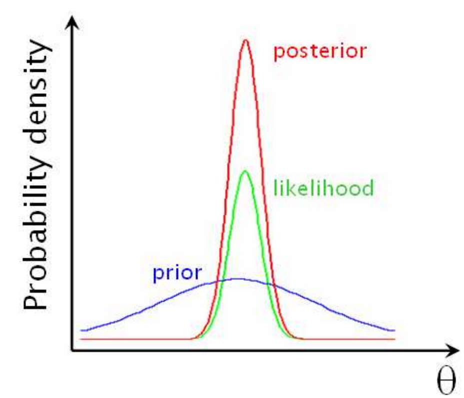

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- usually marginalize over SM (nuisance) parameters ψ $\Rightarrow$ $p(\theta|d)$

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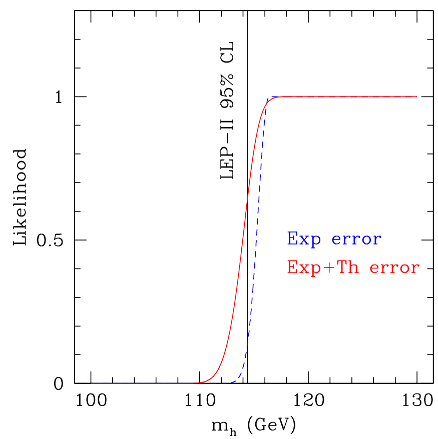
- for several uncorrelated observables (assumed Gaussian):

$$\mathcal{L} = \exp\left[-\sum_i \frac{\chi_i^2}{2}\right]$$

Limits: eg. light Higgs in the CMSSM

LEP: $m_h > 114.4\,\mathbf{GeV}$ (95% CL) - if SM-like

- include both experimental and theoretical error: $\sigma \to s = \sqrt{\sigma^2 + \tau^2}$



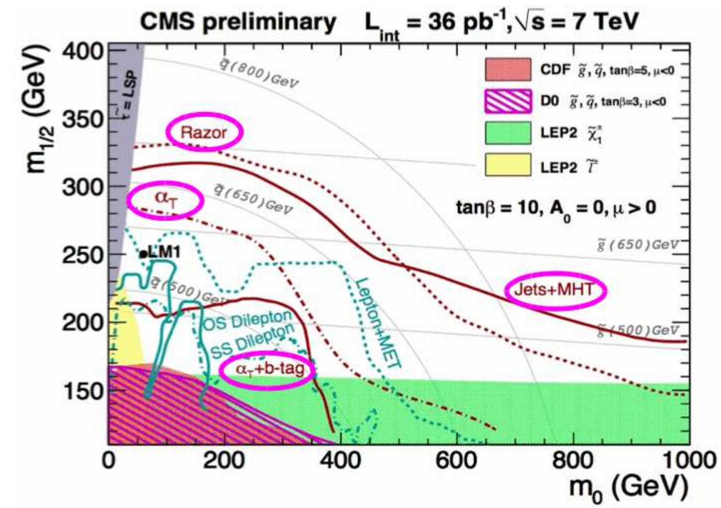
- apply similar way to LHC exclusion limits

compute expected nr of events, compare with data

compute cross sections, efficiency, apply cuts...

E.g., likelihood for the razor

as test case, consider CMS limit from the razor at $\mathbf{35}$ pb^{-1}



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simulate detector efficiency with Pythia

σ with Herwig++

use SoftSusy, SUSY-HIT

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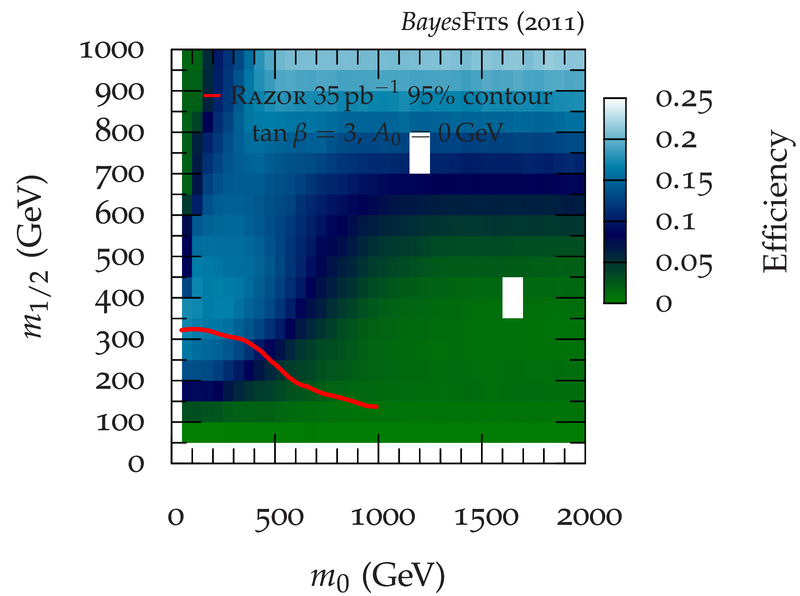

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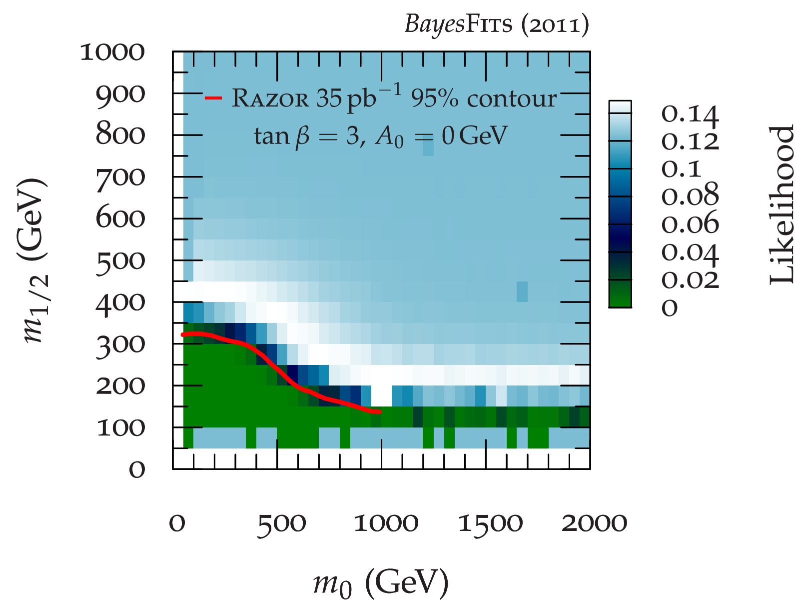

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$\Rightarrow$ very good agreement

encouraging!

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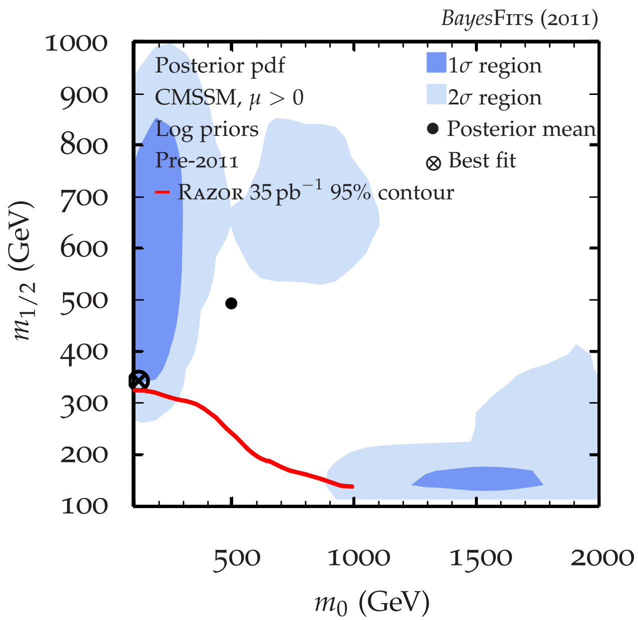
scans and stat analysis: SuperBayeS

"Pre2011": apply $\Omega_\chi h^2$, $b \to s\gamma$, $\mathrm{BR}(\bar{B}_s \to \mu^+\mu^-)$, $(g-2)_\mu$, LEP,...

no limits from LHC



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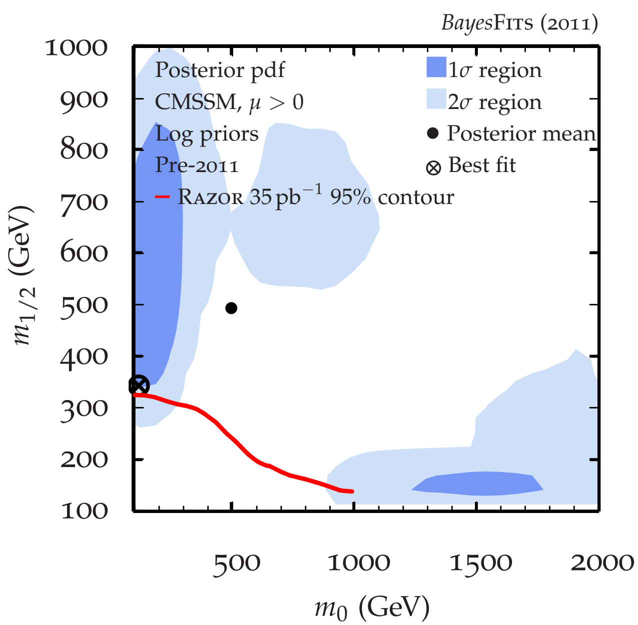
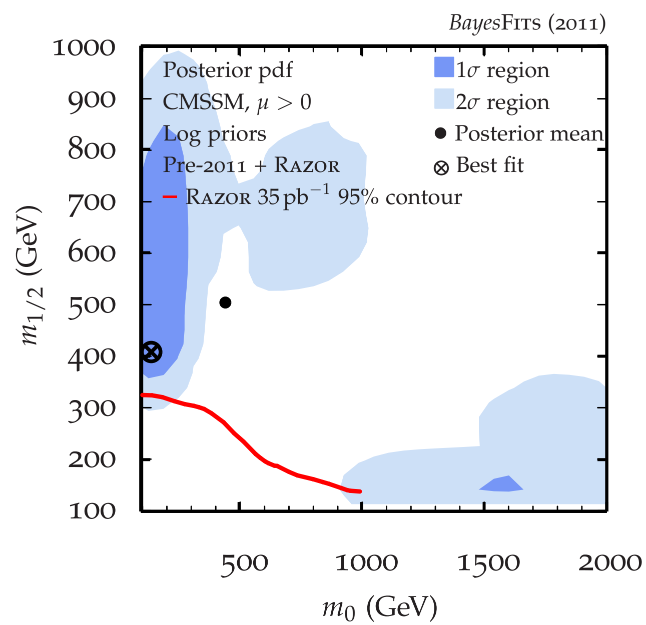


add CMS razor limit



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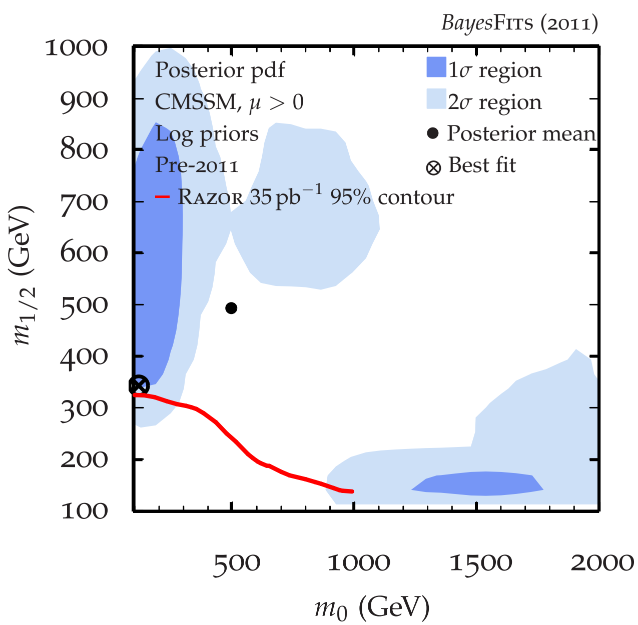

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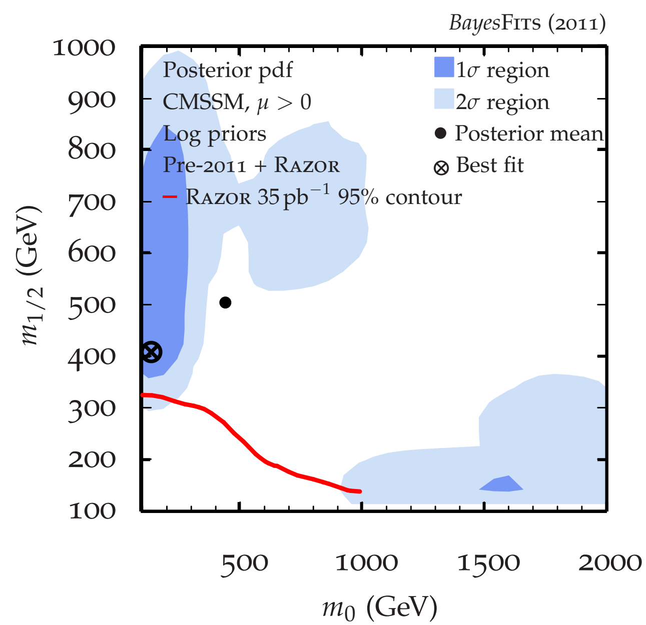

⇒ two favoured regions:

- large $m_{1/2}$ with $m_0 \lesssim m_{1/2}$
- large m_0 and small $m_{1/2}$ (FP, HB,...)

consistent with our previous results (2009-)

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to add: current CMS α_T limit

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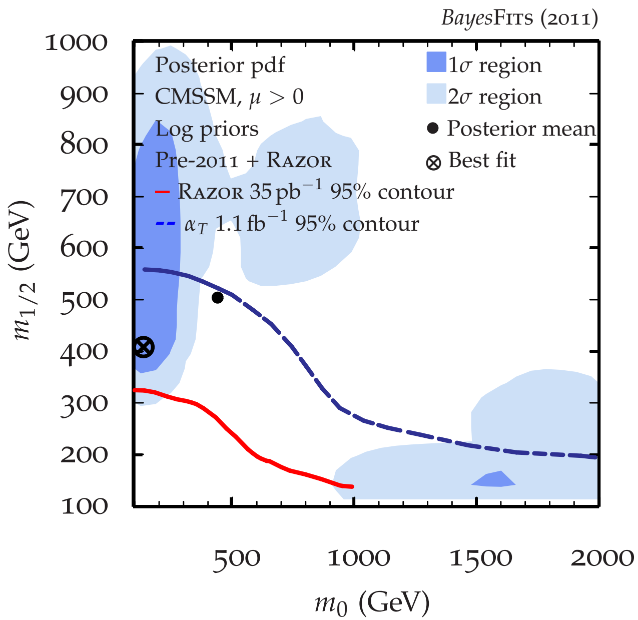

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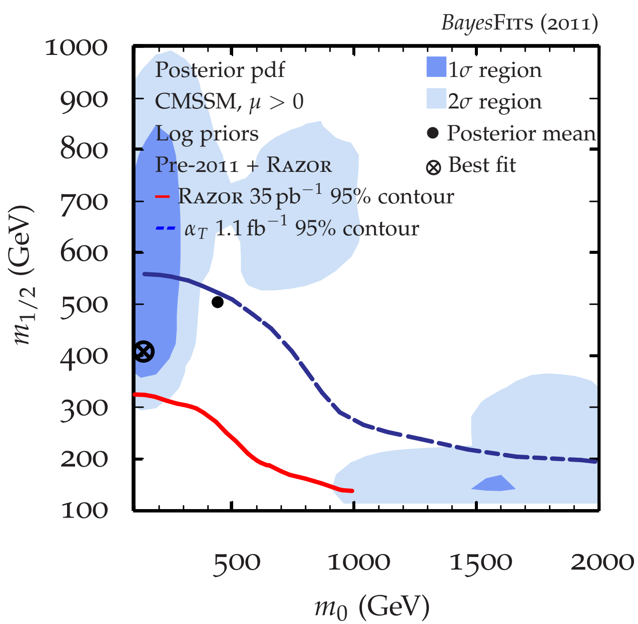

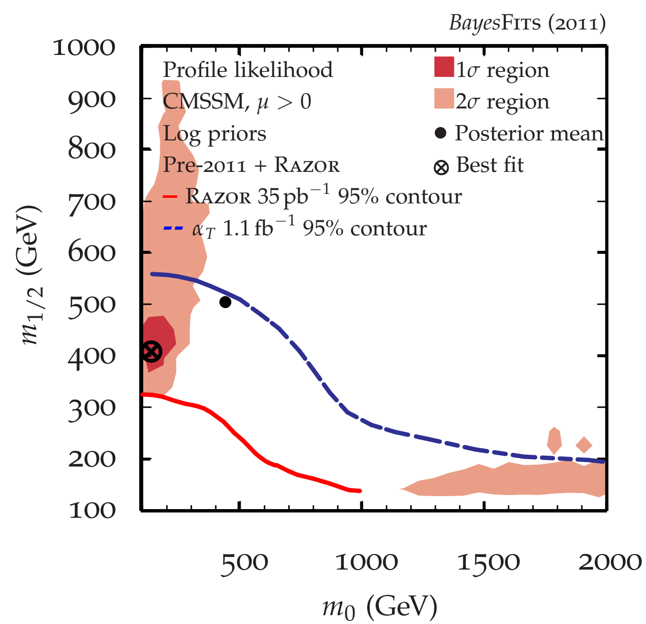

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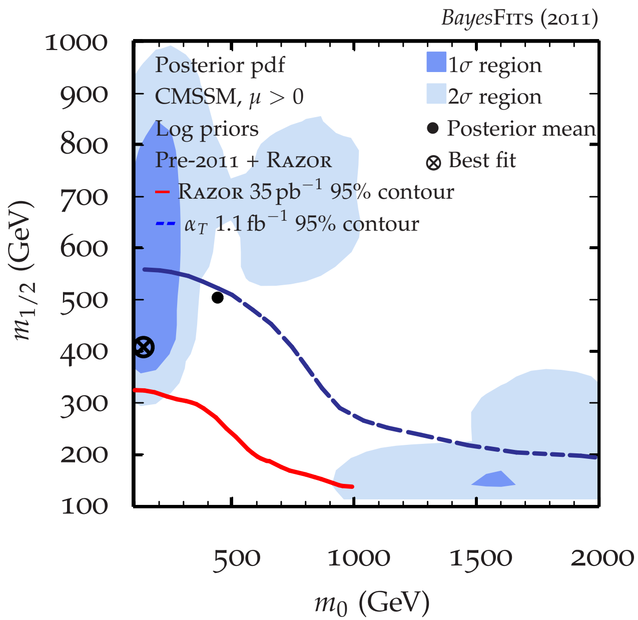

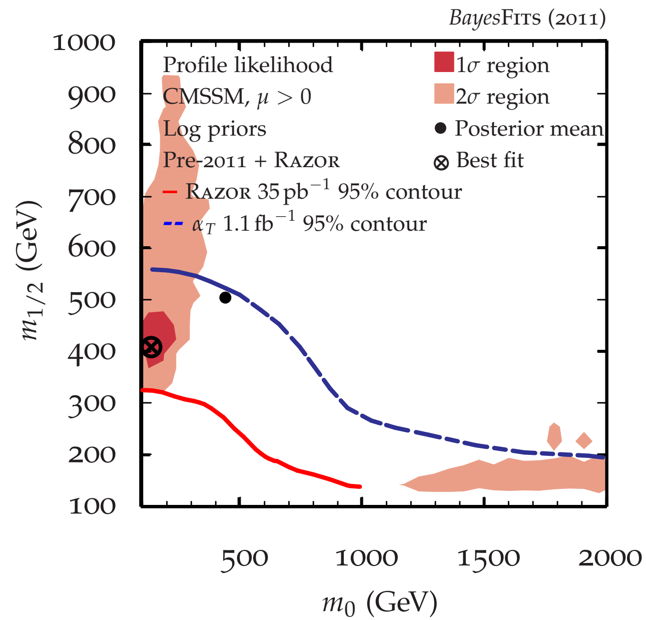

$\Rightarrow$ both favoured regions will get a hard hit

preliminary

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Xenon-100: $\sigma_p^{\rm SI} \lesssim 10^{-8}$ pb at small $m_{1/2}$ (FP)

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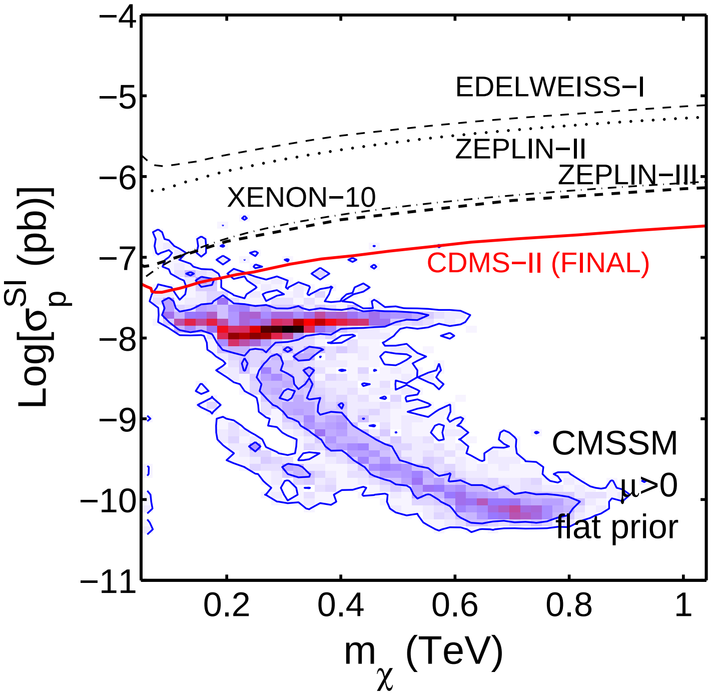

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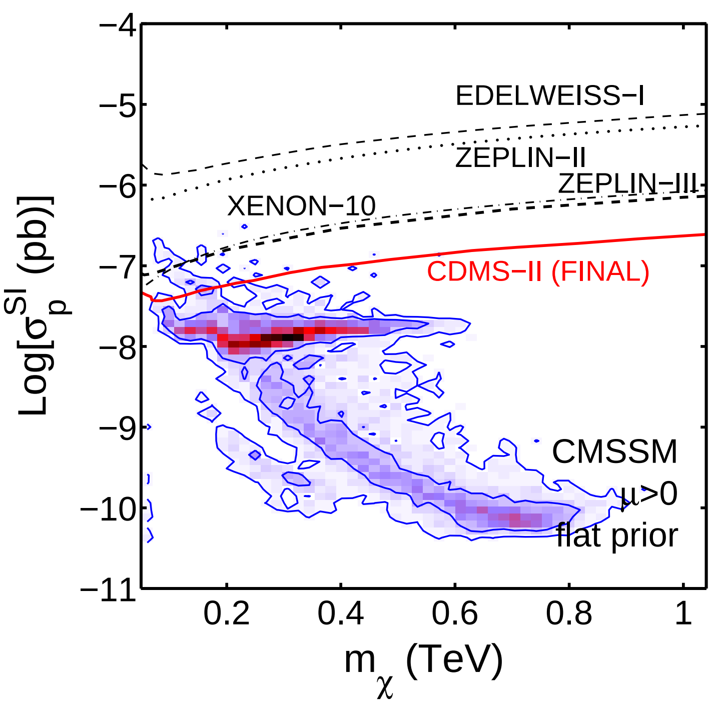

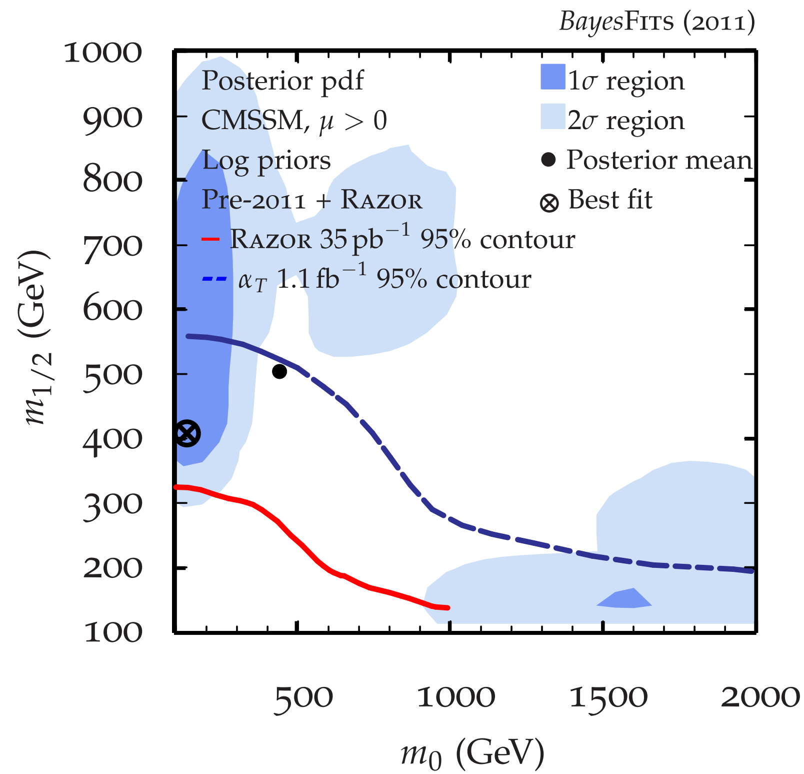

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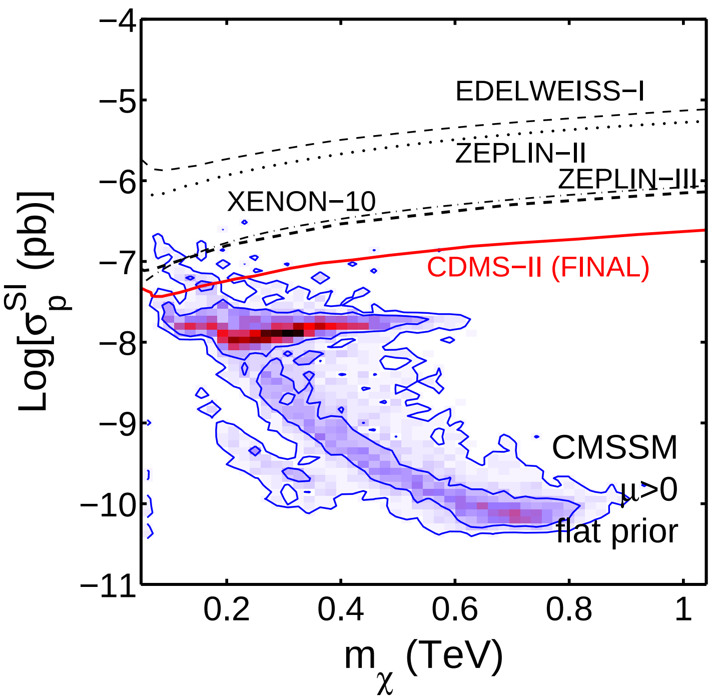
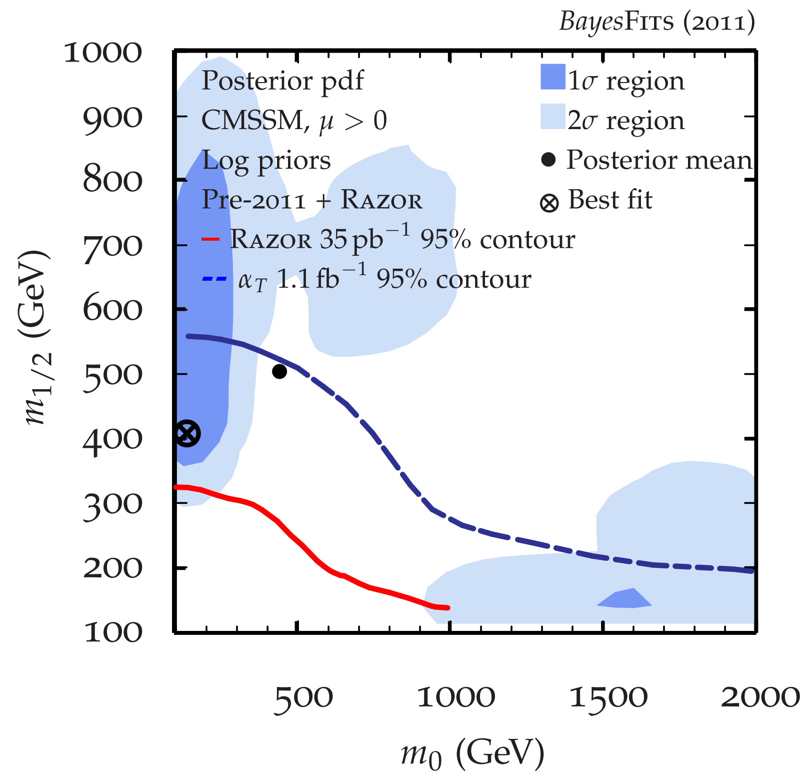


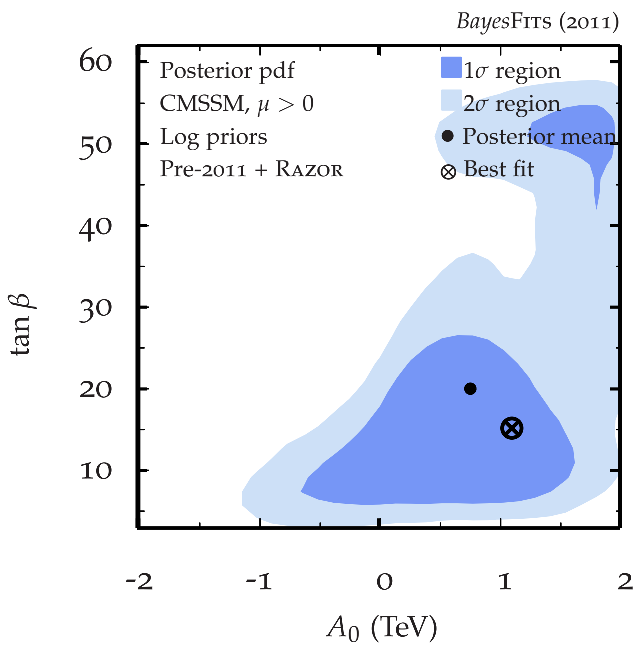


⇒ FP region gets a hard hit, but probably not completely out

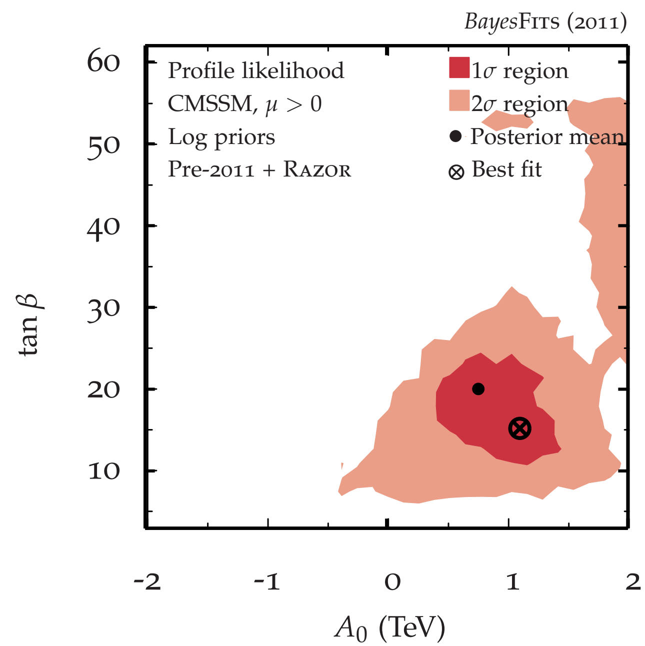
Impact on $\tan\beta$ and A_0

apply CMS razor $\mathbf{36}$/ pb, $\Omega_\chi h^2$, $b \to s\gamma$, $\mathrm{BR}(\bar{B}_s \to \mu^+\mu^-)$, $(g-2)_\mu$, LEP, CDF,...

Bayesian pdf



profile likelihood



- $3 \lesssim \tan\beta \lesssim 25$ favoured but not strongly
- A_0 poorly determined

Summary

- TH vs EXPT: statistical approach is the way to go!
- CMSSM: two favoured regions
- CMS 1 fb^{-1} limits: to strongly constrain both (in prep.)
- XENON-100 limit on DD: constrains (removes?) FP region (in prep.)

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...all this is just a warm-up for dealing with positive measurement(s)

if it ever comes

Backup

Experimental Measurements

(assume Gaussian distributions)

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SM (nuisance) parameter	Mean μ	Error σ (expt)
M_t	172.6 GeV	1.4 GeV
$m_b(m_b)^{\overline{MS}}$	4.20 GeV	0.07 GeV
α_s	0.1176	0.0020
$1/\alpha_{\mathrm{em}}(M_Z)$	127.955	0.030

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Derived observable	Mean	Errors	
	μ	σ (expt)	τ (th)
M_W	$\mathbf{80.398\,GeV}$	$\mathbf{25\,MeV}$	$\mathbf{15\,MeV}$
$\sin^2\theta_{\rm eff}$	$\mathbf{0.23153}$	$\mathbf{16 \times 10^{-5}}$	$\mathbf{15 \times 10^{-5}}$
$\delta a_\mu^{\rm SUSY} \times 10^{10}$	$\mathbf{29.5}$	$\mathbf{8.8}$	$\mathbf{1}$
$\mathbf{BR(\bar{B} \to X_s\gamma) \times 10^4}$	$\mathbf{3.55}$	$\mathbf{0.26}$	$\mathbf{0.21}$
ΔM_{B_s}	17.33	0.12	4.8
$\Omega_\chi h^2$	0.1099	0.0062	$\mathbf{0.1\,\Omega_\chi h^2}$

take w/o error: $M_Z = 91.1876(21)\,\mathrm{GeV}$, $G_F = 1.16637(1) \times 10^{-5}\,\mathrm{GeV}^{-2}$

Experimental Limits

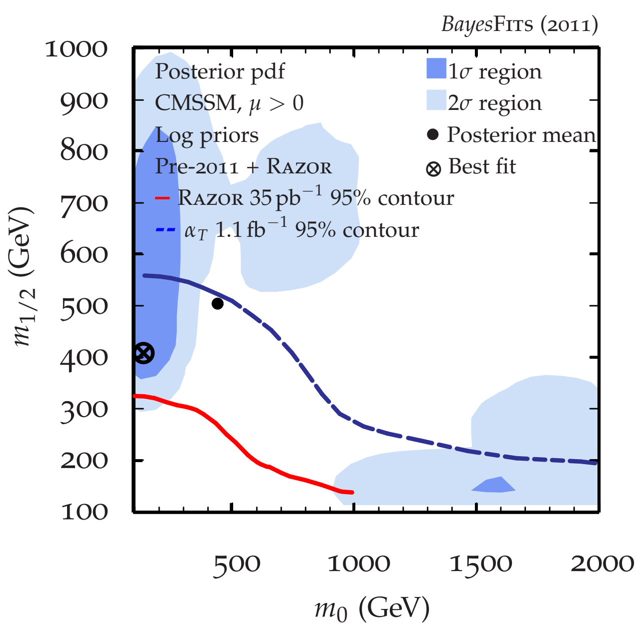
Derived observable	upper/lower limit	Constraints $\xi_{\rm lim}$	τ (theor.)
$BR(B_s \to \mu^+\mu^-)$	UL	1.5×10^{-7}	14%
m_h	LL	114.4 GeV (91.0 GeV)	3 GeV
$\zeta_h^2 \equiv g^2_{ZZh}/g^2_{ZZH_{\rm SM}}$	UL	$f(m_h)$	3%
m_χ	LL	50 GeV	5%
$m_{\chi_1^\pm}$	LL	103.5 GeV (92.4 GeV)	5%
$m_{\tilde{e}_R}$	LL	100 GeV (73 GeV)	5%
$m_{\tilde{\mu}_R}$	LL	95 GeV (73 GeV)	5%
$m_{\tilde{\tau}_1}$	LL	87 GeV (73 GeV)	5%
$m_{\tilde{\nu}}$	LL	94 GeV (43 GeV)	5%
$m_{\tilde{t}_1}$	LL	95 GeV (65 GeV)	5%
$m_{\tilde{b}_1}$	LL	95 GeV (59 GeV)	5%
$m_{\tilde{q}}$	LL	318 GeV	5%
$m_{\tilde{g}}$	LL	233 GeV	5%
($\sigma_p^{\rm SI}$	UL	WIMP mass dependent	$\sim 100\%$)

Note: DM direct detection $\sigma_p^{\rm SI}$ not applied due to astroph'l uncertainties (eg, local DM density)

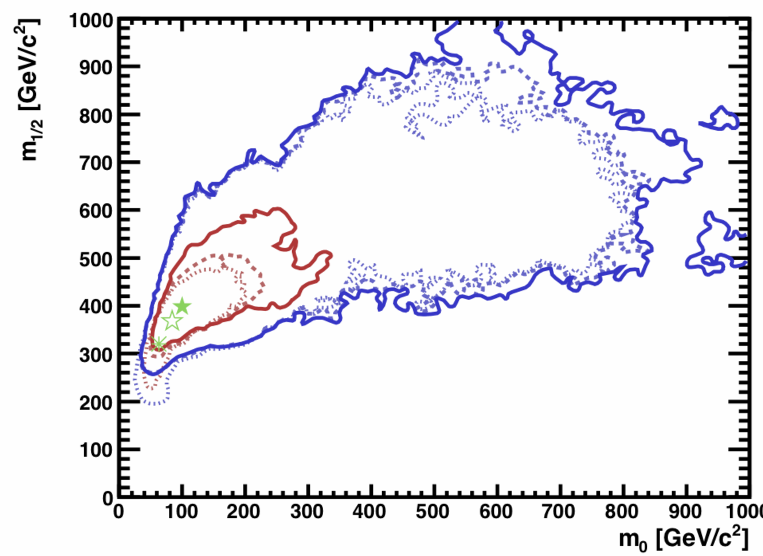
Bayesian vs frequentist

apply CMS razor $36/$ pb, $\Omega_\chi h^2$, $b \rightarrow s\gamma$, $\mathrm{BR}(\bar{B}_s \rightarrow \mu^+\mu^-)$, $(g-2)_\mu$, LEP, CDF,...

Bayesian pdf



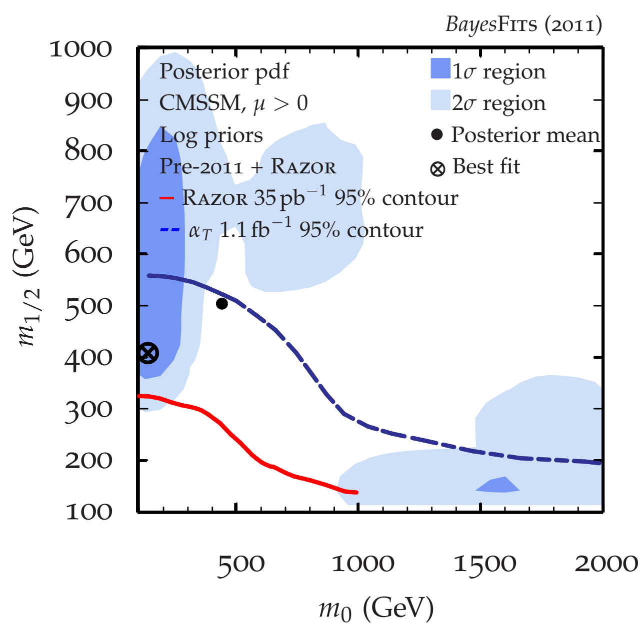
MasterCode



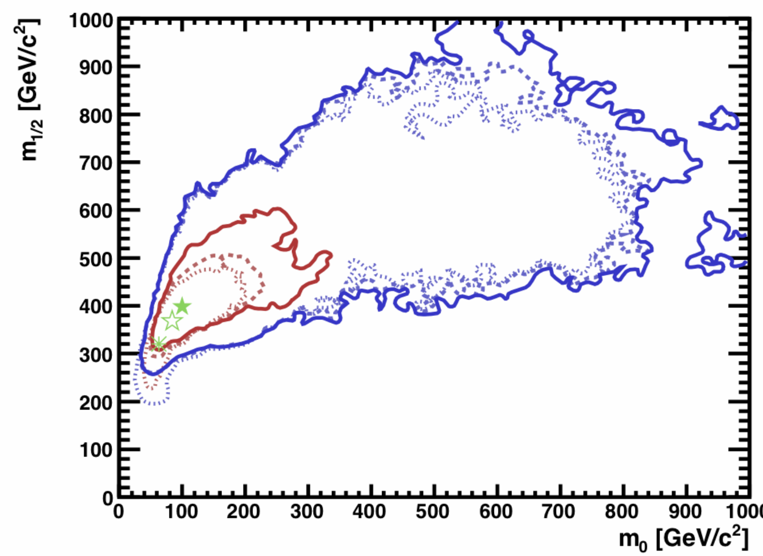
Bayesian vs frequentist

apply CMS razor $36/$ pb, $\Omega_\chi h^2$, $b \to s\gamma$, $\mathrm{BR}(\bar{B}_s \to \mu^+\mu^-)$, $(g-2)_\mu$, LEP, CDF,...

Bayesian pdf



MasterCode



- reasonable agreement in the $m_{1/2} \gtrsim m_0$ region
- disagreement about large m_0 region