The Krakow NLO Parton Shower project

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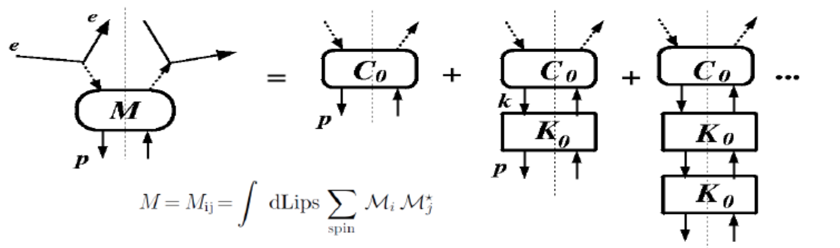
Plan

See http://arxiv.org/abs/1103.5015

- Introduction
- Reorganized Exclusive Collinear Factorization
- New LO Parton Shower scheme
- Exclusive NLO corrections to the hard process
- Exclusive NLO corrections to the ladder
- Summary

EGMPR scheme of collinear factorization (1978)

"Raw" factorization of the IR collinear singularities



$$M = M_{ij} = \int \mathrm{dLips} \sum_{\mathrm{spin}} \mathcal{M}_i \mathcal{M}_j^*$$

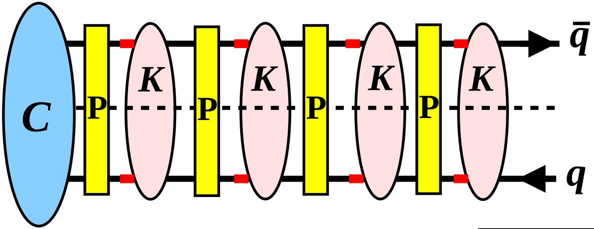
- Cut vertex M: spin sums and Lips integrations over all lines cut across
- C_0 and K_0 and are 2-particle irreducible (2PI)
- C_0 is IR finite, while K_0 encapsulates all IR collinear singularities
- Use of the axial gauge essential for the proof
- Formal proof given in EGMPR NP B152 (1979) 285
- Notation next slide

$$M = C_0(1 + K_0 + K_0^2 + \cdots) = C_0 \frac{1}{1 - K_0} \equiv C_0 \Gamma_0$$

Curci-Furmanski-Petronzio (CFP) collinear factorization scheme (1979)

CFP customized EGMPR to $\overline{MS}$ and exploited it to NLO in practice. Using **casting operator** $\mathbb{P} = P_{spin}\, PP$ they get:

$$F = C \cdot \frac{1}{1-K} = C \cdot (1 + K + K \otimes K + K \otimes K \otimes K + \ldots)$$



where new **finite hard process part** is: $C = C_0 \cdot \frac{1}{1-(1-\mathbb{P})\cdot K_0}$

The ladder encapsulates all collinear singularities.

Reorganized kernel is: $K = \mathbb{P} K_0 \cdot \frac{1}{1-(1-\mathbb{P})\cdot K_0}$

Problems with Monte Carlo based on Collinear Factorization Theorems (CFTs)

Classic CFTs: Ellis, Georgi, Machacek, Politzer, Ross 1979; Curci, Furmanski, Petronzio 1980; Collins, Sterman, Soper, Bodwin... 80-85

- **Why not suited** for the parton shower MC?
  - ⋆ **Non-conservation of the 4-momenta**
  - ⋆ **Over-subtractions**
  - ⋆ **Calculations in *n*-dimensions**
- **How to modify CFT?** for use in psMC:
  - • Redefine projection operators for extracting singular (Coll.) parts, such that 4-mom. is conserved.
  - • Introduce time-ordered exponential earlier, while isolating/subtracting Coll/IR singular parts.
  - • Introduce geometrical regularization instead of dimensional, resign from pole-part prescription.

Over-subtraction problem in translating CFP/EGMPR into Monte Carlo

From renormalization group eqs. (RGE) or explicit LO calc. we know:

$$\Gamma = e^{+\frac{1}{\varepsilon}} = 1 + \frac{1}{\varepsilon} + \frac{1}{2!}\frac{1}{\varepsilon^2} + \ldots$$

Examining up to LO real emissions in CFP scheme we see enormous over-subtractions/cancellations:

$$\Gamma \simeq \frac{1}{1-\left(1-e^{-\frac{1}{\varepsilon}}\right)} = 1 + \left(1-e^{-\frac{1}{\varepsilon}}\right) + \left(1-e^{-\frac{1}{\varepsilon}}\right)^2 + \ldots$$

NO WAY to build Monte Carlo on that!
Need exponent directly from the Feynman diagrams!!!

Translating ε-poles $\frac{1}{\varepsilon} = \int_0^{\mu_F} \frac{dk^T}{k^T} \left(\frac{k^T}{\mu_F}\right)^{\varepsilon}$ into ordinary big logs $\ln \frac{\mu_F}{m_p}$ of EGMPR provides the same picture.

Correcting for over-subtractions
New CFT suited for the psMC?

Over-subtraction eliminated thanks to explicit time ordered exponential= $\exp_{TO}$ in the evolution variable = log of the factorization scale (Soper+Nagy at LO):

$$F = \frac{C_0}{1-K_0} = C_0 \overleftarrow{\mathbb{B}}_\mu \left[\frac{1}{1-K_0}\right] \cdot \exp_{TO}\left(\overleftarrow{\mathbb{P}}'_\mu \left\{ {}^s K_0 \cdot \overleftarrow{\mathbb{B}}_s \left[\frac{1}{1-K_0}\right] \right\} \right)$$

$$\overleftarrow{\mathbb{B}}_\mu \left[\frac{1}{1-K_0}\right] \equiv 1 + \overleftarrow{\mathbb{B}}_\mu [K_0] + \overleftarrow{\mathbb{B}}_\mu [K_0 \cdot K_0] + \overleftarrow{\mathbb{B}}_\mu [K_0 \cdot K_0 \cdot K_0] + \dots$$

Operator $\overleftarrow{\mathbb{B}}$ is defined recursively (similarly as β-functions in Yennie-Frautschi-Suura 1961 subtraction scheme):

$$\overleftarrow{\mathbb{B}}_\mu [K_0] = K_0 - \mathbb{P}'_\mu \{K_0\},$$

$$\overleftarrow{\mathbb{B}}_\mu [K_0 \cdot K_0] = K_0 \cdot K_0 - \mathbb{P}'_\mu \{{}^{s_2} K_0\} \cdot \mathbb{P}'_{s_2} \{{}^{s_1} K_0\} - \mathbb{P}'_\mu \{{}^{s_2} K_0 \cdot \overleftarrow{\mathbb{B}}_{s_2} [K_0]\} - \overleftarrow{\mathbb{B}}_\mu [K_0] \cdot \mathbb{P}'_\mu \{K_0\},$$

$$\overleftarrow{\mathbb{B}}_\mu [K_0 \cdot K_0 \cdot K_0] = K_0 \cdot K_0 \cdot K_0 - \mathbb{P}'_\mu \{{}^{s_3} K_0\} \cdot \mathbb{P}'_{s_3} \{{}^{s_2} K_0\} \cdot \mathbb{P}'_{s_2} \{{}^{s_1} K_0\} - \dots$$

Modified $\mathbb{P}'_\mu$ new projection operator is the key point!

$\mathbb{P} \to \mathbb{P}'_\mu$ conserves four-momentum, contrary to EGMPR.

Specs of $\overleftarrow{\mathbb{P}}'_\mu$ modified projection operator

- $\overleftarrow{\mathbb{P}}'_\mu$ does spin projection as $\mathbb{P}$ of CFP,
- $\overleftarrow{\mathbb{P}}'_\mu(A)$ extracts singular part from integrand A, (not from the integral $\int A$ like CFP!)
- $\overleftarrow{\mathbb{P}}'_\mu$ sets upper limit μ on the phase space for all real partons towards the hadron using kinematic variable $s(k_1, .., k_n) < \mu$
- s: virtuality, max. rapidity $\max(k_i^T/\alpha_i)$, $\max(k_i^T)$, ...
- Notation $\{^{s_i}A\}$ defines $s = s_i = \max(a_1, ..., a_n)$
  e.g. $\mathbb{P}'_\mu\{^{s_3}A\} \cdot \mathbb{P}'_{s_3}\{^{s_2}A\} \cdot \mathbb{P}'_{s_2}\{^{s_1}A\}$
  means $\theta_{\mu>s_3>s_2>s_1}$ instead of $\theta_{\mu>s_3}\ \theta_{\mu>s_2}\ \theta_{\mu>s_1}$ (CFP-like)
- Nesting, like $\overleftarrow{\mathbb{P}}'_\mu[K_0 \cdot (1 - \overleftarrow{\mathbb{P}}'(K_0))]$, is allowed.

Standard inclusive PDFs, evolution eqs. and kernels are recovered after Ph.Sp. integration

Exclusive PDF (ePDF) is the integrand in:

$$D(\mu) = \exp_{TO}\left(\overleftarrow{\mathbb{P}}'_{\mu} \left\{ {}^{s} K_0 \cdot \overleftarrow{\mathbb{B}}_{s} \left[\frac{1}{1-K_0} \right] \right\} \right) = \exp_{TO}(K).$$

LO and NLO truncations of the **exclusive evolution kernel** K_μ are:

$$K_\mu^{LO} = \overleftarrow{\mathbb{P}}'_{\mu} \{ {}^{s}K_0 \}, \quad \text{taken at } \mathcal{O}(\alpha^1),$$

$$K_\mu^{NLO} = \overleftarrow{\mathbb{P}}'_{\mu} \left\{ {}^{s}K_0(s...) + K_0(s...) \cdot (1 - \overleftarrow{\mathbb{P}}'_{s}) \cdot K_0(...) \right\}, \text{ at } \mathcal{O}(\alpha^2).$$

Standard inclusive PDF $D(\mu, x)$, from ph.space integration of ePDF with fixed x, obeys by construction DGLAP evolution equation:

$$\boxed{\partial_\mu D(\mu, x) = \mathcal{P} \otimes D(\mu)(x)}$$

with the DGLAP **standard inclusive kernel**:

$$\mathcal{P}(x) = \int d\mathrm{Lips}\, \delta\left(x = \frac{\sum k_i^+}{E_0} \right) \delta\left(1 - \frac{s}{\mu} \right) \overleftarrow{\mathbb{P}}'_{\mu} \left\{ {}^{s}K_0 \cdot (1 - \overleftarrow{\mathbb{P}}'_{s})[K_0] \right\}.$$

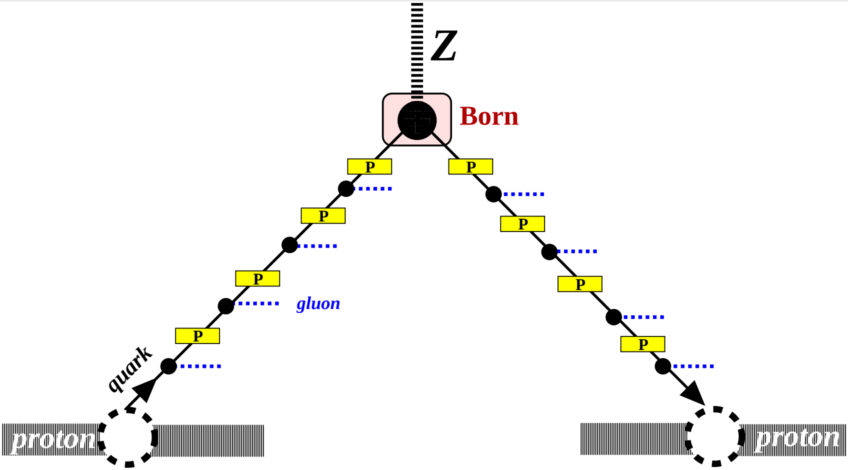
In the following several simplifications are adopted temporarily:

- Non-singlet kernels only,
- Only $q\bar{q} \to W/Z$, omitted $qg \to W/Z$
- non-running α_S
- Initial PDFs at low $\mu = Q$ assumed, but not explicitly shown in the formulae

The gluonstrahlung is our primary target!

LO psMC is (re-)constructed from the scratch, in a way compatible with our new factorization scheme.

LO psMC is (re-)constructed from the scratch



$$\sigma(C_0^{(0)}\Gamma_F^{(1)}\Gamma_B^{(1)}) = \sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\left\{\sigma\left[C_0^{(0)}(\mathbb{P}' K_{0F}^{(1)})^{n_1}(\mathbb{P}'' K_{0B}^{(1)})^{n_2}\right]\right\}_{T.O.}$$

LO psMC for W/Z production: Details

$$\sigma(C_0^{(0)}\Gamma_F^{(1)}\Gamma_B^{(1)}) = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\int d\hat{x}_F\, d\hat{x}_B\, d\Xi$$
$$\times\, e^{-S_F}\int_{\Xi>\eta_{n_1}}\left(\prod_{i=1}^{n_1} d^3\mathcal{E}(\bar{k}_i)\theta_{\eta_i>\eta_{i-1}}\frac{2C_F\alpha_s}{\pi^2}\bar{P}(z_{Fi})\right)\delta_{\hat{x}_F=1-\sum_j\hat{\alpha}_j}$$
$$\times\, e^{-S_B}\int_{\Xi<\eta_{n_2}}\left(\prod_{i=1}^{n_2} d^3\mathcal{E}(\bar{k}_i)\theta_{\eta_i<\eta_{i-1}}\frac{2C_F\alpha_s}{\pi^2}\bar{P}(z_{Bi})\right)\delta_{\hat{x}_B=1-\sum_j\hat{\beta}_j}$$
$$\times\, \delta\left(\Xi - \frac{\eta_{0F}+\eta_{0B}}{2} - \ln\frac{\hat{x}_F}{\hat{x}_B}\right) d\tau_2\Big(P - \sum_{j=1}^{n_1+n_2} k_j; q_1, q_2\Big)\,\frac{d\sigma_B}{d\Omega}(s\hat{x}_F\hat{x}_B,\hat{\theta}),$$

S_F and S_B = Sudakov formfactors, $\bar{P}(z)=\frac{1}{2}(1+z^2)$,
Ξ = Rapidity of Z, division plane between F and B hemispheres.
θ = angle of decay products (leptons) in Z rest frame.
$\hat{s}=s\hat{x}_F\hat{x}_B$ = effective mass of Z boson.

LO psMC for W/Z production: Details

$$\sigma(C_0^{(0)}\Gamma_F^{(1)}\Gamma_B^{(1)}) = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\int d\hat{x}_F\, d\hat{x}_B\, d\Xi$$

$$\times\, e^{-S_F}\int_{\Xi>\eta_{n_1}}\left(\prod_{i=1}^{n_1} d^3\mathcal{E}(\bar{k}_i)\theta_{\eta_i>\eta_{i-1}}\frac{2C_F\alpha_s}{\pi^2}\bar{P}(z_{Fi})\right)\delta_{\hat{x}_F=1-\sum_j\hat{\alpha}_j}$$

$$\times\, e^{-S_B}\int_{\Xi<\eta_{n_2}}\left(\prod_{i=1}^{n_2} d^3\mathcal{E}(\bar{k}_i)\theta_{\eta_i<\eta_{i-1}}\frac{2C_F\alpha_s}{\pi^2}\bar{P}(z_{Bi})\right)\delta_{\hat{x}_B=1-\sum_j\hat{\beta}_j}$$

$$\times\,\delta\left(\Xi-\frac{\eta_{0F}+\eta_{0B}}{2}-\ln\frac{\hat{x}_F}{\hat{x}_B}\right)d\tau_2(P-\sum_{j=1}^{n_1+n_2}k_j;q_1,q_2)\,\frac{d\sigma_B}{d\Omega}(s\hat{x}_F\hat{x}_B,\hat{\theta}),$$

Eikonal phase space for real gluon:
$d^3\mathcal{E}(k) = \frac{d^3k}{2k^0}\,\frac{1}{\mathbf{k}^2} = \pi\frac{d\phi}{2\pi}\frac{d\alpha}{\alpha}d\eta = \pi\frac{d\phi}{2\pi}\frac{d\beta}{\beta}d\eta,$
Lightcone variables: $\alpha = \frac{k^+}{2E},\ \beta = \frac{k^-}{2E}$; rapidity: $\eta = \frac{1}{2}\ln\frac{k^+}{k^-}$,
$d\tau_2(Q;q_1,q_2)$ = two-body phase space element.

LO psMC for W/Z production: Details

$$\sigma(C_0^{(0)}\Gamma_F^{(1)}\Gamma_B^{(1)}) = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\int d\hat{x}_F\, d\hat{x}_B\, d\Xi$$

$$\times\, e^{-S_F}\int_{\Xi>\eta_{n_1}}\left(\prod_{i=1}^{n_1} d^3\mathcal{E}(\bar{k}_i)\theta_{\eta_i>\eta_{i-1}}\frac{2C_F\alpha_s}{\pi^2}\bar{P}(z_{Fi})\right)\delta_{\hat{x}_F=1-\sum_j\hat{\alpha}_j}$$

$$\times\, e^{-S_B}\int_{\Xi<\eta_{n_2}}\left(\prod_{i=1}^{n_2} d^3\mathcal{E}(\bar{k}_i)\theta_{\eta_i<\eta_{i-1}}\frac{2C_F\alpha_s}{\pi^2}\bar{P}(z_{Bi})\right)\delta_{\hat{x}_B=1-\sum_j\hat{\beta}_j}$$

$$\times\,\delta\left(\Xi-\frac{\eta_{0F}+\eta_{0B}}{2}-\ln\frac{\hat{x}_F}{\hat{x}_B}\right)d\tau_2\Big(P-\sum_{j=1}^{n_1+n_2}k_j;q_1,q_2\Big)\frac{d\sigma_B}{d\Omega}(s\hat{x}_F\hat{x}_B,\hat{\theta}),$$

Variables in LO evolution kernels:

$z_{Fi}=\frac{\hat{x}_{Fi}}{\hat{x}_{F(i-1)}}$, $\hat{x}_{Fi}=1-\sum_{j=1}^{i}\hat{\alpha}_i=\prod_{j=1}^{i}z_{Fj}$,

$z_{Bi}=\frac{\hat{x}_{Bi}}{\hat{x}_{B(i-1)}}$ $\hat{x}_{Bi}=1-\sum_{j=1}^{i}\hat{\beta}_i=\prod_{j=1}^{i}z_{Bj}$,

For “mapped” lightcone variables $\hat{\alpha}_i$ and $\hat{\beta}_i$ see next slide.

Mapping entering definition of $\mathbb{P}'$ and $\mathbb{P}''$

Order ALL gluons according to rapidity distance from Ξ,
Define permutation π such that $|\eta_{\pi_i} - \Xi| > |\eta_{\pi_{i-1}} - \Xi|,\ i \in F, B$
Define in a *recursive* way dilatation transformation:

$$k_{\pi_i} = \lambda_i \bar{k}_{\pi_i}, \quad \lambda_i = \frac{s(\bar{x}_{i-1} - \bar{x}_i)}{2(P - \sum_{j=1}^{i-1} k_{\pi_j}) \cdot \bar{k}_{\pi_i}}, \quad i = 1, 2, ..., n_1 + n_2.$$

Rescaling factor λ_i is chosen such that

$$\bar{s}_i = s\bar{x}_i = s \prod_{\pi_j \in F}^{i} \hat{z}_{F\pi_j} \prod_{\pi_j \in B}^{i} \hat{z}_{B\pi_j} = (P - \sum_{j=1}^{i} k_{\pi_j})^2 = (P - \sum_{j=1}^{i} \lambda_j \bar{k}_{\pi_j})^2.$$

Features of the mapping:

– pure **rescaling**
– **preserves angles** i.e. ordering and upper limit
– **preserves soft factors** $(d\alpha/\alpha \dots)$
– **no gaps** in Phase Space

Overview of the LO Monte Carlo algorithm:

- Variables $\hat{z}_F$ and $\hat{z}_B$ are generated by FOAM
- 4-momenta $\bar{k}_i^\mu$ are generated separately in F and B parts of the phase space with constraints $\sum_{j\in F} \hat{\alpha}_j = 1 - \hat{z}_F$ and $\sum_{j\in B} \hat{\beta}_j = 1 - \hat{z}_B$.
- Double ordering permutation π is established.
- Rescaling parameter λ_1 is calculated. $k_{\pi_1} = \lambda_1 \bar{k}_{\pi_1}$ is set, such that $(P - k_{\pi_1})^2 = sx_1$.
- Parameter λ_2 is calculated and $k_{\pi_2} = \lambda_2 \bar{k}_{\pi_2}$ is set, such that $(P - k_{\pi_1} - k_{\pi_2})^2 = sx_2 = sz_{\pi_1} z_{\pi_2}$ and so on.
- In the rest frame of $\hat{P} = P - \sum_j k_{\pi_j}$ 4-momenta of q_1^μ and q_2^μ are generated according to Born angular distribution.

Kinematics of the two hemispheres is interrelated very gently, starting from very collinear gluons and finishing with the least collinear ones.

Exact analytical integration

The above LO MC covers multigluon phase space without any gaps or overlaps.

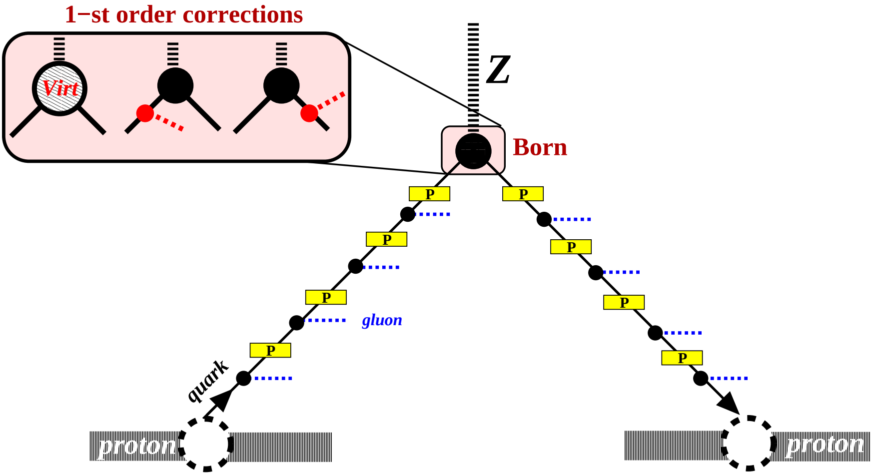
Moreover, EXACT analytical integration of the LO MC distributions over the multigluon phase space is possible:

$$\sigma(C_0^{(0)}\Gamma_F^{(1)}\Gamma_B^{(1)}) = \int_0^1 d\hat{x}_F \; d\hat{x}_B \; D_F(\Xi, \hat{x}_F) \; D_B(\Xi, \hat{x}_B) \; \sigma_B(s\hat{x}_F\hat{x}_B).$$

with two PDFs obeying DGLAP nonsinglet LO evolution equation:

$$\frac{\partial}{\partial \Xi} D_F(\Xi, x) = [\mathcal{P} \otimes D_F(\Xi)](x).$$

NLO in the hard process



NLO Monte Carlo weight
This is Yennie-Frautschi-Suura (YFS) style!

Once LO MC is re-designed, introduction of the complete NLO to hard process part is done with the help of the simple positive MC weight:

$$W_{MC}^{NLO} = 1 + \Delta_{S+V} + \sum_{j\in F} \frac{\tilde{\beta}_1(\hat{s},\hat{p}_F,\hat{p}_B;a_j,z_{Fj})}{\bar{P}(z_{Fj})\, d\sigma_B(\hat{s},\hat{\theta})/d\Omega} + \sum_{j\in B} \frac{\tilde{\beta}_1(\hat{s},\hat{p}_F,\hat{p}_B;a_j,z_{Bj})}{\bar{P}(z_{Bj})\, d\sigma_B(\hat{s},\hat{\theta})/d\Omega},$$

where the IR/Col.-finite real emission part is

$$\tilde{\beta}_1(\hat{p}_F,\hat{p}_B;q_1,q_2,k) = \Big[\frac{(1-\alpha)^2}{2}\frac{d\sigma_B}{d\Omega_q}(\hat{s},\theta_{F1}) + \frac{(1-\beta)^2}{2}\frac{d\sigma_B}{d\Omega_q}(\hat{s},\theta_{B2})\Big] - \theta_{\alpha>\beta}\frac{1+(1-\alpha-\beta)^2}{2}\frac{d\sigma_B}{d\Omega_q}(\hat{s},\hat{\theta}) - \theta_{\alpha<\beta}\frac{1+(1-\alpha-\beta)^2}{2}\frac{d\sigma_B}{d\Omega_q}(\hat{s},\hat{\theta}),$$

and the kinematics independent virtual+soft correction is

$$\Delta_{V+S} = \frac{C_F\alpha_s}{\pi}\left(\frac{1}{3}\pi^2 - 4\right) + \frac{C_F\alpha_s}{\pi}\,\frac{1}{2}$$

Exact analytical integration at NLO

EXACT analytical integration of the NLO MC distributions over the multigluon phase space is again possible(!):

$$\sigma(C_0^{(1)}\Gamma_F\Gamma_B) = \int_0^1 d\hat{x}_F\; d\hat{x}_B\; dz\; D_F(\Xi,\hat{x}_F)\; D_B(\Xi,\hat{x}_B)\; \sigma_B(sz\hat{x}_F\hat{x}_B)$$
$$\times \left\{ \delta_{z=1}(1+\Delta_{S+V}) + C_{2r}^{psMC}(z) \right\}$$

where

$$C_{2r}^{psMC}(z) = \frac{2C_F\alpha_s}{\pi}\left[-\frac{1}{2}(1-z)\right]$$

The above differs from $\overline{MS}$ eq. (90) in Altarelli-Ellis-Martinelli (79)

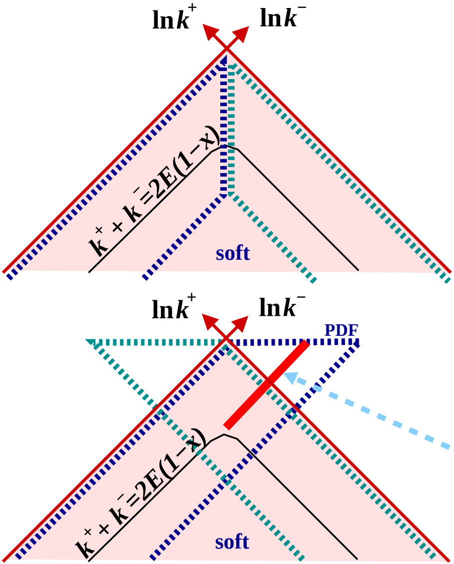
$$C_{2r}^{\overline{MS}}(z) = 2\frac{C_F\alpha_s}{\pi}\left(\frac{\bar{P}(x)}{1-z}\right)_+ [2\ln(1-z) - \ln z]$$

Why? psMC factorization scheme is slightly different from $\overline{MS}$:

$$C_{2r}^{psMC}(z) - C_{2r}^{\overline{MS}}(z) = -2C_{ct}^{psMC}(z) + 2C_{ct}^{\overline{MS}}(z) \simeq 4\frac{C_F\alpha_s}{\pi}\left(\frac{\bar{P}(x)}{1-z}\ln(1-x)\right)_+$$

Difference between $\overline{MS}$ and psMC fact. schemes

Simple kinematics explains $4\ln(1-x)/(1-x)_+$



[Eqs. (98-101) in Altarelli+Ellis+Martinelli (1979)]

psMC fact. scheme:

$$0\,\frac{|\ln(1-x)|}{1-x}$$

$\overline{MS}$ fact. scheme:

$$\int_{1-x}^{1/(1-x)} \frac{1}{1-x}\frac{d\beta}{\beta} = 2\,\frac{|\ln(1-x)|}{1-x}$$

Features of the new psMC scheme

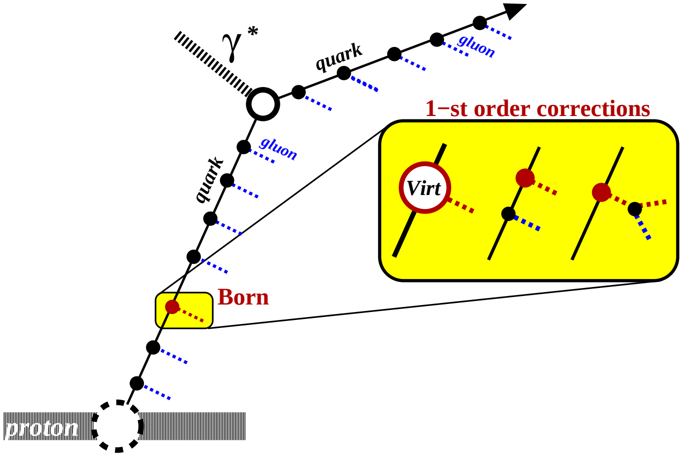
Resulting from close relation to CFTs

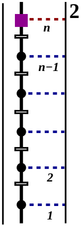
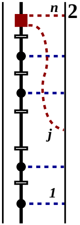
- NLO added on the top of the LO psMC with simple and positive MC weight.
- No need to correct for the difference in collinear counterterms of psMC and $\overline{MS}$ schemes provided PDFs are in psMC scheme.
- Virtual+soft corrections are completely kinematics independent
  – all the complicated $d\Sigma^{c\pm}$ contributions are gone.
- Built in resummation of $\frac{\ln^n(1-x)}{1-x}$ terms (?).

All the above goodies at the price of:

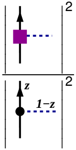
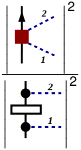
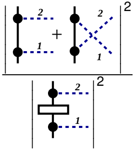
- Rebuilding LO psMC from the scratch
- Factorization scheme in the subtracted hard process ME and in PDFs is changed (in a well defined way)

Again starting from Feynman diagrams of Curci-Furmanski-Petronzio scheme (axial gauge), and recalculating their NLO DGLAP kernels.



$$\bar{D}_B^{[1]}(x,Q) = e^{-S_{ISR}}$$  $$+\, e^{-S_{ISR}} \sum_{j=1}^{n-1}$$  $$= e^{-S_{ISR}} \Big\{ \delta_{x=1} +$$

$$+ \sum_{n=1}^{\infty} \left(\prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \, \frac{2C_F^2\alpha_s}{\pi} \frac{1}{k_i^{T2}} \bar{P}(z_i) \right) \left[\beta_0^{(1)}(z_n) + \sum_{j=1}^{n-1} W(\tilde{k}_n, \tilde{k}_j) \right] \delta_{x=\prod_{j=1}^{n} x_j} \Big\},$$

where $d\eta_i = \frac{d^3k_i}{k_i^0}$, $\beta_0^{(1)} =$  , $W(k_2,k_1) =$  $=$  $-\,1.$

Mapping $k_i \to \tilde{k}_i$ instrumental. S_{ISR} = double-log Sudakov, W is non-singular!

Algebraic crosscheck

Analytical integration of NLO part $\sum_j W(\tilde{k}_n, \tilde{k}_j)$ can be done leading to:

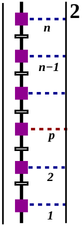
$$\sum_{n=1}^{\infty} \int du \int\limits_{Q>a_n>a_{n-1}} \frac{da_n}{a_n} \mathcal{P}^{(1)}_{qq}(u) \left(\prod_{i=1}^{n-1} \int\limits_{a_{i+1}>a_i>a_{i-1}} \frac{da_i}{a_i} \mathcal{P}^{(0)}_{qq}(z_i) \right) \delta_{x=u\prod_{j=1}^{n-1} z_j}.$$

where we recover precisely NLO part (including virtuals) of standard DGLAP kernel $\mathcal{P}^{(1)}_{qq}(u)$ defined according to:

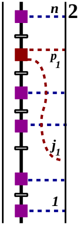
$$\mathcal{P}^{(1)}_{qq}(u) \ln\frac{Q}{q_0} = \int\limits_{Q>a_n>a_0} d^3\eta_n\ \rho^{(1)}_{1B}(k_n)\ \beta^{(1)}_0(z_n)\delta_{u=z_n} + \int\limits_{Q>a_n>a_0} d^3\eta_n \int\limits_{a_n>a_{n'}>0} d^3\eta_{n'}\ \beta^{(1)}_1(\tilde{k}_n, \tilde{k}_{n'})\ \delta_{u=z_n z_{n'}}$$

One NLO standard inclusive kernel of DGLAP truly reproduced, modulo factorization scheme change $\overline{MS}$ →psMC.

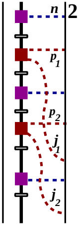
$$\bar{D}_B^{[1]}(x,Q) = e^{-S_{ISR}} \sum_{n=0}^{\infty} \Bigg\{$$



$$+ \sum_{p_1=1}^{n} \sum_{j_1=1}^{p_1-1}$$



$$+ \sum_{p_1=1}^{n} \sum_{p_2=1}^{p_1-1} \sum_{\substack{j_1=1 \\ j_1\neq p_2}}^{p_1-1} \sum_{\substack{j_2=1 \\ j_2\neq p_1, j_2}}^{p_2-1}$$



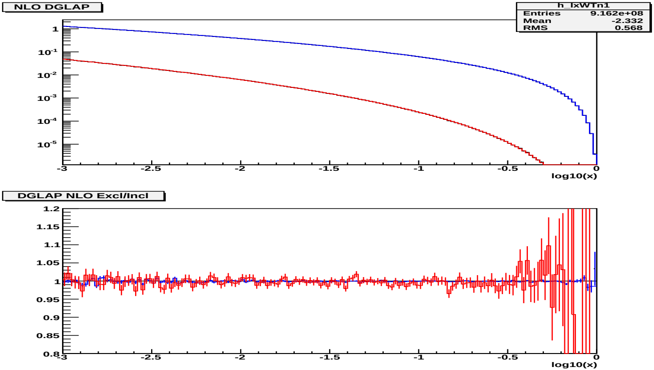
$$+ \dots \Bigg\}$$

$$= e^{-S_{ISR}} \Bigg\{ \delta_{x=1} + \sum_{n=1}^{\infty} \Bigg(\prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \; \rho_{1B}^{(1)}(k_i) \beta_0^{(1)}(z_p) \Bigg) \Bigg[1 + \sum_{p=1}^{n} \sum_{j=1}^{p-1} W(\tilde{k}_p, \tilde{k}_j) +$$

$$+ \sum_{p_1=1}^{n} \sum_{p_2=1}^{p_1-1} \sum_{\substack{j_1=1 \\ j_1\neq p_2}}^{p_1-1} \sum_{\substack{j_2=1 \\ j_2\neq p_1, j_2}}^{p_2-1} W(\tilde{k}_{p_1}, \tilde{k}_{j_1}) W(\tilde{k}_{p_2}, \tilde{k}_{j_2}) + \dots \Bigg] \delta_{x=\prod_{j=1}^{n} x_j} \Bigg\},$$

The above has been tested with 3-digit precision in the MC prototype, see next slide.

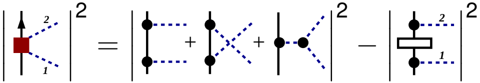
Numerical test of ISR pure C_F^2 NLO MC



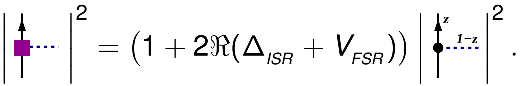
Numerical results for $D(x, Q)$ from inclusive and exclusive **two** Monte Carlos.
Blue curve is single NLO insertion, red curve is double insertion component.
LO+NLO is off scale. Evolution 10GeV$\to$1TeV starting from $\delta(1-x)$.
The ratio demonstrates 3-digit agreement, in units of LO.

The problem with gluon pair component of the NLO kernel, $\sim C_F C_A$ (FSR)

Straightforward inclusion of gluon pair diagram in the previous method would ruin Monte Carlo weight due to presence of Sudakov double logarithmic $+S_{FSR}$ in 2-real correction:



and $-S_{FSR}$ in the virtual correction:

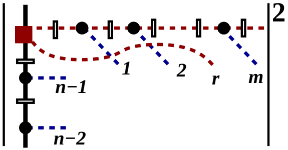


SOLUTION: Resummation/exponentiation of FSR, see next slides for details of the scheme and numerical test of the prototype MC.

NLO FSR corr. at the end of the ladder, $\sim C_F C_A$

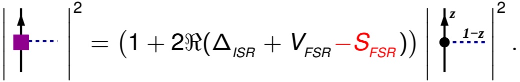
Additional NLO FSR corr. at the end of the ladder:

$$e^{-S_{ISR}-S_{FSR}} \sum_{n,m=0}^{\infty} \sum_{r=1}^{m}$$

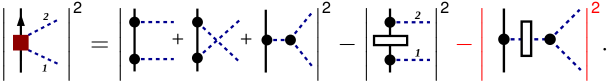


where Sudakov S_{FSR} is subtracted in the virtual part:

$$\left| \text{[diagram]} \right|^2 = \left(1 + 2\Re(\Delta_{ISR} + V_{FSR} - S_{FSR})\right) \left| \text{[diagram]} \right|^2 .$$



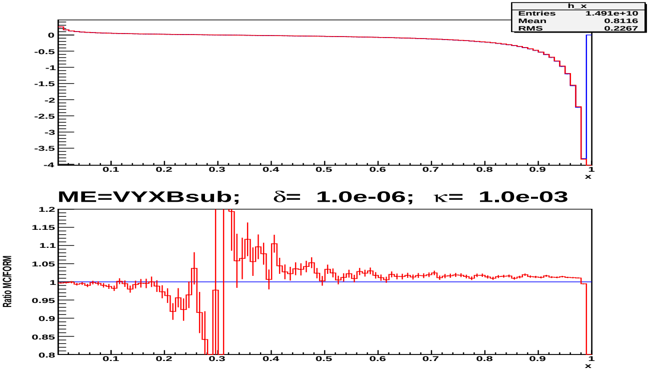
and FSR counterterm is subtracted in the 2-real-gluon part:



The miracle: both are free of any collinear or soft divergence!!!

3-digit precision numerical test of FSR methodology

Numerical test done for single NLO ISR+FSR insertion
for $n = 1, 2$ ISR gluons and infinite no. of FSR gluons:



because in this case analytical integration is feasible.
MC agrees precisely with the analytical result.

Summary and Prospects

- **Extension of the collinear factorization**, better suited for Monte Carlo implementation is defined.
- **NLO parts of *hard process* and *evolution kernels*** are already recalculated in the new scheme (non-singlet NLO exclusive kernels calculated).
- The differences between the new Monte Carlo (MC) and the standard $\overline{MS}$ schemes are understood, keeping the universality (process independence) in mind.
- **New solution has advantages** as compared to the other techniques of adding the NLO corrections to the hard process and it is completely new for the ladder parts.
- **R&D phase (almost) completed**, MC realization for W/Z prod. at LHC/Tevatron and DIS proc. becomes main front.