Status of the antenna approach for $2 \to 2$ NNLO processes at the LHC

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based on work of/with Radja Boughezal, James Currie, Pedro Jimenez Delgado, Aude Gehrmann-De Ridder, Thomas Gehrmann, Gudrun Heinrich, Gionata Luisoni, Pier Francesco Monni, Joao Pires, Mathias Ritzmann, Steven Wells

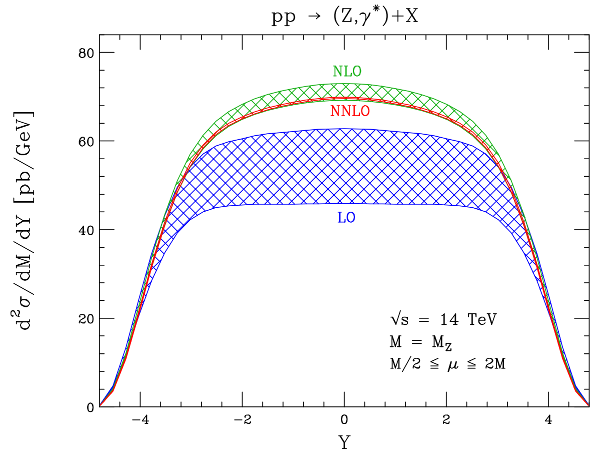
NNLO calculations for $2 \to 2$ processes

$$\mathrm{d}\sigma = \sum_{i,j} \int \frac{d\xi_1}{\xi_1}\frac{d\xi_2}{\xi_2} f_i(\xi_1, \mu_F^2) f_j(\xi_2, \mu_F^2) \mathrm{d}\hat{\sigma}_{ij}(\alpha_s(\mu_R), \mu_R, \mu_F)$$

$$\mathrm{d}\hat{\sigma}_{ij} = \mathrm{d}\hat{\sigma}_{ij}^{LO} + \left(\frac{\alpha_s(\mu_R)}{2\pi}\right) \mathrm{d}\hat{\sigma}_{ij}^{NLO} + \left(\frac{\alpha_s(\mu_R)}{2\pi}\right)^2 \mathrm{d}\hat{\sigma}_{ij}^{NNLO} + \mathcal{O}(\alpha_s^3)$$

Processes of interest

- ✓ $pp \to 2$ jets
- ✓ $pp \to V + jet$
- ✓ $pp \to t\bar{t}$
- ✓ $pp \to VV$
- ✓ $\dots$

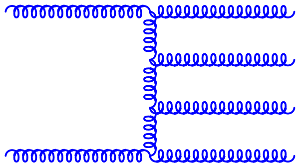


Massively reduced theoretical error

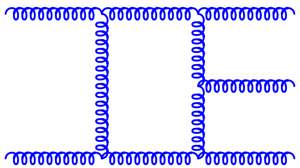
Anastasiou, Dixon, Melnikov, Petriello (04)

Anatomy of a NNLO calculation e.g. $pp \to 2j$

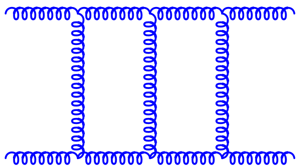
- ✓ double real radiation matrix elements $\mathrm{d}\hat{\sigma}^{R}_{NNLO}$
  - ✓ implicit poles from double unresolved emission



- ✓ single radiation one-loop matrix elements $\mathrm{d}\hat{\sigma}^{V,1}_{NNLO}$
  - ✓ explicit infrared poles from loop integral
  - ✓ implicit poles from soft/collinear emission



- ✓ two-loop matrix elements $\mathrm{d}\hat{\sigma}^{V,2}_{NNLO}$
  - ✓ explicit infrared poles from loop integral
  - ✓ including square of one-loop amplitude



$$\mathrm{d}\hat{\sigma}_{NNLO} \sim \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\hat{\sigma}^{R}_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\hat{\sigma}^{V,1}_{NNLO} + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\hat{\sigma}^{V,2}_{NNLO}$$

- ✓ need method to extract implicit poles - main issue discussed in this talk

Basics of subtraction method - I

- General form of (renormalised) cross section

$$\begin{aligned} d\hat{\sigma}_{NNLO} \equiv & \int_{d\Phi_{m+2}} \left(d\hat{\sigma}^{R}_{NNLO} - d\hat{\sigma}^{S}_{NNLO}\right) + \int_{d\Phi_{m+2}} d\hat{\sigma}^{S}_{NNLO} \\ + & \int_{d\Phi_{m+1}} \left(d\hat{\sigma}^{V,1}_{NNLO} - d\hat{\sigma}^{VS,1}_{NNLO}\right) + \int_{d\Phi_{m+1}} d\hat{\sigma}^{VS,1}_{NNLO} + \int_{d\Phi_{m+1}} d\hat{\sigma}^{MF,1}_{NNLO} \\ + & \int_{d\Phi_{m}} d\hat{\sigma}^{V,2}_{NNLO} + \int_{d\Phi_{m}} d\hat{\sigma}^{MF,2}_{NNLO} \end{aligned}$$

- $d\hat{\sigma}^{S}_{NNLO}$ is the double real radiation subtraction term - subtracted and added back in in integrated form
- $d\hat{\sigma}^{V,S,1}_{NNLO}$ is the real-virtual radiation subtraction term - subtracted and added back in in integrated form
- $d\hat{\sigma}^{MF,1}_{NNLO}$ and $d\hat{\sigma}^{MF,2}_{NNLO}$ are the mass factorisation counter terms

Basics of subtraction method - II

✓ The aim is to recast the NNLO cross section in the form

$$\begin{aligned}
\mathrm{d}\hat{\sigma}_{NNLO} &= \int_{\mathrm{d}\Phi_{m+2}} \left[\mathrm{d}\hat{\sigma}^{R}_{NNLO} - \mathrm{d}\hat{\sigma}^{S}_{NNLO}\right] \\
&+ \int_{\mathrm{d}\Phi_{m+1}} \left[\mathrm{d}\hat{\sigma}^{V,1}_{NNLO} - \mathrm{d}\hat{\sigma}^{T}_{NNLO}\right] \\
&+ \int_{\mathrm{d}\Phi_{m}} \left[\mathrm{d}\hat{\sigma}^{V,2}_{NNLO} - \mathrm{d}\hat{\sigma}^{U}_{NNLO}\right],
\end{aligned}$$

where the terms in each of the square brackets is finite, well behaved in the infrared singular regions and can be evaluated numerically.

$$\begin{aligned}
\mathrm{d}\hat{\sigma}^{T}_{NNLO} &= \mathrm{d}\hat{\sigma}^{VS,1}_{NNLO} - \int_1 \mathrm{d}\hat{\sigma}^{S,1}_{NNLO} - \mathrm{d}\hat{\sigma}^{MF,1}_{NNLO}, \\
\mathrm{d}\hat{\sigma}^{U}_{NNLO} &= -\int_1 \mathrm{d}\hat{\sigma}^{VS,1}_{NNLO} - \int_2 \mathrm{d}\hat{\sigma}^{S,2}_{NNLO} - \mathrm{d}\hat{\sigma}^{MF,2}_{NNLO}.
\end{aligned}$$

Basics of Antenna Subtraction Method - I

- ✓ Colour ordered amplitudes
- ✓ two hard radiators i and k with soft unresolved radiation j between them mapping on to two hard particles I and K
- ✓ consider singularities associated with j in real radiation contribution

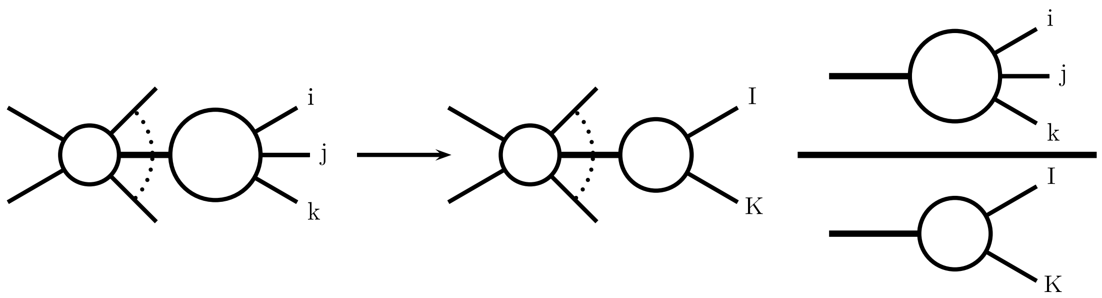
$$\mathrm{d}\hat{\sigma}^{R}_{NLO} \sim |M_{m+1}(\ldots, p_i, p_j, p_k, \ldots)|^2 d\Phi_m(\ldots, p_i, p_j, p_k, \ldots; q)$$

- ✓ construct subtraction term based on
  - A factorisation of squared matrix elements
  - B factorisation of phase space

Basics of Antenna Subtraction Method - II

A factorization of $(m+1)$-particle matrix elements

$$|M_{m+1}(\ldots,p_i,p_j,p_k,\ldots)|^2 \to |M_m(\ldots,p_I,p_K,\ldots)|^2 \times X_{ijk}$$



- ✓ X_{ijk} must have correct limits when $i||j$ or $j||k$ or j soft
- ✓ Different types of hard radiator/unresolved particle accounted for by having different X_{ijk}
- ✓ At NLO, there are five possible antenna - $Qg\bar{Q}$, Qgg, GgG, $Gq\bar{Q}$ and $Qq'\bar{Q}'$

Basics of Antenna Subtraction Method - III

- factorization of single unresolved phase space via antenna phase space mapping $\{p_i, p_j, p_k\} \to \{p_I, p_K\}$ Kosower, (97)

$$d\Phi_{m+1}(\ldots, p_i, p_j, p_k, \ldots; q) \to d\Phi_m(\ldots, p_I, p_K, \ldots; q) \times d\Phi_{X_{ijk}}$$

✓ energy-momentum conservation

✓ particles I and K on-shell

⇒ singularities in $\mathrm{d}\hat{\sigma}^R_{NLO}$ due to j unresolved removed by subtraction term

$$\mathrm{d}\hat{\sigma}^S_{NLO} \sim |M_m(\ldots, p_I, p_K, \ldots)|^2 d\Phi_m(\ldots, p_I, p_K, \ldots; q) \times X_{ijk}\, d\Phi_{X_{ijk}}$$

✓ all dependence on $\{p_i, p_j, p_k\}$ resides in $X_{ijk}\, d\Phi_{X_{ijk}}$

✓ Born-like piece depends only on $\{p_I, p_K\}$

✓ all singularities associated with j unresolved lie in $X_{ijk}\, d\Phi_{X_{ijk}}$

✓ full subtraction term built by summing terms like this

Basics of Antenna Subtraction Method - IV

- ✓ Antenna functions constructed from square of physical matrix elements
  - + singular limits well understood
  - - not completely local - azimuthal terms in single collinear limits averaged (see later)
- ✓ key point of subtraction approach is that the integrated form of the subtraction term has to be added back in,
  i.e. must be able to do the integral over antenna phase space

$$\mathcal{X}_{ijk}(s_{IK}) = \int X_{ijk}\, d\Phi_{X_{ijk}}$$

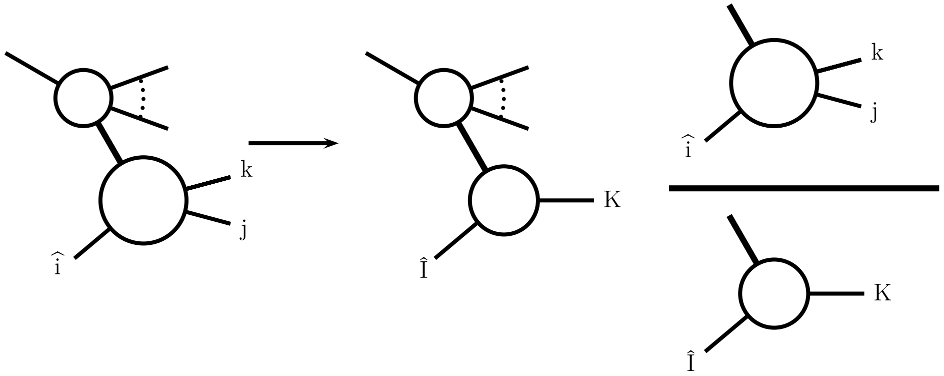
analytically and in $D = 4 - 2\epsilon$

- ✓ first application at NLO, $e^+e^- \to 4$ jets

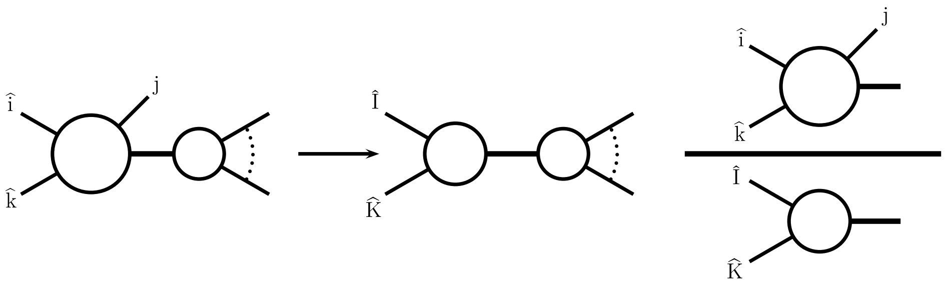
Campbell, Cullen, NG, (98)

Basics of Antenna Subtraction Method - V

Initial-Final Radiators:



Initial-Initial Radiators:



- ✓ Antenna functions obtained by crossing hard radiators into the initial state
- ✓ modified phase space mappings to preserve factorisation of phase space

Daleo, Gehrmann, Maitre, (06)

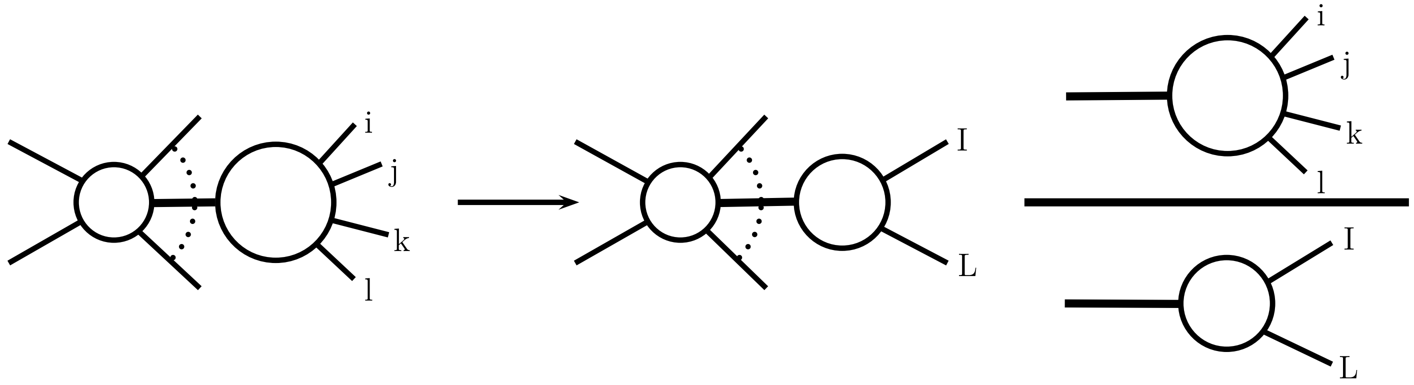
- ✓ Application to massive coloured radiators also studied

Gehrmann-De Ridder, Ritzmann, (09),
Abelof and Gehrmann-De Ridder, (11),
Bernreuther, Bogner, Dekkers, (11)

but here concentrate on massless case

Antenna Subtraction Method at NNLO - I

✓ Double Real radiation factorisation



✓ Similar factorisation as at NLO but now with four-particle tree-level antennae X_4^0 to describe the double unresolved limits e.g. $Qgg\bar{Q}$
Altogether 11 different antennae

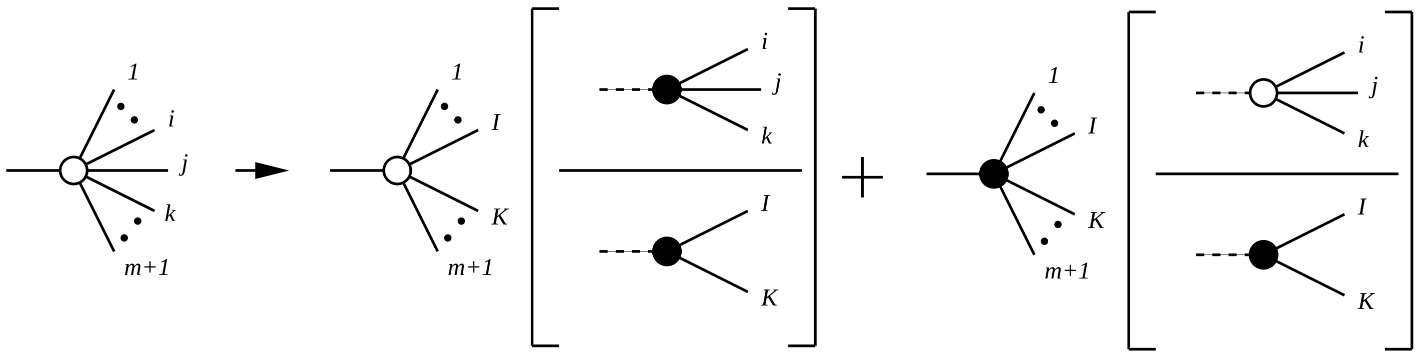
✓ factorization of double unresolved phase space via NNLO antenna phase space mapping that takes the hard radiators i and ℓ separated by colour connected unresolved radiation j and k onto two new on-shell momenta I and L Kosower, (02)

Antenna Subtraction Method at NNLO - II

- ✓ Double real radiation subtraction term $\mathrm{d}\hat{\sigma}^{S}_{NNLO}$ obtained by adding up combinations of
  - ✓ $X_3^0 \times |M_{m+1}|^2$ contributions to describe one unresolved parton but the experimental observable selects only m jets.
  - ✓ $X_4^0 \times |M_m|^2$ contributions to describe the double unresolved limits
  - ✓ $X_3^0 \times X_3^0 \times |M_m|^2$ terms to account for colour disconnected and partially colour connected contributions
  - ✓ wide angle soft correction $S \times X_3^0$ type terms to make sure no overall overcounting of single unresolved limits Weinzierl, (08)
- ✓ check is correct behaviour in singular limits - see later

Antenna Subtraction Method at NNLO - III

✓ Real Virtual factorisation



✓ New type of factorisation with three-particle one-loop antennae X_3^1 to describe the unresolved limits
Altogether 11 different antennae

✓ factorization of single unresolved phase space via phase space mapping
Kosower, (97)

Antenna Subtraction Method at NNLO - IV

- ✓ Real Virtual subtraction term $\mathrm{d}\hat\sigma^{T}_{NNLO}$ obtained by adding up combinations of
  - ✓ $\mathcal{X}_3^0 \times |M_m|^2$ contributions to cancel the explicit infrared poles of the (m+1)-parton one-loop matrix element.
  - ✓ $X_3^1 \times |M_m|^2$ and $X_3^0 \times |M_m^1|^2$ contributions to describe the single unresolved limits of the one-loop contribution
  - ✓ $\mathcal{X}_3^0 \times X_3^0 \times |M_m|^2$ and $\mathcal{S} \times X_3^0 \times |M_m|^2$ type terms coming from $\int_1 \mathrm{d}\hat\sigma^{S}_{NNLO}$
  - ✓ $\mathcal{X}_3^0 \times X_3^0 \times |M_m|^2$ to account for any remaining singularities
- ✓ (after mass factorisation) check is cancellation of explicit infrared poles and correct behaviour in singular limits

⚠ requires full knowledge of integrated wide angle soft contribution $\mathcal{S}$ - see later

$$\left[\mathrm{d}\hat\sigma^{T}_{NNLO} \quad = \quad \mathrm{d}\hat\sigma^{VS,1}_{NNLO} - \int_1 \mathrm{d}\hat\sigma^{S,1}_{NNLO} - \mathrm{d}\hat\sigma^{MF,1}_{NNLO}\right]$$

Antenna Subtraction Method at NNLO - V

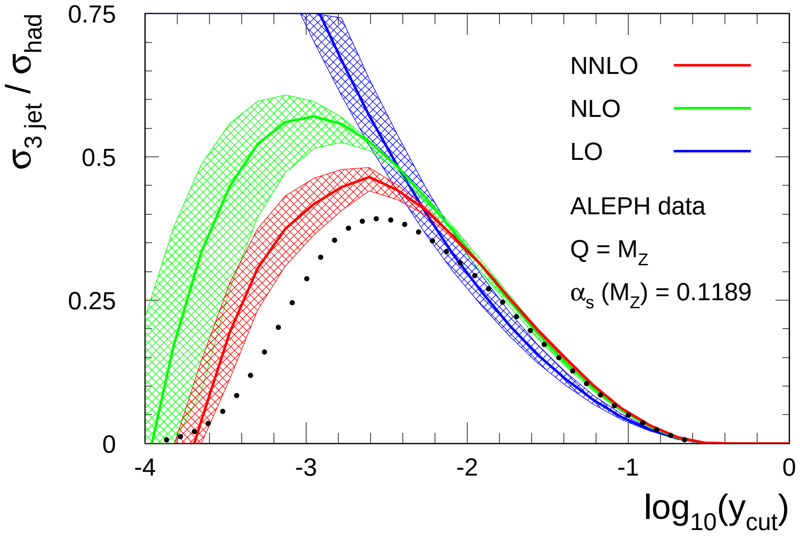
- ✓ Double Virtual subtraction term $\mathrm{d}\hat{\sigma}^{U}_{NNLO}$ fixed by adding up
  - ✓ $\mathcal{X}_4^0 \times |M_m|^2$ contributions coming from $\int_2 \mathrm{d}\hat{\sigma}^{S}_{NNLO}$
  - ✓ $\mathcal{X}_3^1 \times |M_m|^2$, $\mathcal{X}_3^0 \times |M_m^1|^2$ and $\mathcal{X}_3^0 \times \mathcal{X}_3^0 \times |M_m|^2$ contributions coming from $\int_1 \mathrm{d}\hat{\sigma}^{VS,1}_{NNLO}$
- ✓ (after mass factorisation) check is cancellation of explicit infrared poles present in double virtual contribution
- ⚠ requires full knowledge of all integrated subtraction terms, specifically $\mathcal{X}_4^0$ and $\mathcal{X}_3^1$ - see later

$$\left[\mathrm{d}\hat{\sigma}^{U}_{NNLO} \quad = \quad -\int_1 \mathrm{d}\hat{\sigma}^{VS,1}_{NNLO} - \int_2 \mathrm{d}\hat{\sigma}^{S,2}_{NNLO} - \mathrm{d}\hat{\sigma}^{MF,2}_{NNLO}\right]$$

$e^+e^- \to 3$ jets at NNLO

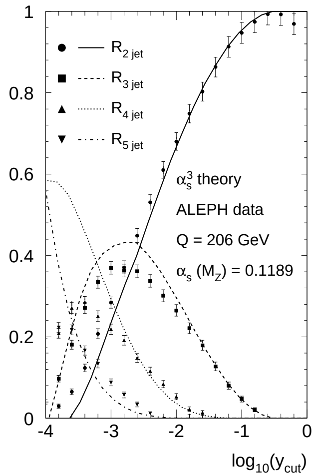
Method thoroughly tried and tested for partons only in the final state

Gehrmann-De Ridder, Gehrmann, Heinrich, NG (07)



- ✓ NNLO corrections to jet rate small
  - ✓ stable perturbative prediction
  - ✓ resummation not needed
  - ✓ theory error below 2%
  - ✓ small hadronisation corrections
- ✓ α_s extraction from jet rates

Dissertori, Gehrmann-De Ridder, Gehrmann, Heinrich, Stenzel, NG (09)

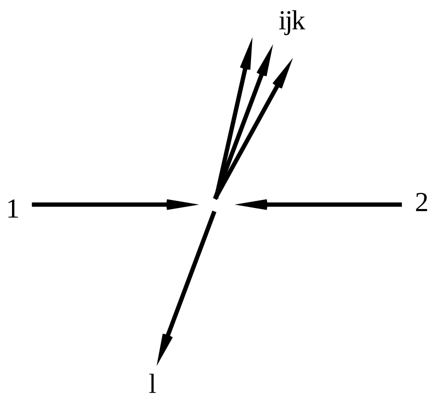


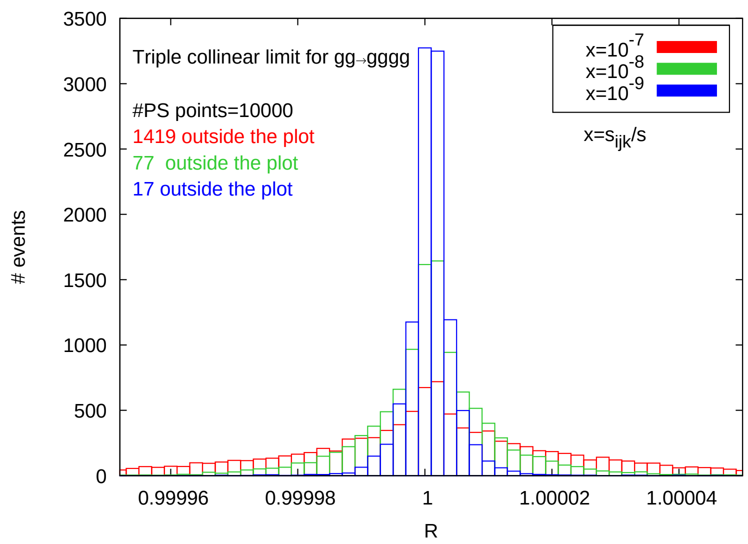
  - ✓ fit at $y_{cut} = 0.02$
  - ✓ consistent results at other y_{cut}

$\alpha_s(M_Z) = 0.1175 \pm 0.0020(exp) \pm 0.0015(th)$

Applications to LHC processes - status

- ✓ All relevant matrix elements for $pp \to 2$ jet and $pp \to V+1$ jet processes available for some time
- ⚠ Aim to push “leading colour gluons-only" $pp \to 2$ jets all the way to the end to demonstrate proof of concept
- ✓ Double unresolved subtraction terms for leading colour six-gluon process tested





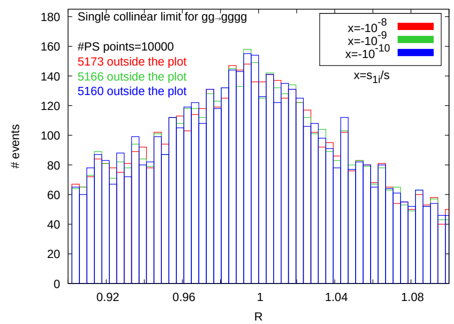
(a) Example configuration of a triple collinear event with $s_{ijk} \to 0$.

(b) Distribution of $\mathrm{d}\hat{\sigma}^R_{NNLO}/\mathrm{d}\hat{\sigma}^S_{NNLO}$ for 10000 triple collinear phase space points.

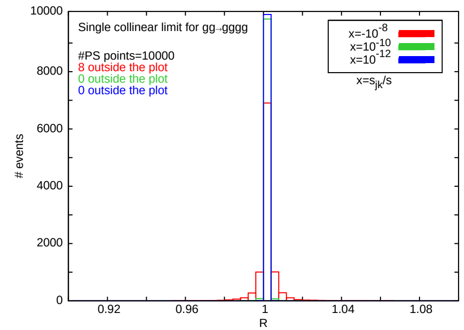
Pires, NG, (10)

Applications to LHC processes - status

✘ Evidence of non-local azimuthal terms in collinear limits e.g. configuration of a single collinear event with $s_{1i} \to 0$.



✓ Solution: Combine events with momenta of collinear pair rotated by 90 degrees



Pires, NG, (10)

⚠ Automatic generation of phase space points related by rotations

Gehrmann

Applications to LHC processes - status

- Real Virtual subtraction terms for one-loop five-gluon process almost complete Gehrmann-De Ridder, Pires, NG
- Few remaining “initial-initial" integrals necessary to complete “leading colour gluons-only" $pp \to 2$ jet (same type of integrals as already encountered)
- ✓ Same integrals are needed for all other processes

Integrated three-parton tree antennae

$\mathcal{X}_3^0$	Final-Final	Initial-Final	Initial-Initial
A	✓ [1]	✓ [2]	✓ [2]
D	✓ [1]	✓ [2]	✓ [2]
E	✓ [1]	✓ [2]	✓ [2]
F	✓ [1]	✓ [2]	✓ [2]
G	✓ [1]	✓ [2]	✓ [2]

[1] Gehrmann-De Ridder, Gehrmann, NG, (05)

[2] Daleo, Gehrmann, Maitre, (06)

$\mathcal{S}$	Final-Final	Initial-Final	Initial-Initial
S	✓ [1]	✓ [2]	

[1] Gehrmann-De Ridder, Gehrmann, NG, Heinrich, (07)

[2] Daleo, Gehrmann-De Ridder, Gehrmann, Luisoni, (09)

Integrated three-parton one-loop antennae

$\mathcal{X}_3^1$	Final-Final	Initial-Final	Initial-Initial
A, $\tilde{A}$, $\hat{A}$	✓ [1]	✓ [2]	✓ [3]
D, $\hat{D}$	✓ [1]	✓ [2]	✓ [3]
F, $\hat{F}$	✓ [1]	✓ [2]	✓ [3]
E, $\tilde{E}$, $\hat{E}$	✓ [1]	✓ [2]	✓ [3]
G, $\tilde{G}$, $\hat{G}$	✓ [1]	✓ [2]	✓ [3]

[1] Gehrmann-De Ridder, Gehrmann, NG, (05)

[2] Daleo, Gehrmann-De Ridder, Gehrmann, Luisoni, (09)

[3] Gehrmann, Monni, (11)

Integrated four-parton tree antennae

$\mathcal{X}_4^0$	Final-Final	Initial-Final	Initial-Initial
A, $\tilde{A}$	✓ [1]	✓ [2]	
B	✓ [1]	✓ [2]	✓ [3]
C	✓ [1]	✓ [2]	
D	✓ [1]	✓ [2]	
E, $\tilde{E}$	✓ [1]	✓ [2]	✓ [3]
F	✓ [1]	✓ [2]	
G, $\tilde{G}$	✓ [1]	✓ [2]	
H	✓ [1]	✓ [2]	✓ [3]

[1] Gehrmann-De Ridder, Gehrmann, NG, (05)

[2] Daleo, Gehrmann-De Ridder, Gehrmann, Luisoni, (09)

[3] Boughezal, Gehrmann-De Ridder, Ritzmann, (10)

Remaining Initial-Initial functions depend on further 20 master integrals

Applications to LHC processes - status

- Real Virtual subtraction terms for one-loop five-gluon process almost complete Gehrmann-De Ridder, Pires, NG
- Few remaining “initial-initial" integrals necessary to complete “leading colour gluons-only" $pp \to 2$ jet (same type of integrals as already encountered)
- ✓ Same integrals are needed for all other processes
- Aim to have “leading colour gluons-only" $pp \to 2$ jet in place in next few months
- In parallel, coding of sub-leading colour contributions, quark processes and $pp \to V + 1$ jet underway