The proper time to the black hole singularity from thermal one point functions

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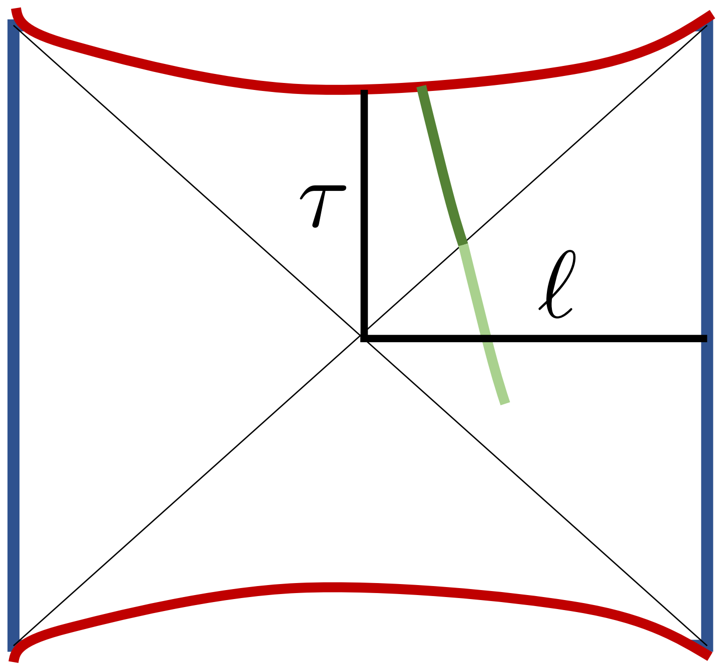
Based on: ArXiv: 2011.01004 by Grinberg and JM

Time to the singularity for black holes in AdS

- This time is of order the AdS radius or shorter.

In $\mathcal{N}$=4 SYM: $$\frac{R}{l_p} \sim N^{1/4} \, , \qquad \frac{R}{l_s} \sim \lambda^{1/4}$$

We need strong coupling



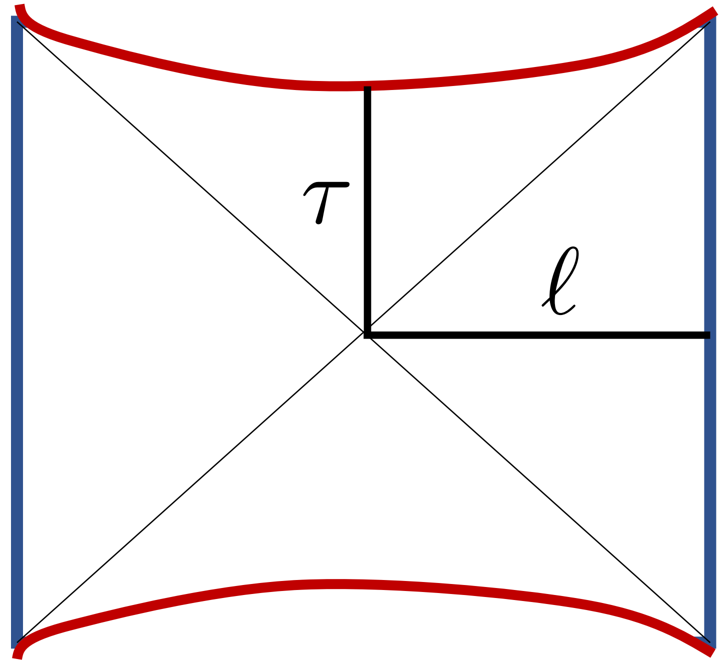
The time between the horizon and the singularity of a Schwarzschild black hole is the maximum amount that an observer can live in the interior.

A clock to measure time

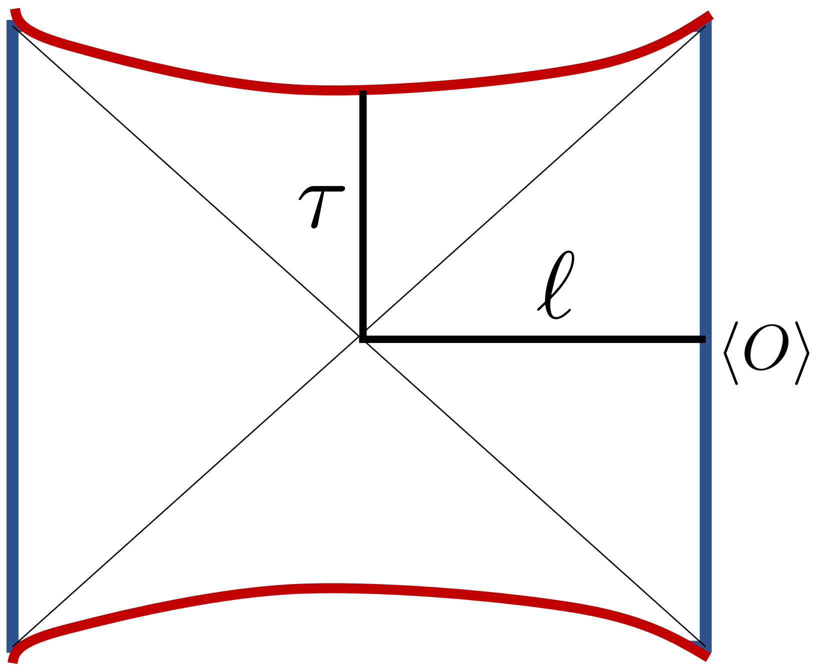
- The phase of a wavefunction acts as a clock. (If you can create and annihilate the particle).

$$\psi \propto e^{-imt}$$

- Can we find an observable sensitive to this?



Time from thermal one point function



We will argue that, under some assumptions, this time is contained in thermal one point functions.

Through the mass dependence:

$$\langle O \rangle \propto (\text{powers of } m) \exp(-im\tau - m\ell)$$

$$\text{Im}(m) < 0$$

We will discuss some fine print later…

Outline

- Thermal one point functions from higher derivative corrections.
- Computation for large mass and geodesics.
- Geodesic for the one point function.
- A simple explicit example for black branes
  - Explicit answer
  - Geodesic approximation
  - Justification of the geodesic approximation.
- Other examples.
- Charged black holes

Minimally coupled scalar field

$$S = \frac{1}{16\pi G_N} \int (\nabla\varphi)^2 + m^2\varphi^2$$

Leads to zero one point functions due to the Z_2 symmetry

Scalar field coupled to gravitons

$$S = \frac{1}{16\pi G_N}\int \underbrace{(\nabla\varphi)^2 + m^2\varphi^2}_{\text{Standard minimal coupling}} + \alpha\varphi W^2 + \cdots$$

Standard minimal coupling

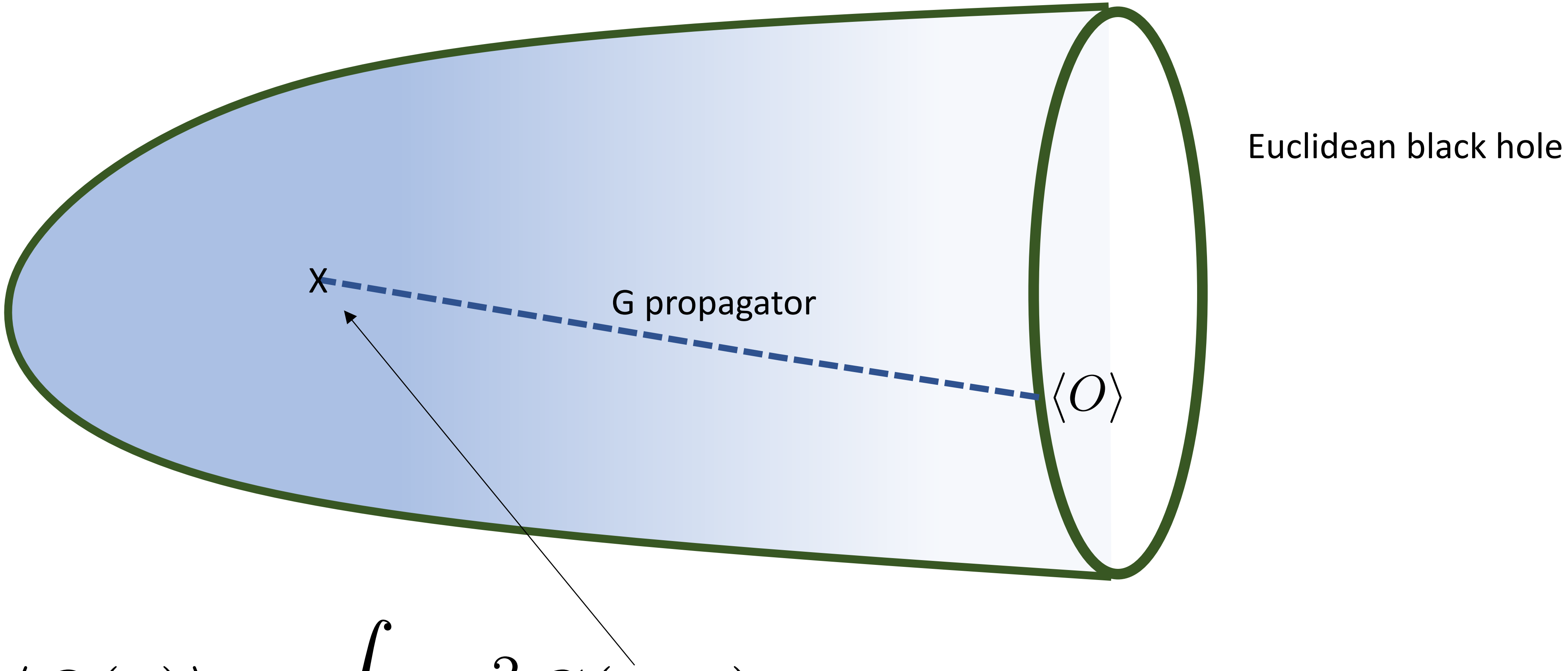
Coupling to two gravitons.

Massive field can decay into two gravitons.

In string theory: present for a generic massive string state. $\alpha \propto \alpha'$

In N=4 SYM, $\frac{\alpha}{R^2} \propto 1/\sqrt{\lambda}$

Thermal one point function

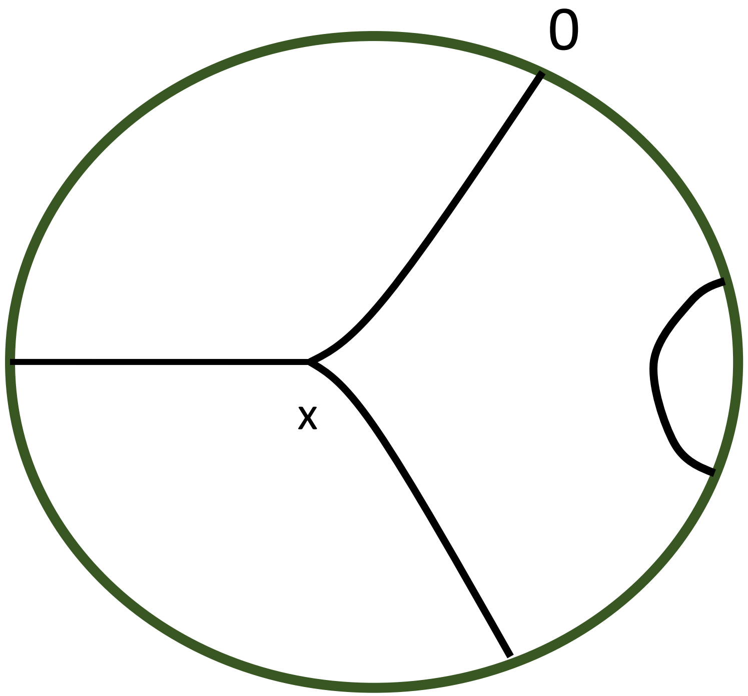


$$\langle O(0) \rangle \propto \int W^2 G(x,0)$$

Large mass and geodesics

- For large mass we can approximate the propagator in terms of a geodesic.

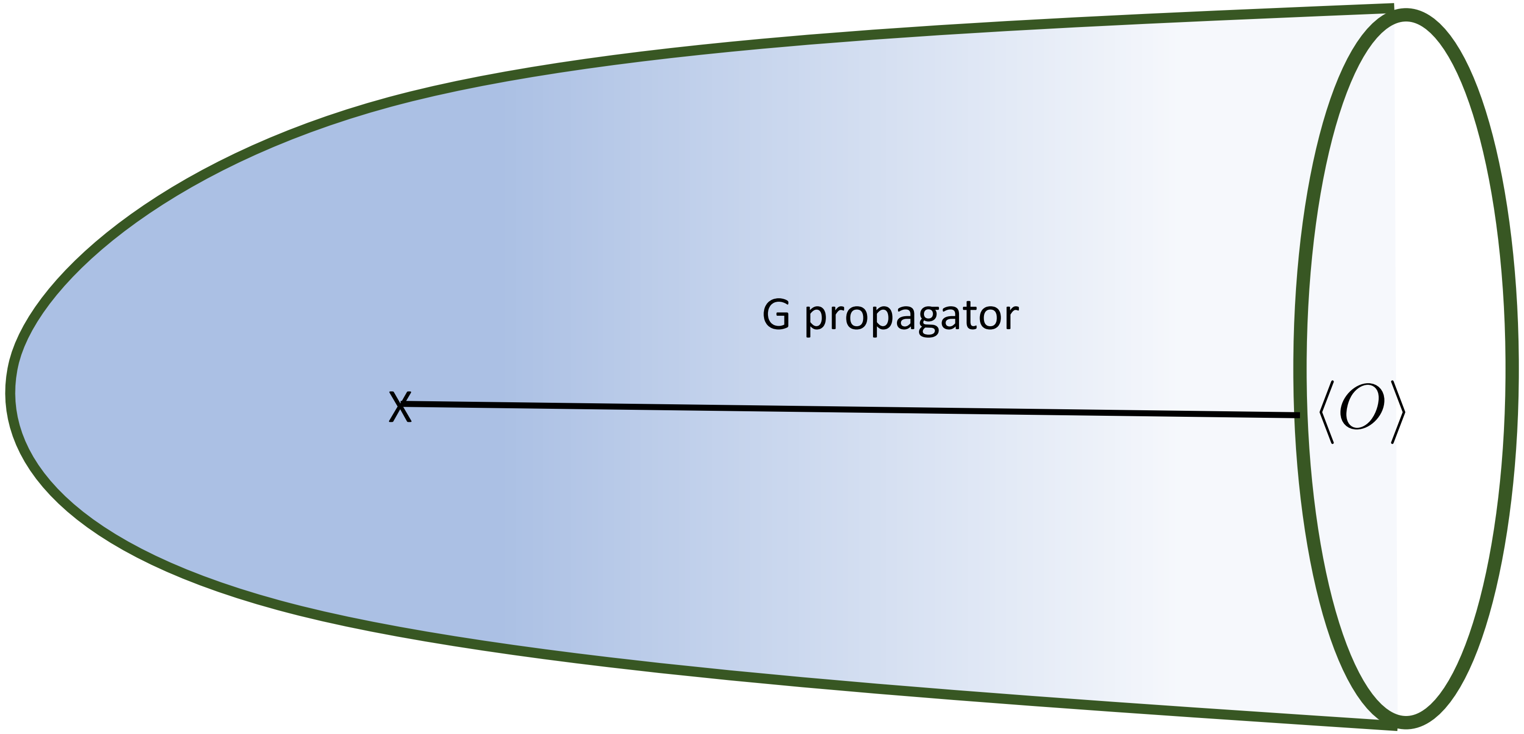
$$G(x,0) \propto e^{-m\ell(x)}$$



Witten diagrams → minimal geodesics.

Integration middle point → also fixed by a saddle point approximation

Thermal one point function



$$\langle O(0)\rangle \propto \int d^D x W^2 G(x,0) \propto \int dr W^2(r) e^{-m\ell(r)}$$

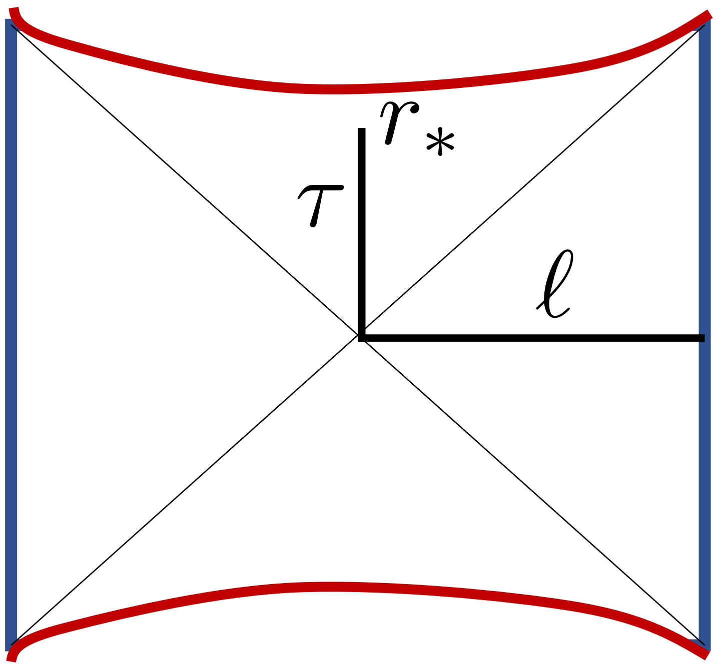
Use saddle point for the x integration,
Or integration over the radial variable

A saddle point

$$-m\partial_r \ell(r) + \partial_r \log W^2 = 0$$

Large coefficient

Must be near the singularity (say r=0)
(complex value in general, but near the singularity)



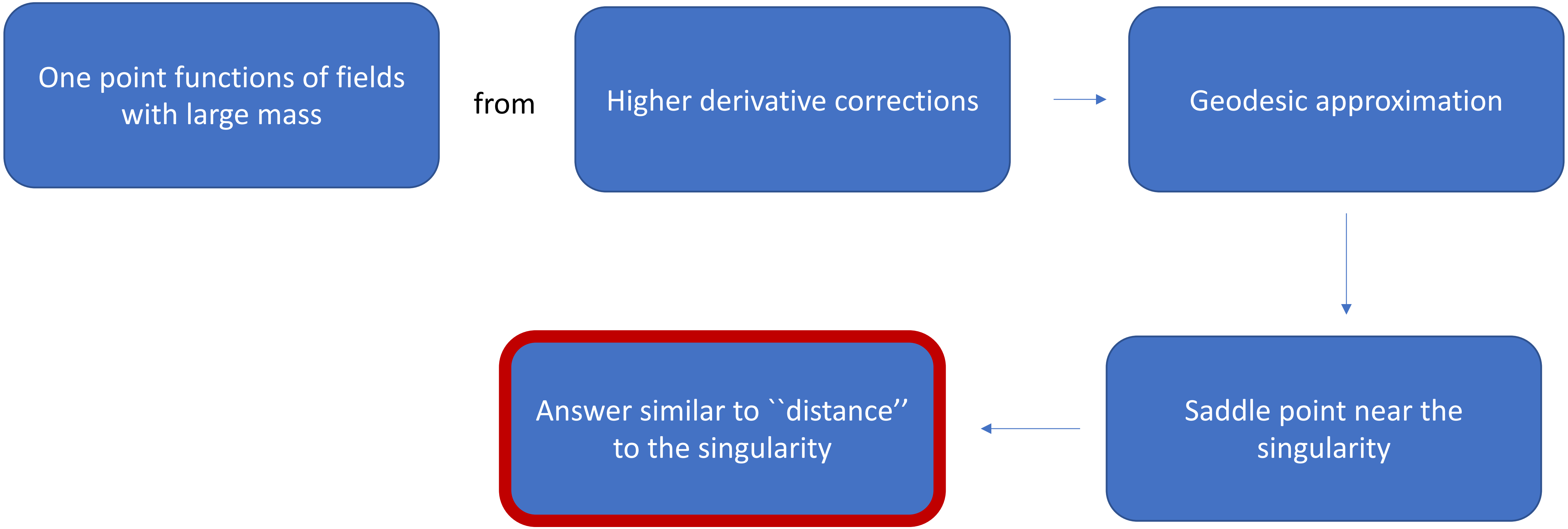
The saddle point is at an imaginary value of r, but very close to the singularity.

Evaluating the integral

$$W^2(r_*)e^{-m\ell(r*)} \propto (\text{powers of } m)e^{-m\ell(r_*)} \propto (\text{powers of } m)e^{-m(i\tau+\ell_{\text{hor}})}$$

→ Gives what we wanted

Logic flow chart



Notice that we are assuming that the higher derivative coupling is suppressed by powers of m, and not exponentials…
This is true in string theory, if we focus on the alpha' dependence.

But…

- Does this saddle contribute if it is not on the original integration contour?
- More details…

We will work out an example in detail:

The planar black brane

The geometry

$$ds^2_{d+1} = R^2 \frac{1}{z^2}\left[-(1-\frac{z^d}{z_0^d})dt^2 + \frac{dz^2}{(1-\frac{z^d}{z_0^d})} + d\vec{x}^2\right]$$

= field theory at finite temperature in flat space

$$\langle O \rangle = T^{\Delta} c(\Delta)$$

Information we want is in the form of this coefficient

Doing the integral

Myers, Sierens and Witczak-Krempa

$$\langle O \rangle = T^{\Delta} (\text{real})^{\Delta} \frac{1}{\sin(\frac{\pi\Delta}{d})} \times (\text{Powers of } \Delta)$$

$$\begin{aligned} \langle O \rangle &= -C_{\mathrm{N}} \sqrt{\frac{16\pi G_{\mathrm{N}}}{R^{d-1}}} \frac{\alpha}{R^2} \frac{(d-2)(d-1)^2}{d} \frac{\Gamma(h)^2}{\Gamma(2h)} \int_0^1 dw\; w^h \; {}_2F_1(h,h,1;1-w) \\ &= -C_{\mathrm{N}} \sqrt{\frac{16\pi G_{\mathrm{N}}}{R^{d-1}}} \frac{\pi\alpha}{R^2} \left(\frac{4\pi T}{d}\right)^{\Delta} \frac{(d-2)(d-1)^2}{d} \frac{\Gamma(h)^2}{\Gamma(2h)} \frac{h(1-h)}{\sin \pi h} . \end{aligned}$$

$$h = \frac{\Delta}{d}$$

Small imaginary part..

$$\langle O \rangle = T^{\Delta} (\text{real})^{\Delta} \frac{1}{\sin(\frac{\pi \Delta}{d})} \times (\text{Powers of } \Delta)$$

Consider

$$\Delta \gg 1 \;, \qquad \text{Im}(\Delta) < 0 \;, \qquad \text{e.g} \quad \Delta = |\Delta|(1 - i\epsilon)$$

$$\frac{1}{\sin \frac{\pi \Delta}{d}} \propto \frac{e^{-i \frac{\pi \Delta}{d}}}{1 - e^{-i \frac{2\pi \Delta}{d}}} \sim e^{-i \frac{\pi \Delta}{d}}$$

Subleading

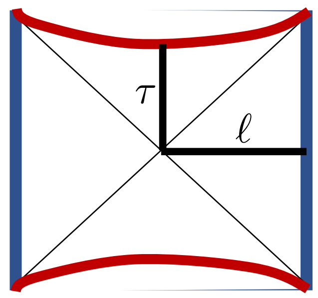
This is how we got a ''phase''.
From the original real number.

This also avoided the zeros in the denominator

Time to the singularity

$$ds^2_{d+1} = R^2 \frac{1}{z^2}\left[-(1-\frac{z^d}{z_0^d})dt^2 + \frac{dz^2}{(1-\frac{z^d}{z_0^d})} + d\vec{x}^2\right]$$

$$\tau = R\int_1^\infty \frac{dz}{z\sqrt{z^d-1}} = \frac{R}{d}\int_1^\infty \frac{dw}{w\sqrt{w-1}} = R\frac{\pi}{d}$$



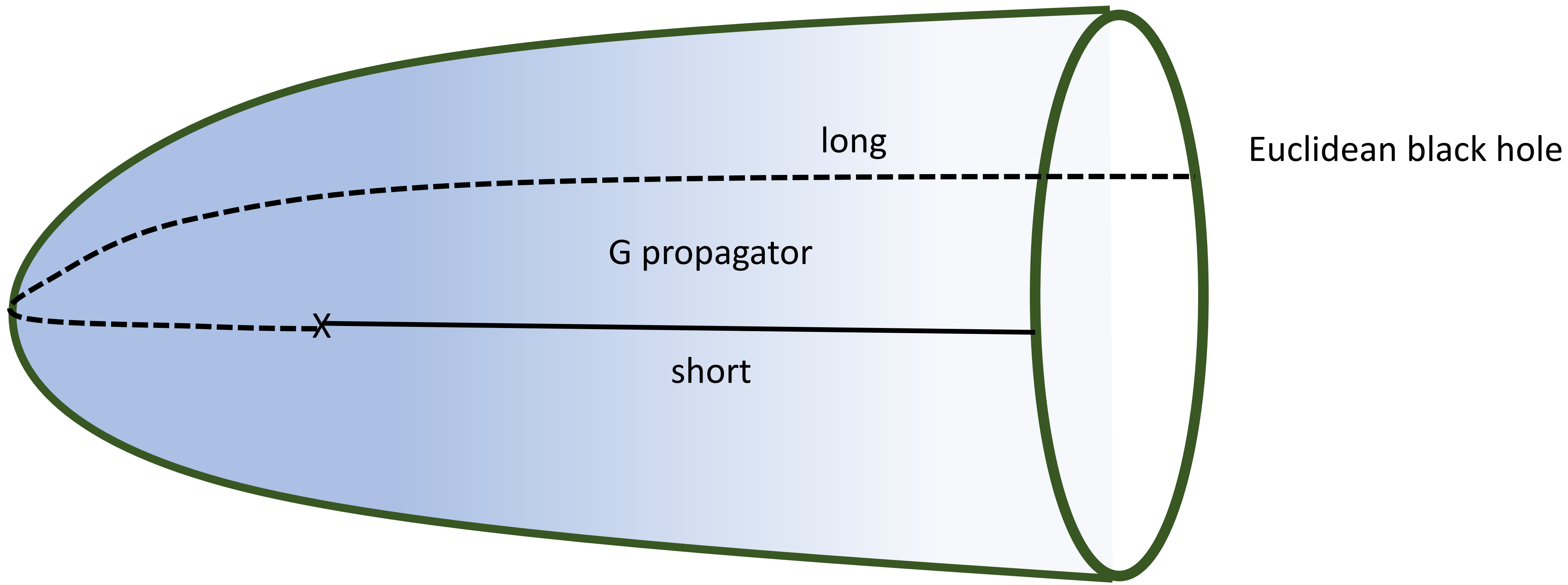
$$\tau m = (mR)\frac{\pi}{d} \sim \frac{\pi\Delta}{d}$$

Indeed what we had in the exponent previously

$$\frac{1}{\sin\frac{\pi\Delta}{d}} \propto \frac{e^{-i\frac{\pi\Delta}{d}}}{1-e^{-i\frac{2\pi\Delta}{d}}} \sim e^{-i\frac{\pi\Delta}{d}}$$

We will now discuss the saddle point approximation to the integral

Saddle point approximation to the propagator

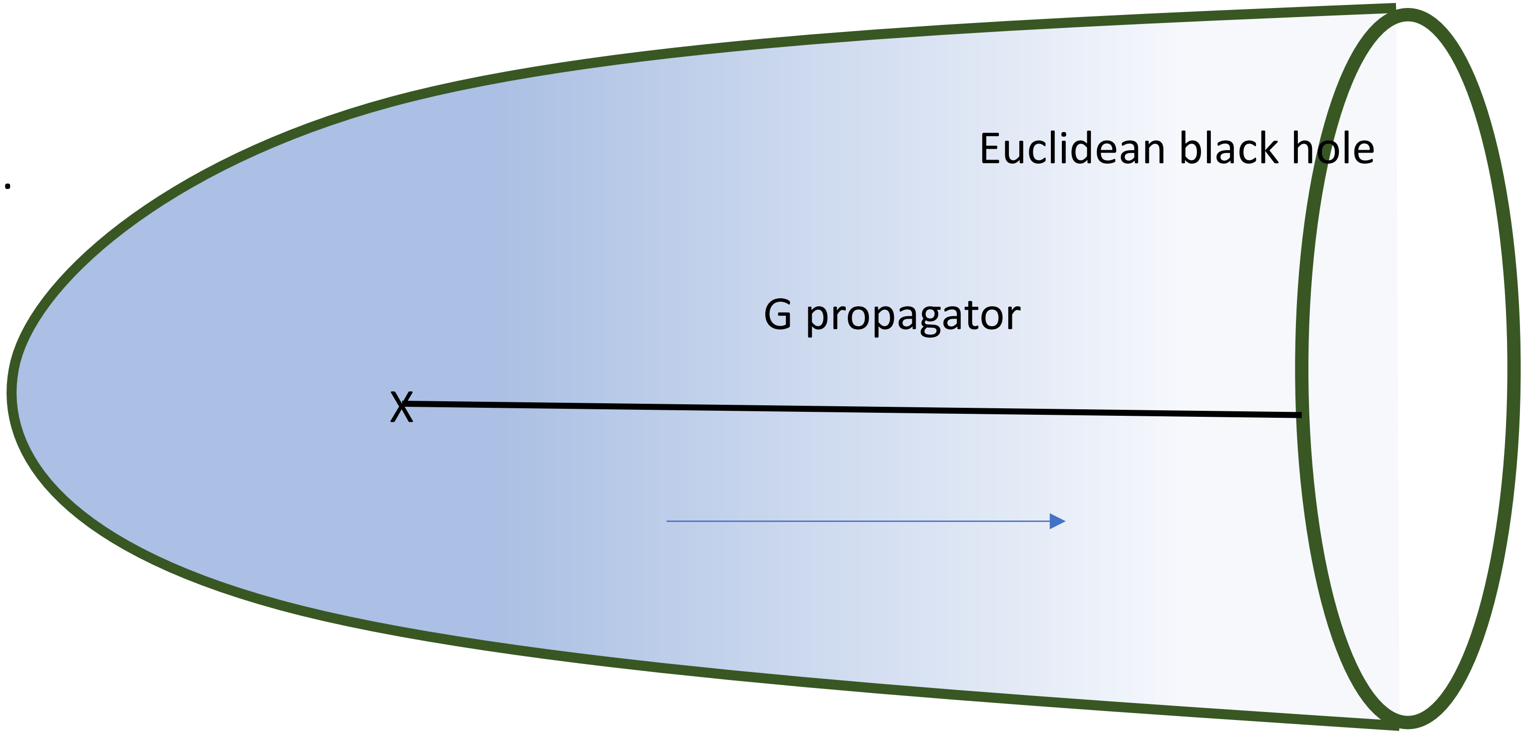


$$\langle O(0)\rangle \propto \int d^D x W^2 G(x,0) \propto \int dr W^2(r) e^{-m\ell(r)}$$

Use saddle point for the x integration,
Or integration over the radial variable

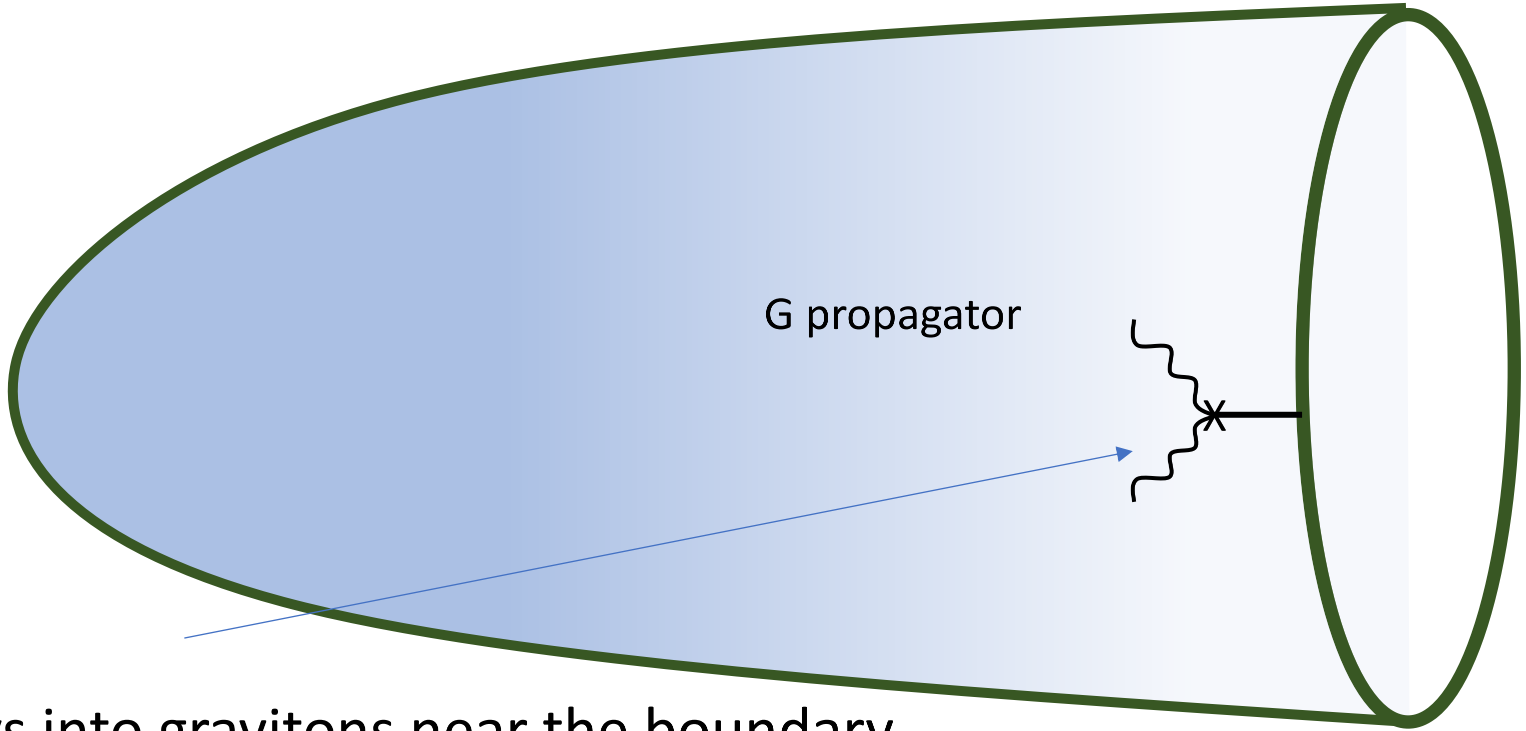
Problem

(For the short geodesic contribution.
The long one is convergent)



$$\langle O(0)\rangle \propto \int d^D x W^2 G(x,0) \propto \int dr W^2(r) e^{-m\ell(r)}$$

Drives the point near the boundary, overwhelming the W^2 contribution



Decays into gravitons near the boundary.

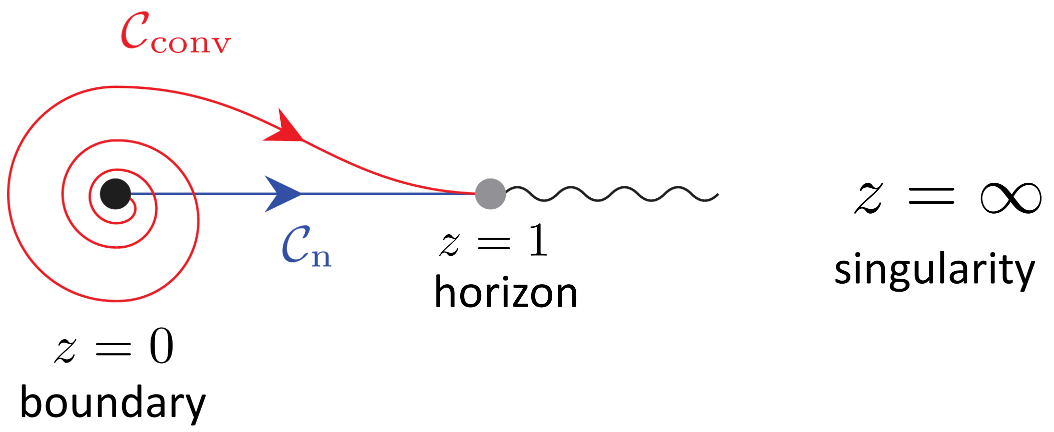
Operator mixing with $(T)^n$, n>1 .

The previous integral is not convergent for $\Delta \geq 2d$

We defined it via analytic continuation. → subtraction of all the lower dimension operators it can mix with.

At $\Delta = nd$ poles due to resonant mixing (at lower order in 1/N perturbation theory).

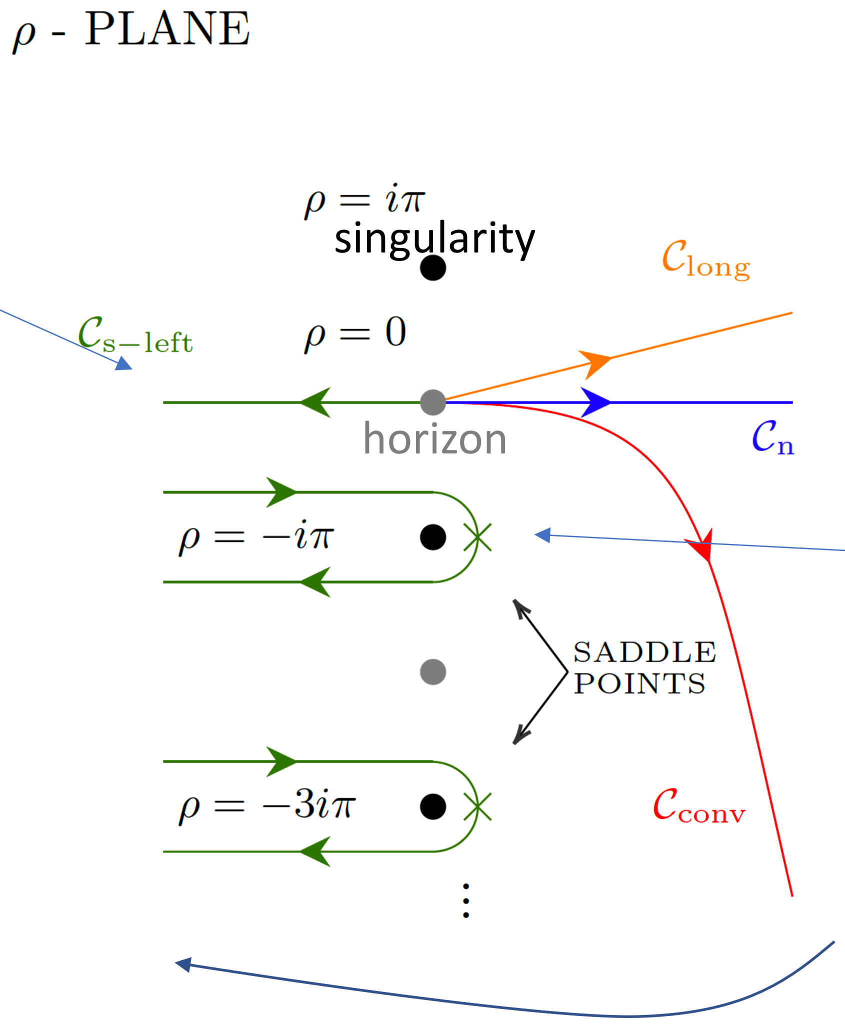
- The integral is naively infinite due to the contribution of multiple gravitons.
- We focus on the operator in question, with $\Delta \neq nd$
- We can define the integral via analytic continuation, when we know it exactly.
- When we do not know it exactly → define it so that it converges.
- New contour near the boundary.
- Convergent after we make Δ complex



Define a new variable ρ = proper length

$$z^d = \frac{1}{\left(\cosh \frac{\rho}{2}\right)^2}$$

Cancels against long geodesic contribution.



First saddle give dominant contribution

This justifies why the saddle points contribute

Rotate the contour

The leading saddle gives

$$e^{-i\frac{\pi\Delta}{d}}$$

which agrees with our general expected answer

The subdominant saddles sum up to

$$\frac{1}{\sin\frac{\pi\Delta}{d}} \propto \frac{e^{-i\frac{\pi\Delta}{d}}}{1-e^{-i\frac{2\pi\Delta}{d}}} \sim e^{-i\frac{\pi\Delta}{d}}$$

(up to some details we did not fully justify…)

Summary

- We considered a specific example: Black brane.
- We did the integral analytically.
- Matched it against the saddle point contribution.
- Analyzed the integral in the saddle for large mass and explained why the saddle point contributes via a contour rotation argument.

- We will now explore saddles for other black holes (but without doing a details analysis on whether they contribute or not, we will assume that they do contribute).

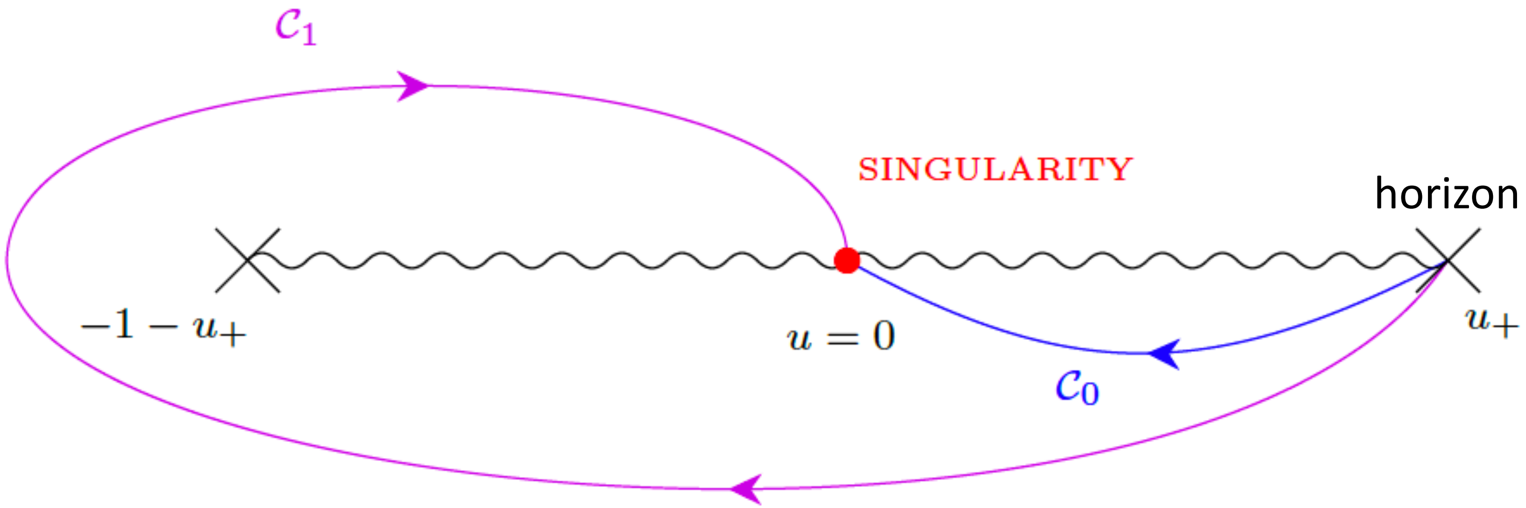
Schwarzschild AdS black holes

$$
\begin{aligned}
ds^2 &= R^2\left(-f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{d-1}^2\right)\,, \\
f(r) &= r^2 + 1 - \frac{\mu}{r^{d-2}}\,.
\end{aligned}
$$

$$
d\hat{\rho} = \frac{dr}{\sqrt{f(r)}}\,,
$$

Proper length

Schwarzschild AdS$_5$ black holes



$u = r^2$

$$\hat{\rho}_0 = -i\chi_0 \,, \qquad \hat{\rho}_1 = -i(\pi - \chi_0) \,, \qquad \sin^2\chi_0 = \frac{u_+}{1+2u_+} \,,$$

Black brane limit:

$u_+ \to \infty \qquad \longrightarrow$ Recover black brane answer by summing over both of them.

$$\langle O \rangle \sim e^{-m\ell_{\rm hor}} e^{-i\pi\Delta/4} \left(\frac{1+e^{-i\pi\Delta/2}}{1-e^{-i\pi\Delta}} \right) \sim \frac{4^{-\Delta/4}}{\sin\left(\frac{\pi\Delta}{4}\right)} \,,$$

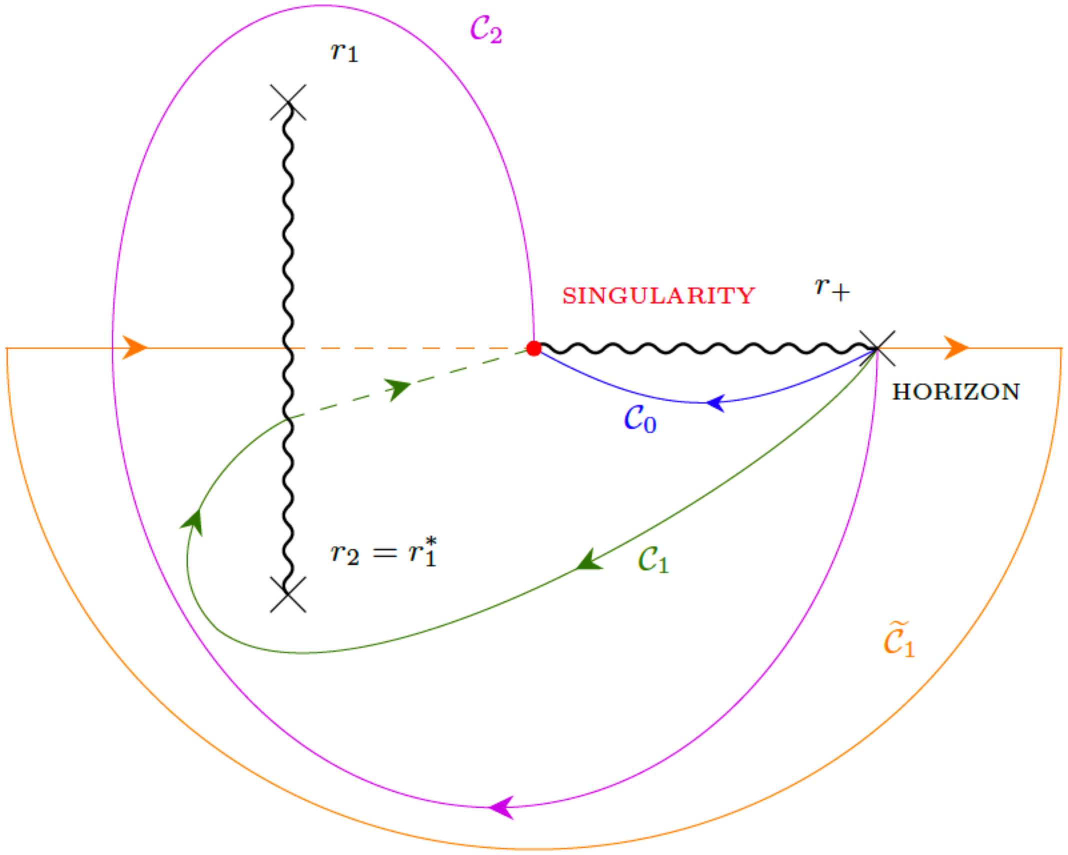
Comment on the poles

$$\langle O \rangle \quad \sim \quad e^{-m\ell_{\rm hor}} e^{-i\pi\Delta/4} \left(\frac{1 + e^{-i\pi\Delta/2}}{1 - e^{-i\pi\Delta}} \right) \quad \sim \quad \frac{4^{-\Delta/4}}{\sin\left(\frac{\pi\Delta}{4}\right)} ,$$

Poles form the denominator at $\Delta = 2n$ come from operators which can mix and have a non-zero vev, such as $T\partial^n T$, T^m

We expect that more information about the black hole comes form the numerator.

Schwarzschild AdS_4 black holes



$$ds^2 = R^2\left(-f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{d-1}^2\right) ,$$
$$f(r) = r^2 + 1 - \frac{\mu}{r^{d-2}} .$$

$$\hat{\rho}_0 = -i\chi_0 , \qquad \chi_0 \equiv \int_0^{r_+} \frac{dr}{\sqrt{-f(r)}} ,$$

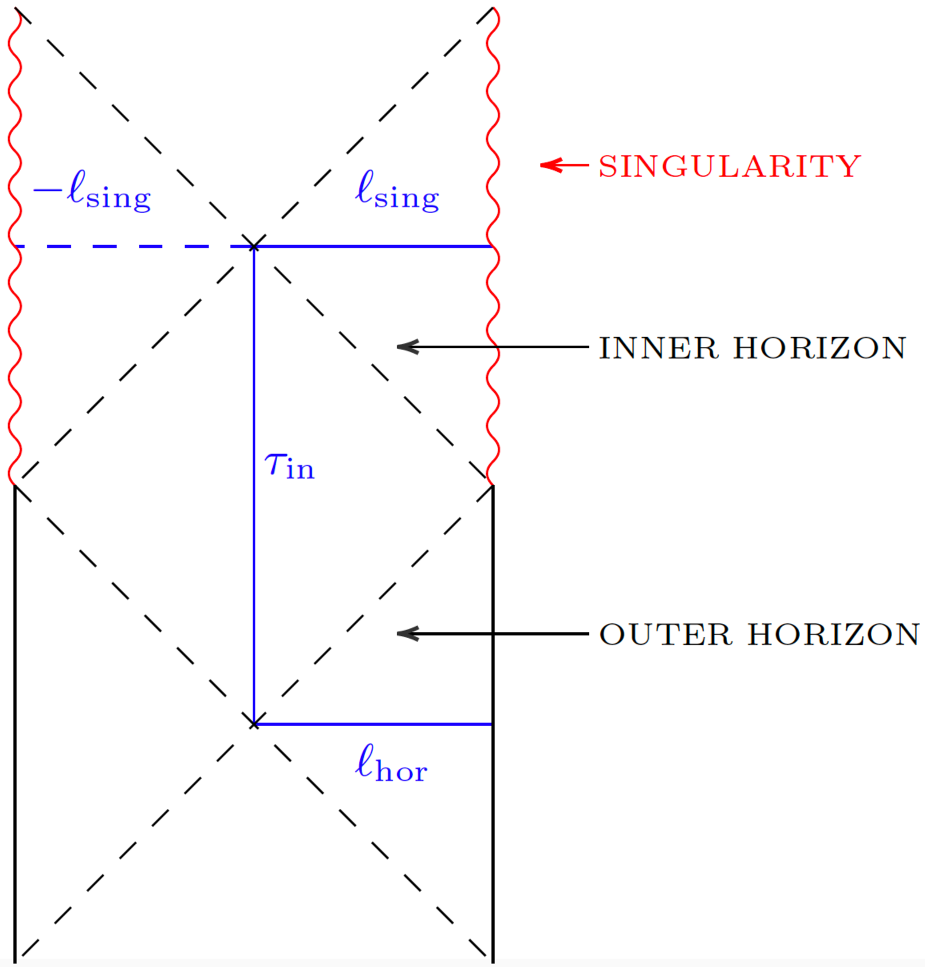
$$\hat{\rho}_1 = -i\pi + \gamma , \qquad \gamma = \lim_{r_c \to \infty}\left[\int_{r_+}^{r_c} \frac{dr}{\sqrt{f(r)}} - \int_{-r_c}^{0} \frac{dr}{\sqrt{f(r)}}\right] .$$

Real part is positive → dominates if Im(Δ) is small.
But not if Im(Δ) is large.

$$e^{m\hat{\rho}_i}$$

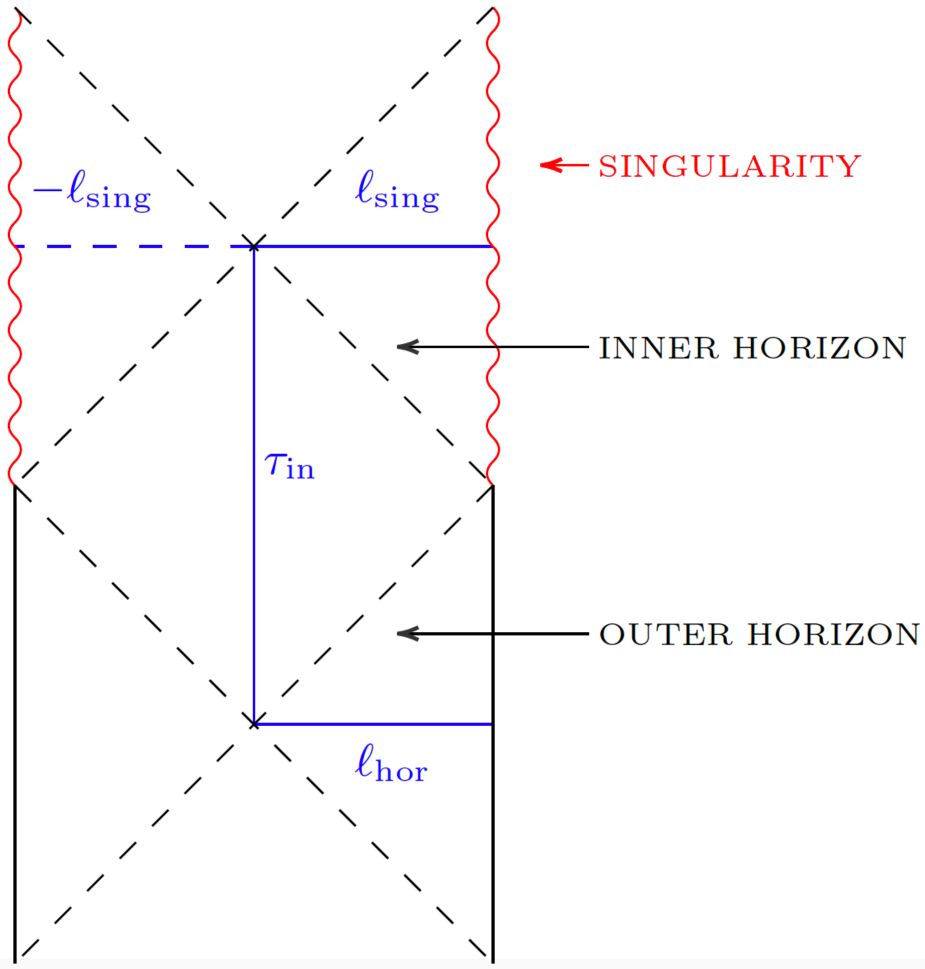
This seems to come from the integral of W^2 far from the black hole but within an AdS radius.

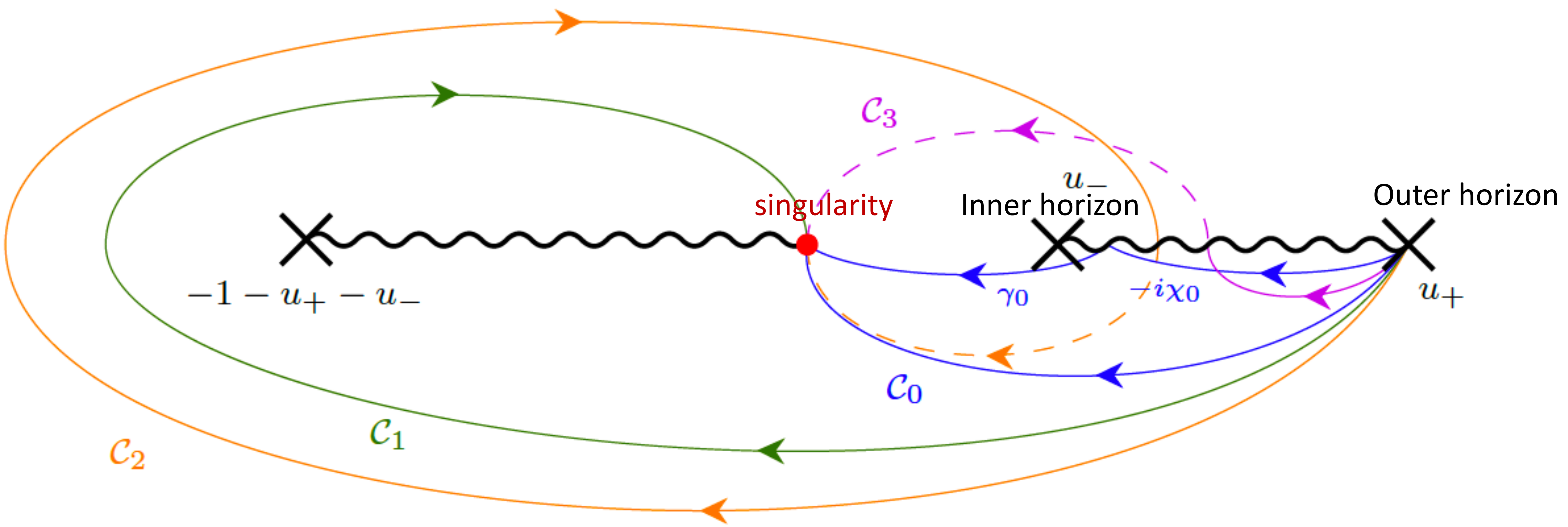
Charged black holes



New position of the singularities.

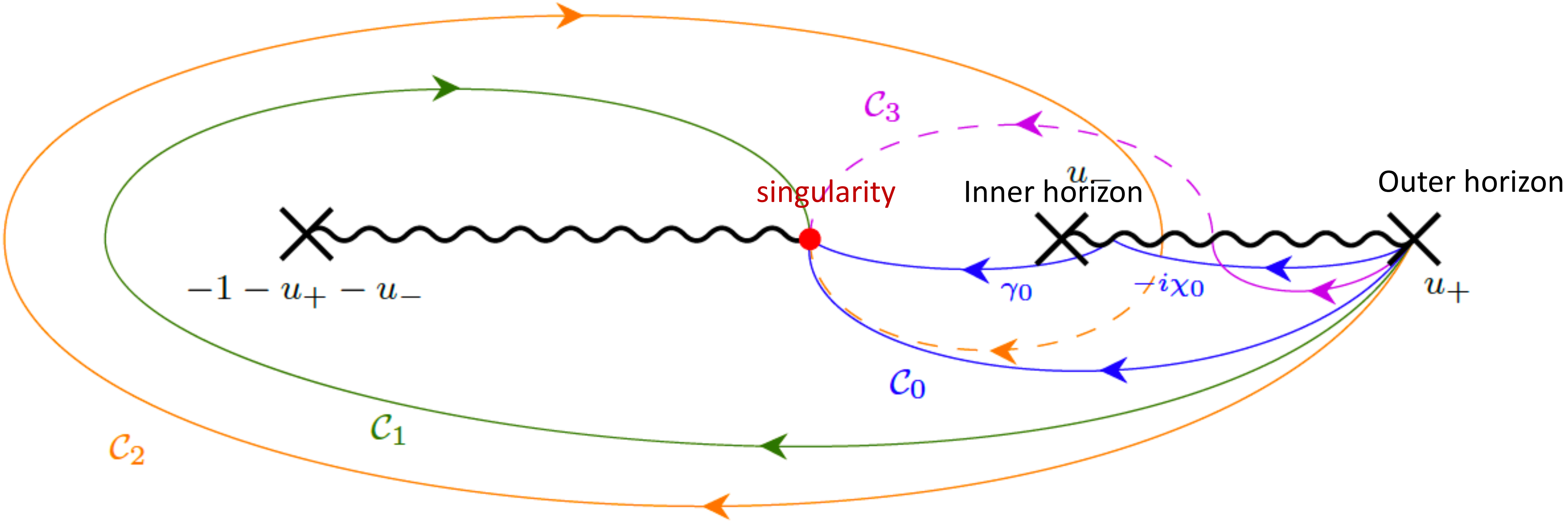
Charged AdS_5 black holes





We can compute the proper distance along all these curves. These are all potential saddle point contributions.

Various candidate saddles



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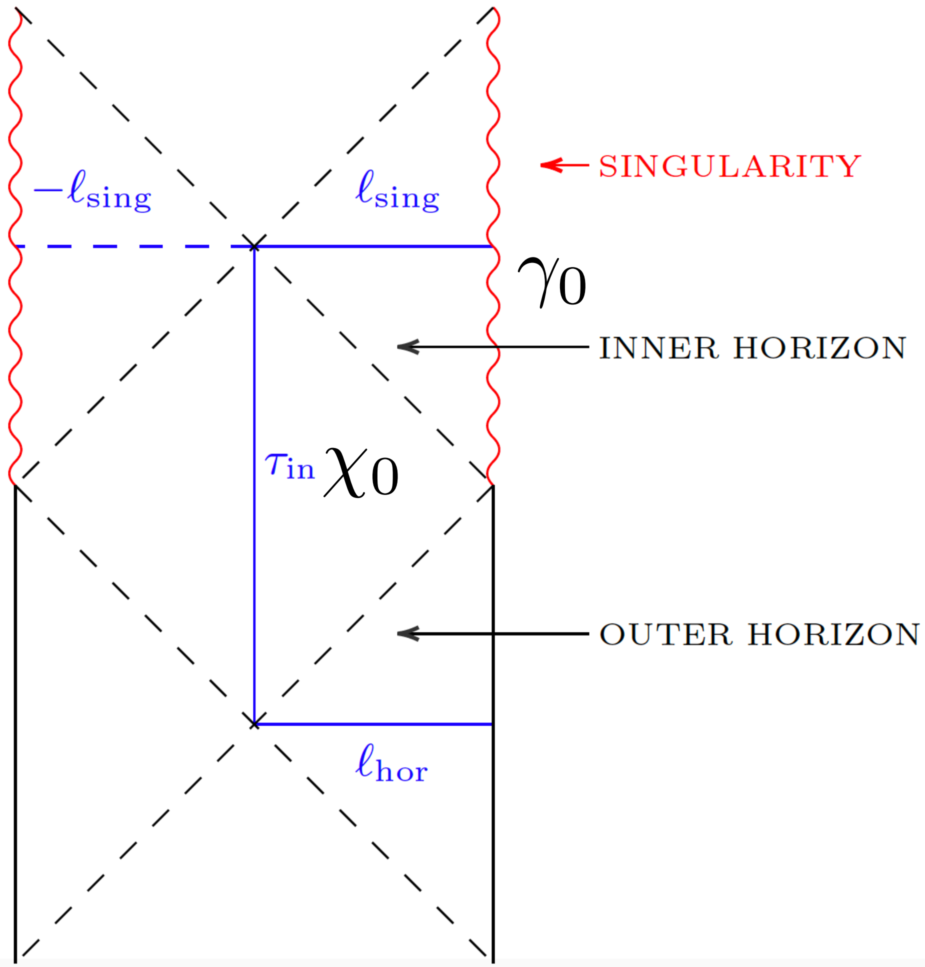
$$\hat{\rho}_0 = -i\chi_0 + \gamma_0$$

Charged AdS_5 black holes



$$\hat{\rho}_0 = -i\chi_0 + \gamma_0$$

$\chi_0 = \tau_{\rm in}$

Time between outer and inner horizons

Positive. Leads to a finite Contribution in the extremal limit → Associated to the exterior connecting region.

Another subleading saddle has

$$\rho = -i\chi_0 - \gamma_0$$

Leads to a part of the one point function going as

$$\langle O \rangle \propto T^{2\Delta'}$$

Should involve the AdS_2 region.

Comments on higher spin operators

- If we consider one point function of higher spin operators, we expect a similar story.
- In particular, they are non-zero and can be calculated in terms of geodesics that go to the singularity.
- At weak coupling, these one point functions are non-zero. They continue to be non-zero at strong coupling, but their contribution to correlators or the OPE of thermal two point functions is suppressed.

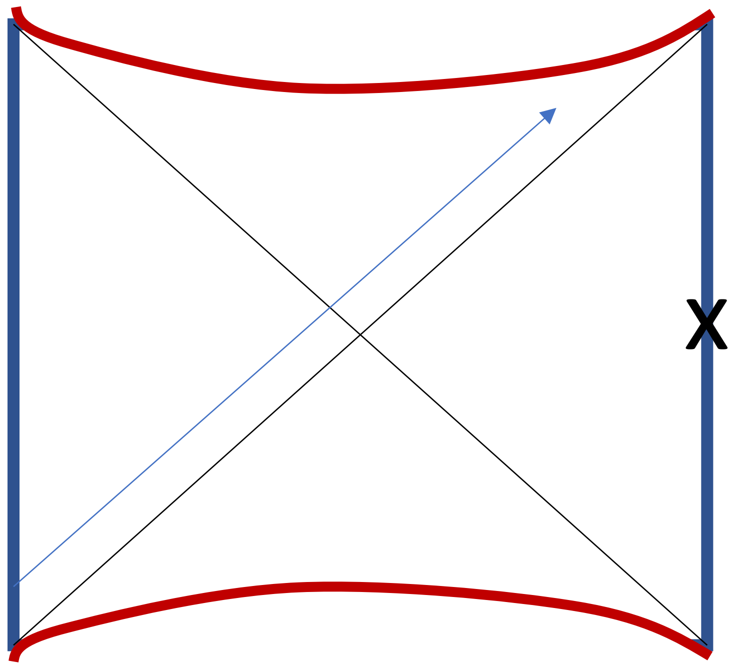
$$\langle O_1 O_2 \rangle_T \sim \sum_n c_{12n} \langle O_n \rangle_T$$

Only multi-stress tensor operators contribute in the gravity regime.

El-Showk, K. Papadodimas; Iliesiu, Kologlu, Mahajan, Perlmutter, Simmons-Duffin; Gobeil, Kulaxizi, Ng, Parnachev; Fitzpatrick, Huang; Li….

Limited interior probe

We should not read too much into these geodesic computations. It is not a very direct probe of the interior.



$\langle O \rangle_T$ is unchanged

Send a perturbation from the left side

Somewhat similar in spirit to

Fidkowski, Hubeny, Kleban, Shenker

The picture we discussed is appropriate for theories in 4 or more dimensions.

In three dimensions there can be a one point function for a BTZ black hole (but not for a black string, which is conformal invariant and has zero one point functions).

This was studied by Kraus and Maloney

Their answer comes from the particle splitting into two others, which circle the black hole horizon.

It seems that it should contain also the phase factor we discussed, but it was apparently not discussed in their paper.

Conclusions

- We discussed thermal one point functions.
- We saw that their dependence on the mass of the field encodes interesting information about the time to the singularity.
- We needed to do some analytic continuation in a parameter that results in an analytic continuation in the mass.
- Hopefully this helps in understanding the singularity…