Global symmetry, Euclidean gravity, and the black hole information problem

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- Requiring $\Lambda < 0$ is clearly too restrictive an assumption for an idea that we hope applies to our universe, but on the other hand we will soon see that *some* kind of nontrivial assumption is necessary: at least in lower dimensions there do exist theories of quantum gravity with global symmetries!

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- We will show that a version of the argument of Harlow/Ooguri can be applied to exclude global symmetries in any theory of quantum gravity where the Page curve calculations of Penington, Almheiri/Engelhardt/Marolf/Maxfield 2019 are valid.

Thus our proposal is reduced to the assumption that this is the only way for black hole evaporation to be unitary and compatible with the Bekenstein-Hawking formula.

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- **String worldsheet:**

$$S = -\frac{1}{4\pi\alpha'} \int d^2x \sqrt{-g} g^{ab} \partial_a X^\mu \partial_b X_\mu,$$

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Neither has black holes, so they are "allowed" to have global symmetry.

A more interesting example is the AdS version of Jackiw-Teitelboim gravity coupled to conformal matter:

$$S = \int_M d^2x\sqrt{-g}\,(\Phi_0 + \Phi(R+2)) + 2\int_{\partial M} dt\sqrt{-\gamma}\,(\Phi_0 K + \Phi(K-1)) + S_{CFT}(\psi_i, g).$$

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Unlike the previous two examples this theory does have black hole solutions, but we will soon see that in the quantum theory their entropy is infinite, and (like inthe CGHS/RST models) their evaporation leads to remnants.

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- Our point of view is that the Euclidean gravity path integral is only compatible with quantum mechanics in theories where a low-energy effective gravity theory emerges from some holographic UV-completion: I'll return to this at the end of the talk.

Pure 2+1 gravity

Our last example is the oriented version of pure gravity in $2+1$ dimensions, which we'll take to have negative cosmological constant:

$$S = \frac{1}{16\pi G}\int_M d^3x\sqrt{-g}\,(R+2) + \frac{1}{8\pi G}\int_{\partial M}\int d^2x\sqrt{-\gamma}(K-1).$$

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This theory has black hole solutions, but they again have infinite entropy Maloney, Kim/Porrati so our proposal allows for global symmetry.

Holography

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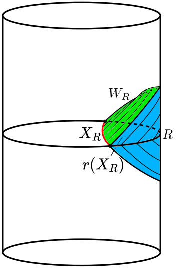
- Instead all UV-complete theories of quantum gravity in $3+1$ dimensions or higher come from string theory, and those which we understand non-perturbatively are all *holographic*: their fundamental description lives in a lower number of spacetime dimensions at some asymptotic boundary. 't Hooft 1993, Susskind 1994

Quantum extremal surfaces

The best-understood example of holography is (of course) the AdS/CFT correspondence, and one of the most important features of that correspondence is the *quantum extremal surface formula*: Ryu/Takayanagi 2006, Hubeny/Rangamani/Takayanagi 2007, Faulkner/Lewkowycz/Maldacena 2013, Engelhardt/Wall 2014



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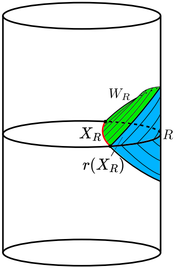

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This implies *entanglement wedge reconstruction*. Dong/Harlow/Wall 2016

Over the last decade it has been understood that the QES formula can be applied to systems other than a piece of a holographic CFT:

- A version of the QES formula holds in rather general tensor network constructions of many-body quantum states Swingle 2009, Pastawski/Yoshida/Harlow/Preskill 2015, Hayden/Nezami/Qi/Thomas/Walter 2016, and more generally in any quantum error correcting code Harlow 2016, Kang/Kolchmeyer 2018, Faulkner 2020. In particular it can be applied to an arbitrary non-gravitational system coupled to a holographic CFT Hayden/Penington 2018.

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- One can use the Euclidean replica method to give a rather general derivation of the QES formula which does not rely on special features of AdS/CFT. Lewkowycz/Maldacena 2013, Faulkner/Lewkwoycz/Maldacena 2013, Dong/Lewkowycz/Rangamani 2016, Dong/Lewkowycz 2017.

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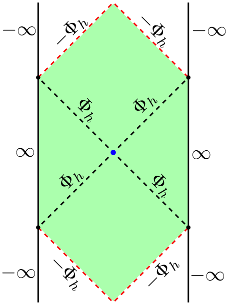
- Last year this idea was applied to a reservoir system coupled to a holographic CFT, giving a remarkable derivation of the Page curve for certain evaporating black holes. Penington, Almheiri/Engelhardt/Marolf/Maxfield 2019

JT black holes

To prepare for our discussion of global symmetry it is useful to first recall how this works for JT gravity coupled to conformal matter:

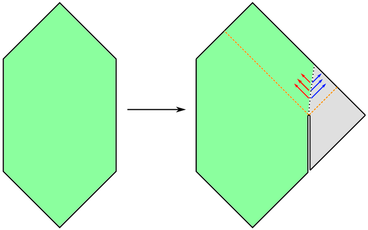


$$\Phi = \phi_b r$$

$$ds^2 = -\left(r^2 - 4\pi^2 T^2\right) dt^2 + \frac{dr^2}{r^2 - 4\pi^2 T^2},$$

$$S = 4\pi\left(\Phi_0 + \Phi(r_s)\right) = 4\pi\left(\Phi_0 + 2\pi\phi_b T\right).$$

Almheiri/Engelhardt/Marolf/Maxfield realized an information problem for this black hole by coupling to a reservoir on one side and allowing the conformal matter to leak out:



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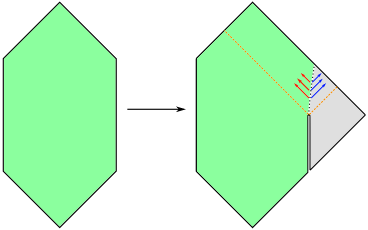

The entropy of the reservoir system in canonical quantization can be computed,

$$S_{res}(t) = 16\pi^2 \phi_b T_1^2 \left(1 - e^{-\frac{ct}{96\pi\phi_b}}\right),$$

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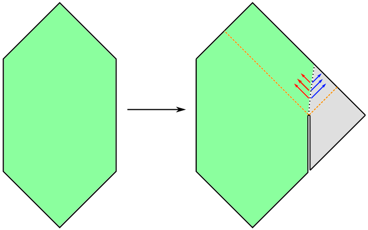

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but for $\phi_b T_0 \gg \Phi_0$ this eventually exceeds the coarse-grained entropy of the remaining black holes! Thus in this theory the entropy formula is wrong (and therefore global symmetry is allowed).

The key point however is that if we only view this theory as a low-energy effective field theory, to be UV completed into some fundamental holographic description, then we should instead use the QES formula!

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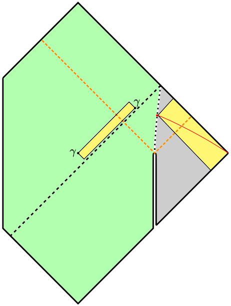

This leads to

$$S_{res} = \min \left[16\pi^2 \phi_b T_1^2 \left(1 - e^{-\frac{ct}{96\pi\phi_b}}\right), 8\pi\Phi_0 + 8\pi^2\phi_b T_0 \left(1 + \frac{T_1}{T_0} e^{-\frac{ct}{96\pi\phi_b}}\right)\right],$$

which never contradicts the entropy formula.

No global symmetries

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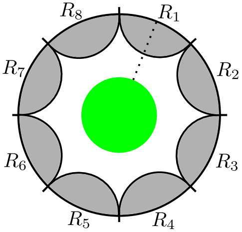

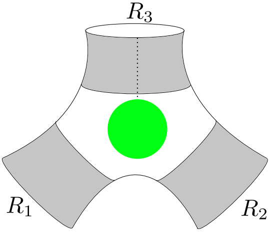

By boundary locality we have

$$U(g) = U(g, R_1)U(g, R_2) \dots U_{edge},$$

but the green charged operator isn't in the entanglement wedge of any of the R_i so it can't actually be charged.

To extend this beyond AdS/CFT, we need a few assumptions. Let S be a quantum gravity system which has black hole solutions whose semiclassical description lives in d dimensions, and let R be a “reservoir” system consisting of weakly interacting quantum fields on $\mathbb{R}^d$ (possibly including linearized gravitons). We then assume the following:

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Roughly speaking, S is a black hole and R is its Hawking radiation.

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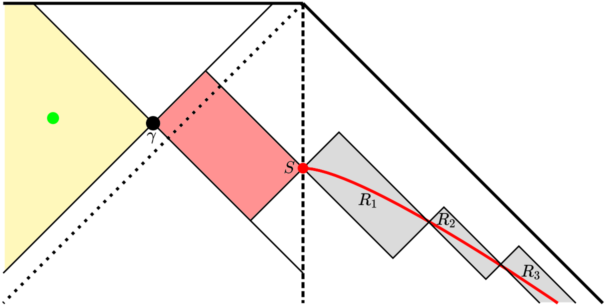

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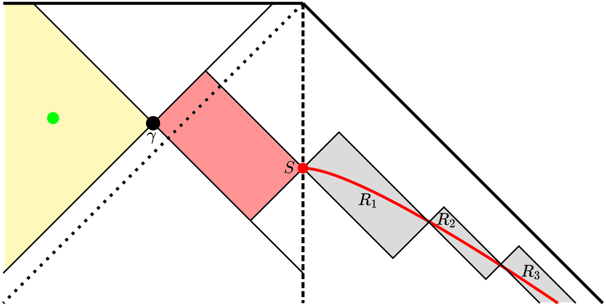

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This contradiction can be quantified by studying the relative entropy $S(\rho_R | U(g, R)^\dagger \rho_R U(g, R))$, which should vanish if there is a (unbroken) global symmetry but can't since $U(g, R)$ can't implement the symmetry on the island (also follows from wormholes). Chen/Lin 2020.

Some comments on Euclidean gravity

We now return to the question of the validity of Euclidean quantum gravity, which we saw gave the wrong answer (via the QES formula) for the entropy of the radiation in a theory obtained by canonical quantization but the right answer (again via the QES formula) in an effective theory which is the low-energy limit of a holographic theory.

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In other words Euclidean gravity and holography are in some sense equivalent.

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This is true for canonically-quantized gravity as well, so any gravitational partition function computed within canonical quantization must also live on $\mathbb{S}^1 \times \Sigma$, possibly with a sum over appropriate Σ.

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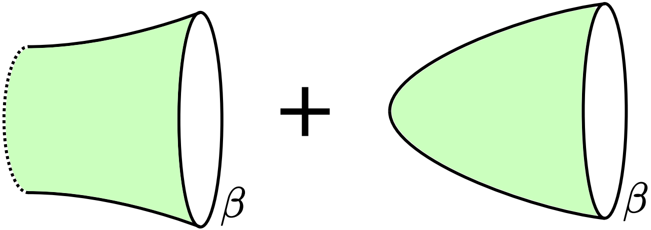
$$Z = \mathrm{Tr} e^{-\beta H}$$

always lives on a Euclidean spacetime of topology $\mathbb{S}^1 \times \Sigma$.
This is true for canonically-quantized gravity as well, so any gravitational partition function computed within canonical quantization must also live on $\mathbb{S}^1 \times \Sigma$, possibly with a sum over appropriate Σ.
There is a simple argument due to Hawking that no such geometry can ever produce an entropy which is of order $1/G$: as long as time-translation around the thermal circle is unbroken, the Euclidean action must obey $I \propto \beta$, and thus at this order

$$S = (1 - \beta\partial_\beta) \log Z \approx -(1 - \beta\partial_\beta) I = 0.$$

The standard fix for this is to also include Euclidean geometries which are not of the form $\mathbb{S}^1 \times \Sigma$, instead requiring only that the *boundary* topology is $\mathbb{S}^1 \times \partial\Sigma$:



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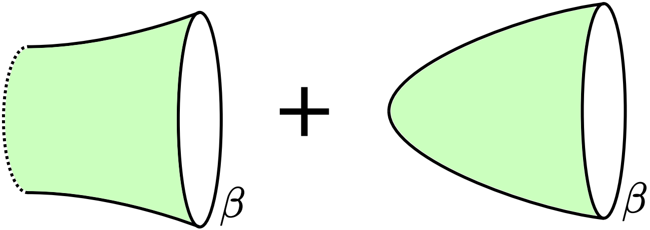

The geometry on the right would not be included by canonical quantization, but if we include it nonetheless then we get $S = A/4G$!

Gibbons/Hawking 1977

But why are we allowed to include the geometry on the right?

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- In a holographic theory the answer seems clear: since the microscopic description lives at the asymptotic boundary, so does the true thermal circle! Away from the boundary there is no particular reason to prevent the $\mathbb{S}^1$ from contracting, and indeed in AdS/CFT if we don't allow this we are unable to reproduce the high-temperature density of states of the dual CFT. Witten 1998

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- This is still somewhat mysterious however: how can mere low-energy effective field theory have access to such deep non-perturbative information about quantum gravity?

- We don't have a complete answer to this question, but in AdS/CFT we can say a bit more. Indeed from the boundary point of view high-temperature/low-temperature duality relates geometries where the thermal circle contracts to geometries where it doesn't. Strominger 1998, Shaghoulian 2015

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Thanks for listening!