Collision in the interior of wormhole

Ying Zhao
arXiv:2011.06016

Outline

- Introduction
- Perturbed thermal field double and quantum circuit
- Collision in the interior of wormhole
- Future directions

Motivation

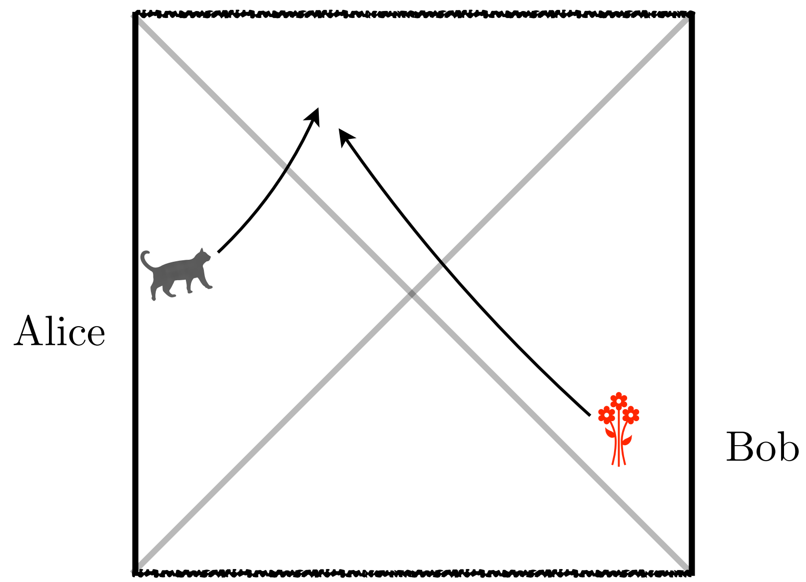
- The wormhole was interpreted as an entangled state.

  ER = EPR

J. Maldacena arXiv: hep-th/0106112

J. Maldacena, L. Susskind arXiv: 1306.0533

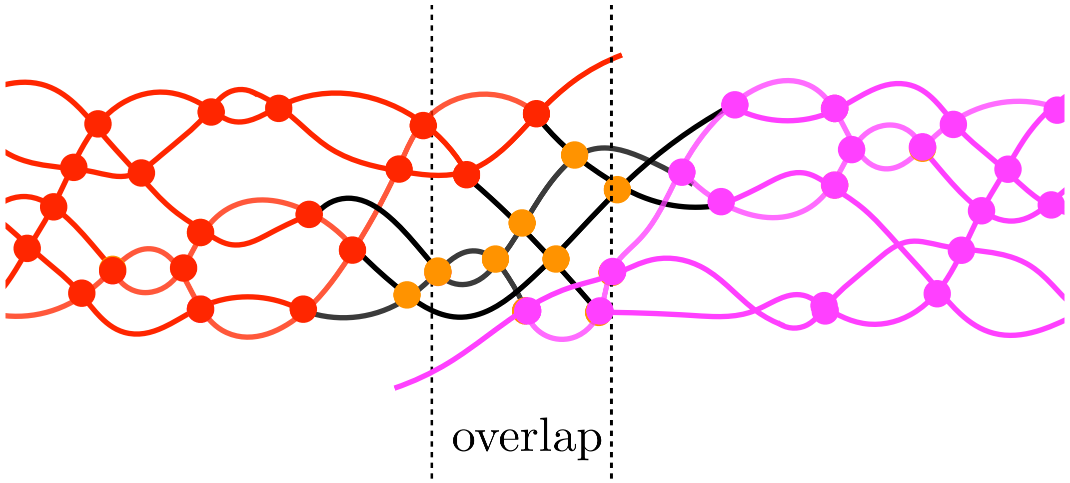
- Two infalling objects can meet in the interior.



- There are no boundary interactions.

- When Alice and Bob send in signals, they create growing perturbations in the quantum circuit.

- The two perturbations can have overlap in the circuit.

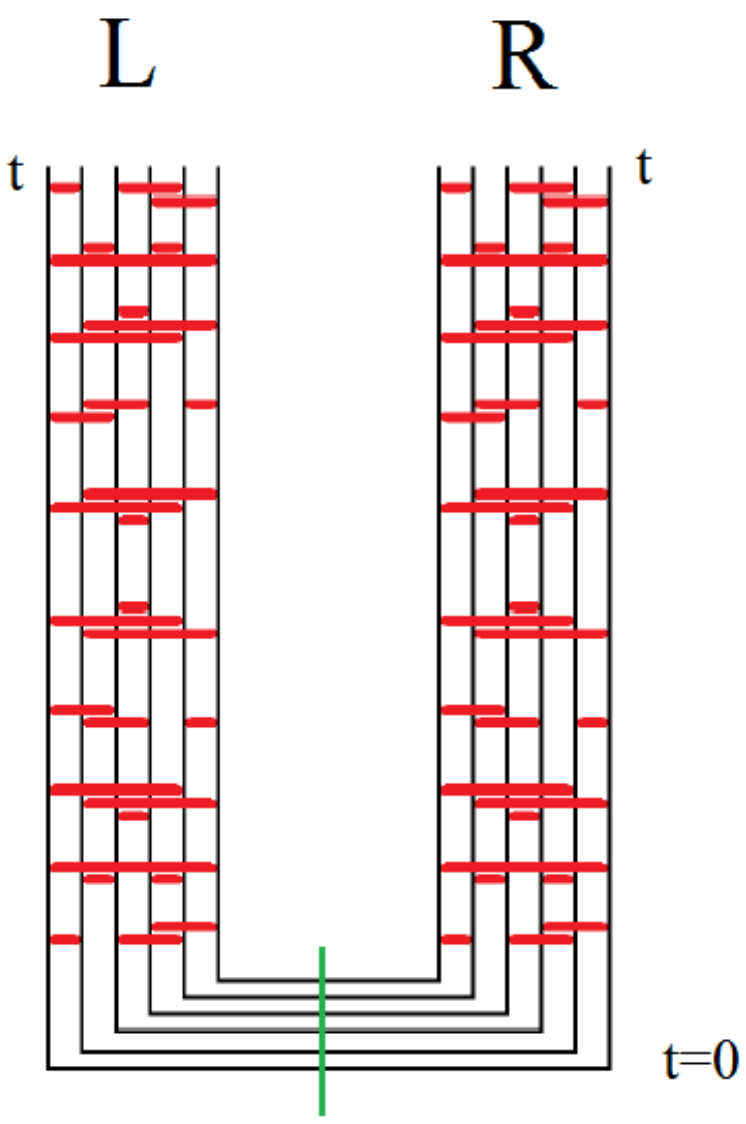


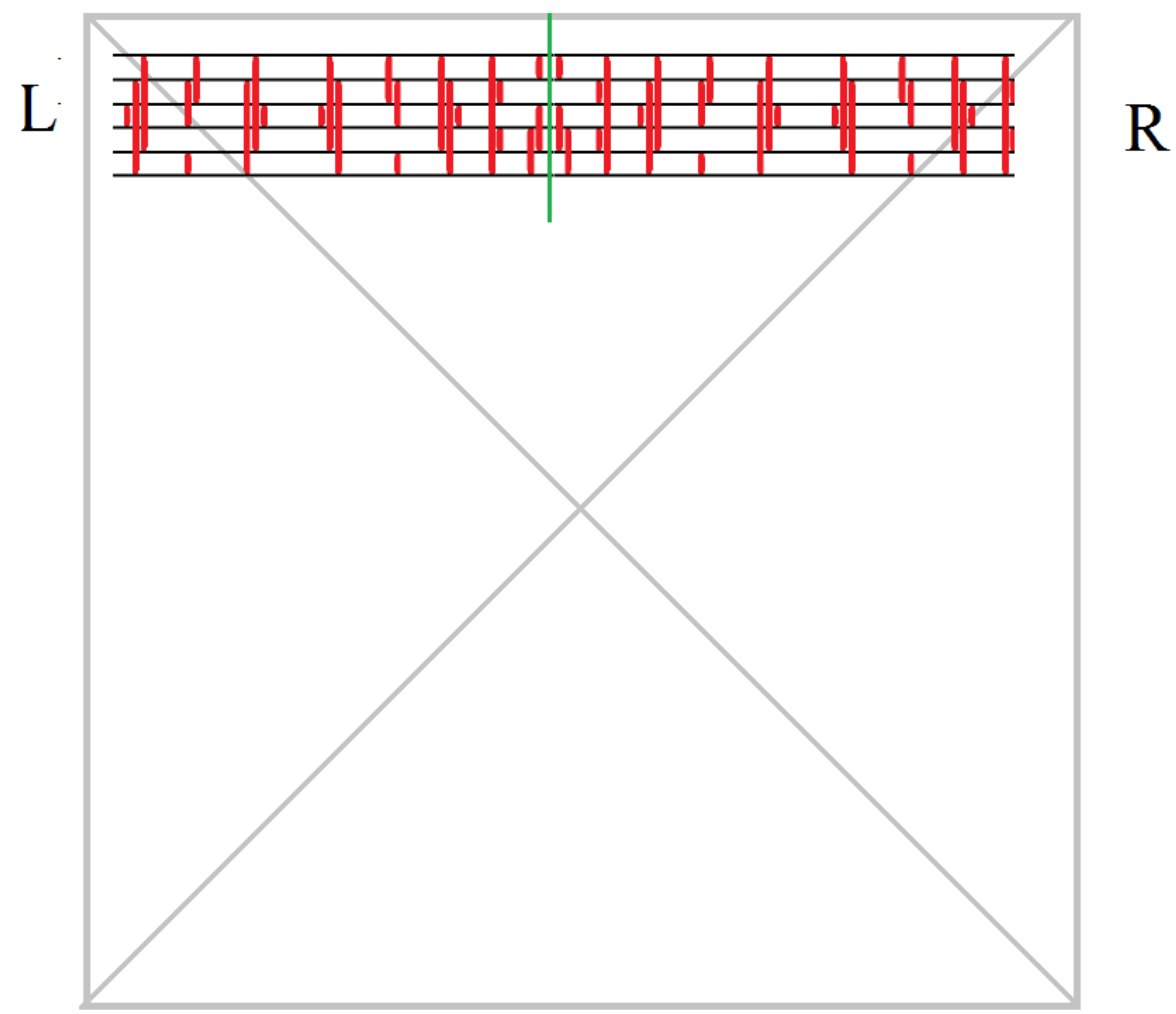
- This overlap represents the meeting of two signals in the interior.

Bulk tensor network and quantum circuit

B. Swingle arXiv:1209.3304
T. Hartman, J. Maldacena arXiv:1303.1080
L. Susskind arXiv:1411.0690

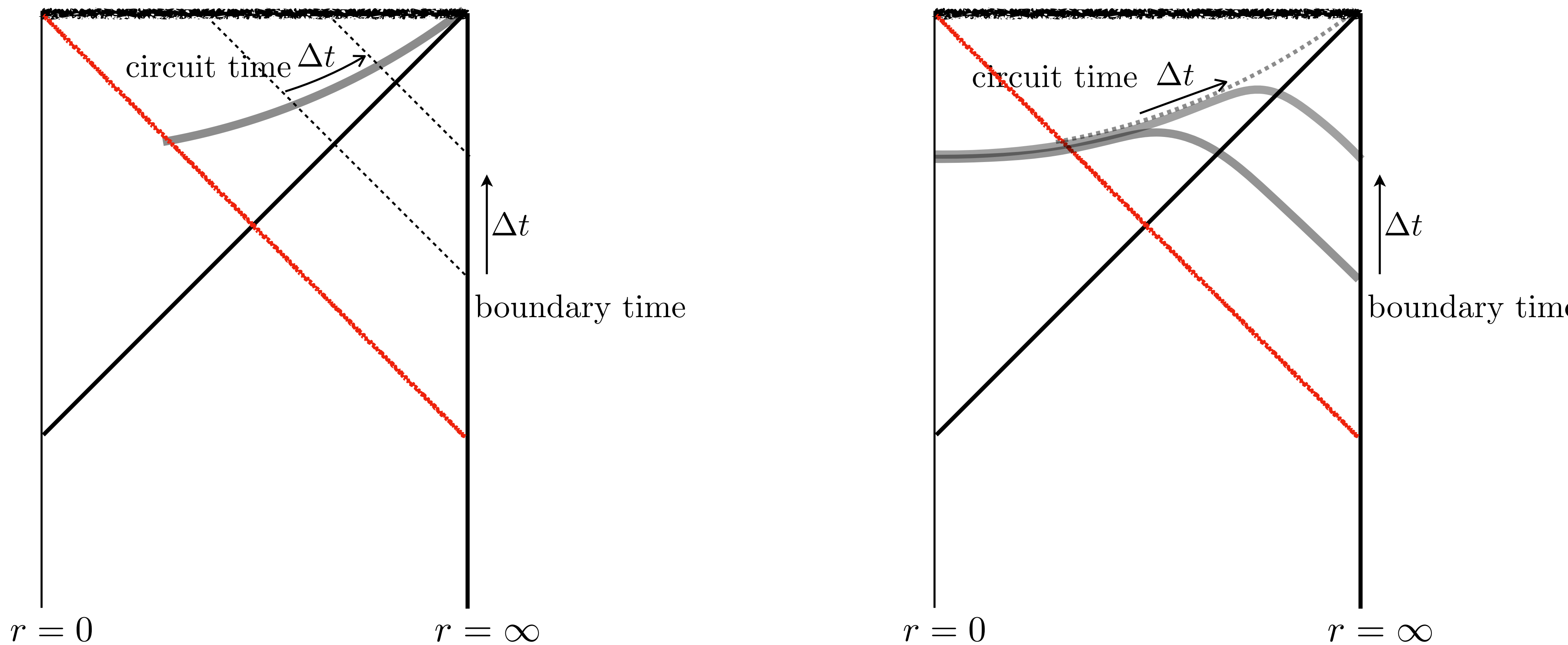
- The bulk geometry reflects the minimal circuit preparing the state.





Pure state black hole

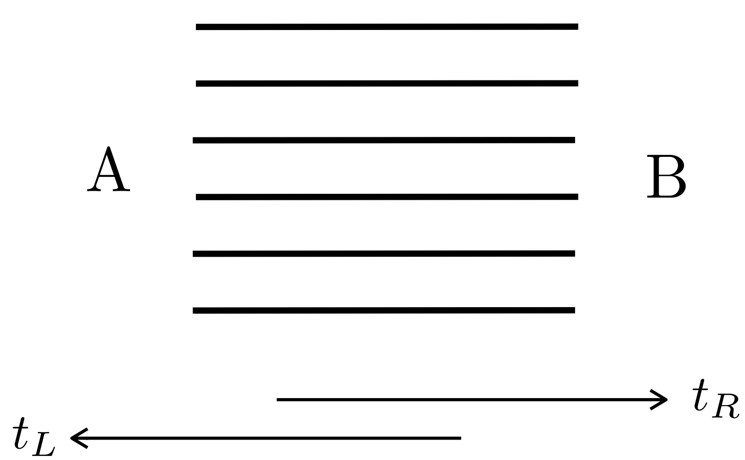
L. Susskind, Y. Z. arXiv:1408.2823



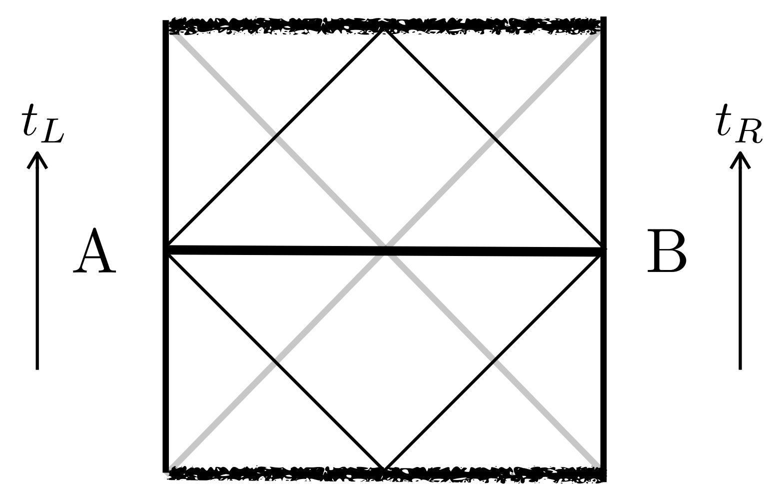
- As we apply unitary time evolution to the circuit, the state gets more complex, the minimal circuit gets longer, and the Einstein-Rosen bridge also gets longer.

- Identify circuit time with boundary time: $d\tau = \frac{2\pi}{\beta} dt$

Thermofield Double



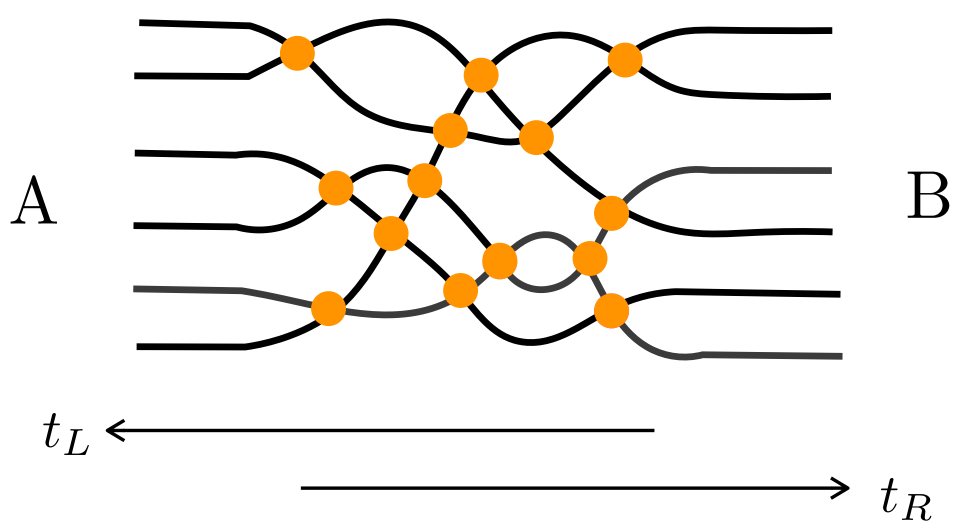
(a) Quantum circuit



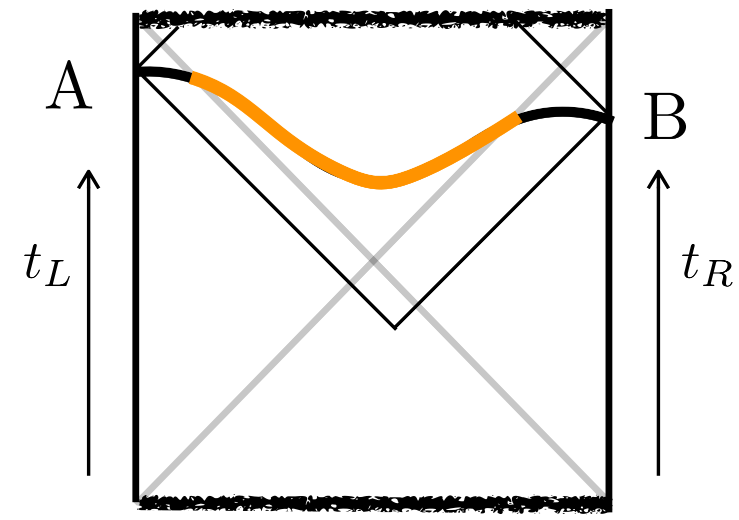
(b) Wormhole geometry

We represent thermofield double by S Bell pairs. The corresponding wormhole geometry has minimal length.

Time-evolved thermofield Double



(a) Quantum circuit



(b) Wormhole geometry

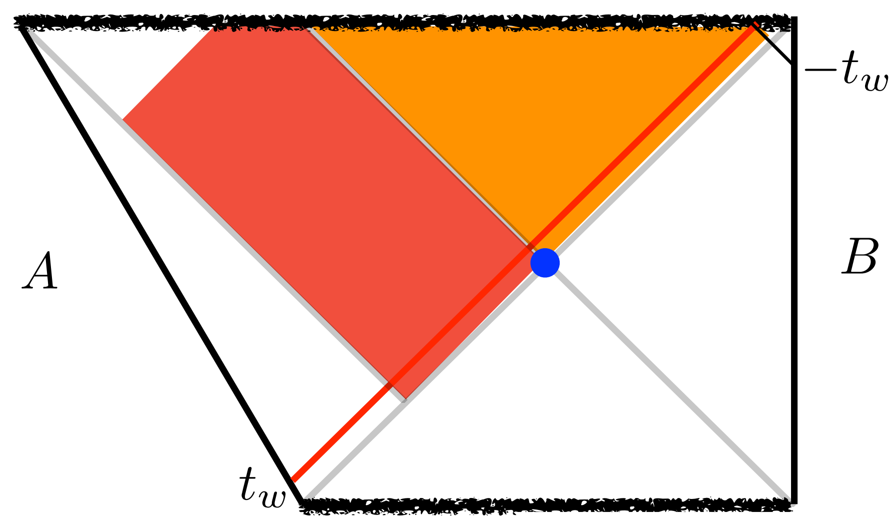
- The orange gates can be undone from either side. They do not belong to subsystem A or B alone.

- The wormhole gets longer but the part that gets longer is outside A's entanglement wedge and also outside B's entanglement wedge.

Perturbed thermofield double

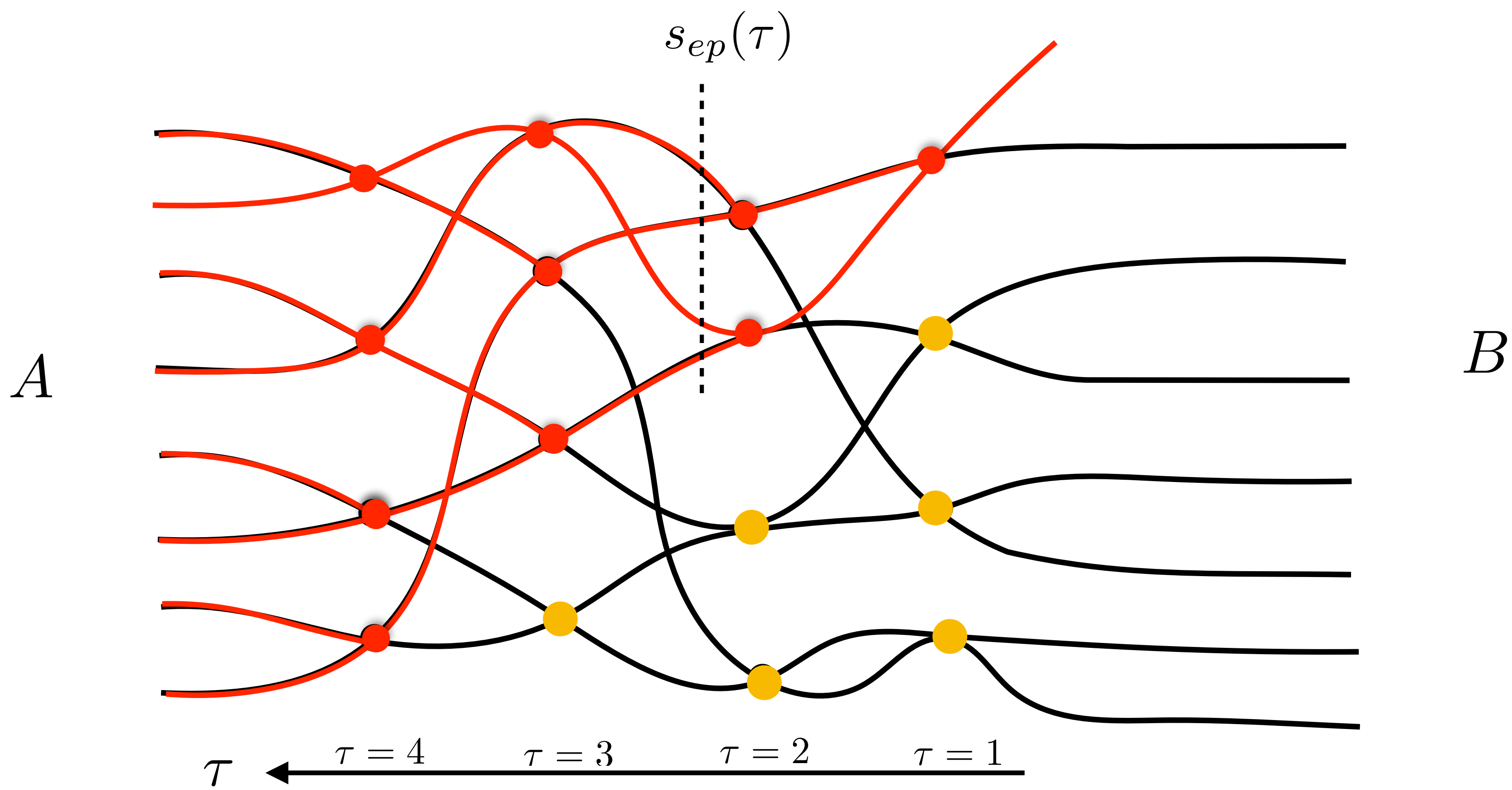
T. Dray, G. 't Hooft, 1985

S. Shenker, D. Stanford arXiv:1306.0622



Epidemic model: the growth of an operator

P. Hayden, J. Preskill arXiv:0708.4025v2
L. Susskind, Y.Z. arXiv:1408.2823
A. Brown, L. Susskind, Y.Z. arXiv:1608.02612



$$\frac{ds_{ep}}{d\tau} = (S + 1 - s_{ep})\frac{s_{ep}}{S} \qquad \frac{s_{ep}(\tau)}{S+1} = \frac{\frac{\delta S}{S}e^{\tau}}{1 + \frac{\delta S}{S}e^{\tau}}, \quad s_{ep}(0) = \delta S$$

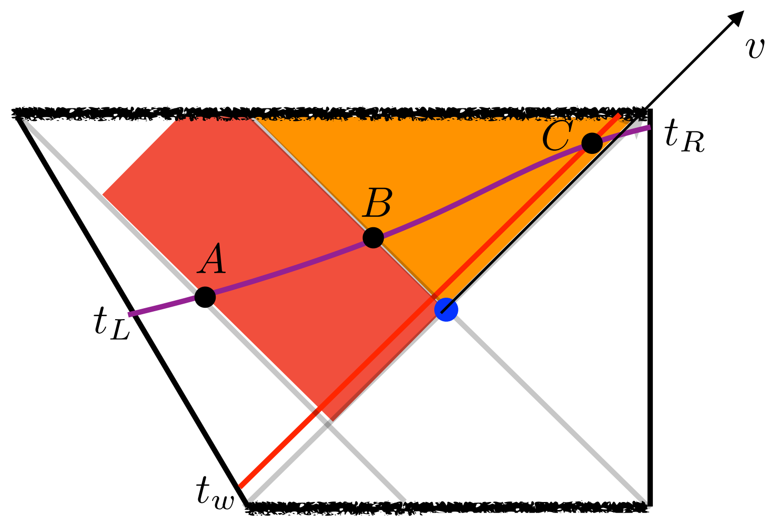
$s_{ep}(\tau):$ gates that can be undone by Alice but not Bob

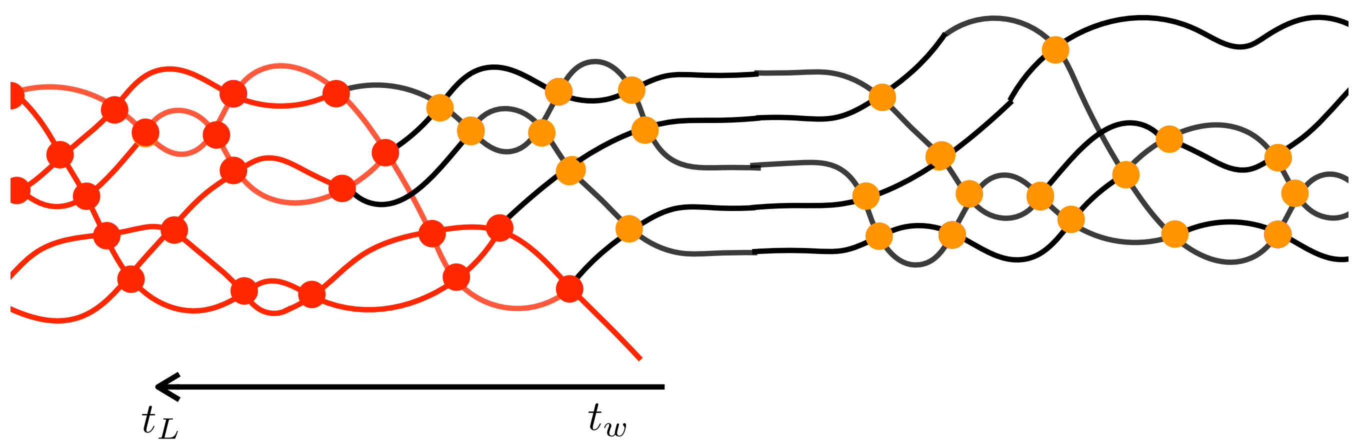
$S - s_{ep}(\tau):$ gates that can be undone by Alice as well as Bob

$$\frac{dN_{\text{sick}}}{d\tau} = s_{ep} \qquad \frac{dN_{\text{healthy}}}{d\tau} = S - s_{ep}$$

Quantum circuit from the point of view of Alice

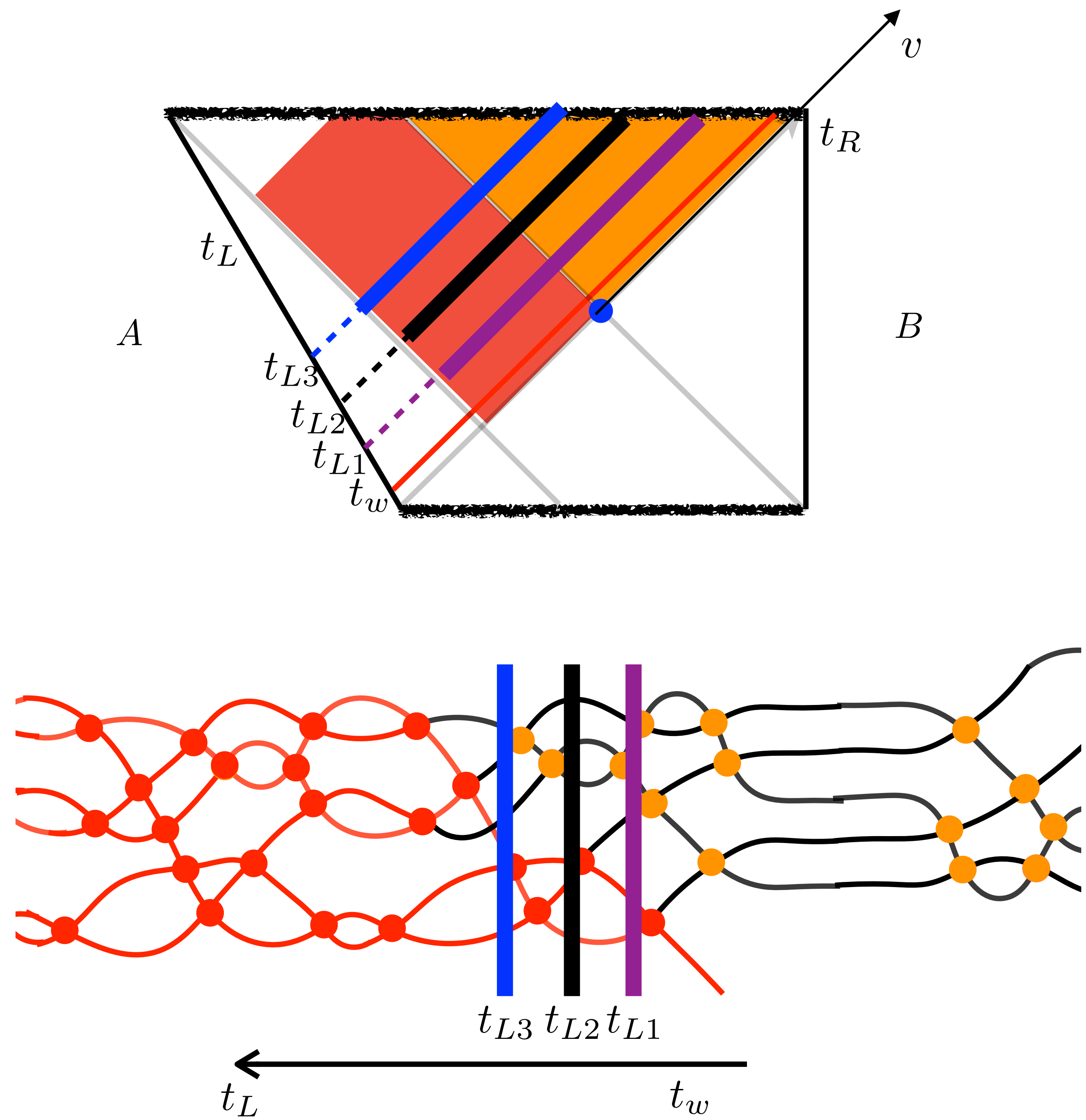
Y.Z. arXiv:1711.03125





$$\frac{d}{dt_L}\left(\frac{d_{AB}}{l}\right) = \frac{2\pi}{\beta}\frac{e^{\frac{2\pi}{\beta}(t_L - t_w - t_*)}}{1+e^{\frac{2\pi}{\beta}(t_L - t_w - t_*)}} = \frac{2\pi}{\beta}\left(\frac{s_{ep}[\frac{2\pi}{\beta}(t_L - t_w)]}{S}\right) = \frac{d}{dt_L}\frac{N_{\text{sick}}}{S}$$

$$\frac{d}{dt_L}\left(\frac{d_{BC}}{l}\right) = \frac{2\pi}{\beta}\frac{1}{1+e^{\frac{2\pi}{\beta}(t_L - t_w - t_*)}} = \frac{2\pi}{\beta}\left(1 - \frac{s_{ep}[\frac{2\pi}{\beta}(t_L - t_w)]}{S}\right) = \frac{d}{dt_L}\frac{N_{\text{healthy}}}{S}$$

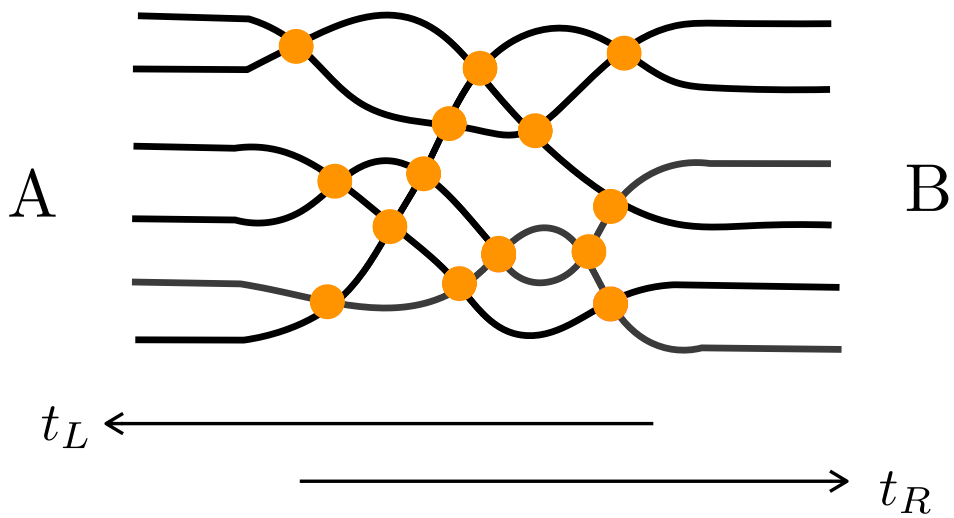


- The gates that can be undone by both Alice and Bob (healthy gates) are stored in the orange region.

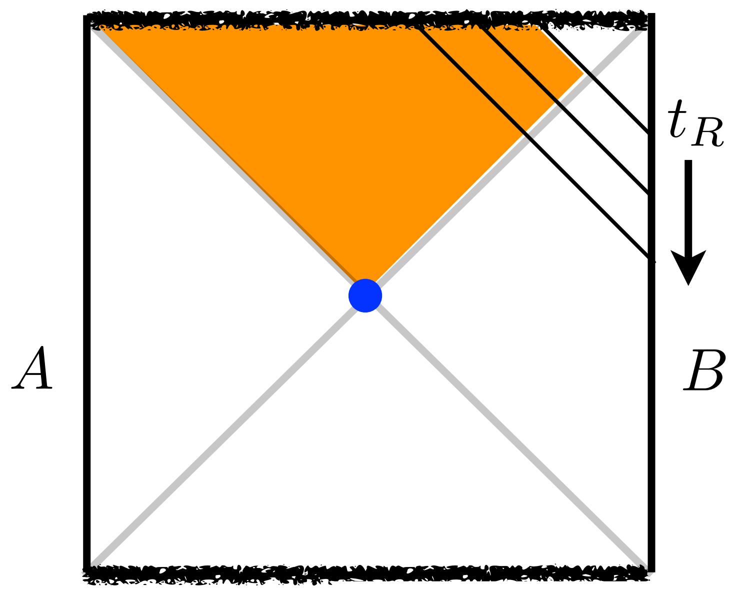
- The gates that can be undone by Alice but not Bob are stored in the red region.

Quantum circuit from the point of view of Bob

Without perturbation



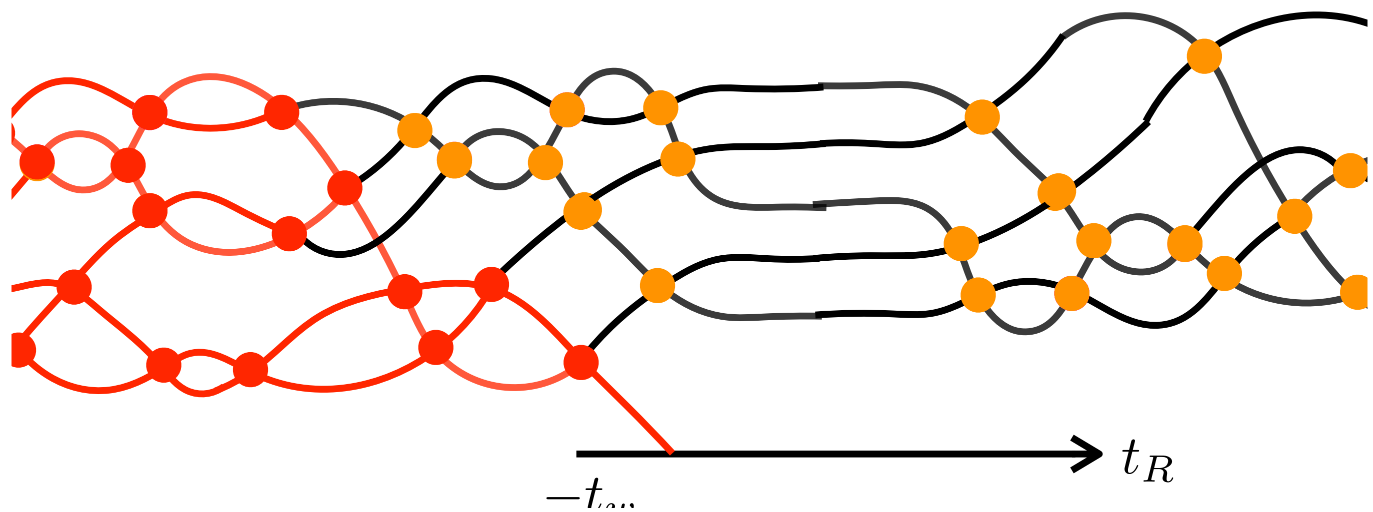
(a) Quantum circuit

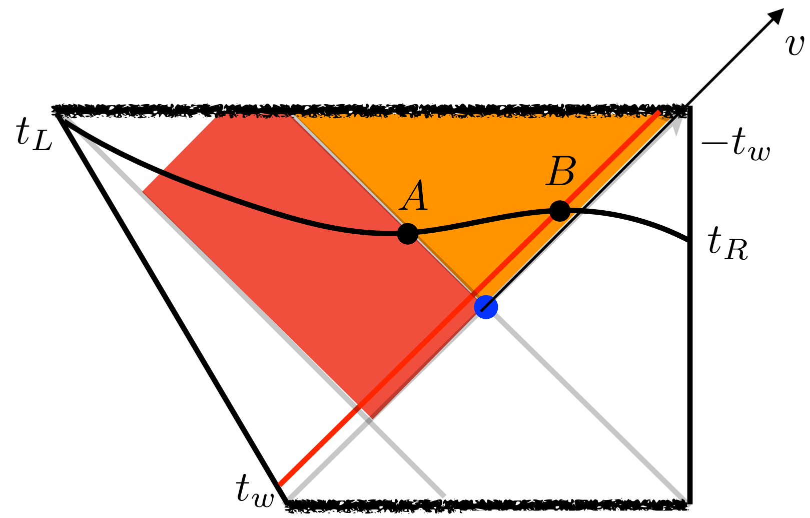


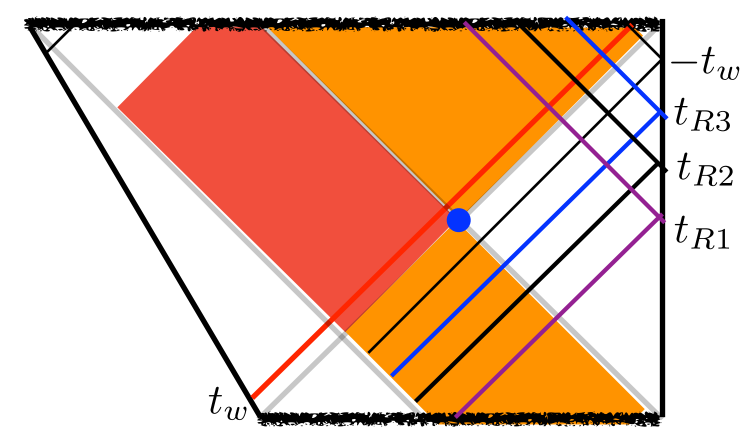
(b) Wormhole geometry

- Without the perturbation, Bob can undo the gates stored in the orange region.

With perturbation



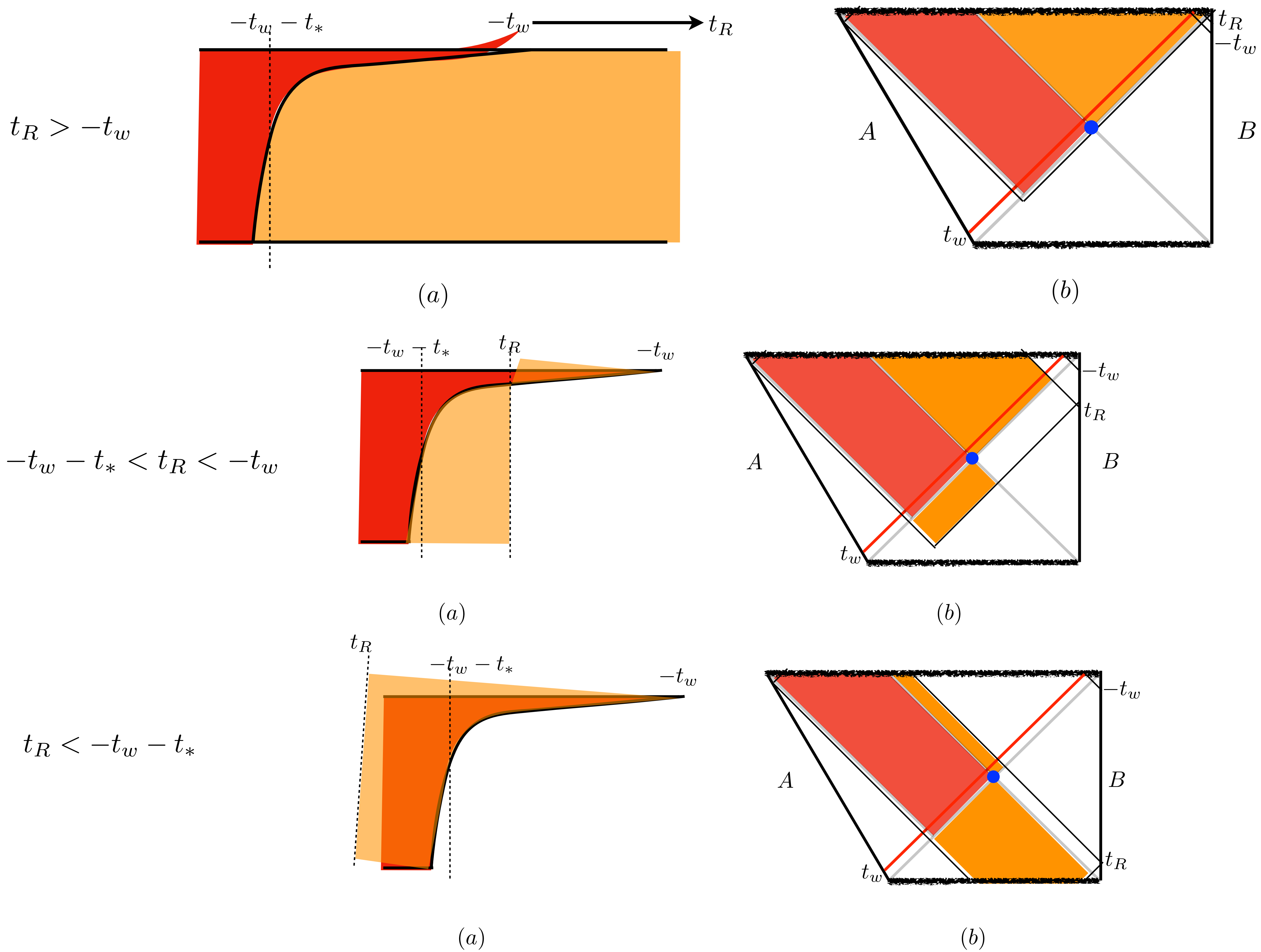




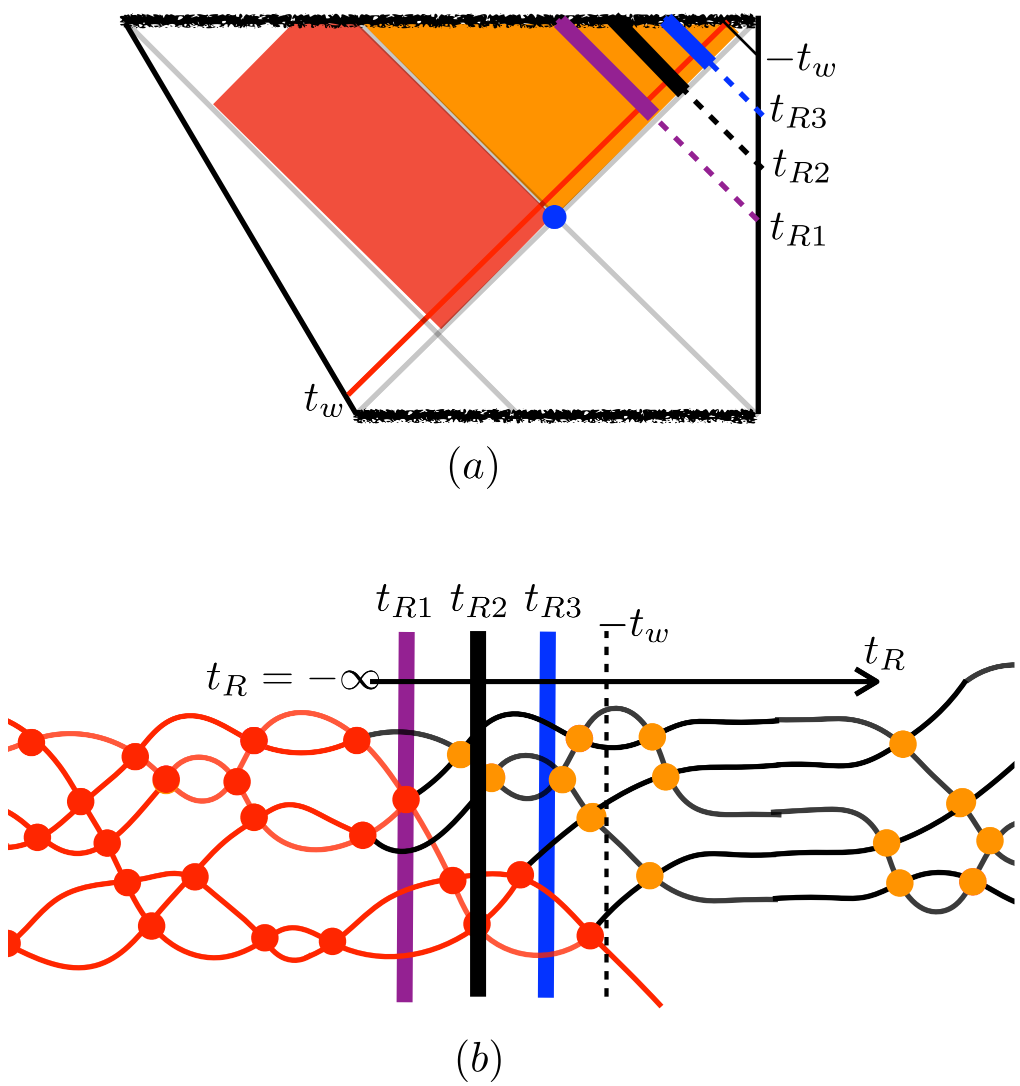
$$\frac{d_{\mathrm{AB}}}{l} = \frac{2\pi}{\beta}(t_R + t_w + t_*) + \log\left(1 + \alpha e^{-\frac{2\pi}{\beta} t_R}\right)$$

$$\frac{d}{dt_R}\frac{d_{\mathrm{AB}}}{l} = \frac{2\pi}{\beta}\left[1 - \frac{e^{\frac{2\pi}{\beta}(-t_w - t_* - t_R)}}{1 + e^{\frac{2\pi}{\beta}(-t_w - t_* - t_R)}}\right] = \frac{2\pi}{\beta}\left(1 - \frac{s_{ep}[\frac{2\pi}{\beta}(-t_w - t_R)]}{S}\right) = \frac{d}{dt_R}\frac{N_{\mathrm{healthy}}}{S}$$

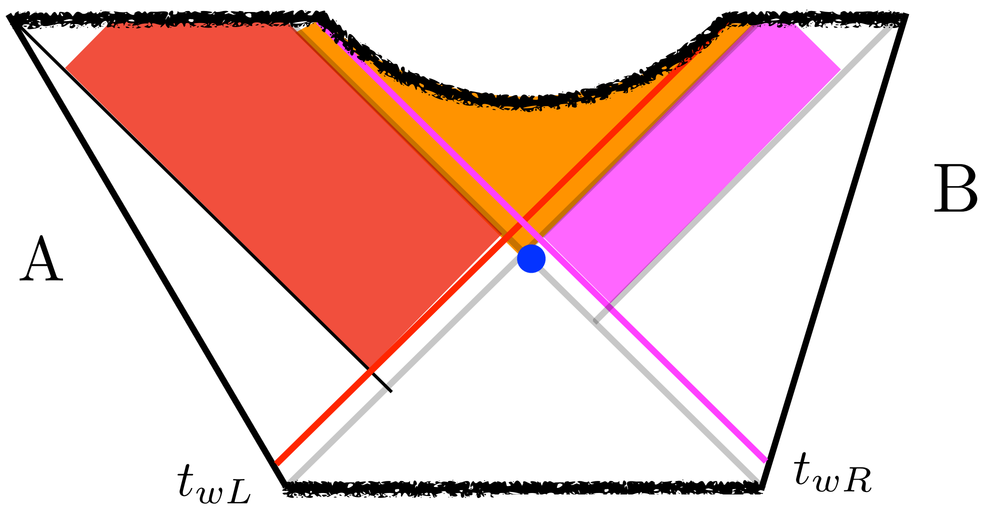
- At each time step, S gates are applied. $S - s_{ep}$ gates are cancelled by the gates in the future interior. s_{ep} gates are stored in the past interior.



$t_R > -t_w$
$-t_w - t_*$
$-t_w$
t_R
A
B
$-t_w$
t_w
(a)
(b)
$-t_w - t_* < t_R < -t_w$
$-t_w - t_*$
t_R
$-t_w$
A
B
$-t_w$
t_R
t_w
(a)
(b)
$t_R < -t_w - t_*$
t_R
$-t_w - t_*$
$-t_w$
A
B
$-t_w$
t_R
t_w
(a)
(b)

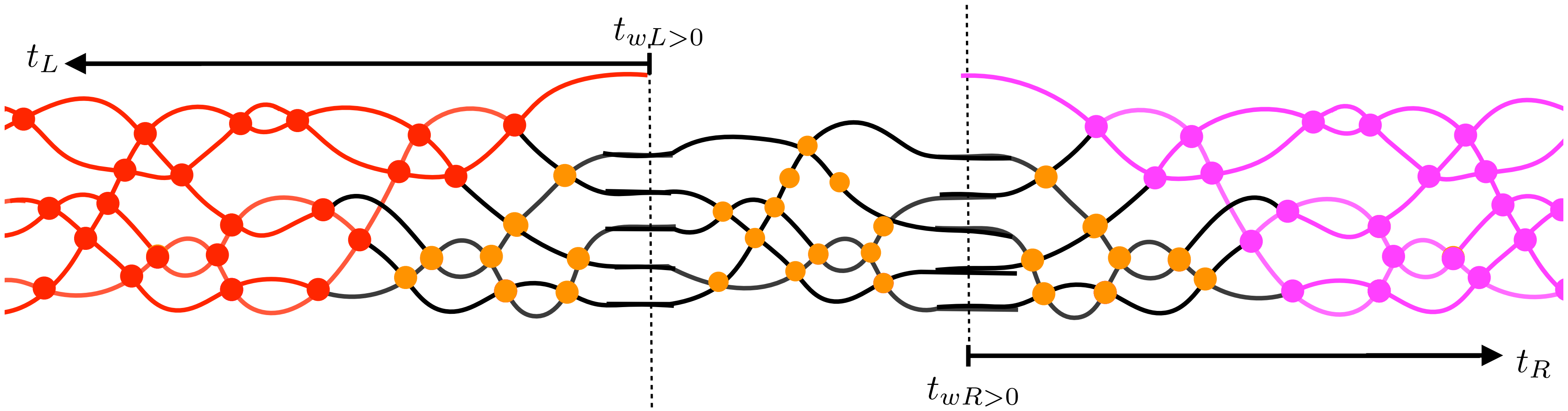

$-t_w$
t_{R3}
t_{R2}
t_{R1}
t_w
(a)
t_{R1} t_{R2} t_{R3} $-t_w$
$t_R = -\infty$
t_R
(b)

Collision in the interior

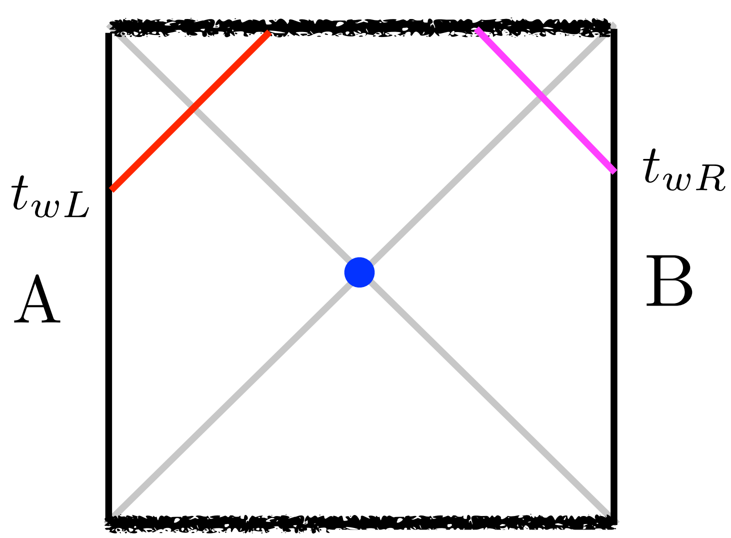


- Infalling objects from the two boundaries can meet in the interior.

$t_{wL} > 0, \quad t_{wR} > 0$

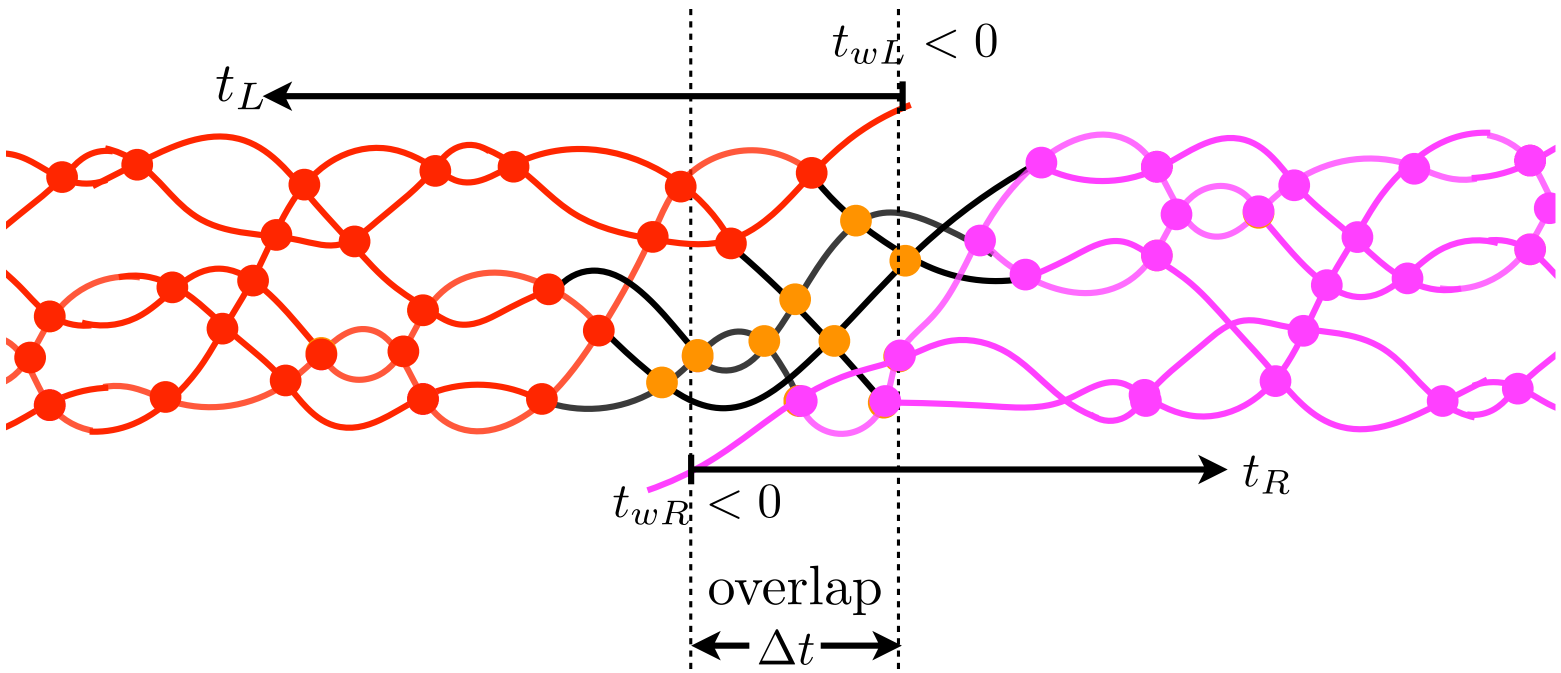


- There are no overlaps between the two perturbations.

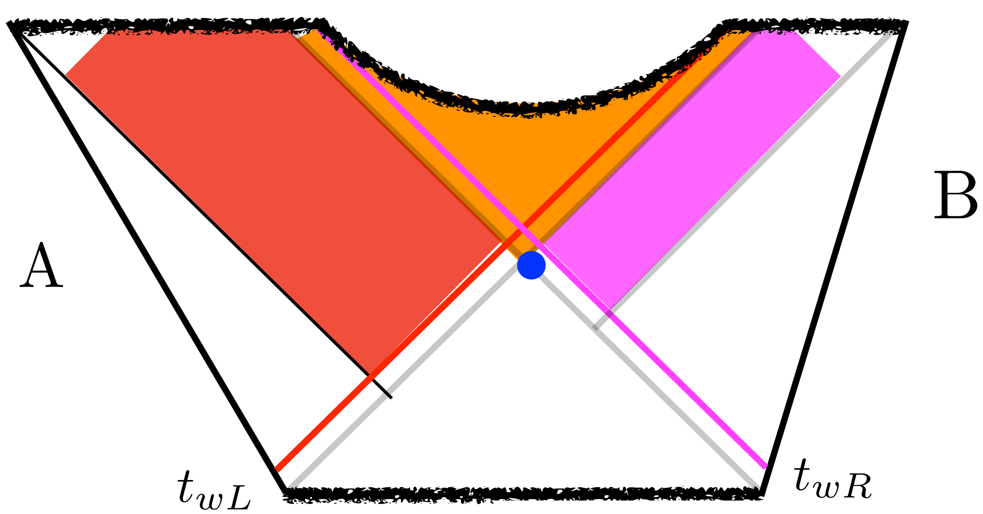


- The two perturbations do not collide in the interior.

$t_{wR} < 0, \quad t_{wL} < 0$

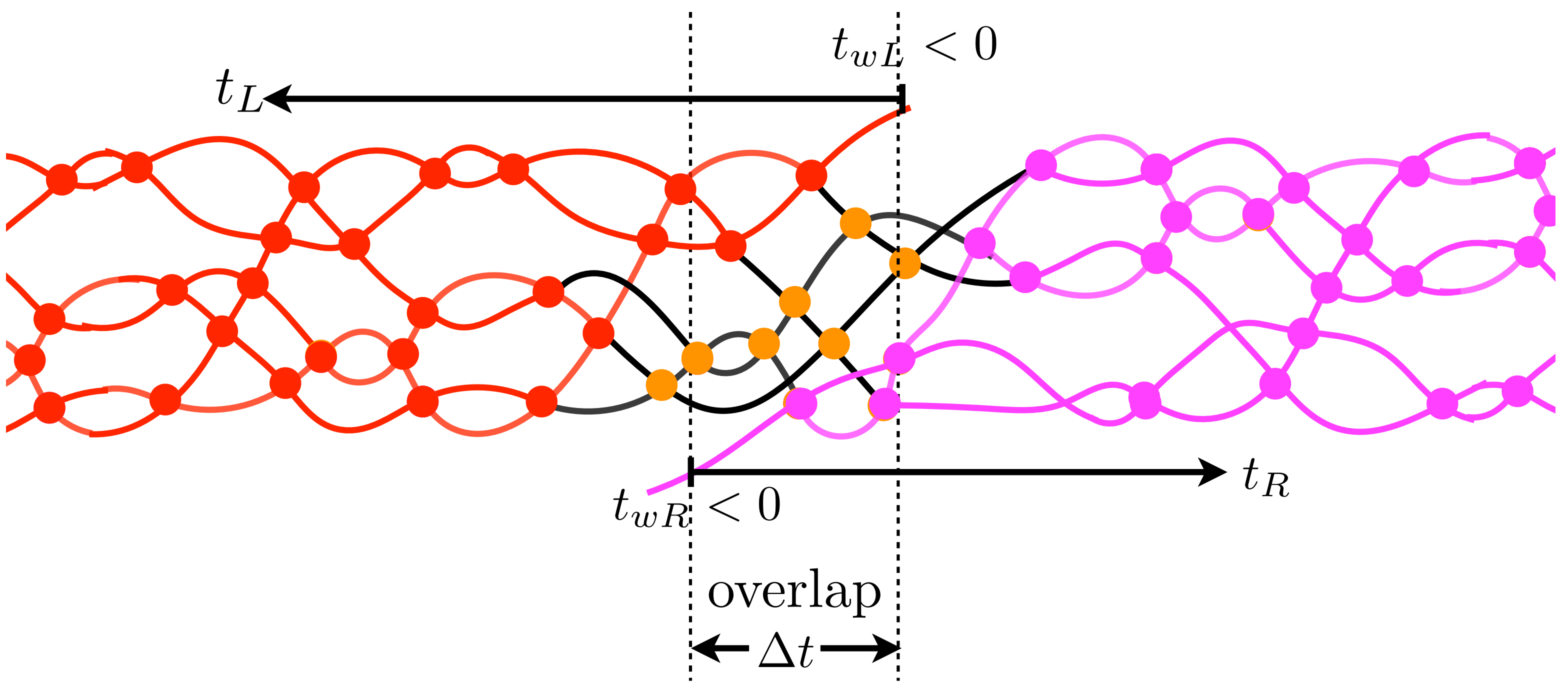


- The two perturbations have overlap in the quantum circuit.



- The two perturbations collide in the interior.

The number of healthy gates in the overlap region



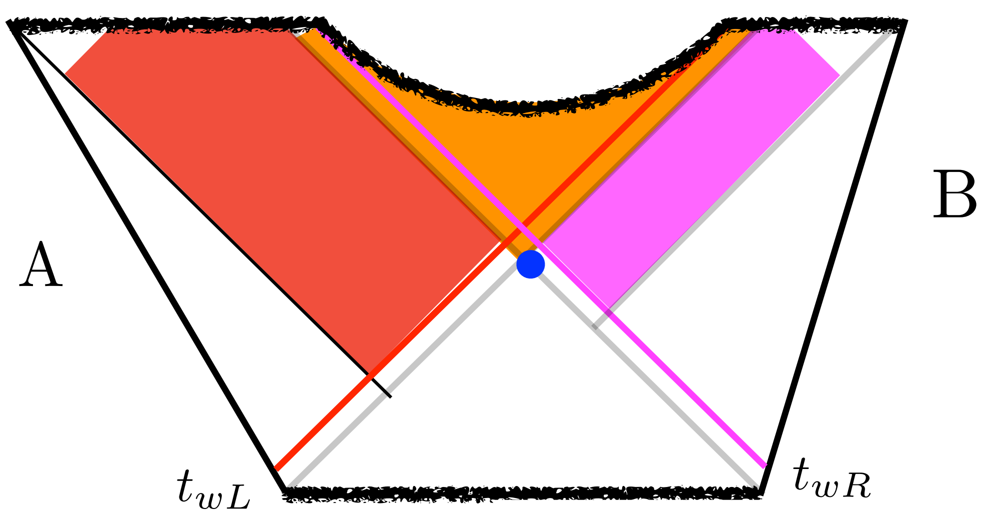
The probability of being healthy from the red epidemic: $1 - \frac{s_{ep}^{red}}{S} = \frac{1}{1 + \frac{\delta S_1}{S} e^{\frac{2\pi}{\beta}(-t_{wL} - t)}}$

The probability of being healthy from the purple epidemic: $1 - \frac{s_{ep}^{purple}}{S} = \frac{1}{1 + \frac{\delta S_1}{S} e^{\frac{2\pi}{\beta}(t - t_{wR})}}$

The probability of being healthy from both epidemic is product of these two.

Number of healthy gates and post-collision region

- The post-collision region stores the healthy gates.



- Post-collision region: A larger black hole forms $\Delta t \equiv -t_{wL} - t_{wR}$

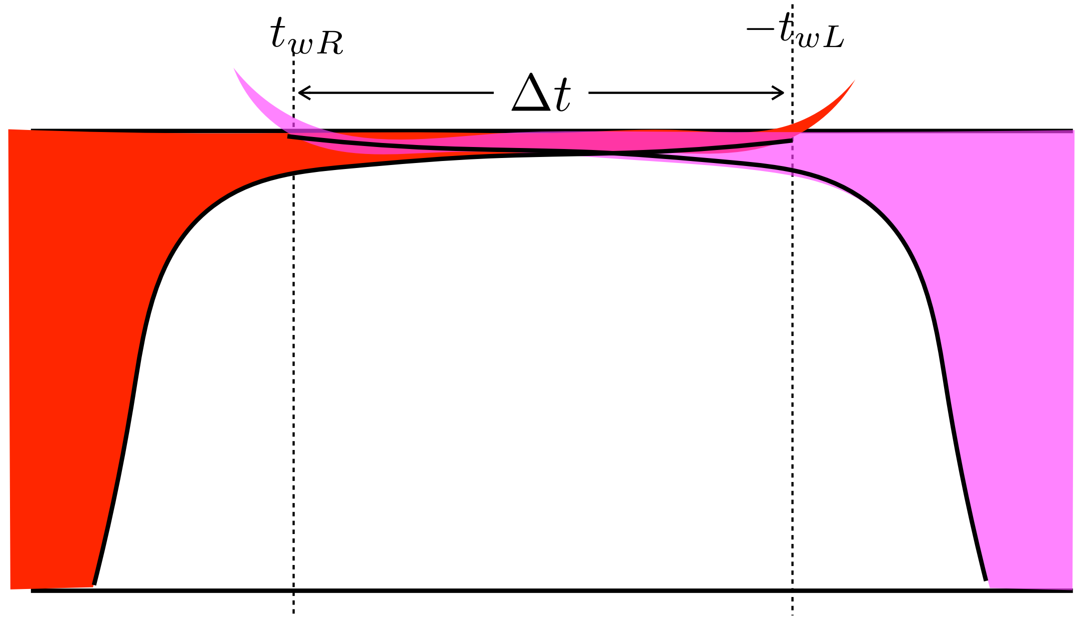
$$\frac{\tilde{r}_h^2}{r_h^2} = 1 + \frac{2\delta S_1}{S} + \frac{2\delta S_2}{S} + \frac{4\delta S_1 \delta S_2}{S^2} \cosh^2\left(\frac{\pi}{\beta}\Delta t\right)$$

T. Dray, G. ’t Hooft 1985

S. Shenker, D. Stanford arXiv:1312.3296

$$V = 2\pi \tilde{r}_h l^2 \left(\frac{1}{2}\log\frac{\tilde{r}_h + r_c}{\tilde{r}_h - r_c} - \frac{r_c}{\tilde{r}_h}\right)$$

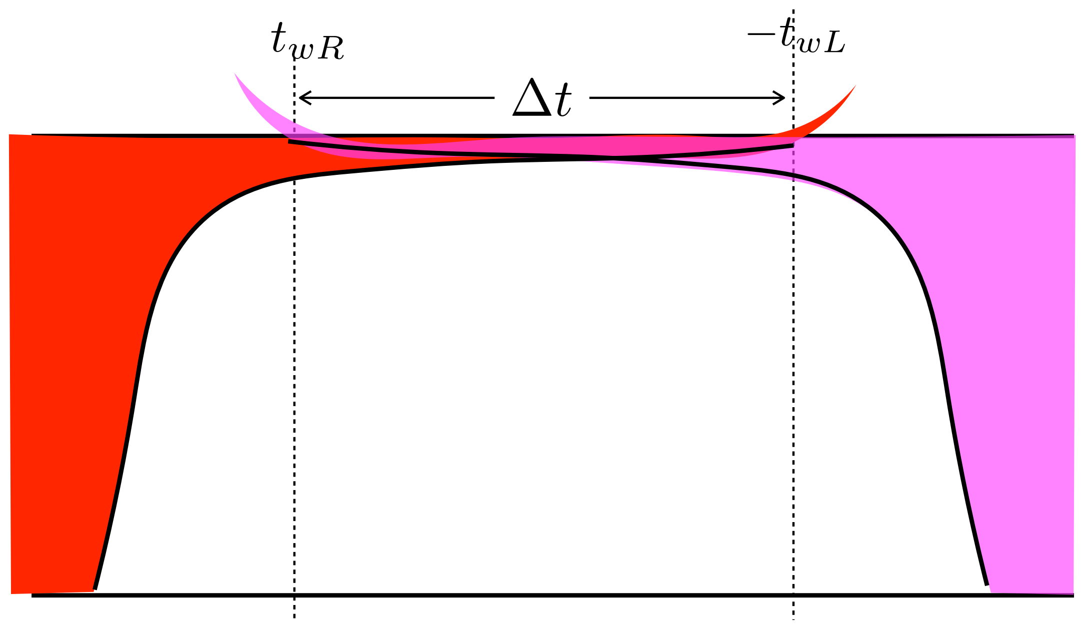
1. Early time $\frac{\beta}{2\pi} \ll \Delta t \ll t_*$

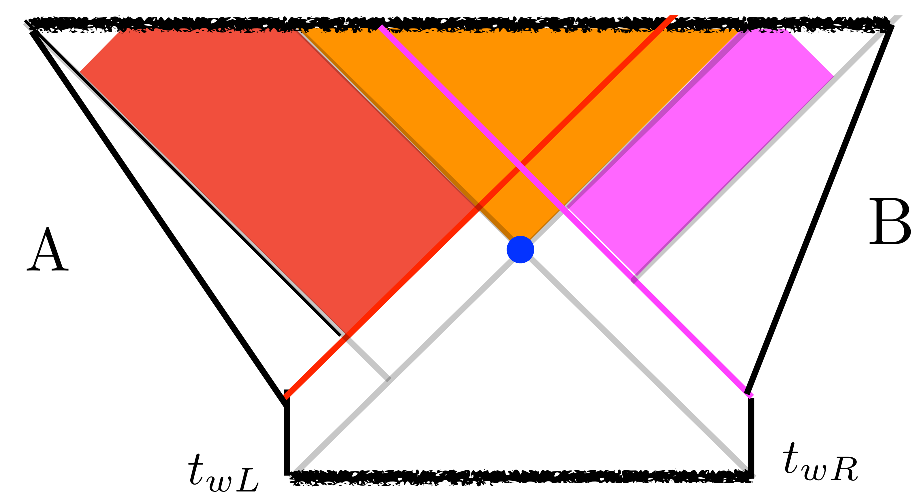


$$\frac{N_{\text{healthy}}}{S} = \int_{t_{wR}}^{-t_{wL}} \frac{2\pi}{\beta} dt \left(\frac{1}{1 + \frac{\delta S_1}{S} e^{\frac{2\pi}{\beta}(-t_{wL}-t)}} \right) \left(\frac{1}{1 + \frac{\delta S_2}{S} e^{\frac{2\pi}{\beta}(t-t_{wR})}} \right)$$

$$= \frac{2\pi}{\beta}\Delta t - \frac{\delta S_1 + \delta S_2}{S} e^{\frac{2\pi}{\beta}\Delta t} + \mathcal{O}\left(\left(\frac{1}{K} e^{\frac{2\pi}{\beta}\Delta t} \right)^2 \right)$$

$$\frac{V}{\pi r_h l^2} \approx \frac{2\pi}{\beta}\Delta t - \frac{\delta S_1 + \delta S_2}{2S} e^{\frac{2\pi}{\beta}\Delta t}$$



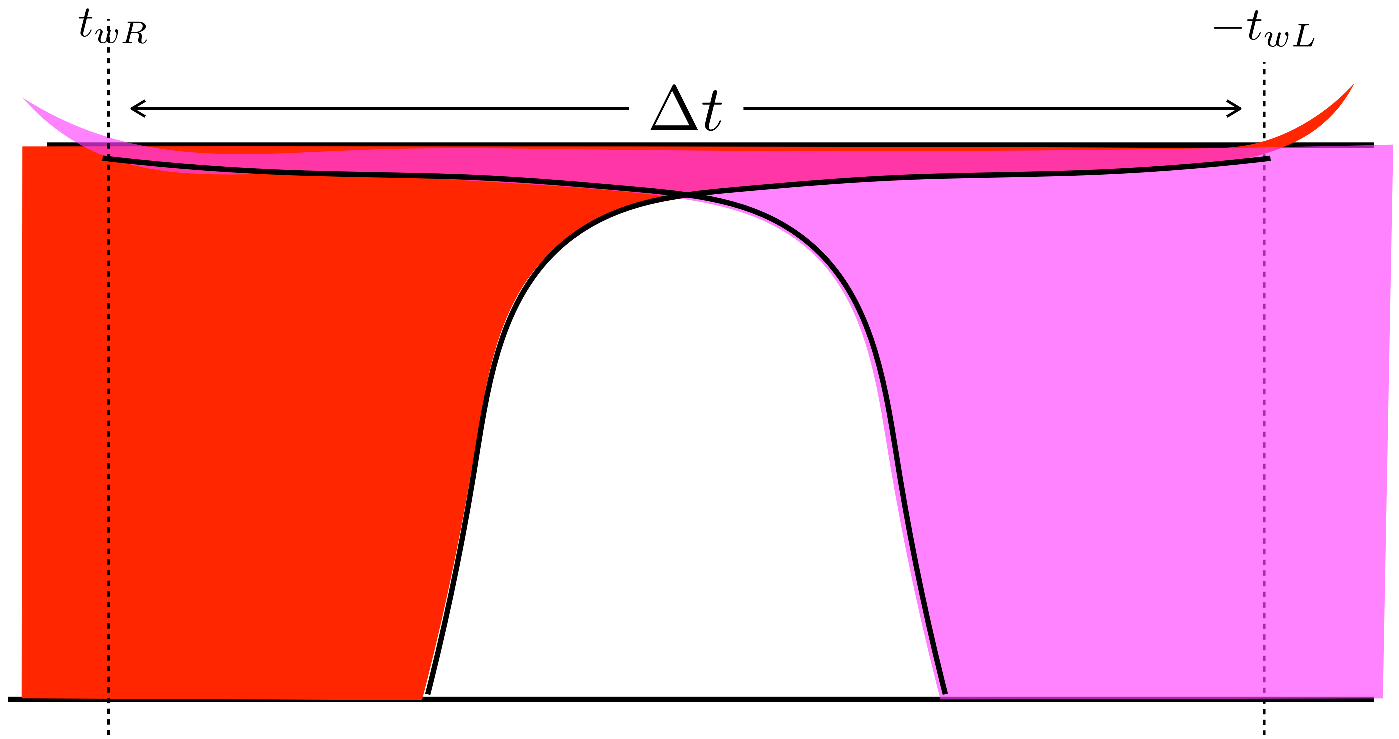


$$\frac{N_{\text{healthy}}^{\text{total}}}{S} = \int_{-\infty}^{+\infty} \frac{2\pi}{\beta} dt \left(\frac{1}{1 + \frac{\delta S_1}{S} e^{\frac{2\pi}{\beta}(-t_{wL} - t)}} \right) \left(\frac{1}{1 + \frac{\delta S_2}{S} e^{\frac{2\pi}{\beta}(t - t_{wR})}} \right)$$

$$\approx \frac{2\pi}{\beta}(2t_* - \Delta t)$$

$$\frac{V}{\pi r_h l^2} \approx \frac{2\pi}{\beta}(2t_* - \Delta t)$$

2.Intermediate time $t_* \ll \Delta t \ll 2t_*$

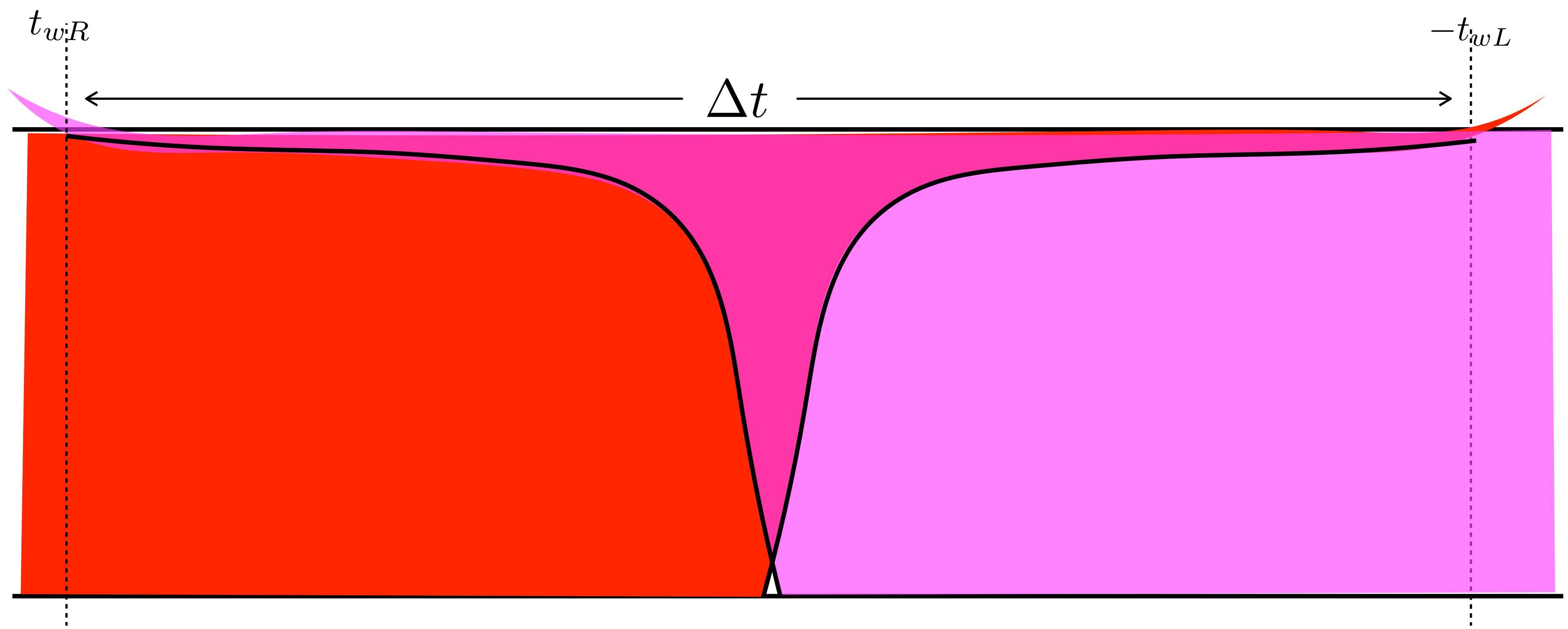


$$\frac{N_{\text{heathy}}}{S} = \int_{t_{wR}}^{-t_{wL}} \frac{2\pi}{\beta} dt \left(\frac{1}{1+\frac{\delta S_1}{S} e^{\frac{2\pi}{\beta}(-t_{wL}-t)}} \right) \left(\frac{1}{1+\frac{\delta S_2}{S} e^{\frac{2\pi}{\beta}(t-t_{wR})}} \right)$$

$$\approx \frac{2\pi}{\beta}(2t_* - \Delta t)\left(1 + \frac{\delta S_1 \delta S_2 e^{\frac{2\pi}{\beta}\Delta t}}{S^2}\right)$$

$$\frac{V}{\pi r_h l^2} \approx \frac{2\pi}{\beta}(2t_* - \Delta t)\left(1 + \frac{\delta S_1 \delta S_2}{2S^2} e^{\frac{2\pi}{\beta}\Delta t}\right)$$

3. Late time $\Delta t \gg 2t_*$



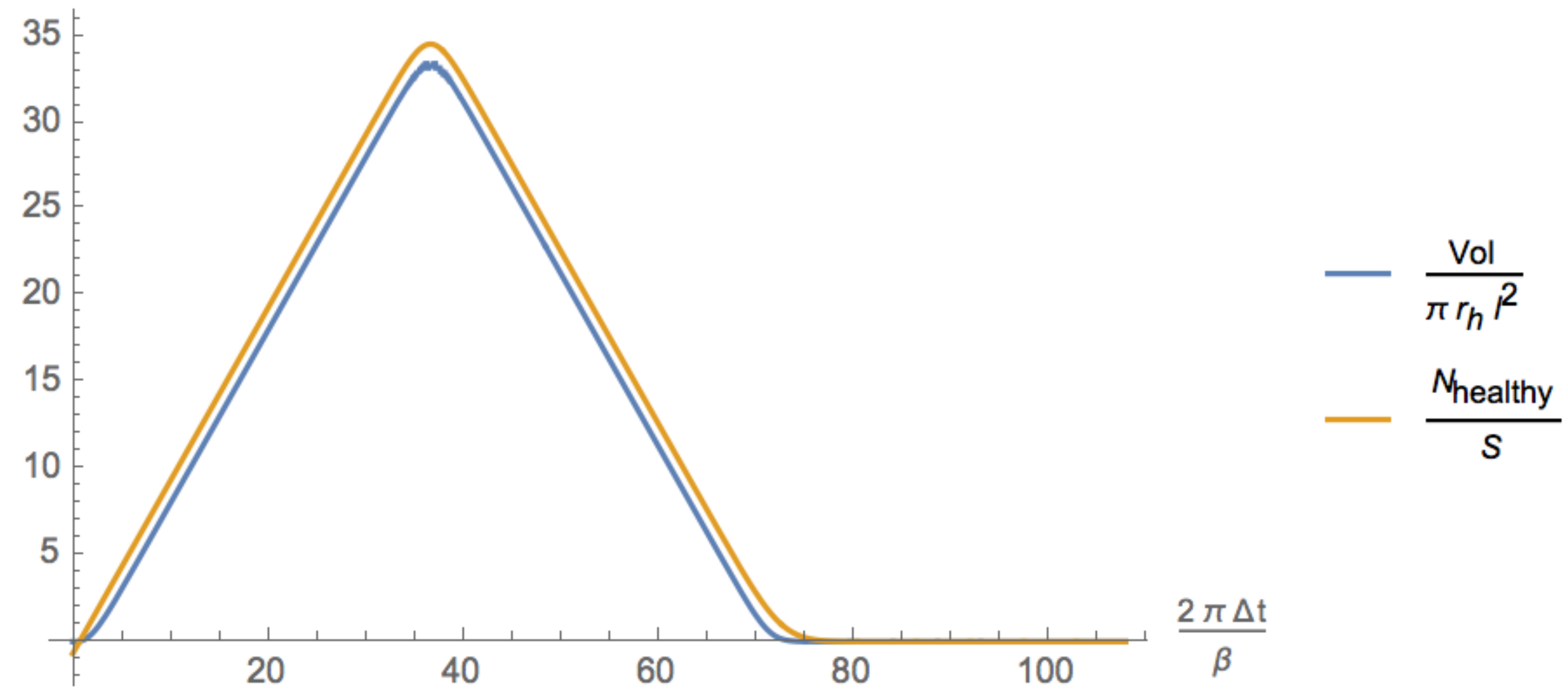
$$\frac{N_{\text{healthy}}}{S} = \int_{t_{wR}}^{-t_{wL}} \frac{2\pi}{\beta} dt \left(\frac{1}{1 + \frac{\delta S_1}{S} e^{\frac{2\pi}{\beta}(-t_{wL}-t)}} \right) \left(\frac{1}{1 + \frac{\delta S_2}{S} e^{\frac{2\pi}{\beta}(t-t_{wR})}} \right)$$

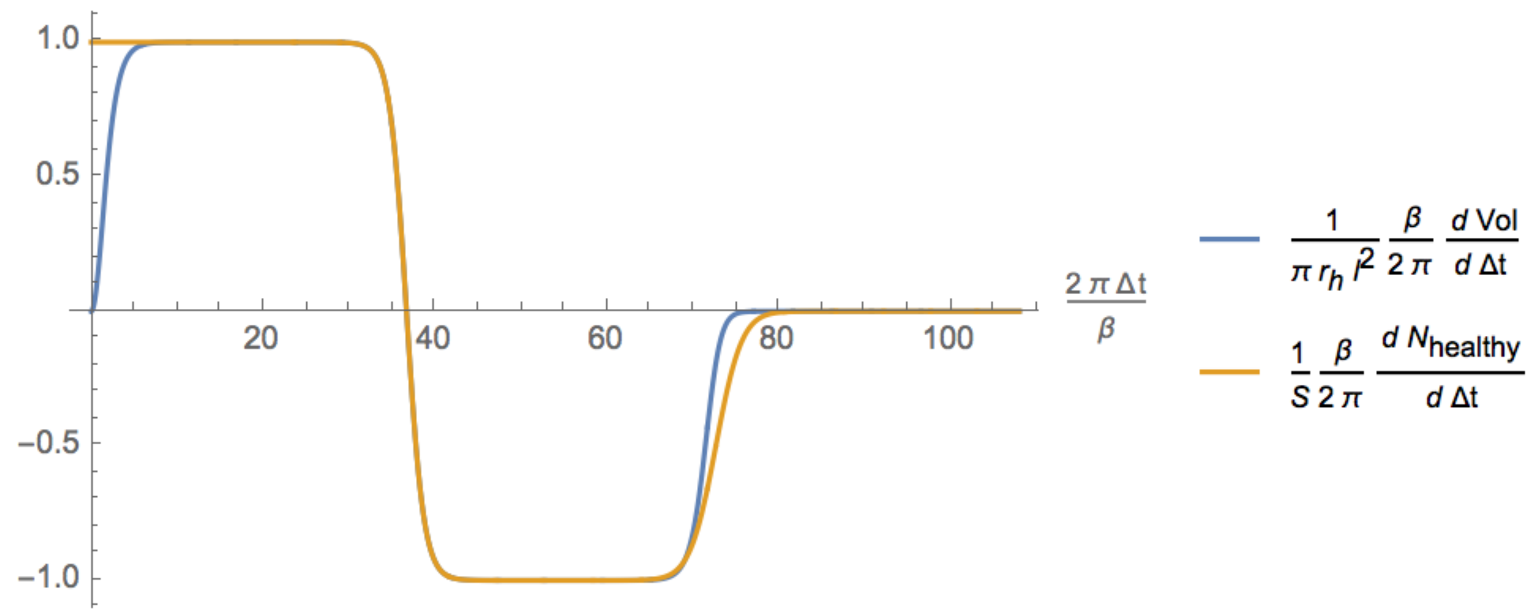
$$\approx e^{-\frac{2\pi}{\beta}(\Delta t - 2t_*)} (\Delta t - 2t_*)$$

$$\frac{V}{\pi r_h l^2} \approx \frac{2}{3} e^{-\frac{2\pi}{\beta}(\Delta t - 2t_*)}$$

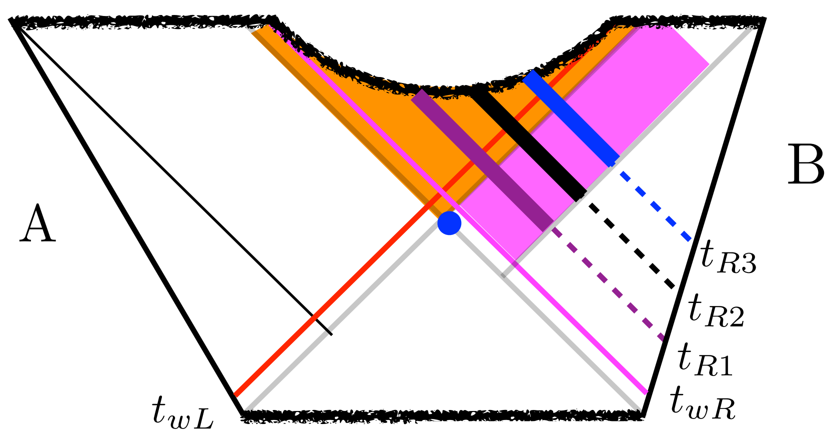
Full comparison

$$\frac{2\pi}{\beta} t_* = 36$$

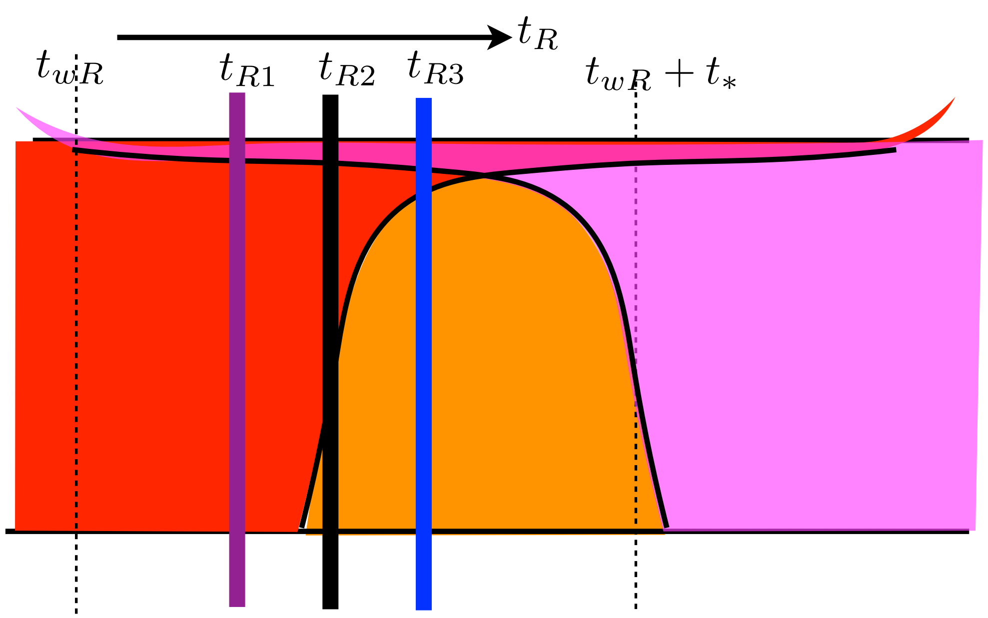




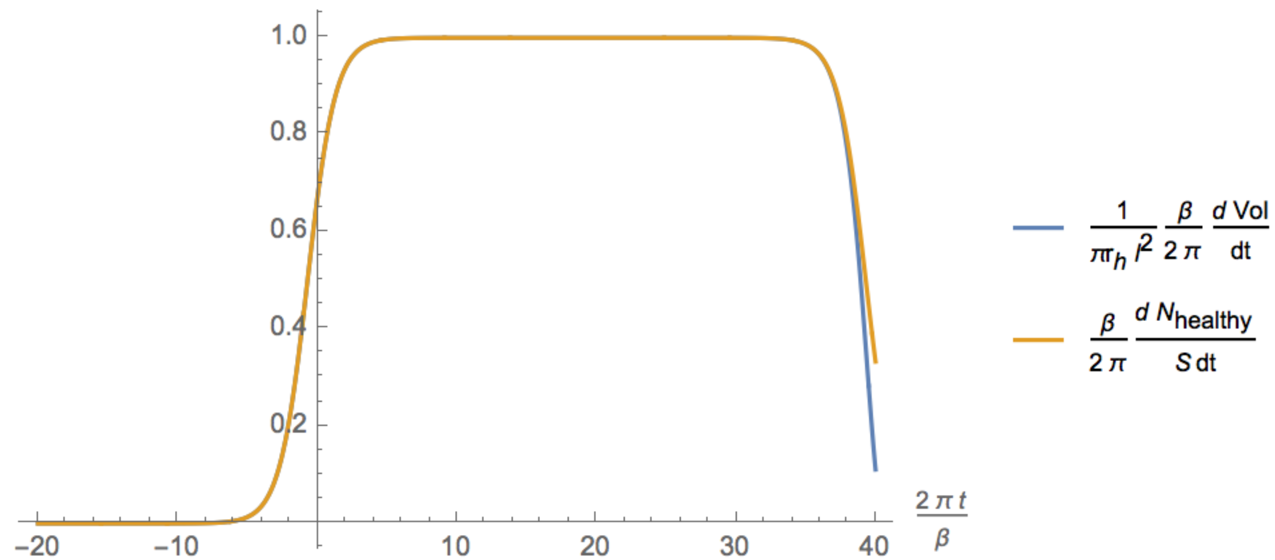
More detailed comparison of time dependence



(a)



(b)



$$\frac{2\pi}{\beta}(t_{wR} + t_*) = 40, \quad \frac{2\pi}{\beta}(-t_{wL} - t_*) = 0$$

Future directions

- Charged black holes

- How to diagnose if two perturbations overlap or not?

$$[W_L, W_R] = 0$$

- Singularity

Thank you.